A Novel Energy Yields Calculation Method for Irregular Wind Farm Layout

Hou, Peng; Hu, Weihao; Soltani, Mohsen; Chen, Zhe

Published in:
Proceedings of the IECON2015, Yokohama, Nov 2015

DOI (link to publication from Publisher):
10.1109/IECON.2015.7392129

Publication date:
2015

Document Version
Accepted author manuscript, peer reviewed version

A Novel Energy Yields Calculation Method for Irregular Wind Farm Layout

Peng Hou, Weihao Hu, Mohsen Soltani, Zhe Chen,
Department of Energy Technology
Aalborg University
Pontoppidanstraede 101, Aalborg DK-9220, Denmark
pho@et.aau.dk, whu@et.aau.dk, sms@et.aau.dk, zch@et.aau.dk

***Abstract*— Due to the increasing size of offshore wind farm, the impact of the wake effect on energy yields become more and more evident. The seafloor topography would limit the layout of the wind farm so that irregular layout is usually adopted in large scale offshore wind farm. However, the calculation for the energy yields in irregular wind farm considering wake effect would be difficult. In this paper, a mathematical model which includes the impacts of the variation of both wind direction and velocity on wake effect is established. Based on the wake model, a binary matrix method is proposed for the energy yields calculation for irregular wind farms. The results show that the proposed wake model is effective in calculating the wind speed deficit. The calculation framework is applicable for energy yields calculation in irregular wind farms.**

***Index Terms*--binary matrix method; calculation framework; energy yields; irregular wind farm; wake model.**

Nomenclature

$C_{p,opt}$	Power coefficient at λ_{opt}
d_{ji}	Distance from O_i to O_j
E_{tol}	Total energy yields
h_{ji}	Length of diagonal line in blue quadrangle
L_{ji}	Distance from the center of upstream wind turbine (WT) to downstream WT's center
M(i, j)	Element of matrix M at row i, column j
N_col	Total number of WTs in a column
N_row	Total number of WTs in a row
O_i	Center of the downstream WT
O_j	Center of the wake that developed from the upstream WT
$P_{m,ij}$	Mechanical power generated by WT at row i, column j
$P_{tol,t}$	Wind farm power production within the corresponding sample time interval t
R	Rotor radius
R_i	Radius of the downstream WT's rotor
R_j	Radius of the wake that generated from the upstream WT rotor
R_0	Radius of the upstream WT's rotor
R(x)	Generated wake radius at x distance along the wind direction
S_i	Fan shaped area of the sweeping area that in downstream WT rotor
S_j	Fan shaped area of the wake area
S_0	Sweeping area of WT's rotor with radius R_0
$S_{partial}$	Blue area in Figure 1(b) which shows the wake effect region of downstream WT
S_q	Blue quadrangle area in Figure 1
v	Injected wind speed
V_{ij}	Wind speed deficit generated by the WT at i^{th} row, j^{th} column of wind farm
$V_j(\alpha,V_0)$	Wind speed of the upstream WT when the inflow wind direction angle is α and velocity is V_0
$V_{n,m}$	Wind velocity at WT at row n, column m
$V_{n,m}(\alpha,V_0)$	Wind speed of the upstream WT when free wind direction angle is α and velocity is V_0
V_0	Free wind velocity or the input wind velocity of the upstream WT
V_w	Wind velocity in the wake at a distance x downstream of the upstream WT
x_i, y_i	Position of the downstream WT in coordinate system
λ_{opt}	Optimal tip speed ratio for the pitch angle β', at which the power coefficient will be maximum
β'	Pitch angle
γ	Chord angle corresponding to S_j
μ	Chord angle corresponding to S_i
ρ	Air density, 1.225kg/m^3 in standard condition

This work has been (partially) funded by Norwegian Centre for Offshore Wind Energy (NORCOWE) under grant 193821/S60 from Research Council of Norway (RCN). NORCOWE is a consortium with partners from industry and science, hosted by Christian Michelsen Research.

I. Introduction

The wake effect can be defined as the impact of the upstream WT to the downstream ones which incurs the reduction of the total energy production of the wind farm [1]. Since the development of wind energy technology, the capacity of the wind farm is increasing a lot. In such a big wind farm, the wake effect becomes particularly evident. The overestimation of energy yields means a higher voltage level selection of electrical equipment and higher capacity of cables are required. This will induce the waste of investment on components' redundancy. Moreover, the wind farm control strategy and operating reserve will be influenced as well [2], [3]. In 1983, Jensen created a wake model which assumes a linear expansion of the wake after the upstream WT based on momentum conservation theory. After that, several wake models are proposed for the wake calculation [4]. In [5], a method to calculate the wake losses by Jensen model is proposed while the Larsen eddy viscosity model is specified in [6]. Presently, there are three wake models that are widely used in Wind Farm Layout Optimization Problem (WFLOP) [7]. The objective of the WFLOP is to find an optimal layout which can meet the requirement of minimizing the investment while maximizing the energy yields [8]. In order to foresee the energy yields better, some works have been done on the wake effect losses calculations by using CFD (Computational Fluid Dynamics) [9]. The authors tried to accurately describe the wake effect by solving differential equations, however, the calculation time is quite long so that it is not an expected way for energy yields calculation. In this paper, a wake model with varying wind speed is established at first. Then, a binary matrix method for calculating the irregular wind farm yields by taking wake effect into account is proposed. The results show that the proposed method is an effective and efficient way for irregular wind farm energy yields calculation.

The analytical equations for the wake model are specified in section II, the calculation framework is presented in Section III. The FINO3 reference wind farm is chosen as the study case to demonstrate the proposed method in Section IV. Finally, conclusions and future work are given In Section V.

II. WIND FARM MODEL

In this section, the Jensen wake model is firstly introduced. Based on which, the wind speed varying wake model which concerns the total wake effects from neighboring WTs as well as the varying wind speed's impacts are proposed. The energy model and calculation framework are presented at last.

A. Wake Effect Model

In this simulation, the Jensen wake model is adopted as the basic wake model to analyze the wake effect for its simplicity.

1) Jensen wake model

The formula for Jensen single wake model can be expressed as [4]:

$$V_w = V_0 + V_0(\sqrt{1-C_t}-1)(\frac{R_0}{R(x)})^2 \tag{1}$$

$$R(x) = R_0 + kx \tag{2}$$

Where, C_t is the thrust coefficient of the WT and k is the wake decay constant. The recommended value of k is 0.04 for offshore environment which is suitable for a free wind condition (turbulence-free, that is to say not affected by any upstream turbine) [10].

In some cases, the center of downstream WT is not aligned with wind velocity. The wind velocity at downstream WTs will be determined by the overlapped area generated by the evolved wake which can be seen in Figure 1 (b). Then equation (1) can be modified as [11]:

$$V_w = V_0 - V_0\left(1-\sqrt{1-C_t}\right)\left(\frac{R_0}{R(x)}\right)^2\left(\frac{S_{partial}}{S_0}\right) \tag{3}$$

B. Multiple wakes

Within the wind farm, there is a probability that one downstream WT would be in the affected region of wake that generated by several upstream WTs. The problem has been solved by sum of squares of velocity deficits method [7]. As a consequence, the wind velocity at WT in row n, column m can be expressed as:

$$V_{n,m} = V_0[1-\sqrt{\sum_{i=1}^{N\_row}\sum_{j=1}^{N\_col}[1-\left(\frac{V_{ij}}{V_0}\right)]^2}] \tag{4}$$

C. Wind speed varying wake model

In reality, the wind direction changed from time to time. The WT would change its nacelle until it faced to the wind direction so that more wind energy can be absorbed. The variation of the wind velocity as well as the direction will both influence the wind speed deficit so does the energy yields. This change can be described by using a modified model with coordinate system. In this model, the wind is considered to be existed in 4 quadrants. In each quadrant two cases are required to be specified as shown in Figure 1. In which, the red line is the distance from the center of the upstream WT to downstream WT. The green area, denoted as $S_{overlap}$ is the overlapped area. The blue quadrangle area is denoted as S_q.

A series of analytical equations for wake velocity calculation in case (a) can be derived as:

$$L_{ji} = \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2} \tag{5}$$

$$d_{ji} = L_{ji}\left|\sin(\alpha+\beta)\right| \tag{6}$$

$$R_j = R_i + kL_{ji}\left|\cos(\alpha+\beta)\right| \tag{7}$$

$$\gamma = 2\cos^{-1}\frac{R_j^2 + d_{ij}^2 - R_i^2}{2R_j d_{ji}} \tag{8}$$

$$\mu = 2\cos^{-1}\frac{R_i^2 + d_{ij}^2 - R_j^2}{2R_i d_{ji}} \tag{9}$$

$$h_{ji} = 2R_i\left|\sin(\mu/2)\right| \tag{10}$$

$$S_0 = \pi R_i^2 \tag{11}$$

$$S_j = \frac{\gamma(R_j)^2}{2} \tag{12}$$

$$S_i = \frac{\mu R_i^2}{2} \tag{13}$$

$$S_q = 0.5h_{ji}d_{ji} \tag{14}$$

$$S_{overlap} = S_j + S_i - S_q \tag{15}$$

The above equations are derived step by step to calculate the green area as illustrated in Figure 1 which is the effective area that used to calculate the speed at the downstream WT's blade. Then the wind velocity at j^{th} WT can be rewritten as:

$$V_j(\alpha, V_0) = V_0[1-\left(1-\sqrt{1-C_t}\right)\left(\frac{R_0}{R_j}\right)^2\left(\frac{S_{overlap}}{S_r}\right)] \qquad (16)$$

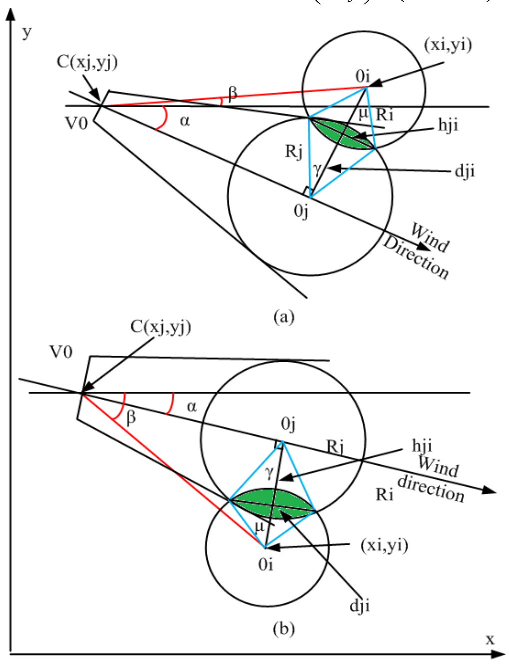

Figure 1. Wind speed varying wake model

Where α is the angle between line CO_j and x axis, in other words, the wind direction while β is the angle between line CO_i and x axis. In case (b), the analytical equations is merely modified by changing all (6) and (7) into (β-α) while keeping all other terms the same.

The model proposed above is valid when the wake and the rotor sweeping area are intersected. In general, three cases should be considered in energy yields calculation as shown in Figure 2.

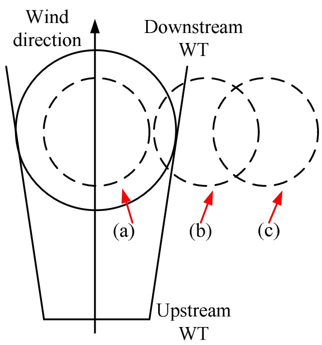

Figure 2. Three cases in wake losses calculation

The dotted circle represents the location of the downwind WT. The wake effect will be receded gradually if the downwind WT is moving from position (a) to (c). As a result, three cases should be specified. The specifications of three cases are summarized in Table I as follow:

TABLE I. JUDGEMENT SPECIFICATIONS

Case	Category	Condition	Analytical equations
(a)	full wake effect	$0 \le d_{ji} \le R_j - R_i$	(1) (2) (4)
(b)	partial wake effect	$R_j - R_i \le d_{ji} \le R_j + R_i$	(5) - (16)
(c)	non-wake effect	$d_{ji} \ge R_j + R_i$	$V_j = V_0$

D. Energy model

The power produced by WT can be calculated using the following equations [12], [13]:

$$P_{m,ij} = 0.5C_{p,opt}(\beta', \lambda_{opt})\rho\pi R^2 v^3 \qquad (17)$$

In the simulation, the power production of each WT is found by assuming a maximum power point tracking (MPPT) control strategy [14]. Hence, the total power production that generated by the WTs at row i, column j can be written as:

$$P_{tol} = \sum_{j=1}^{N\_col}\sum_{i=1}^{N\_row} P_{m,\,ij} \qquad (18)$$

III. Binary Matrix Method for Irregular Wind Farm Energy Calculation

The energy yields calculation for irregular wind farm is difficult since there is no explicit rule to define the distance between the WTs. The problem is solved by introducing a binary matrix in this paper so that the energy calculation can be simplified by using the ordinary way combing this shaping matrix.

A. Binary matrix and modification of equations

As it is known, the wind speed deficit is a function of distance from upstream WT to downstream WT along wind direction. In irregular wind farm it is quite complex to define this distance for each turbine. Hence, a binary matrix is introduced as shown below.

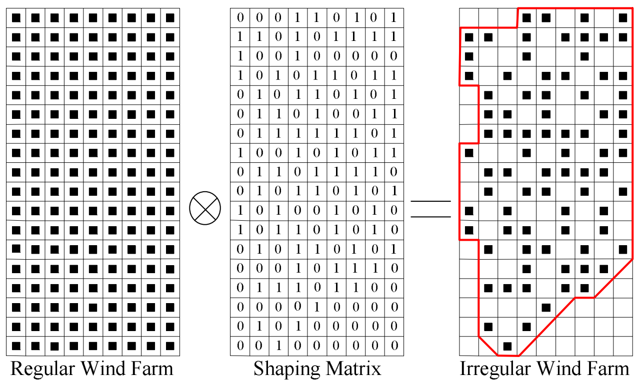

Figure 3. Binary matrix method

The black solid square in Figure 3 represents the WT. Number 1 means there is a WT in this position while 0 means the position is empty. By using the binary matrix, the original full occupied wind farm can be shaped into an irregular wind farm. Combing (4) - (16), the wind velocity and total energy yields can be calculated as:

$$V_{n,m}(\alpha, V_0)=V_0[1-\sqrt{\sum_{i=1}^{N\_row}\sum_{j=1}^{m}[1-M(i,j).\left(\frac{V_{ij}}{V_0}\right)\left(\frac{S_{overlap,ij}}{S_r}\right)^2}] \quad (19)$$

$$E_{tol}=\sum_{t=1}^{TE}P_{tol,t}T \quad (20)$$

Where, TE is the sample time for energy yields calculation. The wind velocity and direction for the calculation is obtained from the Norwegian Meteorological Institute [15], [16]. In their work, the wind speed is sampled every 3 hours. Hence, in this paper, TE is taken as 365*8 which is also the maximum iteration and T is 3 hours in the following simulation.

B. *Calculation framework*

The energy yields calculation for irregular wind farm should be easier solved with the binary matrix method as mentioned above. The calculation framework can be seen in Figure 4.

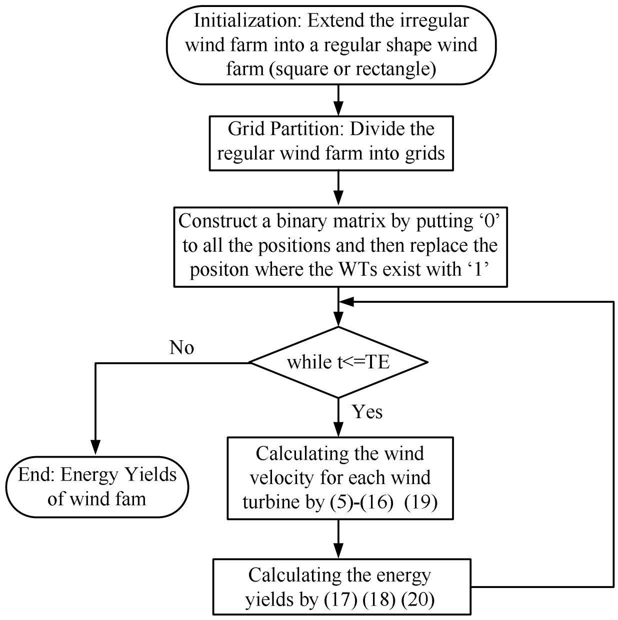

Figure 4. Calculation framework

The wind farm will be partitioned into small grids. Each grid could have one WT or not. The length of the grid is decided by the shortest distance's projection on wind farm side length direction between two WTs.

IV. CASE STUDY

The simulation is implemented on the platform of Matlab R2013a. Two study cases are adopted to verify the feasibility of the proposed method.

A. *Case I*

FINO3 reference wind farm is sited 80km west of German island of Sylt. In the first case, the irregular layout is assumed to be as shown in Figure 5 [17].

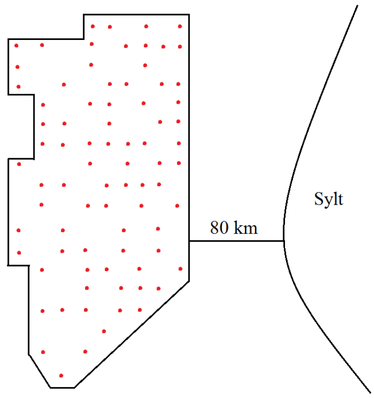

Figure 5. Case I Wind Farm Layout

1) *DTU 10 MW WT*

The DTU 10MW WT is considered as the reference WT in this paper. The specification of which is listed in Table II. The detailed information of C_p and C_t is listed as a lookup table in [18].

TABLE II. DTU 10MW WT SPECIFICATION [15]

Parameter	10 MW DTU WT
Cut-in Wind Speed	4 m/s
Cut-out Wind Speed	25 m/s
Rated Wind Speed	11.4 m/s
Rotor Diameter	178.3 m
Rated Power	10 MW

2) *Simulation and results*

According to the binary matrix method, the layout transformation process is the same as illustrated in Figure 3. The input wind velocity and direction distribution for the simulation are illustrated in Figure 6.

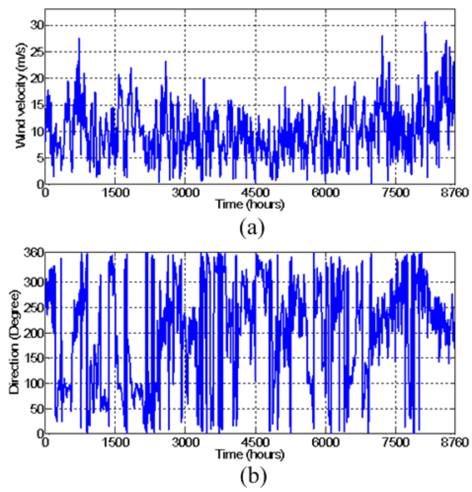

Figure 6. (a) Wind velocity variation every 3 hours (b) Wind direction variation every 3 hours

Following the calculation framework, the energy yields for this case are obtained as in Table III. The energy yields for one year are illustrated in Figure 7.

TABLE III. SIMULATION RESULTS

Name	Binary Matrix Calculation
Duration	8760 h
Wind farm capacity	800 MW
Enegy yields	3460.70 GWh
Capacity Factor	49.38%
Energy yields without wake effect	4164.91 GWh
Capacity Factor (no wakes)	59.43%
Wake losses efficiency	16.91%
Simulation time	162.87 seconds

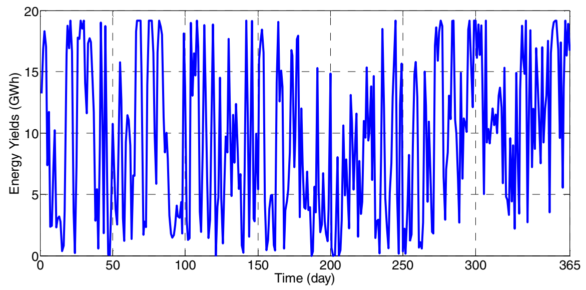

Figure 7. Daily energy yields for irregular wind farm layout in one year

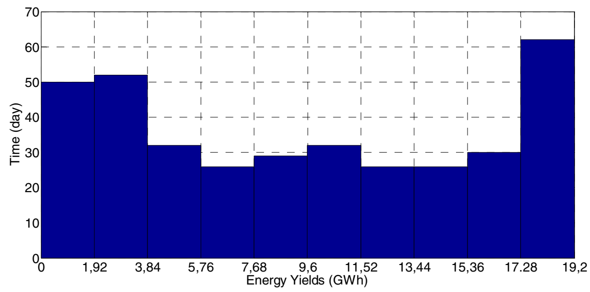

Figure 8. Energy yields distribution for irregular wind farm layout in one year

The program is performed on a computer which is an Intel(R) Core(TM) i7-4800MQ CPU @ 2.70 GHz processor with 8 GB RAM. As illustrated in Figure 7 and 8, in most of time of a year, about 62 days, the wind farm can supply 17.28 to 19.2 GWh electricity to main grid per day. The total energy yields for this case is 3460.70 GWh. Table III shows that the wake effect reduced the energy production by 16.19% and the computational time is around 177 seconds.

B. Case II

In this case, the wind farm layout is assumed to be elliptic as shown in Figure 9. The red stars show the WT locations. Different from case I, the distances between WTs in horizontal or vertical direction are totally different. If the shortest distance's projection on wind farm side length direction between two WTs is still adopted as the length of the grid. Then the energy yields of this wind farm cannot be calculated instance due to out of memory reasons. In order to conquer this problem, the grid partition process could be modified by partitioning grids into different length rectangular instead of square while binary matrix method will be still effective.

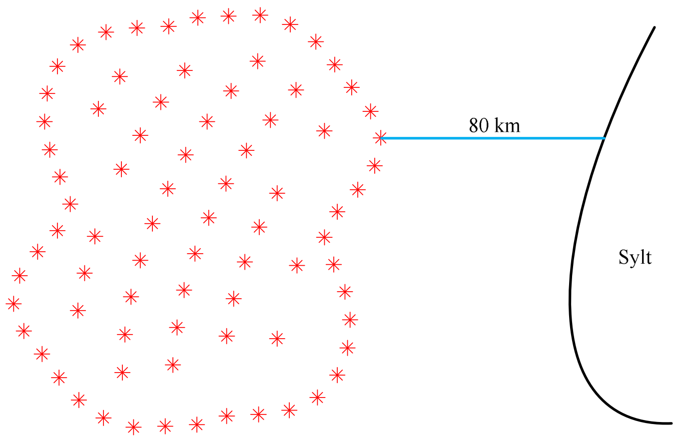

Figure 9. Case II Wind Farm Layout

The energy yields considering either wake effect or not are listed in Table IV. In this case, the wake effect reduced the energy production by 9.03% and the computational time is around 2915 seconds.

TABLE IV. SIMULATION RESULTS

Name	Binary Matrix Calculation
Duration	8760 h
Wind farm capacity	800 MW
Enegy yields	3789.00 GWh
Capacity Factor	54.07%
Energy yields without wake effect	4164.91 GWh
Capacity Factor (no wakes)	59.43%
Wake losses efficiency	9.03%
Simulation time	2915.25 seconds

The energy yields with and without considering wake effects for both cases are also calculated by WAsP (Wind Atlas Analysis and Application software) [19]. The results are included in Table V. The error shows the difference between proposed method and WAsP. It can be seen the results is close to that obtained with WAsP.

TABLE V. ENERGY YIELDS COMPARISON OF TWO CASES

		Energy Yields by Matlab (GWh)	Energy Yields by WAsP (GWh)	Error (%)
Case I	Without considering wake effect	4164.91	4214.40	1.17%
	Considering wake effect	3460.70	3394.46	2.84%
Case II	Without considering wake effect	4164.91	4219.09	0.045%
	Considering wake effect	3789.00	3926.71	3.51%

V. CONCLUSIONS AND FUTURE WORK

The wake effect has a significant contribution to the reduction of offshore wind farms energy yields. The variation in both wind velocity and direction will have impacts on the calculation of wake losses. Since the wind farm layout is usually irregular, it is difficult to calculate the energy yields considering wake effect by traditional ways. In this paper, a new wake model which includes the multiple wakes and the wind speed varying wake is established. Based on which, a new binary matrix method is applied in an irregular wind farm energy yields calculation. The studied cases demonstrate that it is an effective way to calculate any shape wind farm energy yields considering wake effect. In future, the method could be used in finding the optimal locations of WTs in offshore wind farm by taking the components' cost into consideration.

ACKNOWLEDGMENT

Authors would like to thank Norwegian Centre for Offshore Wind Energy (NORCOWE) under grant 193821/S60 from Research Council of Norway (RCN).

REFERENCES

[1] Youjie Ma, Haishan Yang, Xuesong Zhou, Li Ji, "The dynamic modeling of wind farms considering wake effects and its optimal distribution," World Non-Grid-Connected Wind Power and Energy Conference, 2009. WNWEC 2009, pp. 1-4, Nanjing, 24-26 Sept. 2009.

[2] Peng Wang, Goel, L., Yi Ding, Loh Poh Chang, Andrew, A., "Reliability-based long term hydro/thermal reserve allocation of power systems with high wind power penetration," Power & Energy Society General Meeting, 2009. PES '09. IEEE, pp. 1-7, Calgary, AB, 26-30 July 2009.

[3] Erik Ela, Brendan Kirby, Eamonn Lannoye, Michael Milligan, Damian Flynn, Bob Zavadil, "Evolution of Operating Reserve Determination in Wind Power Integration Studies," 2010 IEEE Power & Energy Society General Meeting, 25-29 July, 2010.

[4] N.O. Jensen, "A Note on Wind Generator Interaction," Risø National Laboratory, DK-4000 Roskilde, Denmark, Novermber 1983.

[5] F. González-Longatt, P. Wall, V. Terzija, "Wake effect in wind farm performance: Steady-state and dynamic behavior," Renewable Energy 39 (2012) 329e338, 16 September 2011.

[6] G. C. Larsen, J. Højstrup, H.A. Madsen, "Wind Fields in Wakes," EUWEC '96, Gothenburg, 1996.

[7] WindPRO/PARK, "Introduction wind Turbine Wake Modelling and Wake Generated Turbulence," EMD International A/S.

[8] Serrano Gonzalez, J., Burgos Payan, M. and Riquelme Santos, J. M., "An improved evolutive algorithm for large offshore wind farm optimum turbines layout," PowerTech, 2011 IEEE Trondheim, pp. 1-6, Trondheim, June 2011.

[9] Annette Westerhellweg, Thomas Neumann, "CFD simulations of wake effectsa at the alpha ventus offshore wind farm," EWEA 2011, Brussels.

[10] Philippe Beaucage, Michael Brower, Nick Robinson, Chuck Alonge, "Overview of six commercial and research wake models for large offshore wind farms," Proceedings EWEA 2012, Copenhagen, 2012.

[11] F. González-Longatt, P. Wall, V. Terzija, "Wake effect in wind farm performance: Steady-state and dynamic behavior," Renewable Energy 39 (2012) 329e338, 16 September 2011.

[12] Serrano Gonzalez, J., Burgos Payan, M., Riquelme Santos, J., "Optimum Wind Turbines Operation for Minimizing Wake Effect Losses in Offshore Wind Farms," 2013 13th International Conference on Environment and Electrical Engineering (EEEIC), pp. 188-192, Wroclaw, 1-3 November 2010.

[13] P. Flores, A. Tapia, G. Tapia, "Application of a control algorithm for wind speed prediction and active power generation," Renewable Energy, Vol. 30, Issue 4, pp. 523-536, April 2005.

[14] Wei Qiao, "Intelligent mechanical sensorless MPPT control for wind energy systems," Power and Energy Society General Meeting, 2012 IEEE, pp. 1-8, San Diego, CA, 22-26 July 2012.

[15] Christian Bak, Frederik Zahle, Robert Bitsche, Taeseong Kim, Anders Yde, Lars Christian Henriksen, Anand Natarajan, Morten Hartvig Hansen, "Description of the DTU 10 MW Reference Wind Turbine," DTU Wind Energy, July 2013.

[16] Birgitte R. Furevik and Hilde Haakenstad, "Near-surface marine wind profiles from rawinsonde and NORA10 hindcast," Jounal of Geophysical Research, Vol. 117, 7 Dec. 2012.

[17] FINO3 - research platform in the North Sea and the Baltic No. 3 [Online]. Available: http://www.fino3.de/en/ .

[18] http://met.no/English/, the Norwegian Meteorological Institute.

[19] http://www.wasp.dk/, DTU Wind Energy.