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Power Balancing Aggregator Design for Industrial Consumers Using Direct Control

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Abstract— Demand side management in the future smart grid requires new players in the electricity markets. We assume a player, the so-called *aggregator* which aims to utilize the flexibility in large-scale consumers with thermal energy storage. An aggregator design is proposed to mange the power consumption of flexible demands during a certain period of time. The setup consists of an MPC-like controller to optimally operate the consumption units for upward and downward regulating power provision. To handle the uncertainties arise from model mismatch, a series of feedback loop are also considered along with the central optimization. We formulate the problem for two specific case studies which are supermarket refrigeration systems and chiller systems in conjunction with ice tanks. Simulation results are provided for these specific consumers.

I. INTRODUCTION

Increasing use of renewable resources means that the future electricity system will be faced with more challenges in maintaining balance between production and consumption of electricity due to the unpredictable nature of these kind of resources. Smart grid solutions, in particular, consumer involvement in balancing issues, can be a promising remedy. Flexible consumers can offer different services in various electricity markets such as the day-ahead market, intra-day market or regulating power market and subsequently benefit from these tradings [1].

Participation of the consumers necessitates new infrastructure on both the consumer and grid side. In the smart grid literature, the so-called "Aggregator" has been introduced as a new player in the future market, located between the grid operator and a number of flexible consumers to handle the services that can be derived from the demand these consumers represent [2]. What the aggregator's responsibility is and which entities it has interaction with may be different, however. The aggregator may aim to provide services at the distribution grid level to the DSO (distribution grid operator), at the regulating power markets to the TSO (transmission system operator) or at the balancing power market to the BRPs (balance responsible parties) [3]. Various types of consumers, ranging from household appliances with privacy

issues to large industrial or commercial companies can be aggregated. Moreover, different control strategies can be taken by the aggregator to manage the consumption units. The aggregator may have direct jurisdiction over the consumers or may have negotiations with them. It might also control the units indirectly by broadcasting incentive signals. In [4], VPP aggregators (virtual power plant aggregator) were categorized according to the control strategies, either direct or indirect (acting through price signals). The work in [5] explains an architecture to integrate small-scale resources to the grid. A demonstration environment is also developed to implement and test the whole setup. In the proposed architecture, the aggregator acts as an information center that collects and stores the data it receives from home energy management systems.

In most cases, the aggregator will be a for-profit entity in the future market with the main goal of making money. Thus, in addition to fulfilling the grid operator requirements, it needs to optimize its performance in order to be economically profitable. In general, the objective function at the aggregator should reflect the profit obtained from attending the electricity market trading and accordingly the optimization should attempt to maximize the profit. However, the profit can be formulated differently for different use cases. As an example, in [6], an aggregator design is proposed which utilizes the flexibility in electric vehicles to provide frequency regulation to the grid. A vehicle can provide services as long as it is connected to the grid, except the times it is being charged by its owner's decision. The role of the aggregator is then to control the charging process the rest of the time such that it maximizes its revenue. Another example is given in [7], which proposes a general market model for residential demand response in smart grid. The model comprises three levels: a utility operator at the upper level, a number of residential units at the bottom and a set of competitive aggregators acting as mediators between the utility operator and the users. The utility operator aims to minimize its cost, including the power generation cost as well as the monetary rewards should be given to the aggregators. Each aggregator tries to maximize its own net profit, which is the reward received from the operator minus the compensation that should be paid to the users. Finally the user's problem is to maximize the payoff consisting of the received compensation minus the dissatisfaction caused from changing its own preferred set-point. However, solving this multi-level optimization requires that the utility operator has access to all the information about the end-users. The paper

proposes a practical repeated auction scenario where the aggregator negotiates with the home owner after the operator announces its reward. When they reach an agreement, the aggregator will offer services on behalf of the users. The utility operator also needs to modify its reward at each iteration.

In previous work [8], we proposed an aggregator design based on direct interaction with a few, large-scale consumers. The proposed aggregator will play in a hierarchical market setup as it is shown in figure 1. The setup is similar to the one provided in [7], except that there is no negotiation in our setup. We considered a scenario in which the aggregator is activated by the grid operator, TSO, DSO or BRP, to follow a power reference for a certain period of time. The objective is to optimally split up the power reference between the consumers while respecting their constraints. In [8], we only formulated the problem for down regulation. In this paper, we complete the formulation by providing an up regulation scenario as well. Furthermore, as the main contribution of the paper, we consider the discrepancy inherent in the aggregator/consumer interface, according to which the aggregator utilizes deliberately simplified models of the consumers, although the physical systems are known to be more complicated. Rather than having to increase the model complexity on the aggregator level, we consider the model mismatch as state-dependent uncertainties and propose a simple feedback mechanism for re-distributing power discrepancies among other consumers. We analyze the potential state trajectories subject to said uncertainties, and propose a brute-force approach to determine how large uncertainties the setup can handle over a given activation horizon.
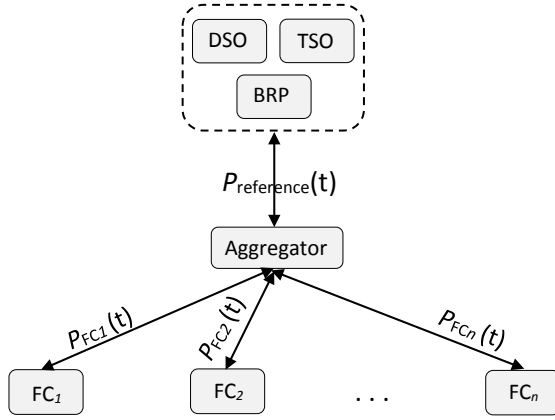

Fig. 1: Hierarchical direct control setup, where $\text{FC}_i (i = 1, ..., n)$ stands for the flexible consumption and $P_{\text{FC}i}(t)$ is the power that is distributed to each consumer

The structure of the rest of the paper is as follows: In section II, we will describe the aggregator setup. In section III, we will provide our solution to compensate for uncertainties in the proposed setup. In section IV, we will examine the constraints for compensation. Simulation results will be presented in section V. Finally we will conclude the paper in section VI.

Nomenclature	
T_s	sampling time
L	latent heat of water
n_{ch}	number of chiller systems
n_{cr}	number of cold rooms in a supermarket
n_r	number of supermarket refrigeration systems
$P_{r_i,\text{base}}$	baseline power consumption of supermarket
$P_{ch_i,\text{base}}$	baseline power consumption of chiller
$P_{r_i,\text{min}}$	minimum power consumption of supermarket
$P_{r_i,\text{max}}$	maximum power consumption of supermarket
$P_{ch_i,\text{min}}$	minimum power consumption of chiller
$P_{ch_i,\text{max}}$	maximum power consumption of chiller
$\text{COP}_{ch,cool}$	COP of chiller system in direct cooling
$\text{COP}_{ch,ice}$	COP of chiller system in ice making
COP_r	COP of supermarket refrigeration system
$m_{\text{ice,base}}$	mass of ice at the baseline consumption
$m_{\text{ice,base}}$	maximum mass of ice in the ice tank
UA_{cr}	heat transfer coefficient of a cold room
m_{food}	mass of refrigerated food in a cold room
c_{food}	specific heat capacity of food
$T_{cr,\text{min}}$	minimum temperature of a cold room
$T_{cr,\text{max}}$	maximum temperature of a cold room
$T_{cr,\text{base}}$	cold room temperature at baseline

II. NOMINAL AGGREGATOR SETUP

A. Optimization Problem

As stated above, we consider the following scenario: "The aggreagtor is paid by the grid operator to follow a power reference it receives within an activation period". The task of the aggregator in this setup is then to optimally distribute the power reference between the consumers. The power reference following service can be of interest to any grid operator in the electricity market, such as BRPs, TSO or DSO. For instance, the aggregator can offer regulating powers, comprising primary, secondary or manual reserves, directly to the TSO or via the BRPs in the regulating power market. The focus in this paper is on thermal energy storage, i.e. excess electrical energy can be stored in form of thermal energy for later use. We assume two general cases at the beginning of the activation period:

$$\text{Scenario 1: } P_{\text{reference}}(t) \geq \sum_{i=1}^{n} P_{\text{FC}i,\text{base}}(t)$$

$$\text{Scenario 2: } P_{\text{reference}}(t) \leq \sum_{i=1}^{n} P_{\text{FC}i,\text{base}}(t)$$

where $P_{\text{reference}}$ denotes the power reference that should be followed by the aggregator and n is the number of flexible consumers under the control of the aggregator. $P_{\text{FC}i,\text{base}}$ represents the baseline power consumption of a flexible consumer. In normal operation, the units consume their baseline consumption as long as there is no activation. Providing downward regulating power and upward regulating power can be examples of Scenario 1 and Scenario 2 respectively. To formulate the optimization problem, we divide the operation time into three time periods: before the activation, activation time and after the activation. Figure 2 shows thermal energy changes and power consumption of a typical thermal storage during these three time periods. In the first case, the aggregator is asked to follow a power reference greater than its baseline power. This requires the aggregator to store some extra energy in thermal storages at its disposal when the activation time starts ($t = t_{\text{start}}$). Right after the
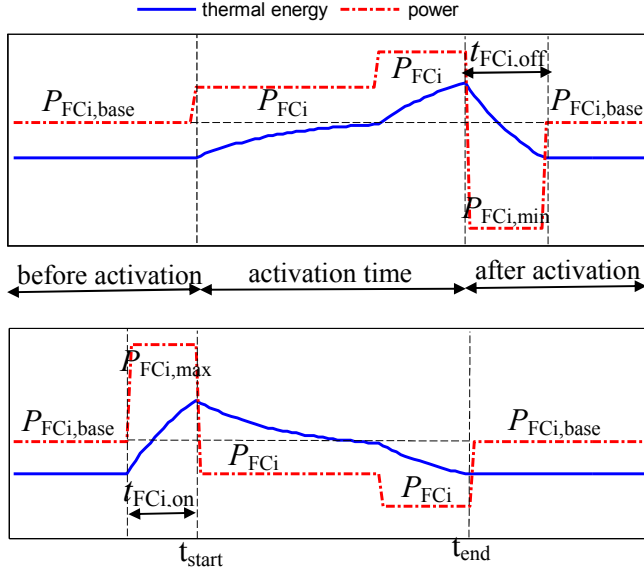
Fig. 2: Thermal energy changes and power consumption of a typical thermal storage during three time periods for two scenarios, Scenario 1: power reference is above the aggregated baseline consumption (top plot), Scenario 2: power reference is below the aggregated baseline consumption (bottom plot)

activation time finishes ($t = t_{\text{end}}$), the consumers can lower the consumption to minimum level and use the stored energy. Reducing the power consumption to the minimum level minimizes the time needed for regaining the stored energy and consequently minimizes the heat loss to the surrounding. $t_{\text{FC}i,\text{off}}$ in the above figure indicates the time period when the consumer consumes its minimum power and uses the stored energy to operate the system within the constraints. In case of no activation, the consumer needs to consume at least the baseline consumption. Thus, the consumer is able to save power corresponding to $P_{\text{FC},\text{base}} - P_{\text{FC},\text{min}}$ at each time instant during $t_{\text{FC}i,\text{off}}$. The second case describes a situation where the aggregator is asked for a power reference below its baseline. In this case, the aggregator needs to store some energy before the activation in order to deliver it during the activation time. $t_{\text{FC}i,\text{on}}$ in figure 2 shows the time period in which the consumer increases its consumption from baseline in order to store the required energy. Note that it is better to increase the power consumption to the maximum level. This minimizes the time required for storing energy and consequently minimizes the energy loss in the saving process. Therefore, the consumer has to consume $P_{\text{FC}i,\text{max}} - P_{\text{FC}i,\text{base}}$ more than its normal operation at each time instant during $t_{\text{FC}i,\text{on}}$.

The objective functions at the aggregator for the two above mentioned scenarios are formulated as below:

$$\max_{P_{\text{FC}i}} \sum_{i=1}^{n} (P_{\text{FC}i,\text{base}} - P_{\text{FC}i,\text{min}}) \times t_{\text{FC}i,\text{off}} \qquad (1)$$

$$\min_{P_{\text{FC}i}} \sum_{i=1}^{n} (P_{\text{FC}i,\text{max}} - P_{\text{FC}i,\text{base}}) \times t_{\text{FC}i,\text{on}} \qquad (2)$$

In the objective function (1), we aim to maximize the total energy saving after the activation while in the objective function (2), we aim to minimize the total energy consumption before the activation. Thus, in addition to power reference following which is the first goal, we aim to minimize the energy losses or, in other words, maximize the energy savings. Needless to say, the above objective functions are not like a standard LQR problem. In general, $t_{\text{FC}i,\text{off}}$ and $t_{\text{FC}i,\text{on}}$ are logarithmic functions of the system state. $P_{\text{FC}i,\text{base}}$, $P_{\text{FC}i,\text{min}}$ and $P_{\text{FC}i,\text{max}}$ are given to the aggregator.

B. Consumer Models

In the previous section, the objective function is provided for the case in which the aggregator aims to exploit the flexibility in consumers with thermal storage. To do the optimization, the aggregator requires a model of each storage that describes thermal energy changes versus input electrical power. Note that, in order to keep the complexity of the problem to be solved at the aggregator level at a manageable level, it is *deliberately* chosen to represent consumers using the simplest model possible. Figure 3 depicts the salient features of such a simplified thermal storage.
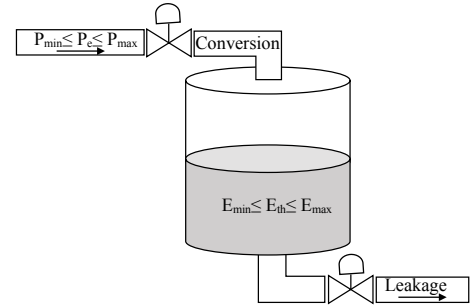

Fig. 3: Salient features of a thermal storage

Each consumer is modeled as an energy collector with constraints on input electrical power ($P_{\text{min}} \leq P_e \leq P_{\text{max}}$) and stored thermal energy ($E_{\text{min}} \leq E_{\text{th}} \leq E_{\text{max}}$). There could be also a leakage to the surrounding. Furthermore, energy conversion and dissipation can be expressed with different dynamics.

We choose two specific consumer types in this work: supermarket refrigeration systems and chillers in air conditioning systems. For the first one, cold rooms or display cases can act as a thermal storage where we store energy in refrigerated foods. For the second one, an ice storage connected to the chiller serves as a thermal storage with water media. In [8], we presented appropriate models of these consumer types for optimization purposes. In brief, cold rooms or display cases at the supermarket can be seen as a storage with state dependent leakage, whereas an ice tank has no leakage because of good insulation. We assumed constant COP (coefficient of performance) for the supermarket, which implies energy conversion from electrical to thermal occurs at a fixed rate. For the chiller, we assumed two constant COPs associated with direct cooling and ice making modes. In the following, the optimization problem for the two scenarios

are presented:

Scenario 1):

$$\max_{U_{r_i},U_{ch_i},\eta_i,Z_{ch_i}} \left(\sum_{i=1}^{n_r} p_{r_i,\text{down}} t_{r_i,\text{off}} + \sum_{i=1}^{n_{ch}} p_{ch_i,\text{down}} t_{ch_i,\text{off}} \right) \tag{3}$$

Scenario 2):

$$\min_{U_{r_i},U_{ch_i},\eta_i,Z_{ch_i}} \left(\sum_{i=1}^{n_r} p_{r_i,\text{up}} t_{r_i,\text{on}} + \sum_{i=1}^{n_{ch}} p_{ch_i,\text{up}} t_{ch_i,\text{on}} \right) \tag{4}$$

where,

$$p_{r_i,\text{down}} = P_{r_i,\text{base}} - P_{r_i,\text{min}}$$
$$p_{ch_i,\text{down}} = P_{ch_i,\text{base}} - P_{ch_i,\text{min}}$$
$$p_{r_i,\text{up}} = P_{r_i,\text{max}} - P_{r_i,\text{base}}$$
$$p_{ch_i,\text{up}} = P_{ch_i,\text{max}} - P_{ch_i,\text{base}}$$

$$t_{r_i,\text{off}} = \frac{-1}{A_{r_i}} \ln \left(1 + \frac{A_{r_i} X_{r_i}(t_{\text{end}})}{-B_{r_i} p_{r_i,\text{down}}} \right) \tag{5}$$

$$t_{ch_i,\text{off}} = \frac{X_{ch_i}(t_{\text{end}})}{B_{ch_i} p_{ch_i,\text{down}}} \tag{6}$$

$$t_{r_i,\text{on}} = \frac{1}{A_{r_i}} \ln \left(1 + \frac{A_{r_i} X_{r_i}(t_{\text{start}})}{B_{r_i} p_{r_i,\text{up}}} \right) \tag{7}$$

$$t_{ch_i,\text{on}} = \frac{X_{ch_i}(t_{\text{start}})}{(B_{ch_i} + D_{ch_i}) p_{ch_i,\text{up}}} \tag{8}$$

and the objective function is subject to the following constraints:

subject to:

$$X_{r_i}(t+1) = (1 + A_{r_i} T_s) X_{r_i}(t) + B_{r_i} T_s U_{r_i}(t) \tag{9}$$
$$X_{r_i,\text{min}} \leq X_{r_i}(t) \leq X_{r_i,\text{max}} \tag{10}$$
$$U_{r_i,\text{min}} \leq U_{r_i}(t) \leq U_{r_i,\text{max}} \tag{11}$$
$$X_{r_i}(t_{\text{start}}) = 0 \quad \text{(Scenario 1)} \tag{12}$$
$$X_{r_i}(t_{\text{end}}) = 0 \quad \text{(Scenario 2)} \tag{13}$$

$$X_{ch_i}(t+1) = X_{ch_i}(t) + B_{ch_i} T_s U_{ch_i}(t) + D_{ch_i} T_s Z_{ch_i}(t) \tag{14}$$

$$p_{ch_i,\text{down}} \eta_i(t) \leq U_{ch_i}(t) + p_{ch_i,\text{down}} \tag{15}$$
$$(p_{ch_i,\text{up}} + \epsilon) \eta_i(t) \leq -U_{ch_i}(t) - \epsilon \tag{16}$$
$$Z_{ch_i}(t) \leq p_{ch_i,\text{up}} \eta_i(t) \tag{17}$$
$$Z_{ch_i}(t) \geq -p_{ch_i,\text{down}} \eta_i(t) \tag{18}$$
$$Z_{ch_i}(t) \leq U_{ch_i}(t) + p_{ch_i,\text{down}}(1 - \eta_i(t)) \tag{19}$$
$$Z_{ch_i}(t) \geq U_{ch_i}(t) - p_{ch_i,\text{up}}(1 - \eta_i(t)) \tag{20}$$
$$X_{ch_i,\text{min}} \leq X_{ch_i}(t) \leq X_{ch_i,\text{max}} \tag{21}$$
$$U_{ch_i,\text{min}} \leq U_{ch_i}(t) \leq U_{ch_i,\text{max}} \tag{22}$$
$$X_{ch_i}(t_{\text{start}}) = 0 \quad \text{(Scenario 1)} \tag{23}$$
$$X_{ch_i}(t_{\text{end}}) = 0 \quad \text{(Scenario 2)} \tag{24}$$

$$U_{r_i}(t) + U_{ch_i}(t) = P_{\text{reference}}(t) - \sum_{i=1}^{n_r} P_{r_i,\text{base}} - \sum_{i=1}^{n_{ch}} P_{ch_i,\text{base}} \tag{25}$$

Equations (9-13) specify the model and constraints for the supermarket refrigeration systems, while equations (14-24) are related to the chillers. Thermal energy changes during the activation are considered as a system state when modeling the consumers (X_{r_i} and X_{ch_i}). Manipulated variables in the above optimization problem are defined as below:

$$U_{r_i}(t) = P_{r_i}(t) - P_{r_i,\text{base}} \tag{26}$$
$$U_{ch_i}(t) = P_{ch_i}(t) - P_{ch_i,\text{base}} \tag{27}$$
$$\begin{cases} \eta_i(t) = 1 \iff U_{ch_i}(t) > 0 \\ \eta_i(t) = 0 \iff U_{ch_i}(t) \leq 0 \end{cases} \tag{28}$$
$$Z_{ch_i}(t) = \eta_i(t) U_{ch_i}(t) \tag{29}$$

As explained in [8], the chiller model is a mixed logical dynamical system that can be described by a binary variable, η_i and an auxiliary variable, Z_{ch_i} such that equations (28) and (29) hold. To solve the optimization problem, we apply the method proposed in [9] in order to convert (28) to the linear inequalities (15-16). Equation (29) should also be used in the inequalities (17-20). ϵ is a small positive value. In a real setup, each consumer should communicate the system parameters and constraints to the aggregator every sample time or whenever they are updated. Thereupon, at the aggregator, an MPC-like controller is run which provides a vector of power consumption for each consumer by the end of activation period. The first sample indicates the desired power consumption each unit is asked to follow at current time. For online optimization, the system parameters and constraints can be identified locally using empirical data. Chiller parameters can for instance be obtained as follows:

$$B_{ch} = \text{COP}_{ch,\text{cool}} \tag{30}$$
$$D_{ch} = \text{COP}_{ch,\text{ice}} - \text{COP}_{ch,\text{cool}} \tag{31}$$
$$X_{ch,\text{min}} = -L m_{\text{ice,base}} \tag{32}$$
$$X_{ch,\text{max}} = L(m_{\text{ice,max}} - m_{\text{ice,base}}) \tag{33}$$

For supermarket refrigeration systems, first we assume a single cold room at the supermarket. According to thermodynamic laws, the system parameters are as follows:

$$A_{cr} = \frac{-UA_{cr}}{m_{\text{food}} c_{\text{food}}} \tag{34}$$
$$B_{cr} = \text{COP}_r \tag{35}$$
$$X_{cr,\text{min}} = m_{\text{food}} c_{\text{food}} (T_{cr,\text{base}} - T_{cr,\text{max}}) \tag{36}$$
$$X_{cr,\text{max}} = m_{\text{food}} c_{\text{food}} (T_{cr,\text{base}} - T_{cr,\text{min}}) \tag{37}$$

Several cold rooms in a supermarket can be lumped together using standard model reduction techniques, yielding an approximate 1st order model:

$$G_r(s) = \frac{(\frac{-B_r}{A_r})}{1 + (\frac{-1}{A_r})s} \tag{38}$$

Since different cold rooms at the supermarket have different time constants, we consider conservative limits for minimum

and maximum thermal energy as shown in equations (39) and (40). This ensures that we will not violate the temperature constraints in all cold rooms.

$$X_{r,\text{min}} = n_{cr} \times \max(X_{cr_i,\text{min}}), \quad i = 1, \ldots, n_{cr} \quad (39)$$

$$X_{r,\text{max}} = n_{cr} \times \min(X_{cr_i,\text{max}}), \quad i = 1, \ldots, n_{cr} \quad (40)$$

III. COMPENSATION FOR UNCERTAINTIES

In the previous section, we proposed an MPC-like controller at the aggregator to provide optimal power distribution. The aggregator does not know a perfect model of the consumers and deliberately utilizes simple models in optimization process. In reality, the physical systems are nonlinear and of high order. This model mismatch may lead to actual power deviation from the desired power reference. In the following, we propose a simple strategy to deal with this mismatch, and examine the potential of ramifications of it. Suppose, there are n consumers in our portfolio, each of which receives a power reference $U_{i,\text{ref}}, i = 1, \ldots, n$ from the aggregator. However, due to model mismatch ect., the actual consumption, $U_{i,\text{act}}, i = 1, \ldots, n$, might be forced to deviate from $U_{i,\text{ref}}$ by some amount ε_i, i.e., $U_{i,\text{act}} = U_{i,\text{ref}} + \varepsilon_i$. It seems reasonable that, although ε_i is considered to be unknown by the agregator, it should be limited to an interval proportional to the current state of the i'th consumer, i.e., $\varepsilon \in [\delta_{i,\text{min}} X_i(t), \delta_{i,\text{max}} X_i(t)]$, where $\delta_{i,\text{min}}$ and $\delta_{i,\text{max}}$ are scalars, consumer-specific constants. Taking advantage of convexity of this set, we will write the actual consumption as

$$U_{i,\text{act}} = U_{i,\text{ref}} + \Delta_i X_i \qquad (i = 1, \ldots, n) \quad (41)$$

This assumption is reasonable since there will be larger deviation for larger energy changes. For instance, in modeling our case studies, we assume the COP to be constant, which is not valid anymore for large deviations from baseline consumption. The goal remains to maintain the combined power consumption of the n units at $P_{\text{reference}}(t)$ for all $t_{\text{start}} \leq t \leq t_{\text{end}}$, which implies that the following constraint has to be imposed:

$$\sum_{i=1}^{n} U_{i,\text{act}} - \sum_{i=1}^{n} U_{i,\text{ref}} = \sum_{i=1}^{n} \Delta_i X_i = 0 \quad (42)$$

Instead of using complicated models or any other solutions such as having negotiation between the entities, we propose a setup consists of a series of feedback loop as illustrated in figure 4. As shown in above figure, we have added $(n-1)$ feedback gains per consumer which propagate the deviation signal to the rest of consumers in order to compensate the deviation. This implies that consumer j will be requested to consume an extra contribution:

$$U_{j,\text{add}} = \sum_{\substack{i=1 \\ i \neq j}}^{n} k_{ij} (U_{i,\text{act}} - U_{i,\text{ref}}) \quad (43)$$

yielding the state equation:
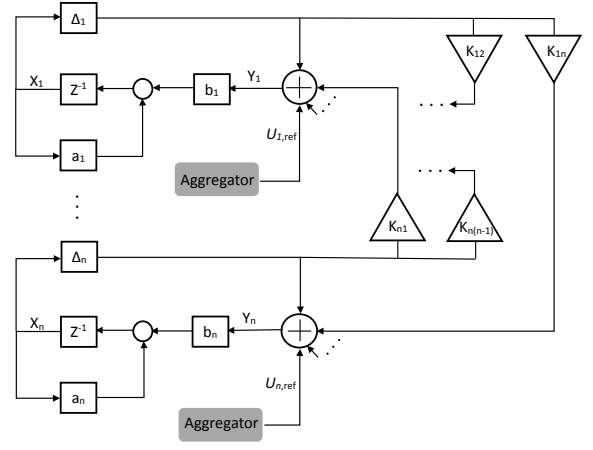
$$X_j(t+1) = a_j X_j(t) + b_j \times$$

Fig. 4: Feedback Controller Design

$$\left(\Delta_j X_j(t) + U_{j,\text{ref}} + \sum_{\substack{i=1 \\ i \neq j}}^{n} k_{ij} \Delta_i X_i(t) \right) \quad (44)$$

Thus, we will have $n \times (n-1)$ proportional controllers in the whole setup. The following state space model describes the whole system:

$$\begin{bmatrix} X_1(t+1) \\ \vdots \\ X_n(t+1) \end{bmatrix} = \left(\bar{A} + \bar{B} \bar{K} \bar{\Delta} \right) \begin{bmatrix} X_1(t) \\ \vdots \\ X_n(t) \end{bmatrix} + \bar{B} \begin{bmatrix} U_{1,\text{ref}}(t) \\ \vdots \\ U_{n,\text{ref}}(t) \end{bmatrix} \quad (45)$$

$$= \left(\bar{A} + \bar{B} \bar{K} \bar{\Delta} \right) \bar{X}(t) + \bar{B} \bar{U}_{\text{ref}}(t)$$

where $\bar{X}(t) \in \mathbb{R}^n$, $\bar{U}_{\text{ref}}(t) \in \mathbb{R}^n$ and

$$\bar{A} = \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix}, \bar{B} = \begin{bmatrix} b_1 & & 0 \\ & \ddots & \\ 0 & & b_n \end{bmatrix}$$

$$\bar{\Delta} = \begin{bmatrix} \Delta_1 & & 0 \\ & \ddots & \\ 0 & & \Delta_n \end{bmatrix}, \bar{K} = \begin{bmatrix} 1 & k_{21} & \ldots & k_{n1} \\ k_{12} & 1 & \ldots & k_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ k_{1n} & k_{2n} & \ldots & 1 \end{bmatrix}$$

In the above expression, we have $\bar{\Delta} \in Co\{\nabla_1, \nabla_2, \ldots, \nabla_{2^n}\}$, where each ∇_i represents a vertex of the hypercube $[\delta_{i,\text{min}}, \delta_{i,\text{max}}]^n$. A natural choice for the feedback gains is $k_{ij} = \frac{-1}{n-1}$, $(i, j = 1, \ldots, n)$ which means to propagate the error signal evenly. Generally, to compensate the error, equation (46) should hold:

$$\sum_{i=1}^{n} \bar{K}(i, j) = 0 \quad (j = 1, \ldots, n) \quad (46)$$

IV. CONSTRAINTS VS. UNCERTAINTIES

As can be seen from the above, the system consisting of several consumers in parallel, combined with the proposed compensating feedback connections, is now a state-constrained linear system in a feedback loop with a structured uncertainty block. Unfortunately, the situation is not

a standard robust control design problem, since the input has already been determined by the MPC scheme of the aggregator. We can safely assume that the *nominal* state trajectory computed by the MPC algorithm will remain within the state constraints, but there is no such guarantee for the uncertain system Eq. (45). As described in Section II, the chillers are modeled at the aggregator level as constrained integrators; this means that the corresponding eigenvalues of $\bar{A}$ have magnitude 1, so $\bar{A} + \bar{B}\bar{K}\bar{\Delta}$ is highly likely to have eigenvalues of magnitude greater than 1 for at least some $\bar{\Delta} \in Co\{\bigtriangledown_1, \bigtriangledown_2, \ldots, \bigtriangledown_{2^n}\}$. This implies that we cannot compute any norm bounds on the system in a meaningful manner. Indeed, the final state at the end of the activation, $\bar{X}_N$, is given by the equation

$$\bar{X}_N = (\bar{A} + \bar{B}\bar{K}\bar{\Delta})^N \bar{X}_0 + \bar{\Gamma} \times \begin{bmatrix} \bar{U}_{\text{ref}}(0) \\ \bar{U}_{\text{ref}}(1) \\ \vdots \\ \bar{U}_{\text{ref}}(N-1) \end{bmatrix} \quad (47)$$

$$\bar{\Gamma} = \begin{bmatrix} (\bar{A} + \bar{B}\bar{K}\bar{\Delta})^{N-1}\bar{B} & (\bar{A} + \bar{B}\bar{K}\bar{\Delta})^{N-2}\bar{B} & \ldots & \bar{B} \end{bmatrix} \quad (48)$$

from which it is seen that $\bar{\Delta}$ appears in a highly nonlinear fashion, making even conservative matrix norm-based estimates difficult to compute. On the other hand, for this specific application, the system only has to remain within the state constraints during the activation time, not indefinitely. Hence, the following question makes sense, even for unstable $\bar{A} + \bar{B}\bar{K}\bar{\Delta}$:

Problem 1: Given finite N and bounded $\bar{U}_{\text{ref}}$, how large uncertainty set $[\delta_{i,\min}, \delta_{i,\max}]^n$ can be permitted, such that $\{\bar{X}_0, \bar{X}_1, \ldots, \bar{X}_N\} \in \mathcal{X}$?

where $\mathcal{X}$ is a hypercube defined by the aforementioned state constraints. Since it does not appear to be possible to find analytical estimates, we choose brute-force simulation-based checking as outlined in Algorithm 1.

$0 < \alpha_i, \beta_i < 1$ are scalars. Algorithm 1 sequentially tries to simulate the system with ever-shrinking uncertainty sets. For each simulation run, it breaks out of the simulation if a test trajectory corresponding to one of the vertices of the uncertainty set violates a state constraint. Algorithm 1 is terminated by the stop criterion if it does not break out of the simulation loop, in which case we may try running it again starting with a larger initial set of $\delta_{i,\min}, \delta_{i,\max}$, of if $|\delta_{i,\max} - \delta_{i,\min}|$ drops below some pre-set threshold for some i. Alternatively, if an estimate of $[\delta_{i,\min}, \delta_{i,\max}]^n$ is known beforehand, it might be possible to incorporate the uncertain description Eq. (45) in the aggregator's optimization problem, which then becomes a robust MPC problem. It is not clear at this point which approach is simpler, however.

V. SIMULATION RESULTS

For simulation, we assume two supermarkets and two chillers under the control of the aggregator. Each supermarket utilizes four of its cold rooms for power reference following program. One of them uses three medium temperature (MT)

Algorithm 1 Numerical algorithm for solving *Problem 1*

1:
Assume a subset of $\bar{\Delta} : \left\{ \bar{\Delta}_1, \bar{\Delta}_2, \ldots, \bar{\Delta}_{2^n} \right\}^*$
for $i = 0$ to $i = 2^n$ **do**
    for $t = 0$ to $t = N$ **do**
        $\bar{X}^i(t+1) = \left(\bar{A} + \bar{B}\bar{K}\bar{\Delta}_i \right) \bar{X}^i(t) + \bar{B}\bar{U}_{\text{ref}}(t)$
        if $\exists \tau < N$ *such that* $\bar{X}^i(\tau) \notin \mathcal{X}$ **then**
            break to 2.
        end if
    end for
end for
2:
$\delta_{j,\min} \leftarrow \alpha_j \delta_{j,\min} \quad j = 1, \ldots, n$
$\delta_{j,\max} \leftarrow \beta_j \delta_{j,\max} \quad j = 1, \ldots, n$
3:
if stop criterion met **then**
    terminate
else
    go to 1
end if

$$* \quad \bar{\Delta}_1 = \begin{bmatrix} \delta_{1,\min} & & 0 \\ & \ddots & \\ 0 & & \delta_{n,min} \end{bmatrix}$$

$$\bar{\Delta}_2 = \begin{bmatrix} \delta_{1,\min} & & 0 \\ & \ddots & \\ 0 & & \delta_{n,\max} \end{bmatrix}$$

$$\vdots$$

$$\bar{\Delta}_{2^n} = \begin{bmatrix} \delta_{1,\max} & & 0 \\ & \ddots & \\ 0 & & \delta_{n,\max} \end{bmatrix}$$

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and one low temperature (LT) cold rooms while the other uses one MT and three LT cold rooms. Maximum power consumption of all four units is equal to $15kW$. Their baseline power consumption are as follows:

$$P_{r_1,\text{base}} = 6.2kW \qquad P_{r_2,\text{base}} = 4.4kW$$
$$P_{ch_1,\text{base}} = 9.4kW \qquad P_{ch_2,\text{base}} = 8.3kW$$

First, we run a simulation with having two identical supermarkets and two identical chillers in our portfolio (r_1 and ch_1). The reason is to investigate the power distribution between two different types of thermal storage. The aggregated baseline consumption is $P_{\text{agg,base}} = 31.2kW$. The result is shown in figure 5.

As shown in figure 5, in the down-regulating scenario, the chiller is mainly utilized in the beginning. The aggregator then switches to the supermarket refrigeration at the end of activation. For the up-regulating scenario, the situation is reversed. The aggregator first depletes the supermarket refrigeration system and chiller is utilized at the end. These
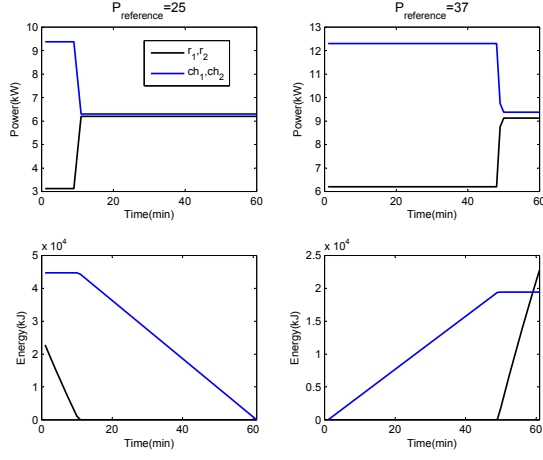
Fig. 5: Identical consumers: power distribution (upper plots) and associated changes in thermal energy (lower plots) during a one-hour activation time. Plots on the left indicate the up-regulating scenario whereas plots on the right indicate the down-regulating scenario)

results are expected. It is reasonable to discharge and charge the leaky unit in the beginning and at the end respectively. Otherwise, depleting energy close to the end or storing energy from the beginning is accompanied with losses.
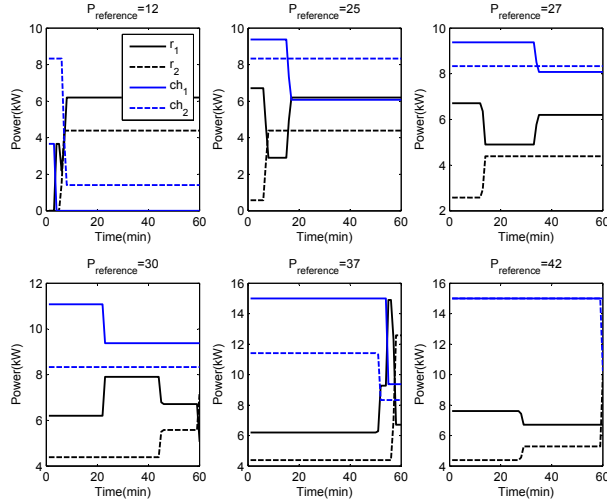

Fig. 6: Various consumers: power distribution for the up-regulating power scenario (upper plots) and the down-regulating power scenario (lower plots) during one-hour activation time

Figure 6 and 7 display the power distributions and energy changes for two different supermarkets and two different chillers. We consider six different values for $P_{\text{reference}}$ which are fixed during the activation. The aggregated baseline consumption is $P_{\text{agg,base}} = 28.3kW$. Thus, the three upper plots represent the up-regulating scenario and the three lower plots represent the down-regulating scenario. As can be seen,
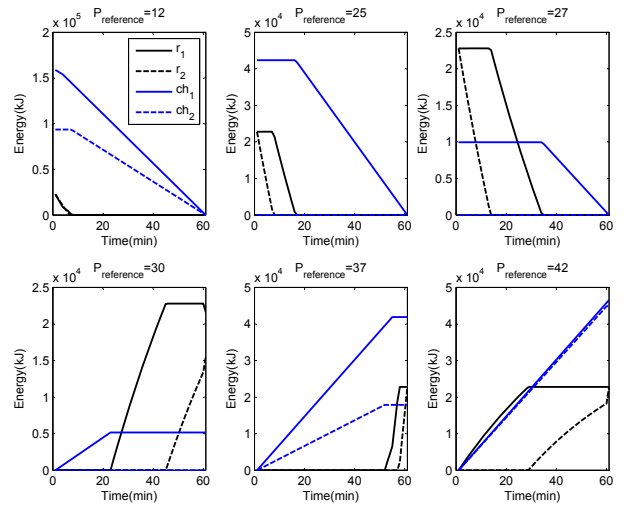

Fig. 7: Various consumers: thermal energy changes for the up-regulating power scenario (upper plots) and the down-regulating power scenario (lower plots) during one-hour activation time

switching from one unit to the other can occur several times when we have various consumers in our portfolio. For instance, for $P_{\text{reference}} = 37kW$, first, there is a switching from ch_2 to r_1. Then, the aggregator switches from ch_1 to r_1. At the end, there is also another switching from r_1 to r_2. Essentially, the form of power distribution is dependent on the power reference values and the consumer characteristics. However, as a general rule, the leaky units are utilized in the beginning and at the end for the up-regulating and down-regulating scenarios, albeit other parameters are also determinant. For example, for $P_{\text{reference}} = 42kW$, one of the supermarkets is utilized from the beginning since there is maximum power constraint for chiller systems. Moreover, we can conclude from the figures that the more deviation from baseline power, the more exploitation of chiller systems occurs. This is due to state-dependent leakage of the supermarket refrigeration systems. For large deviation, the heat loss to the surrounding increases at the supermarkets. Hence, it is better to exploit the chillers.

The above figures show the desired power references that are obtained from the optimization. However, the actual power consumption may be different as we formulated in the previous section. We again consider the case with identical units. Applying Algorithm 1 for $P_{\text{reference}} = 37kW$ provides the following:

$$\delta_{r_1,\max} = \delta_{r_2,\max} = 10^{-5}$$
$$\delta_{ch_1,\max} = \delta_{ch_2,\max} = 10^{-9}$$

In the simulation, we assume $\delta_{i,\min} = -\delta_{i,\max}$ and

$$\bar{K} = \begin{bmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{bmatrix}$$

Figure 8 shows the discrepancy between the actual power, P_{actual}, and the desired power reference. As shown in the figure, without feedback loops, the discrepancy is significant (lower plot). However, with having feedback loops, the difference is almost zero (upper plot). In addition, the system state constraints are not violated. Energy changes for one supermarket and chiller during the activation correspond to $\Delta_{r_1} = 10^{-5}$, $\Delta_{ch_1} = 10^{-9}$ is depicted in figure 9. As shown, system states remain in the cube defined by the constraints during the activation.
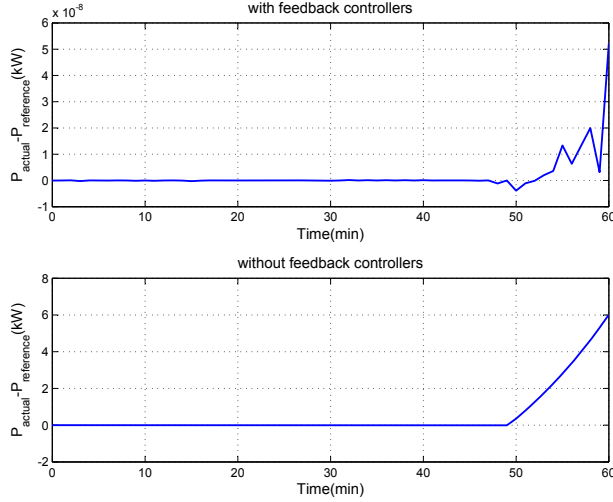

Fig. 8: Discrepancy between the actual power consumption and the power reference for $P_{\text{reference}} = 37kW$, $\Delta_{r_1} = \Delta_{r_2} = 10^{-5}$, $\Delta_{ch_1} = \Delta_{ch_2} = 10^{-9}$. Note the different scale on the ordinate axis
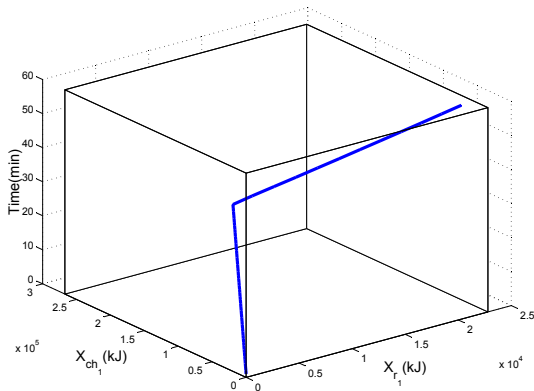

Fig. 9: System states evolution with the feedback loops during the activation for $P_{\text{reference}} = 37kW$, $\Delta_{r_1} = 10^{-5}$, $\Delta_{ch_1} = 10^{-9}$. Note that the time axis is vertical in this plot

VI. CONCLUSION

This paper proposes an aggregator setup to integrate the large-scale flexible consumers with thermal storage to the future smart grid. The aggregator promises to follow a specified power reference which it receives from the grid operator during an activation period. To that end, the proposed setup comprises an MPC controller together with a series of feedback loop. The MPC controller provides optimal power distribution for multiple consumers while respecting their constraints and satisfying the power reference following. The objective function is formulated for both upward and downward regulating power. However, the goal may not be met since the aggregator uses simplified model of the consumers to run the optimization. To compensate the error arises from model mismatch, we propose a feedback mechanism which propagates the error of one unit to the rest of units in our portfolio.

Two case studies are considered in this work: supermarket refrigeration systems and chiller systems with ice storage. Simulation results for these consumers show that the aggregator utilizes supermarkets in the beginning and at the end of activation period for upward and downward regulating power respectively. Moreover, chiller utilization increases for large deviation from baseline consumption. We also simulate a scenario that includes the uncertainties in modeling. The largest uncertainties can be handled in our setup is determined via a brute-force approach. With the feedback loops, we are able to follow the power reference while honoring the constraints in a good manner.

REFERENCES

[1] B. Biegel, L. H. Hansen, J. Stoustrup, P. Andersen and S. Harbo, "Adjustable Consumption Participating in the Electricity Markets", in *IEEE 52th Annual Conference on Decision and Control (CDC)*, Florence, Italy, 2013

[2] J. Medina, N. Muller and I. Roytelman, "Demand Response and Distribution Grid Operations: Opportunities and Challenges", in *IEEE Transactions on Smart Grid*, Vol. 1, No. 2, September 2010

[3] Y. Huang "Market Concepts and Regulatory Bottlenecks for Smart Distribution Grids in EU Countries", in *MSc thesis, KTH Royal Institute of Technology*, Stockholm, Sweden 2011

[4] S. You, C. Traeholt and B. Poulsen, "Generic Virtual Power Plants: Management of Distributed Energy Resources under Lliberalized Electricity Market", in *8th International Conference on Advances in Power System Control, Operation and Management*, Hong Kong, China, 2009

[5] A. Koto, S. Lu, T. Valavaara, A. Rautiainen and S. Repo, "Aggregation of Small-Scale Active Resources for Smart Grid Management", in *2nd IEEE PES International Conference and Exhibition on Innovative Smart Grid Technologies (ISGT Europe)*, Manchester, UK, 2011

[6] S. Han, S. Han and K. Sezaki, "Development of an Optimal Vehicle-to-Grid Aggregator for Frequency Regulation", in *IEEE Transactions on Smart Grid*, Vol. 1, No. 1, June 2010

[7] L. Gkatzikis, I. Koutsopoulos and T. Salonidis, "The Role of Aggregators in Smart Grid Demand Response Markets", in *IEEE Journal on Selected Areas in Communications*, Vol. 37, No. 7, pp. 1247-1257, 2013

[8] S. Rahnama, S. E. Shafiei, J. Stoustrup, H. Rasmussen and J. Bendtsen, "Evaluation of Aggregators for Integration of Large-scale Consumers in Smart Grid", Accepted in *IFAC WC*, Cape Town, South Africa, 2014

[9] A. Bemporad and M. Morari, "Control of Systems Integrating Logic, Dynamics, and Constraints", in *Automatica*, Vol. 35, pp. 407-427, 1999