

Aalborg Universitet



## Papers, volume 5 - 1997-2000

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*Publication date:*  
2006

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Thoft-Christensen, P. (2006). *Papers, volume 5 - 1997-2000*. Aalborg Universitetsforlag.

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## CHAPTER 90

### LIFE-CYCLE COST EVALUATION OF CONCRETE HIGHWAY BRIDGES<sup>1</sup>

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#### ABSTRACT

In this paper life-cycle costs for minor concrete bridges are evaluated on basis of expected costs of design, inspection, repair, and maintenance. Present bridge management systems are in most cases based on a deterministic approach and the assessment of the reliability or the safety is therefore in general based on subjective statements without taking into account the life-cycle costs in a rational way. In future bridge management systems we will see a change to stochastically based systems with rational assessment procedures based on life-cycle costs. Further, recent developments in optimization techniques will make it possible to produce a much better decision tool regarding inspection and repair.

#### 1. INTRODUCTION

For many years it has been accepted that steel bridges must be maintained due to the risk of corrosion of steel girders etc. The situation is a little different for reinforced concrete bridges. Reinforced concrete bridges built in Europe and elsewhere in the past seventy years were designed on the basis of a general belief among engineers that the durability of the composite material could be taken for granted. Although a vast majority of reinforced concrete bridges have performed satisfactorily during their service life, numerous instances of distress and deterioration have been observed in such structures in recent years. One of the most important deterioration processes which may occur in reinforced concrete bridges is reinforcement corrosion, caused by chlorides present in de-icing salts and/or carbonation of the concrete cover zone.

In the paper is also demonstrated how the estimation of the different life-cycle

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<sup>1</sup> Proceedings Structural Engineers World Conference, San Francisco, USA, July 18-23, 1998. Elsevier Sciences Paper T132-6.

cost terms can be used in practice, e.g. as a decision tool in relation with repair. After a structural assessment it must be decided whether the bridge should be repaired and if so, how the repair is to be performed. Solution of this problem requires that all future inspections and repairs are taken into account.

## 2. LIFE-CYCLE COSTS

Estimation of the life cycle cost  $W$  of a bridge is a difficult matter. The usual definition is the sum of the initial costs  $C_I$  (investment costs) and the expected repair costs  $C_R$  (inspection, maintenance and repair costs) and the expected failure costs  $C_F$ ; see e.g. Ellingwood [1]

$$W = C_I + C_R + C_F \quad (1)$$

It is important to observe that in this definition the benefits of having a bridge are not included. Therefore, in this paper it is proposed to maximize the benefits minus the costs, instead of minimize the life-cycle costs.

## 3. CORROSION

In this paper only one deterioration mechanism is considered namely chloride induced corrosion of the reinforcement, Thoft-Christensen [2] and Thoft-Christensen et al. [3]. When concrete is exposed to chloride it has become normal practice to describe the response of the concrete to the chloride exposure by its chloride profile, i.e. the distribution of the chloride content of the concrete in its near-to-surface layer or by the concentration-distance curve.

The information given by the exposure time and the achieved chloride profile of the concrete may be simplified by a few parameters sufficient to determine the shape of the chloride profile from a mathematical point of view. The controlling parameters with regard to the corrosion initiation time are the initial chloride content  $C_i$ , the chloride content at the surface  $C_0$ , and the chloride diffusion coefficient  $D_c$ . After corrosion has been initiated then the controlling parameter is the rate of corrosion  $i_{corr}$ .

Corrosion of the reinforcement is supposed to take place when the chloride concentration at the site of the reinforcement reaches a critical level  $C_{cr}$ . The corrosion due to chloride ingress will usually be pitting corrosion, i.e. a much localized corrosion of the reinforcement. When corroded rebars become pitted their properties change. Pitting is particularly vicious because it is a localized and intense form of corrosion, and failures of the bar in question often occur with extreme suddenness. *For structures where ductility is needed the initiation stage of corrosion can be taken as the service lifetime*; Thoft-Christensen [4]. Furthermore, the cost of rehabilitation of a structure during the initiation period is generally small compared with the rehabilitation during the corrosion propagation period.

For a reinforced concrete slab bridge pitting corrosion of a single rebar or a few rebars will not drastically change the ductility due to the "parallel" behavior of the rebars. Therefore, in this paper it is considered acceptable to model the corrosion as a uniform corrosion of the rebars to avoid the difficult task of including pitting corrosion. In the modeling of the deterioration sufficient ductility is assumed preserved to justify application of the yield line theory.

The rate of chloride penetration into concrete is modelled by Fick's law of diffusion

$$\frac{\delta c(x,t)}{\delta t} = D_C \frac{\delta^2 c(x,t)}{\delta x^2} \quad (2)$$

where  $D_C$  is the chloride diffusion coefficient,  $x$  is the distance from the surface and  $t$  is the time. The solution of the equation (3) is

$$C(x,t) = C_0 \left\{ 1 - \operatorname{erf} \left( \frac{x}{2\sqrt{D_C t}} \right) \right\} \quad (3)$$

where  $C(x,t)$  is the chloride content at the distance  $x$  from the surface and at the time  $t$ .  $C_0$  is the chloride content on the surface. The corrosion initiation period is

$$T_I = \frac{(d_1 - D_1 / 2)^2}{4D_C} (\operatorname{erf}^{-1}(\frac{C_{cr} - C_0}{C_i - C_0}))^2 \quad (4)$$

where  $C_i$  is the initial chloride concentration,  $C_{cr}$  is the critical chloride concentration, and  $d_1 - D_1 / 2$  is the concrete cover.

The diameter  $D_I(t)$  of the reinforcement bars at the time  $t$  after initiation of corrosion can then be modelled by

$$D_I(t) = D_1 - C_{corr} i_{corr} t \quad (5)$$

where  $D_1$  is the initial diameter,  $C_{corr}$  is a corrosion coefficient, and  $i_{corr}$  is the rate of corrosion.

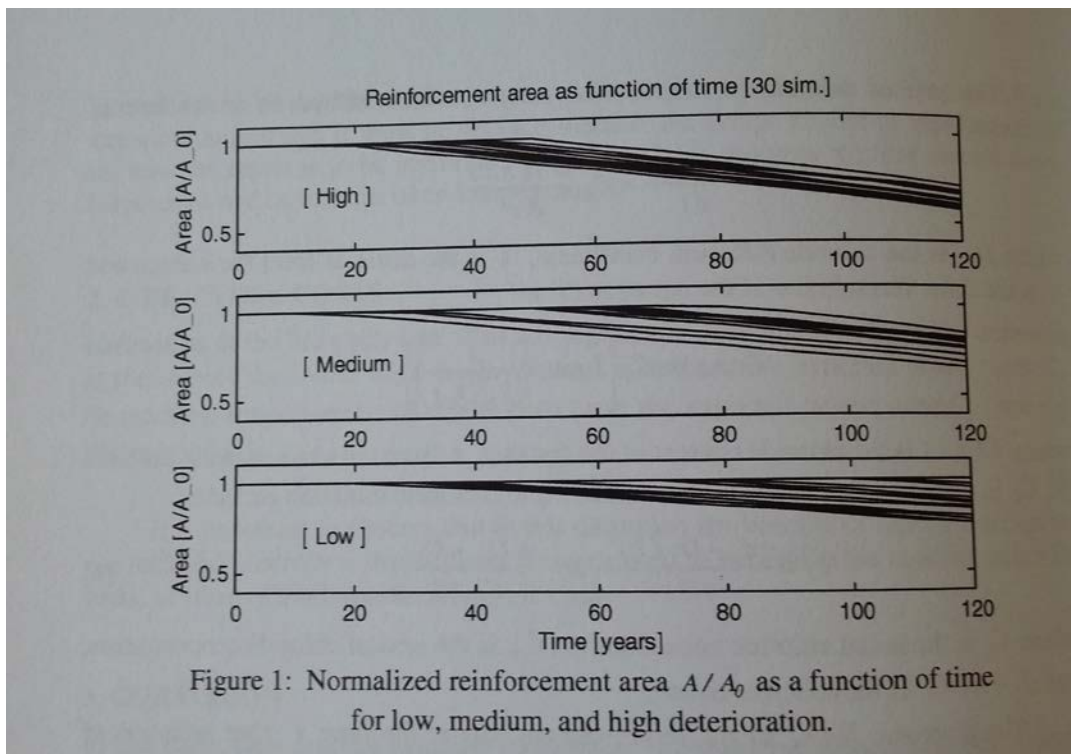
It is for practical applications convenient to use three levels of deterioration: low deterioration, medium deterioration and high deterioration.

|                |                          |                                                      |
|----------------|--------------------------|------------------------------------------------------|
| <i>Low:</i>    | Diffusion coefficient    | $D_C$ : N(25.0, 5.0)[mm <sup>2</sup> /year]          |
|                | Chloride conc. , surface | $C_0$ : N(0.575, 0.038) [%]                          |
|                | Corrosion density        | $i_{corr}$ : Uniform[2.0, 3.0] [μA/cm <sup>2</sup> ] |
| <i>Medium:</i> | Diffusion coefficient    | $D_C$ : N(30.0, 5.0) [mm <sup>2</sup> /year]         |
|                | Chloride conc., surface  | $C_0$ : N(0.650, 0.038) [%]                          |
|                | Corrosion density        | $i_{corr}$ : Uniform[3.0, 4.0] [μA/cm <sup>2</sup> ] |
| <i>High:</i>   | Diffusion coefficient    | $D_C$ : N(35.0, 5.0) [mm <sup>2</sup> /year]         |
|                | Chloride conc. , surface | $C_0$ : N(0.725, 0.038) [%]                          |
|                | Corrosion density        | $i_{corr}$ : Uniform[4.0, 5.0] [μA/cm <sup>2</sup> ] |

Realizations of these three models are shown in figure 1. The area of reinforcement is modelled deterministically using the equation

$$A(t) = \begin{cases} nD_i^2 \frac{\pi}{4} & \text{for } t \leq t_a \\ n(D(t))^2 \frac{\pi}{4} & \text{for } t_a \leq t \leq t_a + D_i / \alpha \\ 0 & \text{for } t > t_a + D_i / \alpha \end{cases} \quad (6)$$

where  $D(t) = D_i - \alpha(t - t_a)$



#### 4. REPAIR STRATEGY

After a structural assessment at time  $T_0$  the problem is to decide if the bridge (or a part of it) should be repaired and if so, how and when should it be repaired. Solution of this optimization problem requires that all future inspections and repairs are taken into account. However, the numerical calculations can clearly become very time consuming. Therefore, in the decision model proposed in the European research project BREU 3091 (1993) some approximations are introduced. After each structural assessment the total expected benefits minus expected repair and failure costs in the remaining lifetime of the bridge are maximized with considering only the repair events in the remaining lifetime of the bridge.

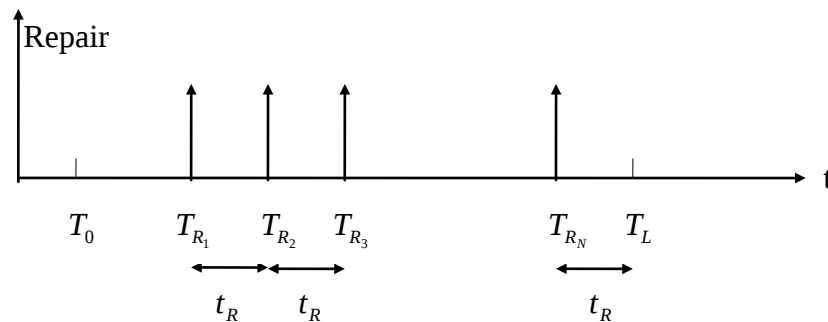


Figure 2: Repair plan.

In order to simplify the decision problem it is assumed that  $N_R$  repairs of the same type are performed in the remaining lifetime  $T_L$  of the bridge. The first repair is performed at time  $T_{R_1}$ , and the remaining are performed at equidistant times with the

time interval  $t_R = (T_L - T_{R_1}) / N_R$ , see figure 2.

The above decision model can be used in an adaptive way if the stochastic model is updated after each structural assessment or repair and a new optimal repair decision is taken. Therefore, it is mainly the time of the first repair after a structural assessment which is of importance.

In order to decide which repair type is optimal after a structural assessment, the following optimization problem is considered for each repair technique; Thoft-Christensen [4]:

$$\begin{aligned} \max_{T_R, N_R} W(T_R, N_R) &= B(T_R, N_R) - C_R(T_R, N_R) - C_F(T_R, N_R) \\ \text{s.t. } \beta^U(T_L, T_R, N_R) &\geq \beta^{\min} \end{aligned} \quad (7)$$

where the optimization variables are the expected number of repair  $N_R$  in the remaining lifetime and the time  $T_R$  of the first repair.  $W$  is the total expected benefits minus costs in the remaining life-time of the bridge.  $B$  is the benefit.  $C_R$  is the repair cost capitalized to the time  $t = 0$  in the remaining lifetime of the bridge.  $C_F$  is the expected failure costs capitalized to the time  $t = 0$  in the remaining lifetime of the bridge.  $T_L$  is the expected lifetime of the bridge.  $\beta^U$  is the updated reliability index.  $\beta^{\min}$  is the minimum reliability index for the bridge (related to a critical element or to the total system).

## 5. MODELLING OF BENEFITS

The inspection costs are not included in the optimization problem since they do not influence the choice of repair action in the present modeling.

The benefits are modelled by

$$B(T_R, N_R) = \sum_{i=[T_0]+1}^{[T_L]} B_i (1+r)^{T_0-T_{ref}} \frac{1}{(1+r)^{T_i-T_0}} \quad (8)$$

where  $[T]$  signifies the integer part of  $T$ . measured in years and  $B_i$  is the benefits in year  $i$  (time interval  $[T_{i-1}, T_i]$ ).  $T_i$  is the time from the construction of the bridge. The  $i$ th term in (8) represents the benefits from  $T_{i-1}$  to  $T_i$ . The benefits in year  $i$  is modelled by

$$B_i = k_0 V(T_i) \quad (9)$$

$k_0$  is a factor modeling the average benefits for one vehicle passing the bridge. It can be estimated as the price of rental of an average vehicle/km times the average detour length. The reference year for  $k_0$  is  $T_{ref}$ . It is assumed that bridges are considered in isolation. Therefore, the benefits are considered as marginal benefits by having a bridge (with the alternative that there is no bridge but other nearby routes for traffic).  $V$  is the traffic volume per year which is estimated by

$$V(T) = V_0 + V_1(T - T_{ref}) \quad (10)$$

where  $V_0$  is the traffic volume per year at the time of construction,  $V_1$  is the increase in traffic volume per year, at  $T$  is the actual time (in years).

## 6. MODELLING OF EXPECTED REPAIR COSTS

The expected repair costs capitalized to time  $t = 0$  are modelled by

$$C_R(T_R, N_R) = \sum_{i=1}^{N_R} (1 - P_F^U(T_{R_i})) C_{R_i}(T_{R_i}) \frac{1}{(1+r)^{T_{R_i}-T_0}} \quad (11)$$

$P_F^U(T_R)$  is the updated probability of failure in the time interval  $[T_0, T_R]$ . The updating is based on a no failure event and the available inspection data at time  $T_0$ . The factor  $(1 - P_F^U(T_{R_i}))$  models the probability that the bridge is not failed at the time of repair.  $r$  is the discount rate.  $C_{R_i}(T_{R_i})$  is the cost of repair modelled by

$$C_{R_i}(T_{R_i}) = C_{R,func} + C_{R,fixd} + C_{R,unit} \quad (12)$$

where the three terms are the functional repair costs, the fixed repair costs, and the unit dependent repair costs respectively. The first term in (12) represents the functional costs and the last two terms represent the direct repair costs.

The functional repair costs are modelled by

$$C_{R,func} = \frac{n}{365} \frac{n_L}{t_{n_L}} k_1 V(T_{R_i}) (1+r)^{T_0-T_{ref}} \quad (13)$$

where  $n$  is the duration of the repair in days,  $n_L$  is the number of lanes closed for the repair,  $t_{n_L}$  is the total number of lanes, and  $k_1$  is a factor used to model the marginal functional repair costs for one vehicle. If the bridge is totally closed then  $k_1 = k_0$ .

The fixed costs are modelled by

$$C_{R,fixd} = k_{F_i} L_B + R_i r_{R_i} \quad (14)$$

$k_{F_i}$  is a coefficient modelling the costs due to the distance from the headquarter [ECU/km],  $L_B$  is the distance from the headquarter [km],  $R_i$  is the roadblock costs for a period of 8 hours per lane, and  $r_{R_i}$  is the number of 8 hours periods needed to perform the repair of the bridge.

The unit costs are modelled by

$$C_{R,unit} = f_D f_T Q_i (k_{L_i} c_h + k_{M_i}) \quad (15)$$

$f_D$  is a factor which depends on how easy the defect is to repair (1.0, 1.3 or 1.5),  $f_T$  is a factor which depends on the time needed to perform the repair (1.0, 1.3 or 1.5),  $Q_i$  is a quantity describing the extent of the repair using the relevant repair technique,  $k_{L_i}$  is the man hours needed per unit of parameter  $Q_i$  for the repair technique considered [hours/unit],  $c_h$  is man hour cost [ECU/h],  $k_{M_i}$  is the material/equipment cost per unit of parameter  $Q_i$ .

## 7. MODELLING OF EXPECTED FAILURE COSTS

The capitalized expected costs due to failure is determined by

$$C_F(T_R, N_R) = \sum_{i=1}^{N_R+1} C_F(T_{R_i}) (P_F^U(T_{R_i}) - P_F^U(T_{i-1_i})) \frac{1}{(1+r)^{T_{R_i}}} \quad (16)$$

where  $T_{R_0} = T_0$  is the time of the structural assessment and  $T_{R_{N_R+1}} = T_L$  is the expected lifetime. The  $i$ th term in (16) represents the expected failure costs in the time interval

$[T_{R_{i-1}}, T_{R_i}]$ .  $C_F(T)$  is the cost of failure at time  $T$

$$C_F(T) = (C_{F_0} + \frac{n_r}{365} k_0 V(T))(1+r)^{T_0 - T_{ref}} \quad (17)$$

where  $C_{F_0}$  is the direct failure costs, and  $n_r$  is the number of days needed for replacement of the failed bridge. The first part of (17) represents the direct failure costs and the second part represents the functional failure costs modelled by loss of benefit.

## 8. IMPLEMENTATION

The optimization problem formulated above has been implemented in the project BREU P3091 [9]. An expert system called BRIDGE2 developed within BREU P3091 contains a number of submodules which can be used to find solutions to the optimization problem formulated in this paper. Some aspects of BRIDGE2 are briefly presented below. A more detailed presentation is given by Thoft-Christensen [4].

As an example of the input needed for using the developed software is shown in Figure 3 how some of the cost data are entered.

The screenshot shows a terminal window titled 'BREU P3091 - BRIDGE2 - Bridge Analysis Based On Reliability'. The main menu is 'BRIDGE ANALYSIS'. A sub-menu 'FACTORS THAT MODEL THE COSTS' is selected. The input data is as follows:

- Factor used to model the benefits (k0): 3 ECU/vehicle
- Factor used to model the functional repair costs (k1): 8 ECU/vehicle
- Direct failure costs (CF0): 30000000 ECU
- Number of days needed for replacement of a failed bridge (nr): 700
- Distance from the headquarters of the bridge owner (LB): 100 km

At the bottom right, it says 'Press F6 for HELP'.

Figure 3. Input of some cost data.

The screenshot shows the same terminal window. The main menu is 'BRIDGE ANALYSIS'. The results of the reliability analysis are displayed:

- Bridge: 153-0002
- The reliability index is 2.38 calculated before any inspection was performed
- The value of the reliability index taking into account the results of the inspections is: BETA = 2.86
- \*\*\* STRUCTURAL ASSESSMENT IS TO BE PERFORMED BEFORE THE NEXT CURRENT OR DETAILED INSPECTION \*\*\*

At the bottom, it says 'Press F8 to Continue'.

Figure 4. Reliability analysis before and after an inspection.

The reliability of the bridge is estimated using the reliability index  $\beta$  for a single

failure mode or the systems reliability index  $\beta^S$  for the structural system (the bridge), see Thoft-Christensen & Baker [5] and Thoft-Christensen & Murotsu [6]. When new information from e.g. an inspection becomes available the estimates of the reliability of the bridge is updated using Bayesian statistical theory, Lindley [7] and Aitchison & Dunsmore [8]. The reliability indices before and after the inspection are used together with expert knowledge to decide whether a structural assessment is needed or not, see figure 4.

The solution of the optimization problem above is performed for a large number of relevant repairs techniques to determine the optimal repair technique, the optimal time for the repair the benefits minus repair and the repair costs, figure 5.

| Repair technique        | Time | Number | Benefits-costs | Repair Cost |
|-------------------------|------|--------|----------------|-------------|
| R_C02 Concrete Patching | 1995 | 1      | 26431713       | 5228        |
| R_D02 Concrete Patching | 1995 | 1      | 26303962       | 145988      |
| R_D01 Concrete Patching | 1995 | 1      | 26118570       | 366800      |

More ↓  
Press F8 to Continue

Figure 5. Optimal repair plan for the “rust stain” defect.

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