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# Coordinated Control for Flywheel Energy Storage Matrix Systems for Wind Farm Based on Charging/Discharging Ratio Consensus Algorithms

Qian Cao, Y. D. Song, *Senior Member, IEEE*, Josep M. Guerrero, *Fellow, IEEE* and Shulin Tian\*

**Abstract**—Abstract - This paper proposes a distributed algorithm for coordination of flywheel energy storage matrix system (FESMS) cooperated with wind farm. A simple and distributed ratio consensus algorithm is proposed to solve FESMS dispatch problem. The algorithm is based on average consensus for both undirected and unbalanced directed graphs. Average consensus is guaranteed in unbalanced digraphs by updating the weight matrix with both its row sums and column sums being 1. Simulation examples illustrate the effectiveness of the proposed control method.

**Index Terms**—distributed coordinated control, average consensus, ratio consensus, dispatch problem, FESMS.

## I. INTRODUCTION

WIND energy is an environmental friendly renewable energy source, yet its large-scale utilization and integration into the main electrical grid involves great challenges, due to the wind power fluctuation caused by the stochastic wind in a wind farm. [1] With the rapid development of the industrial technology, the energy storage attracts increasing attention as a promising approach to address this issue. [2] As a relatively attractive storage method, flywheel energy storage system (FESS) has been widely applied in power smoothing, quality regulating and voltage restoring for wind turbines generator system (WTGS). [3] [4] Compared with batteries, pump storage system and other energy storage methods, FESS presents several merits such as long life cycles, rapid response and environmental friendly. These attracting features make FESS an ideal option to coordinate wind power generation.

Flywheel Energy Storage Matrix System (FESMS) is an aggregation structure with multiple flywheel units connected together to coordinate the fluctuated generators such as wind turbine generators. Several investigations have been carried out on the coordination of the FESMS and the WTGS [5] [6] [7]. However, to the authors' best knowledge, little work has been done on the research of power dispatch within the FESMS. Previous efforts to solve the dispatching problem have been made by implementing various numerical methods including the lambda-iteration method [8] and the gradient

search method [9]. Further more sophisticated techniques have been employed such as genetic algorithms (GA) [10], particle swarm optimization (PSO) [11] and so on. However, the above mentioned methods require for a central controller that can access the state of the entire system. This centralized control framework has many performance limitations. First, in order to control the whole system, some priori global knowledge has to be known, such as the number of the devices and the states of every units under control. Second, the centralized framework imposes great computational burden on the central controller to complete the whole dispatching algorithm, especially when the number of agents is large. Moreover, the centralized control scheme is sensitive to single-point failure. Once the central controller or any communication line collapses, the system may malfunction. Furthermore, the centralized scheme is not as flexible as the distributed one in respect of system reconfiguration, units plug-and-play. These problems may occur in the FESMS due to the massive installation of FESS units. In contrast, the distributed control scheme exhibits several merits. The distributed scheme disperses computational burden into the distributed controllers. It is more robust to communication failures. It also allows for flexible reconfiguration. Therefore, this paper proposes a dispatching method that uses a distributed algorithm.

Consensus problem is a fundamental issue in distributed algorithms. It has been widely and deeply studied in the literature (see [12] [13] [14] [15] for broad overview and [16] [17] [18] [19] [20] for various application). The system achieving a consensus indicates that all the agents in a multi-agent system reach to a common state by exchanging information between them through sparse communication network. Recently, there are several researches on the application of consensus algorithms in dispatching problem. Most of them consider the communication network with undirected graph or balanced graph [17] [21] [22] [23]. Other works address the unbalanced issue. In [24], the authors construct a row-stochastic and a column-stochastic matrix, but the feedback gain needs to be sufficient small. [25] studies the average consensus in general digraph, but the approach also needs a small perturbation to avoid unexpected equilibrium point. [26] investigates average consensus in unbalanced digraph, using three different algorithms. [27] proposed a ratio-based dispatch method using a distributed algorithm with communication failures. However, either these algorithms need two or more auxiliary variables, or do they require ultra-iterations.

In this paper, we explore a distributed dispatch scheme for

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the FESMS based on ratio consensus algorithm. The main features and contributions of this work can be summarized as follows. First of all, a fully distributed dispatching algorithm for FESMS is proposed. The approach does not rely on central controller and requires no prior knowledge of the communication network topology. Secondly, for the dispatch in unbalanced network, the proposed algorithm does not involve complicated process of determining control parameters, which makes it more favorable for practical application. Moreover, the proposed method is able to guarantee that in the network corresponding to an unbalanced graph, the weight matrix  $P$  converges with a certain rate. Last but not the least, the relationship between balanced graph and unbalanced graph is established, based on which the proposed algorithm is made adaptive to the variation of network topologies, as long as the corresponding graph is strongly connected.

The rest of the paper is organized as follows. In Section II, the problem of coordinated control of the networked FESMS is stated and the overall distributed control structure for the FESMS is proposed. In Section III, the distributed control scheme based on ratio consensus algorithm is explicitly addressed in both undirected and unbalanced directed graphs. Then a simulation of a FESMS with wind farm is established. The performance using the proposed control algorithm is illustrated in Section IV. Finally, conclusion remarks are made in Section V.

## II. FESMS CONFIGURATION AND OPERATION PRINCIPLE

Flywheels can be driven to acceleration or deceleration so as to generate or release energy depending on different requirements, thus smoothing the power fluctuation. In the wind farm with a large number of wind turbines, the power transmitted to the grid is considerable. Apparently, a single flywheel unit is unable to smooth the power fluctuation in this case due to the limit of its capacity. This calls for a Flywheel Energy Storage Matrix System (FESMS) with massive installation of FESS units. The FESMS and the wind farm cooperating together can provide relative smoothing power to the main grid, reducing the pollution of power fluctuation caused by the intermittent and waving wind speed.

The imbalance between the grid demanded power and the actual output power of the WTGS is given by:

$$\Delta P = P^* - P_W \quad (1)$$

where  $P_W$  is the active power generated by WTGS.  $P^*$  is the total reference output power of WTGS-FESMS, which is set and regulated by the relevant authorities according to the operation conditions. In Eq. (1),  $\Delta P > 0$  indicates that FESMS needs to be discharged and release extra power to compensate the power generated by WTGS. Likewise,  $\Delta P < 0$  indicates that the FESMS needs to be charged to smooth the peak of the power generated by WTGS.

*Remark 1:*  $\Delta P$  is constantly changing over time. To tackle this problem, it is assumed that  $\Delta P$  remains nearly constant during one dispatching period. This assumption is based on the timescale relationship between the relatively steady wind speed and the short dispatching period, which, according to

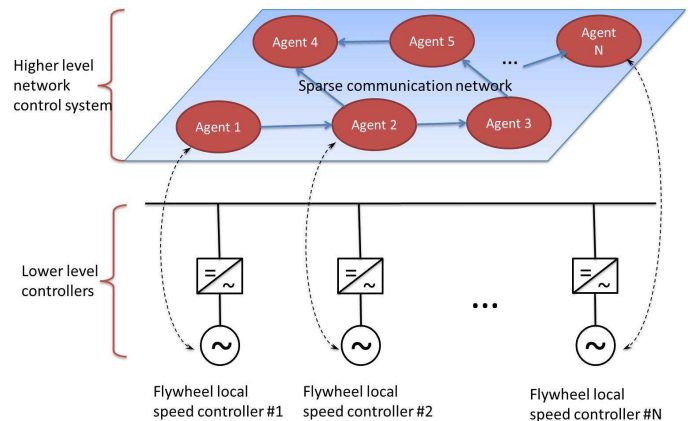


Fig. 1. Distributed control framework for FESMS

[28], is reasonable and accurate enough for the problem under consideration. In fact, our dispatching period can be less than a few seconds, within which the variation of the steady wind speed can be regarded small enough.

As we know from (1),  $\Delta P$  is the reference power for the whole FESMS. However, it still remains a problem on how to distribute this power difference among all the FESS units. In a traditional centralized framework, the central controller provides all the FESS units their charging and discharging power references. In this paper, a decentralized framework is adopted for the cooperative control of FESMS. Figure 1 illustrates the hierarchical distributed method for FESMS, where all the FESS units are network-connected without a central controller. This hierarchical structure has also been applied in microgrid control [29]. The units calculate and update their references by exchanging information to one another in the higher control level. Then the FESS unit realizes the charging and discharging process according to the given power reference, by regulating the flywheel machine's rotate speed in the lower control level. The acceleration and deceleration are controlled independently by the local controller in each FESS unit.

## III. DISTRIBUTED CONTROL BASED ON RATIO CONSENSUS ALGORITHMS

### A. Power Ratio Consensus Algorithm with Undirected Graph

In this section, we propose a distributed dispatching method for the FESMS based on the charging and discharging capacity of each FESS unit. First, we consider the networked FESMS with  $n$  FESS units as an undirected graph  $G = \{V, E\}$ , which implies that the information exchanging in the network is balanced. In the undirected graph,  $V = \{v_1, \dots, v_n\}$  and  $E \subseteq V \times V$  represent the set of FESS units and the set of edges respectively.  $A = [a_{ij}] \in R^{N \times N}$  is the associated adjacency matrix. If  $(v_j, v_i) \in E$ , then unit  $v_j$  is a neighbor of unit  $v_i$  and  $a_{ij} = 1$ , otherwise,  $v_j$  is not a neighbor of unit  $v_i$  and  $a_{ij} = 0$ . The set of neighbors of unit  $i$  is denoted as  $N_i = \{v_j | (v_j, v_i) \in E\}$ . The degree matrix  $D \in R^{N \times N}$  is a diagonal matrix with the  $i$ th element being  $\deg(v_i)$ . The Laplacian matrix is defined as  $L = D - A \in R^{N \times N}$ . Define a matrix  $P$  as  $P = [p_{ij}] = I - \pi L \in R^{N \times N}$ , where

$0 < \pi < 1/\deg(v_i)$  so that  $0 < p_{ij} < 1$  for  $i, j = 1, \dots, n$ . The weight matrix  $P$  is crucial in preparation for calculating the reference power of each FESS in the following part. As for the Laplacian matrix  $L$ , we have the following lemma.

*Lemma 1:* [30] All eigenvalues of  $L$  have non-negative real parts from Gershgorins disc theorem, and zero is a simple eigenvalue of  $L$  if and only if the graph has a spanning tree.

The main principle of this dispatching method is that the charge or discharge power reference for each flywheel is decided by its current charging or discharging capacity. The current maximum charge and discharge power for FESS unit  $i$  are denoted as  $\overline{P}_i^{char}, \overline{P}_i^{disc}$  and given by the following equations respectively:

$$\overline{P}_i^{char} = \frac{\overline{E}_i - E_{i0}}{t}, \overline{P}_i^{disc} = \frac{E_{i0} - \underline{E}_i}{t} \quad (2)$$

where  $\overline{E}_i, \underline{E}_i$  are the maximum and minimum storage level of FESS unit  $i$ .  $E_{i0}$  is the current storage level of FESS unit  $i$ , and  $\underline{E}_i \leq E_{i0} \leq \overline{E}_i$ .  $t$  represents the time duration of one dispatching period.

Here, we propose the charging and discharging ratio factors of FESS unit  $i$  at the  $k$ th time iteration as follows:

$$r_i^{char}[k] = \frac{y_i^{char}[k]}{z_i^{char}[k]}, r_i^{disc}[k] = \frac{y_i^{disc}[k]}{z_i^{disc}[k]} \quad (3)$$

where  $y_i^{char}[k], z_i^{char}[k], y_i^{disc}[k], z_i^{disc}[k]$  are the auxiliary variables for further updating.

Then we set the initial values for the four variables in  $r_i^{char}[k]$  and  $r_i^{disc}[k]$ . Without loss of generality, we suppose that the FESS units in FESMS that are directly connected to the wind farm are indexed from 1 to  $l$ . These FESS units are called leader FESS units and only they have access to  $\Delta P$  in FESMS, while other units do not have such access. The initial values for the leader FESS units and other units are set respectively as follows:

$$\begin{cases} y_i^{char}[0] = \Delta P/l, z_i^{char}[0] = \overline{P}_i^{char} \\ y_i^{disc}[0] = \Delta P/l, z_i^{disc}[0] = \overline{P}_i^{disc} \end{cases} \quad (4)$$

for  $i = 1, \dots, l$ .

$$\begin{cases} y_i^{char}[0] = 0, z_i^{char}[0] = \overline{P}_i^{char} \\ y_i^{disc}[0] = 0, z_i^{disc}[0] = \overline{P}_i^{disc} \end{cases} \quad (5)$$

for  $i = l + 1, \dots, n$ .

The following iteration protocols are designed to update the ratio factors using the weight matrix  $P$ . For simplicity, we take  $y_i^{char}[k]$  as an example.

$$y_i^{char}[k+1] = p_{ii}y_i^{char}[k] + \sum_{j \in N_i} p_{ij}y_j^{char}[k] \quad (6)$$

The corresponding matrix form can be given as

$$\mathbf{y}_{char}[k+1] = P\mathbf{y}_{char}[k] \quad (7)$$

where  $\mathbf{y}_{char}[k] = \{y_i^{char}[k]\}$ , for  $i = 1, \dots, n$ .

Given the initial values, the four auxiliary variables are all updated according to (7) over time. The following lemma is given on some basic knowledge for later use.

*Lemma 2:*  $P$  is a doubly stochastic matrix, and  $v = \frac{1}{n}\mathbf{1}_n^T$  is the left Perron-Frobenius eigenvector of  $P$ , where  $\mathbf{1}_n$  is an  $n$ -dimension column vector with all ones.

**Proof:** It is easy to know that all the row-sums and the column-sums of matrix  $P$  are 1, which makes  $P$  a doubly stochastic matrix. The graph is connected, which means  $P$  is irreducible. According to Perron-Frobenius theorem [31], 1 is the largest and simple eigenvalue of  $P$ . Define  $v$  as the left eigenvector of  $P$  associated with eigenvalue 1, satisfying  $v^T \mathbf{1}_n = 1$ . Then it is obvious that  $v = \frac{1}{n}\mathbf{1}_n^T$ .

With lemma 2, the following theorem can be obtained for the ratio factors  $r_i^{char}[k]$  and  $r_i^{disc}[k]$ .

*Theorem 1:* If the undirected graph has a spanning tree, the charging and discharging ratio factors of all FESS units converge to  $r_{char}^* = \Delta P / \sum_{i=1}^n \overline{P}_i^{char}$  and  $r_{disc}^* = \Delta P / \sum_{i=1}^n \overline{P}_i^{disc}$  respectively, given the initial values as in Eq. (4) and (5) and using the iteration protocols in Eq. (7).

**Proof:** To address the ratio consensus, we first argue the convergence of the four auxiliary variables defined in Eq. (3). According to the well-known result in Markov chains [31], one has  $\lim_{k \rightarrow \infty} P^k = \mathbf{1}_n v^T$ , where from lemma 2  $v = \frac{1}{n}\mathbf{1}_n^T$  is the left Perron-Frobenius eigenvector of  $P$ . Taken  $y_i^{char}[k]$  as an example, we have,

$$\begin{aligned} \lim_{k \rightarrow \infty} \mathbf{y}_{char}[k] &= \lim_{k \rightarrow \infty} P^k \mathbf{y}_{char}[0] = \mathbf{1}_n \frac{1}{n} \mathbf{1}_n^T \mathbf{y}_{char}[0] \\ &= \left( \frac{1}{n} \sum_i y_i^{char}[0] \right) \mathbf{1}_n \end{aligned} \quad (8)$$

Eq. (8) implies that the auxiliary variable  $y_i^{char}[k]$  reaches an average consensus for  $i = 1, \dots, n$ . In the same way, variables  $z_i^{char}[k], y_i^{disc}[k]$  and  $z_i^{disc}[k]$  can also achieve average consensus respectively. Therefore, we have the following convergence values of the four variables as time goes to infinity:

$$\begin{cases} y_{char}^* = \frac{1}{n} \sum_{i=1}^n y_i^{char}[0] = \frac{\Delta P}{n} \\ z_{char}^* = \frac{1}{n} \sum_{i=1}^n z_i^{char}[0] = \frac{1}{n} \sum_{i=1}^n \overline{P}_i^{char} \end{cases} \quad (9)$$

$$\begin{cases} y_{disc}^* = \frac{1}{n} \sum_{i=1}^n y_i^{disc}[0] = \frac{\Delta P}{n} \\ z_{disc}^* = \frac{1}{n} \sum_{i=1}^n z_i^{disc}[0] = \frac{1}{n} \sum_{i=1}^n \overline{P}_i^{disc} \end{cases} \quad (10)$$

Eq. (9) and (10) implies that the charging and discharging ratio factors of all the FESS units converge to the common values respectively. The convergence values are given as:

$$r_{char}^* = \frac{y_{char}^*}{z_{char}^*} = \frac{\Delta P}{\sum_{i=1}^n \overline{P}_i^{char}} \quad (11)$$

$$r_{disc}^* = \frac{y_{disc}^*}{z_{disc}^*} = \frac{\Delta P}{\sum_{i=1}^n \overline{P}_i^{disc}} \quad (12)$$

This completes the proof of Theorem 1.

*Remark 2:* The necessary and sufficient conditions for the iteration in Eq. (7) to reach an average consensus are: (a)  $P$  has a simple eigenvalue at 1, with both left eigenvector and right eigenvector  $\mathbf{1}_n^T$ , and (b) all the other eigenvalues of  $P$  have magnitude strictly less than 1. In an undirected network, the fixed doubly stochastic weight matrix  $P$  satisfies the conditions naturally thus ensuring the agents asymptotically reach an average consensus.

### B. Power Ratio Consensus Algorithm with Unbalanced Directed Graph

In the following, we consider another scenario where the communications network is denoted as an unbalanced digraph. That means the exchanging of information among agents is not symmetrical, thus making the average consensus problem much more challenging. We consider the networked FESMS with  $n$  FESS units as a directed graph  $G = \{V, E\}$ , where  $V = \{v_1, \dots, v_n\}$  and  $E \subseteq V \times V$  are the set of FESS units and the set of edges respectively. The set of agents which can receive information from unit  $i$  is denoted as  $N_i^+ = \{v_j | (v_i, v_j) \in E\}$ . Likewise, the set of agents which can send information to unit  $i$  is denoted as  $N_i^- = \{v_j | (v_j, v_i) \in E\}$ . Since the Laplacian matrix of an unbalanced directed graph is not symmetrical, we therefore define a new weight matrix  $P$  at iteration  $k$  as  $P[k] = [p_{ij}[k]] \in R^{N \times N}$ . The weights of node  $i$  is set as follows:

$$p_{ij}[k] = \begin{cases} 1 - \theta_i[k] & j = i \\ \bar{p}_{ij}\theta_j[k] & j \in N_i^- \\ 0 & j \notin N_i^- \end{cases} \quad (13)$$

where the parameter  $\bar{p}_{ij}$  is set by node  $j$  and satisfies  $\bar{p}_{jj} = 0$ ,  $0 < \bar{p}_{ij} < 1$ , and  $\sum_{i, i \neq j} \bar{p}_{ij} = 1$  (This can be achieved under the condition that each agent knows its outgoing degree).  $0 < \theta_i[k] < 1$  is a local variable for further updating.

Then we can write the weight matrix as

$$P[k] = \bar{P}\Theta[k] + (I_n - \Theta[k]), \quad (14)$$

where  $\bar{P} = [\bar{p}_{ij}]$ ,  $\Theta[k] = \text{diag}\{\theta_i[k]\}$ .

*Remark 3:* The idea of the weight setting is that each node (node  $j$  for example) separates its information into two parts. One part is left to itself  $(1 - \theta_j)$ , and the other part  $(\theta_j)$  is fully split among its neighbors, which is represented by matrix  $\bar{P}$ . Then node  $i$  calculates its ingoing weights as (13). It can be noticed that the column sum of  $\bar{P}$  is 1. This is a collaborative efforts of all  $j$ th outgoing neighbors by setting their ingoing weights as  $\bar{p}_{ij}\theta_j[k]$ . As a result, the row sum of matrix  $P$  is not necessary 1, which is addressed in the following analysis.

To achieve average consensus, a natural thought is to drive the row sum of weight matrix  $P[k]$  to 1 by updating  $\theta[k]$ . Define  $\delta_i[k] \triangleq s_i[k] - 1 = \sum_j p_{ij}[k] - 1$ , where  $s_i[k]$  is the  $i$ th row sum of  $P[k]$ . Then we have:

$$\delta[k] = (P[k] - I)\mathbf{1}_n^T = (\bar{P} - I)\Theta[k]\mathbf{1}_n^T = (\bar{P} - I)\theta[k] \quad (15)$$

where  $\delta[k] = \{\delta_i[k]\}$ .

We propose the following update algorithm:

$$y_i^{char}[k+1] = p_{ij}[k]y_i^{char}[k] + \frac{1}{2}\theta_i[k] - \frac{1}{2}\bar{p}_{ij}\theta_j[k] \quad (16)$$

$$\theta_i[k+1] = \frac{1}{2}\theta_i[k] + \frac{1}{2}\sum_j \bar{p}_{ij}\theta_j[k] \quad (17)$$

Eq. (17) and (16) can be written in the vector form as:

$$\mathbf{y}_{char}[k+1] = P[k]\mathbf{y}_{char}[k] - \frac{1}{2}(\bar{P} - I)\theta[k] \quad (18)$$

$$\theta[k+1] = \frac{1}{2}(\bar{P} + I)\theta[k] \quad (19)$$

The closed-loop dynamics of the system is obtained as:

$$\begin{pmatrix} \mathbf{y}^{char}[k+1] \\ \theta[k+1] \end{pmatrix} = M \begin{pmatrix} \mathbf{y}^{char}[k] \\ \theta[k] \end{pmatrix} \quad (20)$$

where

$$M = \begin{pmatrix} P[k] & I - \frac{1}{2}(\bar{P} + I) \\ \mathbf{0} & \frac{1}{2}(\bar{P} + I) \end{pmatrix} \quad (21)$$

Notice that the row sums of  $M$  are always 1. So by left multiplying  $\mathbf{1}_n^T$  on both sides of Eq. (20), we obtain that  $\sum_i (y_i^{char}[k] + \theta_i[k])$  is invariant over time.

In the following, we will analyse the convergence properties by giving out theorem 2.

*Theorem 2:* A multi-agent system under a strongly connected digraph network reach average consensus using Eq. (16) and (17). Furthermore, the row sum of the weight matrix  $P[k]$  converges to  $\mathbf{1}_n^T$  asymptotically, and the convergence rate is given by:

$$\max_i \left\{ \left| \lambda_i \left( \frac{\bar{P} - I}{2} \right) \right| \right\}$$

**Proof.** Consider system (20), it can be shown that  $(y_a^T, \theta_a^T)^T = (\frac{1}{n} \sum_i y_i^{char}[0] \mathbf{1}_n^T, v^T)^T$  is a unique equilibrium point, where  $v$  is the right eigenvector of  $\bar{P}$  corresponding to eigenvalue 1, normalized by  $\sum_i v_i = \sum_i \theta_i[0]$ . To prove this, we consider the steady state of system (20). From Eq. (19), we have  $\theta = \gamma v$ , where  $\gamma \neq 0$ . According to Perron-Frobenius Theorem,  $\bar{P}$  has a simple eigenvalue 1. On the other hand, from Eq. (19), since  $\frac{1}{2}(\bar{P} + I)$  is column stochastic, the sum of  $\theta_i$  is constant over time. That means the steady state of  $\theta$  is unique as  $v$ .

Substituting  $\theta = v$  into (15), we obtain that the row sum of  $P[k]$  is 1. It implies from (18) that  $y_i^{char}[k] = \frac{1}{n} \sum_i y_i^{char}[k]$ . Because from Eq. (14), the column sum of  $P[k]$  is 1, combining that  $\frac{1}{2}(\bar{P} - I)\theta[k] = 0$ , we know that the sum of  $y_i^{char}[k]$  is also invariant over time. Then we have  $y_i^{char}[k] = \frac{1}{n} \sum_i y_i^{char}[0]$  and thus average consensus is achieved.

Furthermore, combining (15) and (19), we have  $\delta[k+1] = \frac{\bar{P} - I}{2} \delta[k]$ . Then it is obvious that the convergence rate is the maximum absolute value of the eigenvalue of matrix  $\frac{\bar{P} - I}{2}$ .

This completes the proof of 2.

*Remark 4:* Theorem 2 shows that the weight matrix  $P$  converge to a matrix where both the column sums and the row sums are 1. The column guarantees the invariance of the sum of  $y_i^{char}[k]$ , and the row sum guarantees the consensus of  $y_i^{char}[k]$ . However, the matrix does not need to be a doubly stochastic one (where all the entries are non-negative) to possess these functions.

*Remark 5:* It is seen that the proposed algorithm is simple in structure and inexpensive in computation. In particular,

compared with the works [26], [27] and [32], the choice of  $\bar{p}_{ij}$  and  $\theta_i[0]$  has been significantly simplified, the matrix  $P$  does not need to be doubly stochastic. Furthermore, the proposed algorithm involves only one auxiliary variable  $\theta_i^{char}[k]$  and no super-iteration is needed, which obviously reduces the complexity of the algorithm and makes it more favorable for practical application.

*Remark 6:* It is worth noting that the scenario with undirected graph can be considered as a special case of unbalanced directed graph with the entries of  $\bar{P}$  as  $\bar{p}_{ii} = 0$ ,  $\bar{p}_{ij} = \frac{a_{ij}}{\sum_i a_{ij}}$  (for  $i \neq j$ ) and  $\theta_i[0] = \pi \sum_i a_{ij}$ . If the topology of the graph is not known, one can simply assume that it is an unbalanced directed graph, and set  $\bar{p}_{ij}$  and  $\theta_i[0]$  as aforementioned. Average consensus can be achieved in both scenarios. In other words, as long as the graph is strongly connected, the algorithm is adaptive to the topology switches caused by communication failures.

To clarify the whole process, we summarized the algorithm into the following three steps.

*step 1:* At iteration  $k$ , agent  $j$  spreads its share  $\theta_j[k]$  and the sharing weight  $\bar{p}_{ij}$  to its outgoing neighbor  $i$ .

*step 2:* Agent  $i$  obtains information from its neighbor  $j$  and set its ingoing weights as in Eq. (13).

*step 3:* Each agent update its  $\theta[k]$ , as well as the four variables used for ratio consensus. One iteration is over and then go back to *step 1*.

### C. Distributed Cooperative Control of FESMS

As we can see from the above section, charging and discharging ratio factors have the same signs. However, the FESS units can only operate in one of the charging or discharging modes in one dispatching period. That means only one kind of the ratio factors can be used to calculate the actual power references for flywheels, while the other is not applied in this particular allocated period. According to the definition of  $\Delta P$ , when  $\Delta P > 0$ , the flywheels should operate in the charging mode, otherwise, they need to operate in the discharging mode. In other words, the auxiliary variables  $y_i^{disc}[k]$  and  $y_i^{char}[k]$  reflect the sign of  $\Delta P$ . Therefore, we use  $r_i[k]$  to unify the charging and discharging ratio factors. The saturations are added to prevent the power reference exceed the rated power.

$$r_i[k] = \begin{cases} -1, & y_i^{disc}[k] \leq -1 \\ y_i^{disc}[k]/z_i^{disc}[k], & -1 < y_i^{disc}[k] < 0 \\ y_i^{char}[k]/z_i^{char}[k], & 0 < y_i^{char}[k] < 1 \\ 1, & y_i^{char}[k] \geq 1 \end{cases} \quad (22)$$

Then the power reference is calculated as

$$\Delta P_i[k] = \begin{cases} r_i[k] \bar{P}_i^{disc}, & r_i[k] < 0 \\ r_i[k] \bar{P}_i^{char}, & r_i[k] > 0 \end{cases} \quad (23)$$

So far, we have obtained the reference charging or discharging power for each FESS unit by using a distributed ratio consensus algorithm. From the above analysis, we know that the amount of charging or discharging power of each FESS unit is proportional with its current charge or discharge capacities. The proportion is decided by the power imbalance

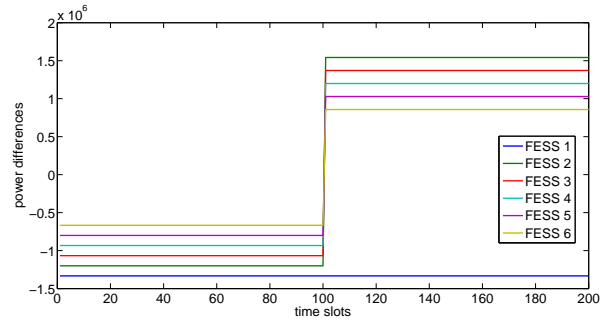


Fig. 2. Trajectory of the power references using centralized control with communication failures

and the total charge and discharge capacities of FESMS. By iteration computation, all the charging and discharging ratios converge to two same values respectively as in (11) and (12). The charge or discharge mode and the final power references are determined by (23).

## IV. SIMULATION RESULTS

A simulation model of FESMS integrated in a wind farm is built by using Matlab/Simulink software. The FESMS consists of six FESS units with the minimum speeds as  $2000 \text{ r/min}$  ( $209 \text{ rad/s}$ ) and the maximum speed as  $10000 \text{ r/min}$  ( $1047 \text{ rad/s}$ ). The moment of inertia are  $J = [10, 9, 8, 7, 6, 5] \text{ kgm}^2$ . The current rotate speed of the FESS units are  $6000 \text{ r/min}$ .

### A. case study 1: centralized control with communication failures

Before providing simulation studies using the proposed distributed algorithm, we first provide a case study with centralized control. To prevent the comparison from being comparing apple with orange, we need to set up the similar condition for both methods. As centralized control is based on global information while the proposed distributed one only uses local information, we thus consider the case that involves communication failures such that only local information is available for both control schemes. In this centralized case, FESS 1 is disconnected from the central controller in the first dispatch period (between time slot 0~100). Figure 2 shows the power references of the six FESS units. As can be seen from this figure, due to the communication fault in FESS 1, the output power reference of FESS 1 does not change as  $\Delta P$  varies at the second dispatch period. This leads to the result shown in Figure 3 that the total power of FESMS can not compensate for the power imbalance any more.

### B. case study 2: undirected graph with spanning trees

In this case, the 6 FESS units are connected by undirected network. The topology is shown in Figure 4. The power imbalance  $\Delta P$  is  $-6 \text{ MW}$ . That means the FESMS should operate in discharging mode and the total power references should be  $-6 \text{ MW}$ . Figure 5 shows the consensus of the ratio factors. The horizontal axis is number of iterations. Figure 6 shows the power references of the 6 units. The power references

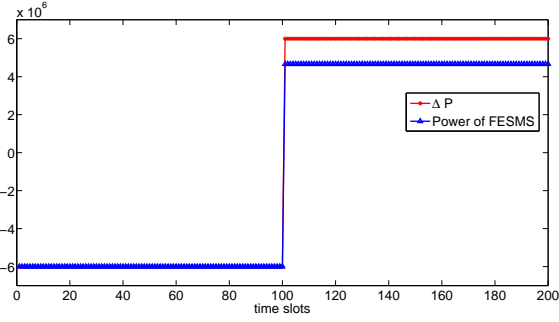


Fig. 3. The total power of FESMS and the power imbalance using centralized control with communication failures

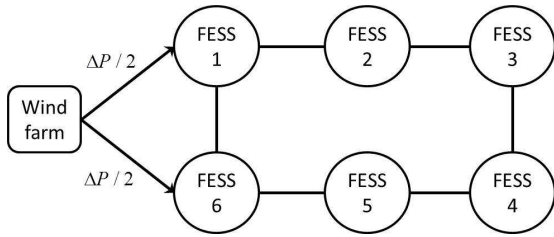


Fig. 4. Topology of FESMS under undirected communication network

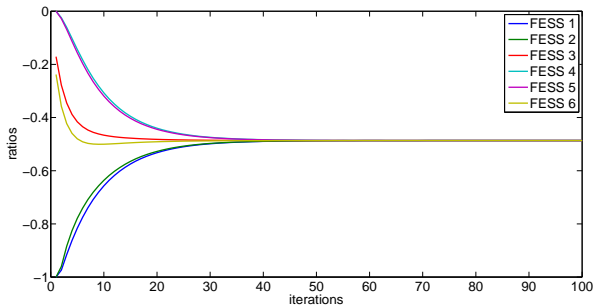


Fig. 5. Trajectory of ratio factors under undirected graph

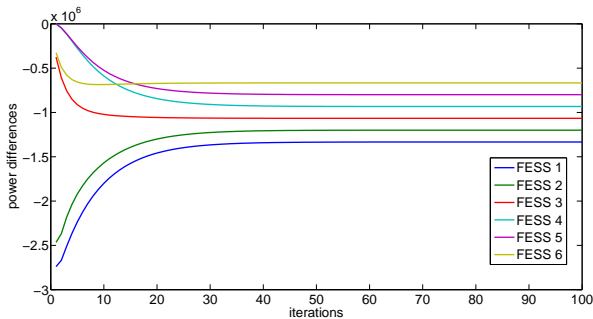


Fig. 6. Trajectory of the power references under undirected graph

are  $[-1.33, -1.2, -1.06, -0.93, -0.8, -0.67]MW$ , which are proportional to their capacities. Figure 7 shows the total power of the 6 units and the power imbalance. It can be seen that the total power of FESMS is converge to the power imbalance asymptotically.

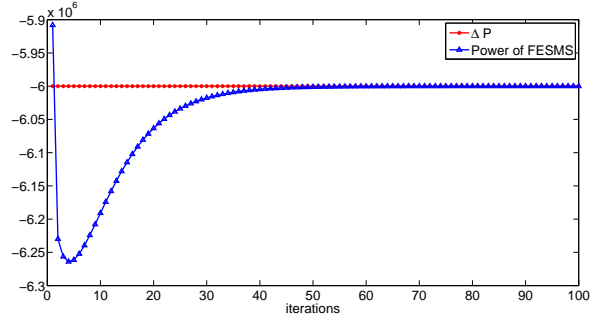


Fig. 7. The total power of FESMS and the power imbalance under undirected graph

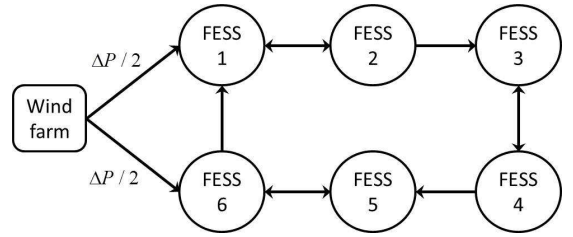


Fig. 8. Topology of FESMS under unbalanced directed communication network

### C. case study 3: strongly connected unbalanced digraph

In this case, the network of FESMS is not balanced, as shown in Figure 8. We set  $\bar{p}_{ij} = \frac{1}{D^+}$  and  $\theta[0] = \frac{1}{2}\mathbf{1}_n^T$ . According to the definition in (13), the matrix  $\bar{P}$  can be obtained as

$$\bar{P} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure 9 shows the consensus of the ratio factors under unbalanced digraph. Figure 10 shows the power references of the 6 units. The final values are the same as in *case 1*. Figure 11 shows the sum of the 6 units and the power imbalance. It shows that the total power reference of FESMS is equal to the power imbalance when FESS units are connected through unbalanced digraph. Figure 12 is the trajectory of the row sums error  $\delta[k]$ . The figure shows that all the row sums tend to 1 when time goes to infinity. Figure 13 is the trajectory of  $\theta[k]$ . The stable state of  $\theta[k]$  in this case is  $[0.75, 0.375, 0.375, 0.375, 0.375, 0.75]^T$ , which verifies the theoretical results that  $\sum_i \theta_i[k] = \sum_i \theta_i[0] = 3$ , and  $\bar{P}\theta[k] = \theta[k]$ .

### D. case study 4: time-varying power imbalance with topology changes

In this case, the network topology changes between two dispatch periods from a balanced graph in Figure 4 to an unbalanced digraph in Figure 8. The power imbalance also varies from  $-6MW$  to  $6MW$ . To avoid unnecessary large overshoot between two dispatching periods, we make a slight

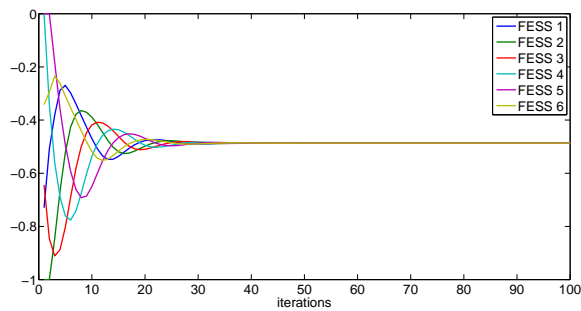


Fig. 9. Trajectory of ratio factors under unbalanced digraph

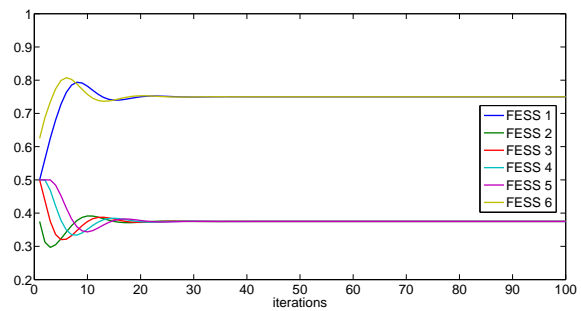


Fig. 13. Trajectory of  $\theta[k]$

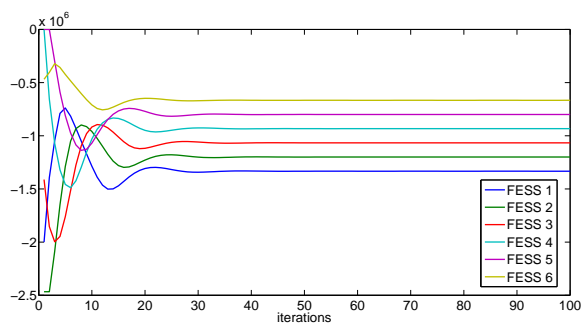


Fig. 10. Trajectory of the power references under unbalanced digraph

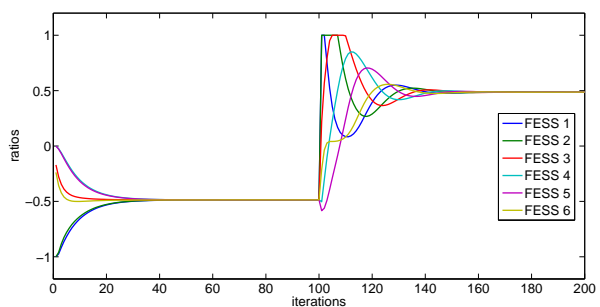


Fig. 14. Trajectory of ratio factors with varying imbalance under switching topologies

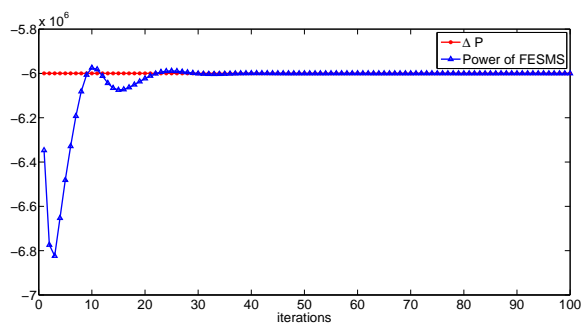


Fig. 11. The total power of FESMS and the power imbalance under unbalanced digraph

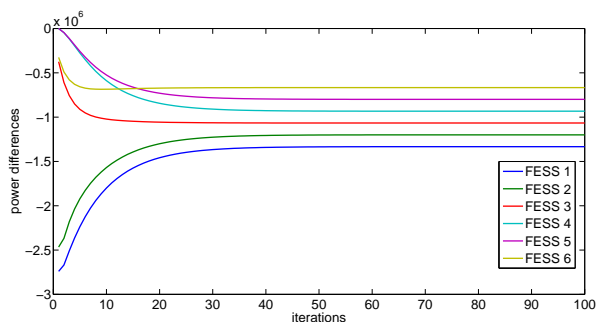


Fig. 15. Trajectory of the power references with varying imbalance under switching topologies

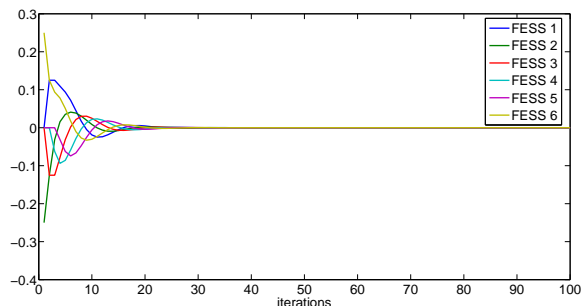


Fig. 12. Trajectory of row sum errors  $\delta[k]$

modification when setting the initial values. Instead of setting the initial values as (4) and (5), we keep the followers' values unchanged as the last period, and set the leaders initial value as  $y_i[ck + 1] = y_i[ck] + (\Delta P_{c+1} - \Delta P_c)/l$ , where  $y_i[ck + 1]$  is the initial value of  $(c + 1)$ th dispatch period, and  $y_i[ck]$  is

the value at the end of the  $c$ th dispatch period.  $\Delta P_{c+1}$  and  $\Delta P_c$  represent the power imbalance of the  $(c + 1)$ th and the  $c$ th periods respectively. Figure 14 shows the convergence of the ratio factors in two dispatch periods. Compared with case study 1, this example shows that under unexpected communication faults and varying power imbalance, centralized control is unable to maintain power balance, while this undesirable situation does not occur in the proposed distributed control.

## V. CONCLUSION

This paper investigates the distributed control scheme for FESMS cooperated with wind farm. The control method solves the dispatch problem within FESMS using only local and neighboring information. Each FESS unit in FESMS calculates its own charging and discharging power reference according to the same ratio. The dispatch scheme is applicable in both undirected and directed network topologies. In the



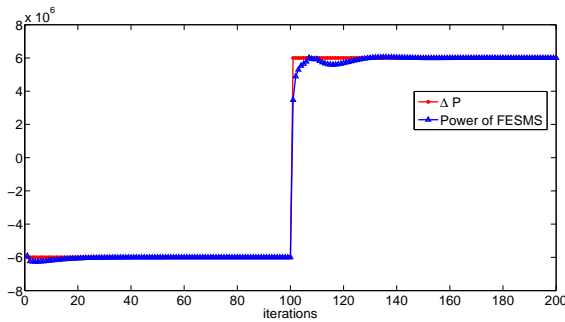


Fig. 16. The total power of FESMS and the power imbalance under switching topologies

network with undirected topology, the weight matrix is doubly stochastic. The auxiliary states reach average consensus, and the ratio factor can be obtained by calculation. In the network with unbalanced topology, the weight matrix is updated to a matrix with both row sums and column sums being 1 using the proposed distributed algorithm. Average consensus of the auxiliary states and consensus the ratio factors are accordingly achieved. Simulation results show that the proposed scheme is effective for FESMS dispatch problem under general strongly connected graphs. Nevertheless, for a power system to be reliable, perhaps both centralized and distributed units are needed to meet critical safety requirements during its operation, and this is certainly an interest topic for future research.

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