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CHAPTER 99

ESTIMATION OF BRIDGE RELIABILITY DISTRIBUTIONS[1]

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1. INTRODUCTION

In this paper it is shown how the so-called reliability distributions for structures can be estimated using crude Monte Carlo simulation. The main purpose is to demonstrate the methodology. Therefore very exact data concerning reliability and deterioration are not needed. However, it is intended in the paper to use realistic data everywhere. The methodology is illustrated using UK data for reinforced concrete overbridges.

The reliability distributions derived in this paper are used in development of optimum strategies for highway bridges, see Frangopol et al. [1]. However, reliability distributions can be used for several other purposes. A number of definitions are used in this paper for single bridges and for a group of bridges.

Single bridge:

$\beta(t)$, where β is a measure of the reliability e.g. the reliability index, and where t is the time. Is called the reliability profile for the structure in question, see figure 1.

Group of bridges:

$\beta(0)$ is called the initial reliability distribution, see figure 3.

$f(t_I)$, where t_I, a measure of the deterioration initiation time for a bridge, is called the deterioration initiation distribution, see figure 5.

$g(\alpha)$, where α, a measure of the deterioration rate of a bridge, is called the

[1] Proceedings from Conference on “Bridge Design, Construction, and Maintenance”, Singapore, October 1999. Thomas Telford, London, 1999 pp. 15-25.

deterioration rate distribution.

$f(t)$ is called the fundamental reliability distribution at the time t.

$a(t_{cr})$, where t_{cr} is defined by $\beta(t_{cr}) = \beta_{cr}$ and where β_{cr} is the critical reliability index, is called the rehabilitation time distribution. If no maintenance has taken place it is called the first rehabilitation time distribution. If maintenance has taken place it is called the rehabilitation time distribution after maintenance.

In the paper it is shown how all these distributions can be estimated for reinforced concrete bridges using Monte Carlo simulation.

2. RELIABILITY PROFILES

In this paper two straight lines as shown in figure 1 are used to model the reliability profile for a reinforced concrete overbridge. The time t = 0 is the year when the bridge in question is built. $\beta(0)$ is the reliability index at the time t = 0. $\beta(t)$ is the reliability index at the time t. Deterioration is assumed to be initiated at the time t_I . The deterioration rate as defined in (1) is α,. $\beta(t)$ = 4.6 is used here as the critical (target) reliability index.

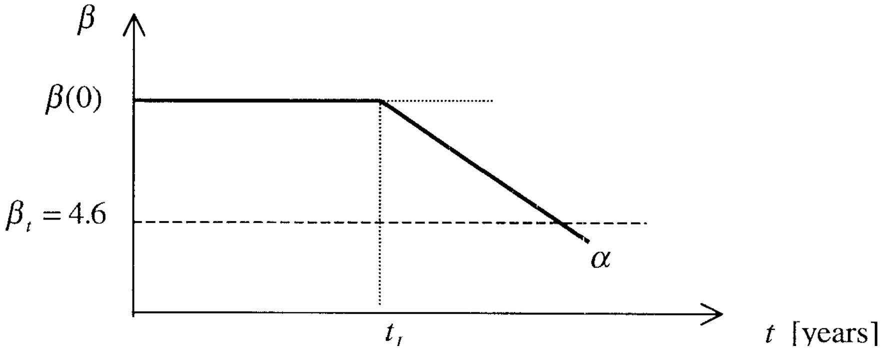

Figure 1. Modelling of a reliability profile.

The reliability index $\beta(t)$ as a function of the time t is

$$\beta(t) = \begin{cases} \beta(0) & for \quad 0 \le t \le t_I \\ \beta(0) - (t - t_I)\alpha & for \quad t_I \le t \end{cases} \tag{1}$$

The stochastic modelling is based on three stochastic variables modelling the reliability index $\beta(0)$ at the time t = 0, the deterioration (corrosion) initiation time t_I [years], and the deterioration (corrosion rate) α [years^{-1}], respectively.

These three stochastic variables are discussed below for reinforced concrete overbridges with the conclusion that the following distribution types can be used:

α $\beta(0)$ is modelled using a log-normal distribution

α t_I is modelled using a Weibull distribution

α α is modelled using a normal distribution.

3. INITIAL RELIABILITY DISTRIBUTION $\beta(0)$

Only little information is available on the reliability index of reinforced concrete bridges as built (t = 0). In Thoft-Christensen & Jensen [2] the reliability index $\beta(0)$ of 15 "good" bridges in UK is evaluated, see figure 2.

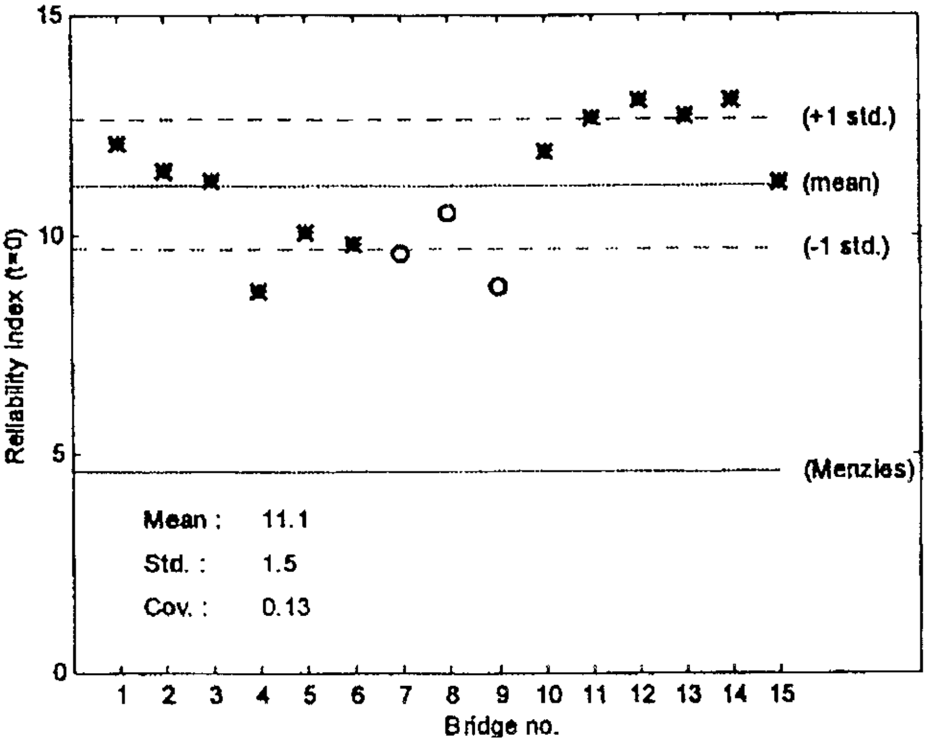

Figure 2. Reliability index at the time t = 0 for good bridges. Slab bridges are indicated by an asterisk and beam/slab bridges are indicated by a circle. Taken from [2].

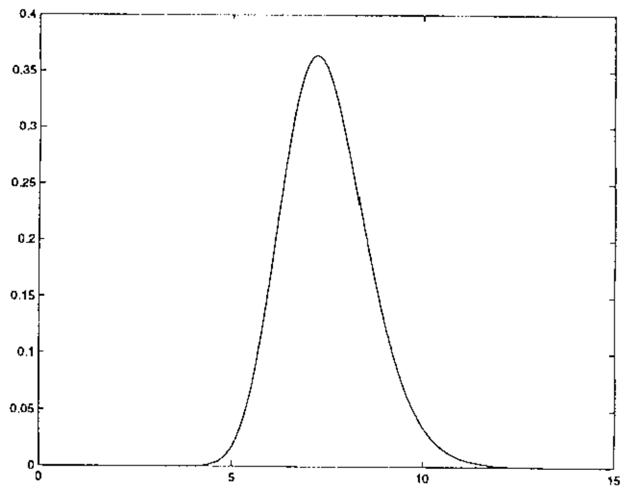

Figure 3. Modelling of the /3(0) distribution using a log-normal distribution LN(2.0,0.15).

The β-values shown in figure 2 cannot be used directly as a basis for modelling the initial distribution at the time t = 0, since they can only be expected to model the upper part of the distribution. However, a log-normal distribution with a small probability for β -values smaller than the critical value, here chosen as 4.6, and with an upper tail corresponding to the information shown in figure 2 is chosen for the simulation study. The log-normal distribution is shown in figure 3. The parameters are LN(2.0,0.15).

4. DETERIORATION INITIATION DISTRIBUTION

The deterioration considered for the reinforced concrete overbridges is here limited to corrosion of the reinforcement. The corrosion initiation time is based on information obtained by P. Thoft-Christensen [3], see figure 4.

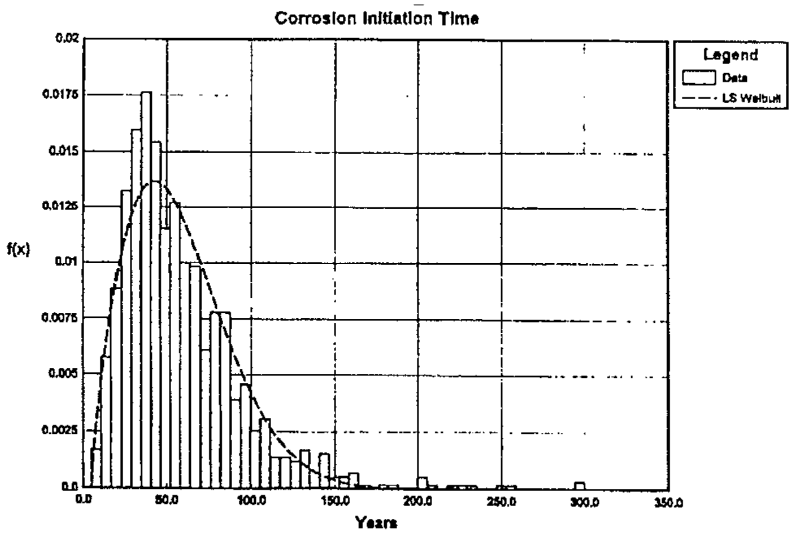

Figure 4. Density function of the corrosion initiation time t_I, Taken from [2].

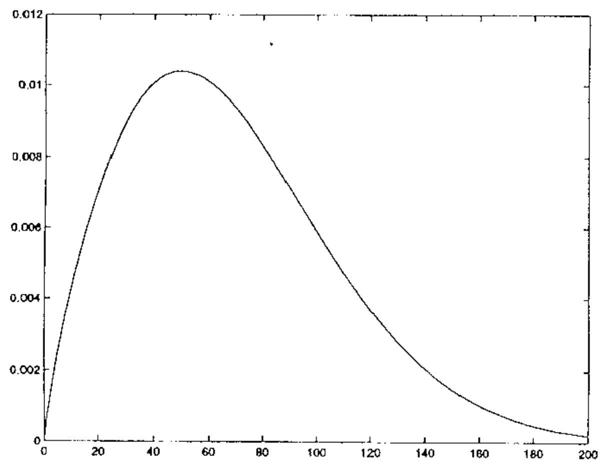

Figure 5. Modelling of the t[distribution using a Weibull distribution with a = 0.00037952 and b =1.81.

The distribution of the corrosion initiation the time t[(for "high" corrosion) as indicated in figure 4 is well approximated by the Weibull distribution shown in figure 5. The parameters of that distribution are a = 0.00037952 and b = 1.81 (or μ = 63.67, k = 1.81 and ε = 0).

5. DETERIORATION RATE DISTRIBUTION

The corrosion rate a used in this study is based on information obtained by Thoft-Christensen & Jensen [1] for 15 "good" bridges in the UK and "high" corrosion, see figure 6.

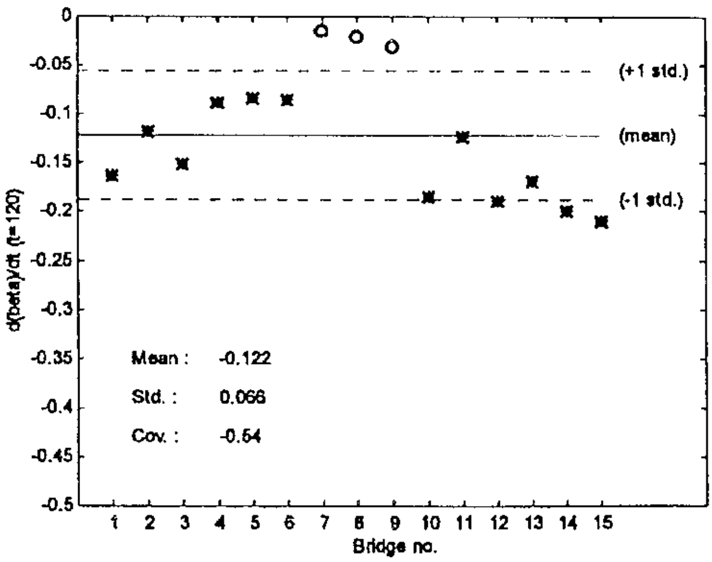

Figure 6. Corrosion rates for "high" deterioration. Slab bridges are indicated by an asterisk and beam/slab bridges are indicated by a circle. Taken from [2].

On the basis of the information in figure 6 a uniform distribution U[0.01,0.20] is chosen for α .

6. TIME PROFILE OF RELIABILITY DISTRIBUTIONS (FUNDAMENTAL DISTRIBUTIONS)

In this section the variation with time of the reliability distributions (fundamental reliability distributions) is estimated on the basis of simulations. The simulation results are fitted to normal distributions. The resulting fundamental distributions are shown in figure 7. As expected, the mean value decreases with time and the standard deviation increases with time:

$$\mu(t) = -0.000635t^2 + 0.00035t + 7.4888$$

$$\sigma(t) = 0.000253t^2 + 0.0108t + 0.9920 \quad (2)$$

t = 0 years corresponds to the construction year of a given bridge. In figure 7, N = 0 indicates the time 0 years, N = 10 indicates the time 10 years, etc. The expressions in (2) are obtained by curve fitting.

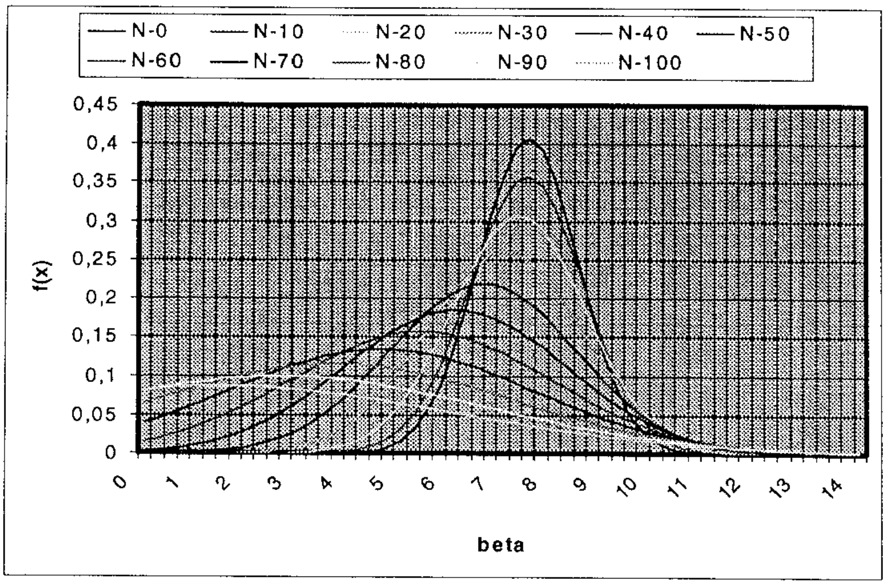

Figure 7. The variation of the fundamental reliability distribution with time *t*.

7. RELIABILITY STATE ESTIMATION FOR NON-MAINTAINED BRIDGES

This section shows how to estimate the number of bridges in different states (reliability states) as a function of the time t. For illustration, reinforced concrete overbridges in the UK are used. The bridge states as functions of the reliability index β are defined in table 1.

State	State no:	β
Extremely good	5	>10
Very good	4	[8;10]
Good	3	[6;8]
Acceptable	2	[4.6;6]
Non-acceptable	1	<4.6

Table 1. Definition of bridge states.

Year	Number
1950	2
1960	114
1970	291
1980	316
1990	247
1995	0
Total	**970**

Table 2. Simplified data for number of bridges and construction years.

To simplify the calculations the construction years and numbers of reinforced concrete overbridges are shown in table 2. In the table the 291 bridges built in 1970 were actually built in the time interval 1965-1975, etc.

It is seen in figure 7 that the upper tails of the normal distributions for increasing time *t* show an unrealistic behaviour due to these approximations, namely an increasing probability for high β values. This has to be counteracted by truncation when the number of bridges in the different states is estimated. However, this problem is of less importance for a maintenance methodology since bridges with very high β values will not need any maintenance in the period of time considered.

Bridges built in each of the five time intervals [1945 , 1955],..., [1985 ,1995] apply the distributions in figure 7 analysed as a group, and not individually, to simplify the calculations. The above-mentioned correction for unrealistic predictions in the upper tails of the fundamental distributions is performed manually. Using this procedure the number of bridges in each of the states are estimated for years 2000, 2005,..., 2030. Maintenance is not taken into account, so the predictions apply to bridges without any maintenance. The result of this estimation is shown in figure 8.

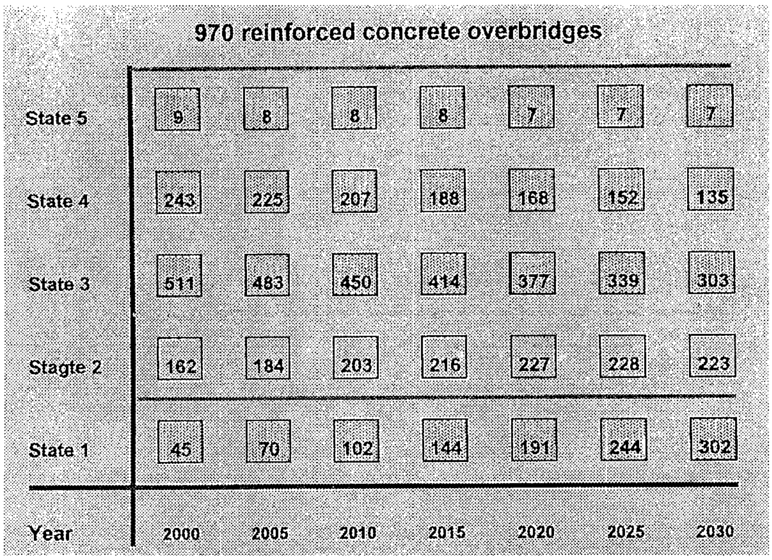

Figure 8. The distribution of the 970 non-maintained bridges on reliability states as a function of time.

8. FIRST REHABILITATION TIME DISTRIBUTION

In this section it is shown how the distribution indicated by “?” in figure 9 can be estimated using the data obtained. This distribution, called the first rehabilitation time distribution, is e.g. essential for the design of optimal maintenance strategies.

When bridges deteriorate without any intervention such as preventive maintenance, the reliability level will decrease and at a certain time it will cross the critical level (say, 4.6). This is the time when essential rehabilitation is needed. The variability at the age when bridges of a particular class require essential rehabilitation is the first rehabilitation time distribution.

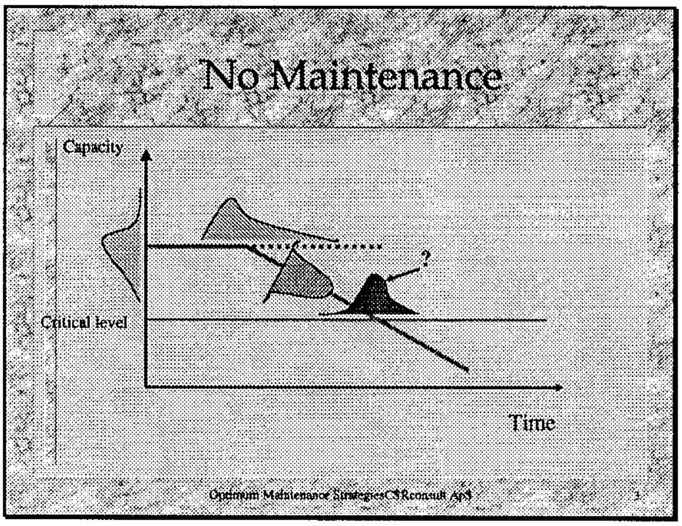

Figure 9. Definition of the first rehabilitation time distribution.

The first rehabilitation time distribution can easily be calculated using the fundamental distributions shown in figure 7. The result of the estimation is shown in figure 10.

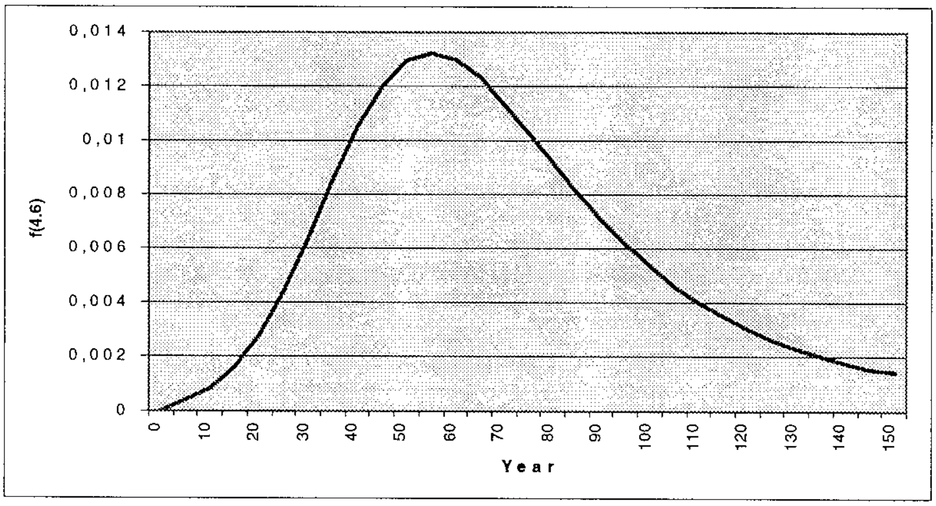

Figure 10. The first rehabilitation time distribution.

9. RELIABILITY STATE ESTIMATION FOR MAINTAINED BRIDGES

The main effect of preventive maintenance is a delay in the deterioration (the corrosion). Therefore, it is in this study assumed that a bridge undergoing preventive maintenance will remain in the current state but that the corrosion rate will be reduced (improved). This modelling is illustrated in figure 11.

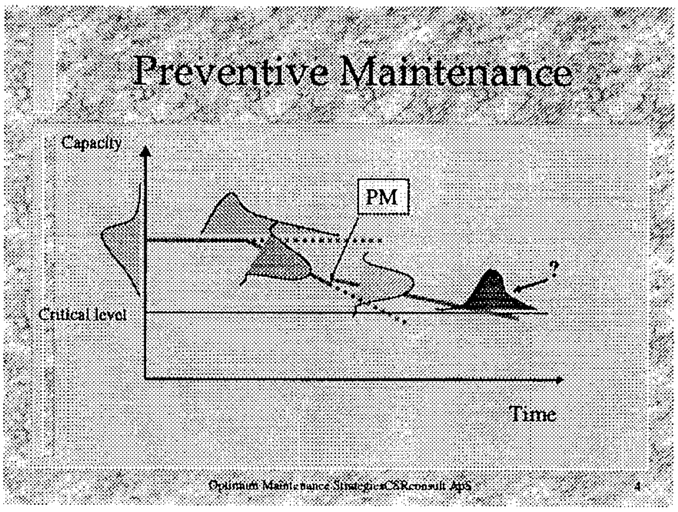

Figure 11. Stochastic modeling of preventive maintenance.

The main effects of essential maintenance of a bridge are not only a delay in the deterioration rate (the corrosion rate), but the bridge reliability is at the same time increased. The bridge moves up to a higher state perhaps even to its initial state or even higher. For the sake of simplicity it is here assumed that a bridge undergoing essential maintenance will return to the initial state and will, from a deterioration point of view, behave like a new bridge, i.e. with the same corrosion initiation time and corrosion rate distributions as new bridges. This modelling is illustrated in figure 12.

The main purpose of this section is to obtain the data needed to estimate the distribution indicated by “?” in figure 11 and figure 12 when preventive and/or essential maintenance is performed. This distribution is called the rehabilitation time distribution after maintenance.

The bridge distribution on states depends significantly on the maintenance strategy chosen. Since the state of a bridge at the time t depends not only on the state at t - 5 years, but also on the state at earlier points of time, a Baysian approach will probably be too uncertain an approximation. However, it is not necessary to follow the state development for each individual bridge. It is sufficient to treat similar bridges as a group. This conclusion will simplify the analysis considerably, but it is still not a trivial matter to estimate bridge distribution on states, when preventive and essential maintenance is taken into account.

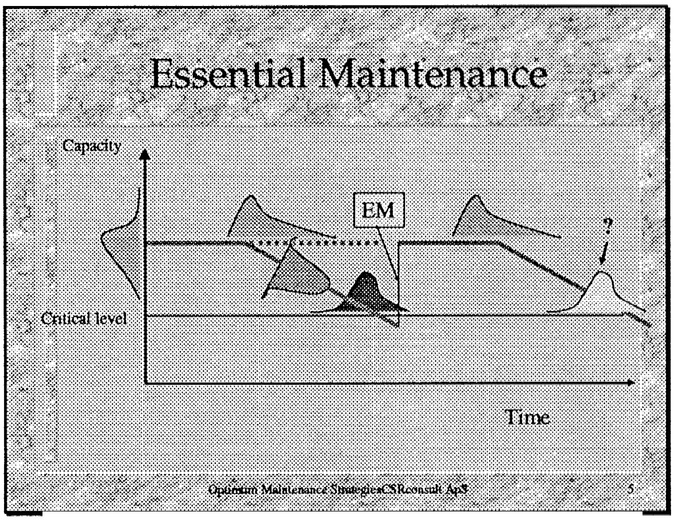

Figure 12. Stochastic modelling of essential maintenance.

To illustrate the proposed methodology the following artificial, but hopefully not too unrealistic, maintenance strategy is chosen:

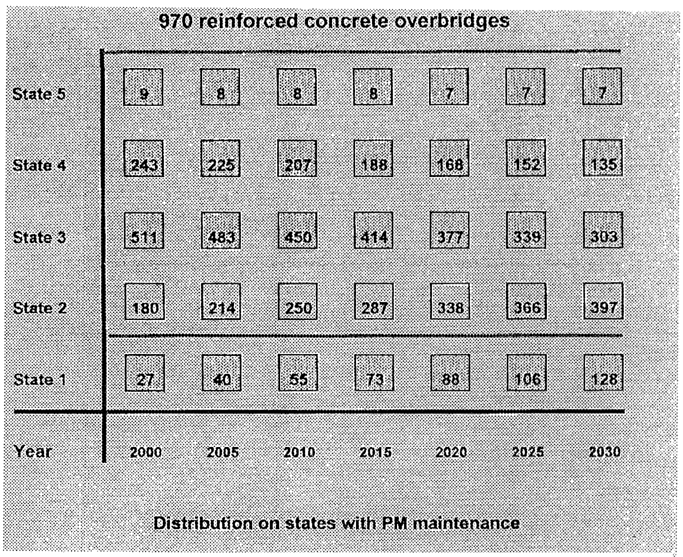

Figure 13. The distribution of the 970 maintained bridges on reliability states as a function of time.

- 25 % of all bridges that are in state 2 at the time *t* will undergo preventive maintenance between the time *t* and the time *t* + 5 years. The bridge will remain in state 2 but the new corrosion rate a PM will be smaller, since no data equal to 0.04 are available.
- 50 % of all bridges that are in state 1 at the time *t* will undergo essential maintenance between the time *t* and the time *t* + 5 years. For the sake of simplicity, it is assumed that the state of the bridge will change from state 1 to state 3.
- Bridges in states 3-5 will not undergo any maintenance.

This very simplified maintenance strategy will drastically increase the number of bridges in state 2 and decrease the number of bridges in state 1 as time goes on. The reliability estimates after preventive maintenance are made using a method similar to the one used in the making of figure 8, but simulation is not needed, since only one deterministic variable is involved. The result of this analysis is with some minor manual corrections shown in figure 13.

It is interesting to compare this figure with figure 8. The number of bridges in state 1 is in year 2030 reduced from 302 to 128 bridges due to preventive maintenance. This is a substantial reduction that should be calibrated against real data. As expected, the number of bridges in state 2 in year 2030 will increase, namely from 223 to 397 bridges. The number of bridges in the states 3-5 is unchanged, since the maintenance strategy used does not affect these states.

10. REHABILITATION TIME AFTER MAINTENANCE

In this section IT is shown how the lower tail of the rehabilitation time distribution after maintenance indicated by "?" in figure 12 can be estimated using the same procedure as applied for estimation of the first rehabilitation time distribution shown in figure 10. Table 3 shows a simple estimation of the distribution based on the number of bridges in state 1 for the years 2000, 2005,..., 2030 (see figure 13) divided by the total number of bridges in the year in question. Further, values of the density function $f(\cdot)$ are calculated by numerical differentiation.

Year no:	Year	No. of bridges	In state 1	F(.)	f(.)
0	1950	2	0	0.0000	0
5	1955	2	0	0.0000	0
10	1966	116	0	0.0000	0
15	1965	116	0	0.0000	0.000491
20	1970	407	2	0.0049	0.000491
25	1975	407	2	0.0049	0.0002
30	1980	723	5	0.0069	0.000615
35	1985	723	8	0.0111	0.000546
40	1990	970	12	0.0124	0.000852
45	1995	970	19	0.0196	0.001546
50	2000	970	27	0.0278	0.002165
55	2005	970	40	0.0412	0.002887
60	2010	970	55	0.0567	0.003402
65	2015	970	73	0.0753	0.003402
70	2020	970	88	0.0907	0.003402
75	2025	970	106	0.1093	0.004124
80	2030	970	128	0.1320	0.004536

Table 3. Calculation of values of the distribution function $F(\cdot)$ and the density function $f(\cdot)$.

With the maintenance strategy chosen in this study, few bridges will cross the critical level in the considered time interval. This is of course one of the reasons for performing maintenance. This is reflected in the estimation of $f(\cdot)$ values as function of *t*. Only the lower tail of the rehabilitation time distribution after maintenance is obtained, see figure 14.

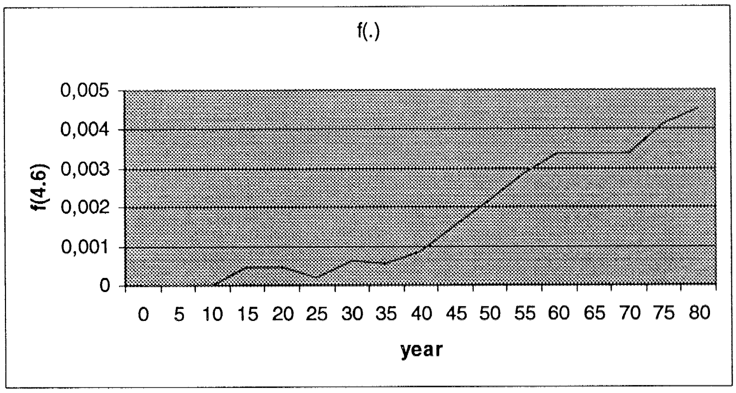

Figure 14. Lower tail part of the rehabilitation time distribution after maintenance.

11. CONCLUSIONS

A number of important bridge reliability distributions are defined in the paper. Simplified modelling of the controlling distributions is proposed. A procedure for estimation of the so-called fundamental distributions, the first rehabilitation time distribution and the rehabilitation time distribution after maintenance is described. The procedure is applied to 970 reinforced concrete overbridges in UK.

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