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Multiagent System-Based Distributed Coordinated Control for Radial DC Microgrid Considering Transmission Time Delays

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Abstract—**This paper focuses on a multi-agent based distributed coordinated control for radial DC microgrid considering transmission time delays. Firstly, a two-level multi-agent system is constructed, where local control is formulated based on local states and executed by means of the first-level agent, and distributed coordinated control law is formulated based on wide-area information and executed by means of the secondary-level agent in order to improve the voltage control performances. Afterwards, the research mainly focuses on designing the local controller and the distributed coordinated controller. For purpose of robust stability, the local control is designed as local state-feedback based H∞ robust controller. It is worth mentioning that the distributed coordinated control consists of local state feedback control and decoupling coordinated control law that only come from adjacent DER units. Moreover, the distributed coordinated controller is designed by means of delay-dependent H∞ robust control method taking into account transmission time delays. Finally, the validity of the proposed control scheme is demonstrated by means of simulation results.**

Index terms—**DC microgrid, multi-agent-system, distributed coordinated control, time delay, robust control**

I. INTRODUCTION

THE increasing penetration of DC renewable energy sources and increasing use of DC loads have motivated a growing interest in DC microgrid (MG) [1], [2]. The DC network can integrate various kinds of small DC or non-60Hz frequency distributed energy resources (DERs) such as photovoltaic arrays, wind turbine and battery etc. to supply DC loads more efficiently. DC MGs have several advantages over their AC counterparts [3]. For instance, the control regarding reactive power and frequency is not an issue. So DC MGs have been widely regarded as one of the best means of using renewable energy resources to meet DC load demand.

The major concern in the stabilization operation problem of DC MGs is how to design a feasible and efficient control scheme to maintain bus voltage stabilization. In the last a few years, many researches considered the decentralized "peer to peer" control, which means that each DER subsystem is controlled independently based on its local available information, for example local droop control scheme [4]. This control mode is obviously feasible and efficient [5], [6]. However, the bus voltage of each DER subsystem usually changes with the DER operation state. Therefore, the decentralized "peer to peer" control scheme may lead to a large steady voltage deviation between two DER units. In order to solve this problem, many researchers proposed hierarchical coordinated strategies [7]-[9]. The secondary coordinated strategies are ordinarily divided into three types: the centralized communication and control scheme [10], centralized communication and decentralized control scheme [11] and distributed coordinated control scheme [12]-[17]. Due to huge dimensionality of MG and communication pressure, the first scheme is neither applicable nor economic. Although the second scheme implements secondary adjustment in a decentralized manner, but it still depends on global information, which leads to communication pressure and decreased reliability. On the contrary, as an effective means, the third scheme recently has attracted more attentions. In the distributed coordinated control scheme, a local controller of each DER unit at the first level is helped by a distributed coordinated controller at the secondary level. The secondary-level controller uses remote transmission signals that only come from the adjacent DER unit to synthesize decoupling control law in order to improve control performance. So this kind of scheme only depends on low bandwidth communication. However, In a MG, DERs are very dispersive and loosely connected to each other, so even two adjacent DERs are far away. When the control signals that come from adjacent DER unit are transmitted through the transmission line, communication delay is unavoidably introduced [18]. In general, the communication delay can vary from tens to several hundred milliseconds or more [18], [19]. The existence of time delays would deteriorate the control performance. Therefore, when designing the distributed coordinated control scheme, the time delay should be taken into consideration. The time delay stabilization problem can be classified into two types: delay-independent stabilization [20] and delay-dependent one [21].The delay-independent stabilization is considered more conservative in general than the delay-dependent one, especially for the system where time delay is actually small. Due to communication delays in MG are within the range of several hundred milliseconds [19], so the less conservative delay-dependent stabilization approach is more suitable for designing the distributed coordinated control scheme.

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Moreover, with respect to the two-level distributed coordinated scheme, the multi-agent system (MAS) technique is one of the best choices to make the control much intelligent and flexible [22]. MAS is composed of multiple agents, where each agent is not only autonomous and intelligent, but also interacts with each other in a cooperation manner, so that MAS is able to handle complex problems in a more flexible and intelligent way. So far, the researches regarding management and control of MGs had been done in multi-agent approach [23], [24]. In this paper, a two-level MAS based distributed coordinated control scheme is proposed, where each DER unit is associated with a unit control agent at the first level to deal with local control for its unit system, also where there is a distributed coordinated agent at the secondary level to implement the distributed coordinated control.

The research is decomposed into the following steps: (i) The MAS based distributed coordinated control scheme is constructed. Also the structures of two levels of agents are designed, respectively. (ii) According to the bus voltage deviation between any two adjacent DER units, the secondary-level agent determines what kind of control mode it needs to execute. (iii) Both local and distributed coordinated controllers in four kinds of control modes are designed. For purpose of robust stability, the local control is designed as local state-feedback based H_∞ robust controller. It is worth mentioning that, the distributed coordinated control consists of local state-feedback control and decoupling coordinated control law that only come from adjacent DER units, so that communication pressure is decreased largely. Moreover, for improving control performance, the distributed coordinated control is designed by means of delay-dependent H_∞ robust control method taking into account the communication time delays. The sufficient conditions for controller existence in four kinds of control modes are transformed into line matrix inequality (LMI) convex optimization problem. (iv) The validity of the proposed control scheme is demonstrated by means of simulation study.

II. THE MAS BASED DISTRIBUTED COORDINATED CONTROL SCHEME OF DC MG

The proposed MAS based distributed coordinated control scheme for radial DC MG is depicted in Fig.1. The radial DC MG contains multiple DER units connected with each other in proper order through a DC line. And each DER unit supplies a local load connected on the point of common coupling (PCC) node. Here, a hierarchical MAS that consists of first-level unit control agents and secondary-level distributed coordinated control agents, is proposed to implement distributed coordinated control for the radial DC MG.
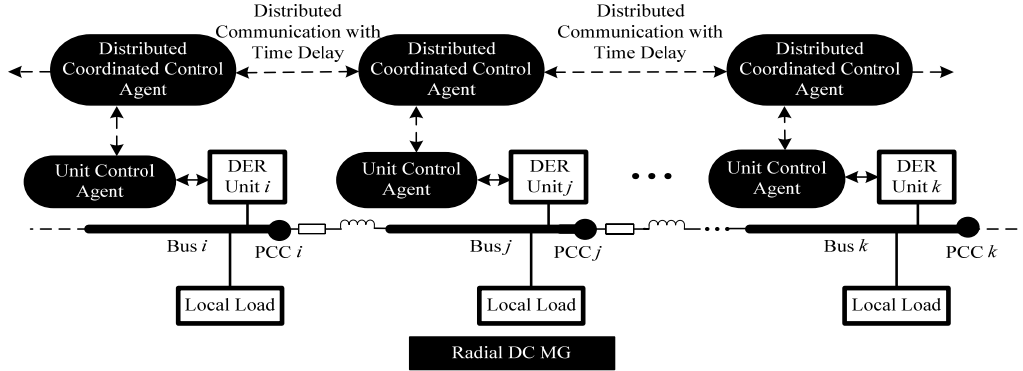

Fig.1 MAS based distributed coordinated control scheme

Corresponding to Fig.1, each first-level unit control agent is mainly responsible for formulating and implementing the local control for its DER unit. It is designed as hybrid agent, which is composed of reactive layer and deliberative layer as shown in Fig.2. The reactive layer is defined as "recognition, perception and action" and has priority to respond quickly to emergencies of environment. The deliberative layer is defined as "belief, desire and intention", and has high intelligence to control the dynamic behavior of its DER unit.
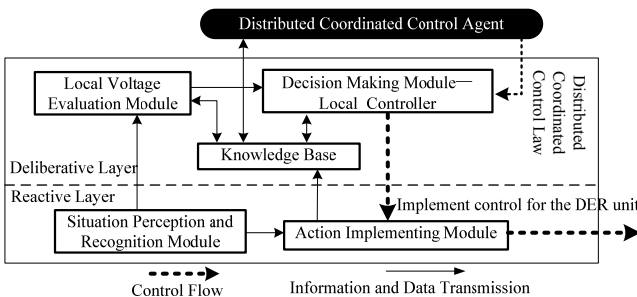

Fig.2 Structure of the first-level unit control agent

Each secondary-level distributed coordinated control agent is mainly responsible determining what kind of control mode and implementing the distribution coordinated control law for its DER unit to improve control performance. The distributed coordinated control law is formulated by using wide-area states that only come from the adjacent DER unit. Each secondary-level agent is designed as a deliberative agent. The voltage deviation between any two adjacent DER units is estimated in voltage assessment module. The distributed coordinated control law is formulated in decision-making module. When the voltage deviation is larger than a specified threshed, the distributed coordinated control law is executed through action implementation module.

In the hierarchical MAS, the interactive manner among agents divides into two categories [25]: no fixed master-slave mode and master-slave mode. For example, when the request of one unit agent is responded by another unit agent, this kind of interaction is no fixed master-slave mode. While when the distributed coordinated control agent requests to send the coordinated control law to the unit agent, the request must be

responded by the unit agent, in this case this kind of interaction is master-slave mode. The request of the coordinated control agent has the higher priority than that of any unit agent.
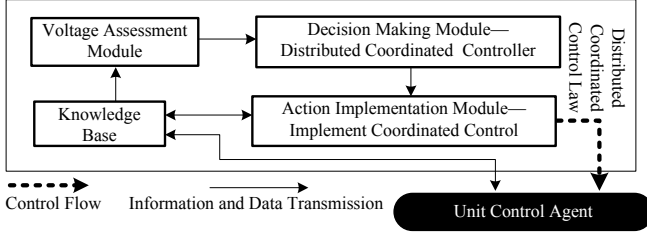

Fig.3 Structure of the secondary-level distributed coordinated control agent

III. CONTROL MODE AND DYNAMIC MODELING

Corresponding to Fig.1, the dynamic model of any three adjacent DER units is built as shown in Fig.4. In Fig.4, the pth DER is connected among both qth and rth DERs through DC lines with different impedances specified by parameters $R_{pq} = R_{qp} > 0$, $L_{pq} > 0$, $R_{pr} = R_{rp} > 0$ and $L_{pr} > 0$. In each DER unit, a Buck converter is presented to supply a local DC load connected to the PCC through a LC filter. The local load is unknown and is treated as current disturbance.
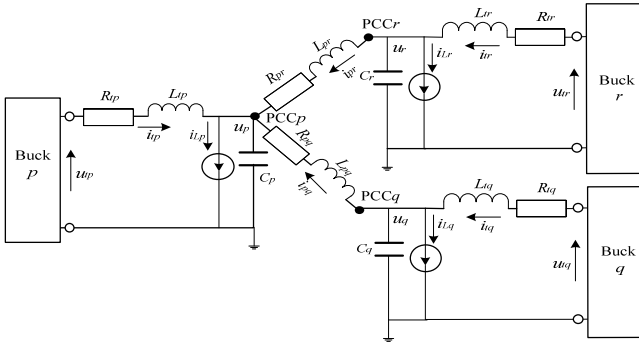

Fig.4 Dynamic model of three adjacent DER units

Corresponding to Fig.4, according to Kirchoff's voltage law and Kirchoff's current law, the following set of equations is obtained

$$\text{DER } p \begin{cases} \dfrac{du_p}{dt} = \dfrac{1}{C_i}i_{tp} - \dfrac{1}{C_i}i_{Lp} + \dfrac{1}{C_i}(i_{pq} + i_{pr}) \\ \dfrac{di_{tp}}{dt} = -\dfrac{1}{L_{tp}}u_p - \dfrac{R_{tp}}{L_{tp}}i_{tp} + \dfrac{1}{L_{tp}}u_{tp} \end{cases} \quad (1)$$

$$\text{DER } k \in \{q,r\} \begin{cases} \dfrac{du_k}{dt} = \dfrac{1}{C_k}i_{tk} - \dfrac{1}{C_k}i_{Lk} + \dfrac{1}{C_k}i_{kp} \\ \dfrac{di_{tk}}{dt} = -\dfrac{1}{L_{tk}}u_k - \dfrac{R_{tk}}{L_{tk}}i_{tk} + \dfrac{1}{L_{tk}}u_{tk} \end{cases} \quad (2)$$

As [25], since i_{pq} and i_{pr} are currents on DC transmission line, $di_{pq}/dt = -di_{qp}/dt = 0$ and $di_{pr}/dt = -di_{qr}/dt = 0$. Therefore, the following equations are given

$$i_{pq} = -i_{qp} = (u_q - u_p)/R_{pq} \quad (3)$$

$$i_{pr} = -i_{rp} = (u_r - u_p)/R_{pr} \quad (4)$$

Remark 3.1: According to Eqs. (3) and (4), when $|u_q - u_p| \le \varepsilon_{pq}$ and $|u_r - u_p| \le \varepsilon_{pr}$, where ε_{pq} and ε_{pr} are small specified thresholds, $|i_{pq}|$ and $|i_{pr}|$ are very small, since R_{pq} and R_{pr} are con-

stant. In this case, $i_{pq} + i_{pr}$, i_{pq} and i_{pr} are regarded as current disturbances, then the three DER units can be described as follows

$$\text{DER}i: \dot{x}_i(t) = A_i x_i(t) + B_i v_i(t) + D_i \omega_i(t), i \in \{p,q,r\} \quad (5)$$

where $x_i(t) = [u_i(t), i_{ti}(t)]^{\mathrm{T}}$ is state vectors; $v_i(t) = u_{ti}(t)$ is input variables; $\omega_p(t) = [i_{Lp}(t), (i_{pq}(t) + i_{pr}(t))]^{\mathrm{T}}$, $\omega_q(t) = [i_{Lq}(t), i_{qp}(t)]^{\mathrm{T}}$ and $\omega_r(t) = [i_{Lr}(t), i_{rp}(t)]^{\mathrm{T}}$ are disturbance vectors of the three DER units, respectively; the coefficient matrices are as follows

$$A_i = \begin{bmatrix} 0 & 1/C_i \\ -1/L_{ti} & -R_{ti}/L_{ti} \end{bmatrix}; \quad B_i = \begin{bmatrix} 0 \\ 1/L_{ti} \end{bmatrix}; \quad D_i = \begin{bmatrix} -1/C_i & 1/C_i \\ 0 & 0 \end{bmatrix}.$$

Since there is not coupling in Eq.(5), the three DER units can be respectively controlled only by means of their own local controller in the first-level unit agent. The local controller design will be discussed in Section IV. A. According to Remark 3.1, the control mode of the three DER units is defined as control mode 1.

Remark 3.2: When $|u_q - u_p| > \varepsilon_{pq}$ and $|u_r - u_p| \le \varepsilon_{pr}$, $|i_{pq}|$ is no longer very small, thus $i_{pq} + i_{pr}$ and i_{pq} are also no longer deemed as current disturbances. In this case, taking into account transmission time delay, both pth and qth DER units are described in the following form

$$\text{DER}i: \dot{x}_i(t) = \widetilde{A}_{ij} x_i(t) + B_i v_i(t) + A_{ij} x_j(t - \tau_{ij}) + \widetilde{D}_i \omega_i(t) \quad (6)$$

where $i \in \{p,q\}, j \in \{p,q\}, i \ne j$; $\tau_{pq} = \tau_{qp} \le \overline{\tau}, (\overline{\tau} > 0)$ is the transmission time delay between pth DER and qth DER, $x_j(t - \tau_{ij}) = [u_j(t - \tau_{ij}), i_{tj}(t - \tau_{ij})]^{\mathrm{T}}$, B_i is in the same form of Eq.(5),

$$\widetilde{A}_{ij} = \begin{bmatrix} -1/R_{ij}C_i & 1/C_i \\ -1/L_{ti} & -R_{ti}/L_{ti} \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} 1/R_{ij}C_i & 0 \\ 0 & 0 \end{bmatrix}, \quad \widetilde{D}_i = \begin{bmatrix} -1/C_i & 0 \\ 0 & 0 \end{bmatrix}.$$

In Eq.(6), since there is coupling between pth DER and qth DER, the two DER units need to be controlled by means of the local controller combined with the decoupling coordinated control law. The MAS based delay-dependent distributed coordinated controller will be studied in Section IV. B. While the rth DER units is still described in the same form of Eq.(5), where only $i = r$. Therefore, the rth DER unit can be controlled only by means of the local controller in the first-level unit agent. According to Remark 3.2, the control mode of the three DER units is defined as control mode 2.

Remark 3.3: When $|u_r - u_p| > \varepsilon_{pr}$ and $|u_q - u_p| \le \varepsilon_{pq}$, for the same reason, both pth and kth DER units are described in a similar form of Eq.(6), where only $i \in \{p,r\}, j \in \{p,r\}, i \ne j$, $\tau_{pr} = \tau_{rp} \le \overline{\tau}, (\overline{\tau} > 0)$ is the transmission time delay between the pth DER and the rth DER. In this case, the two DER units need to be controlled by means of the MAS based delay-dependent distributed coordinated controller. While the qth DER units can be described in the same form of Eq.(5), where only $i = q$. Therefore, the qth DER unit can be controlled only by means of the local controller in the first-level unit agent. According to Remark 3.3, the control mode of the three DER units is defined as control mode 3.

Remark 3.4: When $|u_q - u_p| > \varepsilon_{pq}$ and $|u_r - u_p| > \varepsilon_{pr}$, all currents including $i_{pq} + i_{pr}$, i_{pq} and i_{pr} are no longer dealt as current disturbances. In this case, both qth and kth DER units are de-

scribed in a similar form of Eq.(6) where only $i \in \{q,k\}$, $j = p$. While the pth DER unit is described as

$$\text{DER}p: \dot{x}_p(t) = \widetilde{A}_{pqr} x_p(t) + B_p v_p(t) \tag{7}$$

$$+ A_{pq} x_q(t - \tau_{pq}) + A_{pr} x_r(t - \tau_{pr}) + \widetilde{D}_p \omega_p(t)$$

where, $\widetilde{A}_{pqr} = \begin{bmatrix} -1/R_{pq}C_p - 1/R_{pr}C_p & 1/C_p \\ -1/L_{tp} & -R_{tp}/L_{tp} \end{bmatrix}$, other coefficient

matrices are in a similar form of Eq.(6) only $i = p$, $j \in \{q,r\}$.

The three DER units need to be controlled by means of the MAS based delay-dependent distributed coordinated controller. According to Remark 3.4, the control mode of the three DER units is defined as control mode 4.

No matter the local controller or the distributed coordinated controller, their control goal is to guarantee the state variables (voltage and current) to track their reference values. Therefore, here, the tracking reference model is given for the pth, qth and rth DER units as follows

$$\dot{x}_{ri}(t) = A_{ri} x_{ri}(t) + r_i(t), i \in \{p,q,r\} \tag{8}$$

where $x_{ri}(t)$ represents a desired trajectory for $x_i(t)$ to follow. A_{ri} is a specific asymptotically stable matrix, and $r_i(t)$ denotes bounded reference input.

IV. THE CONTROLLER DESIGN

A. Local Controller Design in First-level Unit Agent

When one DER unit (including each DER unit in remark 3.1, the rth DER unit in remark 3.2, and the qth DER unit in remark 3.3) is modeled in the form of Eq.(5), it can be controlled by means of only local controller in the first-level unit control agent. The local controller is designed as

$$v_i(t) = K_i [x_i(t) - x_{ri}(t)], i \in \{p,q,r\} \tag{9}$$

where K_i is local controller parameter matrix of ith DER unit. Then, by using Eq (5) and Eq.(8), the ith DER tracking control system under the local controller (9) is written in the following form

$$\text{DER}i: \dot{\hat{x}}_i(t) = \hat{A}_i \hat{x}_i(t) + \hat{D}_i \hat{\omega}_i(t) \tag{10}$$

where, $\hat{x}_i(t) = \begin{bmatrix} x_i^T(t), & x_{ri}^T(t) \end{bmatrix}^{\mathrm{T}}$, $\hat{\omega}_i(t) = \begin{bmatrix} \omega_i^T(t), & r_i^T(t) \end{bmatrix}^T$, and

$$\hat{A}_i = \begin{bmatrix} A_i + B_i K_i & -B_i K_i \\ 0 & A_{ri} \end{bmatrix}, \quad \hat{D}_i = \begin{bmatrix} D_i & 0 \\ 0 & I \end{bmatrix}.$$

H_∞ control performance regarding the tracking error is given in the following form

$$\frac{\int_0^{t_f} [(x_i(t) - x_{ri}(t))^T Q_i (x_i(t) - x_{ri}(t))] dt}{\int_0^{t_f} \hat{\omega}_i(t)^T \hat{\omega}_i(t) dt} \le \rho_i^2 \tag{11}$$

where t_f denotes terminal time of control, and the weighting matrix $Q_i = Q_i^T > 0$. Physical meaning of (11) is that effect of any $\hat{\omega}_i(t)$ on tracking error $x_i(t) - x_{ri}(t)$ must be attenuated below a desire level ρ_i from viewpoint of energy. H_∞ control performance with a prescribed attenuation level is useful for a robust control design without knowledge of $\omega_i(t)$ and $r_i(t)$.

Considering the initial condition, H_∞ control performance can be rewritten as follows:

$$\int_0^{t_f} \hat{x}_i^T(t) \hat{Q}_i \hat{x}_i(t) dt \le \rho_i^2 \int_0^{t_f} \hat{\omega}_i^T(t) \hat{\omega}_i(t) dt + \hat{x}_i^T(0) P_i \hat{x}_i(0) \tag{12}$$

where $\hat{Q}_i = \begin{bmatrix} Q_i & -Q_i \\ -Q_i & Q_i \end{bmatrix}$, and weighting matrix $P_i = P_i^T > 0$.

According to the following Theorem 4.1, the first-level unit control agent can determine the local controller (9) for the ith tracking system (10) with the guaranteed H_∞ control performance (12) for $\forall \hat{\omega}_i(t)$.

Theorem 4.1 The ith DER unit tracking control system (10) is asymptotically stable with the guaranteed H_∞ control performance in (12) for $\forall \hat{\omega}_i(t)$ if there exists $P_i = P_i^T > 0$, satisfying the following matrix inequality

$$\begin{bmatrix} \hat{A}_i^T P_i + P_i \hat{A}_i + \hat{Q}_i & P_i \hat{D}_i \\ \hat{D}_i^T P_i & -\rho_i^2 I \end{bmatrix} \le 0. \tag{13}$$

The proof is given in Appendix A

The inequality (13) can be transformed into linear matrix inequality (LMI) according to the following procedures
(1) Denote a new matrix

$$W_i = \begin{bmatrix} \overline{W}_i & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} P_i^{-1} & 0 \\ 0 & I \end{bmatrix}$$

where $\overline{W}_i^T = \overline{W}_i = P_i^{-1} > 0$, left and right multiplying it into (14), it can be obtained

$$\begin{bmatrix} \overline{W}_i \hat{A}_i^T + \hat{A}_i \overline{W}_i + \overline{W}_i \hat{Q}_i \overline{W}_i & \hat{D}_i \\ \hat{D}_i^T & -\rho_i^2 I \end{bmatrix} \le 0 \tag{14}$$

(2) Define

$$\overline{W}_i = \begin{bmatrix} \overline{W}_{i1} & 0 \\ 0 & \overline{W}_{i1} \end{bmatrix} = \begin{bmatrix} P_{i1}^{-1} & 0 \\ 0 & P_{i1}^{-1} \end{bmatrix}, \quad \hat{K}_i = K_i \overline{W}_{i1}, \quad \breve{Q} = \overline{W}_1 Q \overline{W}_1,$$

it is easy to gain

$$\hat{A}_i \overline{W}_1 = \begin{bmatrix} A_i \overline{W}_{i1} + B_i \hat{K}_i & -B_i \hat{K}_i \\ 0 & A_{ri} \overline{W}_{i1} \end{bmatrix}.$$

Hence, the inequality (14) is equivalent to LMI.

Remark 4.1 The local controller design in (9) is transformed into the following LMI convex optimization problem

$$\min_{\overline{W}_i, \hat{K}_i} \rho_i^2,$$

subject to $\overline{W}_i = \overline{W}_i^T > 0$ and (14). $\tag{15}$

B. MAS based Distributed Coordinated Controller Design

1. Distributed coordinated controller design between two DER units

When two DER units (including both pth and qth DER units in remark 3.2, as well as both pth and rth DER units in remark 3.3) are modeled in the form of Eq.(6), they need to be controlled by means of the two-level MAS. The distributed coordinated controller is designed as

$$v_i(t) = K_i [x_i(t) - x_{ri}(t)] + K_{ij} x_j(t - \tau_{ij}) \tag{16}$$

where $i \in \{p,q\}, j \in \{p,q\}, i \ne j$; or $i \in \{p,r\}, j \in \{p,r\}, i \ne j$; K_i is local controller parameter matrix of the ith DER unit, K_{ij} is decoupling coordinated control law that comes form jth DER unit.

By Eqs (6) and (8), the ith tracking control system under the controller (16) is given as follows

$$\text{DER}i: \dot{\hat{x}}_i(t) = \overline{A}_{ij} \hat{x}_i(t) + \overline{A}_{ij} \hat{x}_j(t - \tau_{ij}) + \overline{D}_i \hat{\omega}_i(t) \tag{17}$$

where $i \in \{p,q\}, j \in \{p,q\}, i \ne j$; or $i \in \{p,r\}, j \in \{p,r\}, i \ne j$,

$$\overline{A}_{ij} = \begin{bmatrix} \widetilde{A}_{ij} + B_i K_i & -B_i K_i \\ 0 & A_{ri} \end{bmatrix}, \overline{A}_{ij} = \begin{bmatrix} A_{ij} + B_i K_{ij} & 0 \\ 0 & 0 \end{bmatrix}, \overline{D}_i = \begin{bmatrix} \widetilde{D}_i & 0 \\ 0 & I \end{bmatrix}.$$

Then augmented system by integrating the ith DER with the jth DER is described as

$$\text{DER}ij\text{: } \dot{\widetilde{x}}(t) = \widetilde{A}\widetilde{x}(t) + \widehat{A}\widetilde{x}(t - \tau_{ij}) + \widetilde{D}\widetilde{\omega}(t) \quad (18)$$

where $i = p$, $j \in \{q, r\}$; $\widetilde{x}(t) = \left[x_i^T(t), x_{ri}^T(t), x_j^T(t), x_{rj}^T(t) \right]^T$ is the state vector of the augmented system, $\widetilde{v}(t) = \left[v_i(t), v_j(t) \right]^T$ is the input vector and $\widetilde{\omega}(t) = \left[\omega_i^T(t), \ \omega_j^T(t) \right]^T$ is the disturbance vector of the augmented system. Moreover,

$$\widetilde{A} = \begin{bmatrix} \overline{A}_{ij} & 0 \\ 0 & \overline{A}_{ji} \end{bmatrix}, \widehat{A} = \begin{bmatrix} 0 & \overline{A}_{ij} \\ \overline{A}_{ji} & 0 \end{bmatrix}, \widetilde{D} = \begin{bmatrix} \overline{D}_i & 0 \\ 0 & \overline{D}_j \end{bmatrix}.$$

Corresponding to the augmented system (18), H_∞ control performance can be given as follows:

$$\int_0^{t_f} \widetilde{x}^T(t) \widetilde{Q} \widetilde{x}(t) dt \le \rho^2 \int_0^{t_f} \widetilde{\omega}^T(t) \widetilde{\omega}(t) dt + \text{V}(0) \quad (19)$$

where $\widetilde{Q} = \begin{bmatrix} Q_i & -Q_i & 0 & 0 \\ -Q_i & Q_i & 0 & 0 \\ 0 & 0 & Q_j & -Q_j \\ 0 & 0 & -Q_j & Q_j \end{bmatrix}$, V(0) is Lyapunov function

initial value.

By mans of the following Theorem 4. 2, the two-level agents can determine the distributed coordinated controller (16) for the augmented system (18) with the guaranteed H_∞ control performance (19) for $\forall \widetilde{\omega}(t)$.

Theorem 4.2 Given allowable upper bound $\overline{\tau}$ of the time delays, the controlled system (18) is asymptotically stable for all time-delays satisfying $\tau_{ij} \in [0, \overline{\tau}]$ with the H_∞ control performance in (19), if there exist symmetric positive definite matrices P, S, Z, X satisfying the following matrix inequalities

$$\begin{bmatrix} \Theta & P\widehat{A} - Y & P\widetilde{D} & \overline{\tau}\widehat{A}^T Z \\ * & -S & 0 & \overline{\tau}\widehat{A}^T Z \\ * & * & -\rho^2 & \overline{\tau}\widetilde{D}^T Z \\ * & * & * & -\overline{\tau}Z \end{bmatrix} \le 0 \quad (20)$$

and $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0$.

where, $i = p$, $j \in \{q, r\}$, $\Theta = \widetilde{Q} + S + P\widetilde{A} + \widetilde{A}^T P + \overline{\tau} X + Y + Y^T$.
The proof is given in the Appendix B.
The inequality (20) is not linear, so left multiply and right multiply matrix $diag\{P^{-1}, I, I, Z^{-1}\}$, then define

$$P^{-1} = \begin{bmatrix} P_1^{-1} & 0 & 0 & 0 \\ 0 & P_1^{-1} & 0 & 0 \\ 0 & 0 & P_1^{-1} & 0 \\ 0 & 0 & 0 & P_1^{-1} \end{bmatrix}, P_1 = P_1^T > 0,$$

$\widetilde{K}_i = K_i P_1^{-1}$, $\widetilde{K}_j = K_j P_1^{-1}$, $\widetilde{K}_{ij} = K_{ij} P_1^{-1}$, $\widetilde{K}_{ji} = K_{ji} P_1^{-1}$,
$\widetilde{X} = P^{-1}XP^{-1}$, $\widetilde{S} = P^{-1}SP^{-1}$, $\widehat{Q} = P^{-1}\widetilde{Q}P^{-1}$, $\widetilde{Y} = P^{-1}YP^{-1}$,

it is easy to gain,

$$\widetilde{A}P^{-1} = \begin{bmatrix} \widetilde{A}_{ij}P_1^{-1} + B_i\widetilde{K}_i & -B_i\widetilde{K}_i & 0 & 0 \\ 0 & A_{ri}P_1^{-1} & 0 & 0 \\ 0 & 0 & \widetilde{A}_{ji}P_1^{-1} + B_j\widetilde{K}_j & -B_j\widetilde{K}_j \\ 0 & 0 & 0 & A_{rj}P_1^{-1} \end{bmatrix}.$$

Then the inequality (20) is transformed to LMI (21).

$$\begin{bmatrix} \widetilde{\Theta} & \widehat{A} - P^{-1}Y & \widetilde{D} & \overline{\tau}P^{-1}\widehat{A}^T \\ * & -S & 0 & \overline{\tau}\widehat{A}^T \\ * & * & -\rho^2 & \overline{\tau}\widetilde{D}^T \\ * & * & * & -\overline{\tau}Z^{-1} \end{bmatrix} \le 0 \quad (21)$$

where $\widetilde{\Theta} = \widehat{Q} + \widetilde{S} + \widetilde{A}P^{-1} + P^{-1}\widetilde{A}^T + \overline{\tau}\widetilde{X} + \widetilde{Y} + \widetilde{Y}^T$.

Remark 4.2 The distributed coordinated controller design in (16) is transformed into the following LMI convex optimization problem

$$\min_{P^{-1}, \widetilde{K}_i, \widetilde{K}_j, \widetilde{K}_{ij}, \widetilde{K}_{ji}} \rho^2,$$

subject to $P^{-1} = P^{-T} > 0$, $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0$, and (21) $\quad (22)$

where $i = p$, $j \in \{q, r\}$.

2. Distributed coordinated controller design among three DER units

When three DER units are described as remark 3.4, the two-level MAS needs to implement the distributed coordinated control among three DER units. In this case, the distributed coordinated controller is designed as

$$v_i(t) = K_i[x_i(t) - x_{ri}(t)] + K_{ij}x_j(t - \tau_{ij}), \quad i \in \{q, r\}, j = p \quad (23)$$

$$v_p(t) = K_p[x_p(t) - x_{rp}(t)] + K_{pq}x_q(t - \tau_{pq}) + K_{pr}x_r(t - \tau_{pr}) \quad (24)$$

where the definition regarding controller parameters is similar to Eq.(16).

By Eqs (6) and (8), the ith tracking control system under the controller (23) is given as follows

$$\text{DER}i\text{: } \dot{\widehat{x}}_i(t) = \overline{A}_{ij}\widehat{x}_i(t) + \overline{A}_{ij}\widehat{x}_j(t - \tau_{ij}) + \overline{D}_i\widehat{\omega}_i(t), i \in \{q, r\}, j = p \quad (25)$$

where the coefficient matrices are in a similar form of Eq.(17), only $i \in \{q, r\}$, $j = p$.

By Eqs (7) and (8), the pth tracking control system under the controller (24) is given as follows

$$\text{DER}p\text{: } \dot{\widehat{x}}_p(t) = \overline{A}_{pqr}\widehat{x}_i(t) + \overline{A}_{pq}\widehat{x}_q(t - \tau_{pq}) + \overline{A}_{pr}\widehat{x}_r(t - \tau_{pr}) + \overline{D}_p\widehat{\omega}_p(t) \quad (26)$$

where $\overline{A}_{pqr} = \begin{bmatrix} \widetilde{A}_{pqr} + B_p K_p & -B_p K_p \\ 0 & A_{rp} \end{bmatrix}$, other coefficient matrices are

in a similar form of Eq.(17) only $i = p$, $j \in \{q, r\}$.
Then augmented system by integrating the three DER units is described as

$$\text{DER}pqr\text{: } \dot{\widetilde{x}}(t) = \widetilde{A}_1\widetilde{x}(t) + \widehat{A}_1\widetilde{x}(t - \tau) + \widetilde{D}_1\widetilde{\omega}(t) \quad (27)$$

where $\widetilde{x}(t) = \left[x_p^T(t), x_{rp}^T(t), x_q^T(t), x_{rq}^T(t), x_r^T(t), x_{rr}^T(t) \right]^T$ is the state vector, $\widetilde{x}(t - \tau) = \left[x_p^T(t), x_{rp}^T(t), x_q^T(t - \tau_{pq}), x_{rq}^T(t), x_r^T(t - \tau_{pr}), x_{rr}^T(t) \right]^T$ $\widetilde{v}(t) = \left[v_p(t), v_q(t), v_r(t) \right]^T$ is the input vector of the augmented system, and $\widetilde{\omega}(t) = \left[\omega_p^T(t), \ \omega_q^T(t), \ \omega_r^T(t) \right]^T$ is the disturbance vector. Moreover,

$$\widetilde{A}_1 = \begin{bmatrix} \overline{A}_{pqr} & 0 & 0 \\ 0 & \overline{A}_{qp} & 0 \\ 0 & 0 & \overline{A}_{rp} \end{bmatrix}, \quad \widehat{A}_1 = \begin{bmatrix} 0 & \overline{A}_{pq} & \overline{A}_{pr} \\ \overline{A}_{qp} & 0 & 0 \\ \overline{A}_{rp} & 0 & 0 \end{bmatrix}, \quad \widetilde{D}_1 = \begin{bmatrix} \overline{D}_p & 0 & 0 \\ 0 & \overline{D}_q & 0 \\ 0 & 0 & \overline{D}_r \end{bmatrix}.$$

Remark 4.3 According to the similar design process of Theorem 4.2, the distributed coordinated controller design in (23) and (24) is transformed into the following LMI convex optimization problem

$$\min_{P^{-1}, \widetilde{K}_p, \widetilde{K}_q, \widetilde{K}_r, \widetilde{K}_{pq}, \widetilde{K}_{qp}, \widetilde{K}_{pr}, \widetilde{K}_{rp}} \rho^2,$$

subject to $P^{-1} = \begin{bmatrix} P_1^{-1} & 0 & \cdots & 0 \\ 0 & P_1^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_1^{-1} \end{bmatrix}^{6 \times 6}$, $P_1 = P_1^T > 0$,

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0, \text{ and } \begin{bmatrix} \widetilde{\Theta}_1 & \widehat{A}_1 - P^{-1}Y & \widetilde{D}_1 & \overline{\tau} P^{-1} \widetilde{A}_1^T \\ * & -S & 0 & \overline{\tau} \widehat{A}_1^T \\ * & * & -\rho^2 & \overline{\tau} \widetilde{D}_1^T \\ * & * & * & -\overline{\tau} Z^{-1} \end{bmatrix} \leq 0 . \quad (22)$$
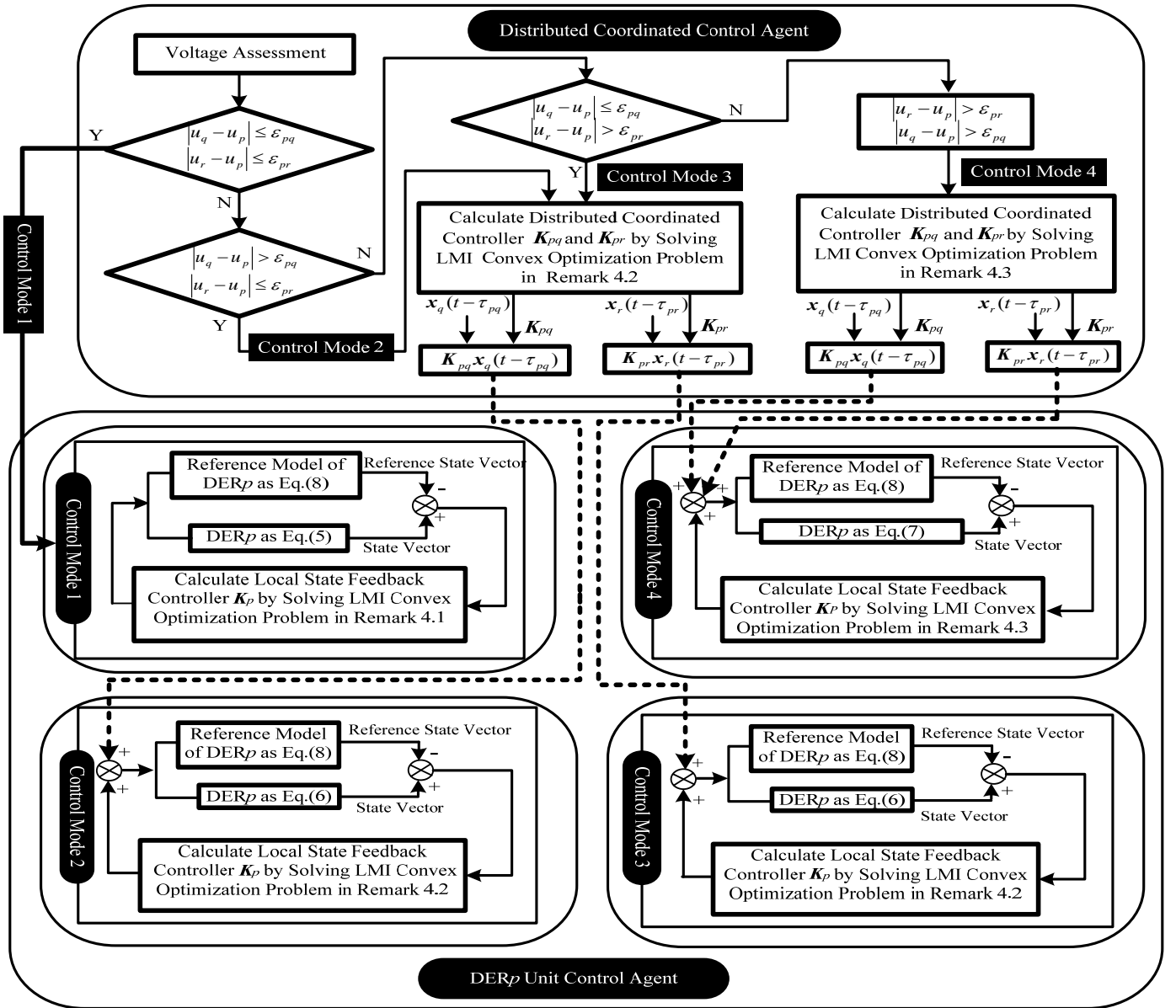
where $\widehat{Q}_1 = P^{-1} \begin{bmatrix} \widehat{Q}_p & 0 & 0 \\ 0 & \widehat{Q}_q & 0 \\ 0 & 0 & \widehat{Q}_r \end{bmatrix}$, $\widehat{Q}_i = \begin{bmatrix} Q_i & -Q_i \\ -Q_i & Q_i \end{bmatrix}$, $i \in \{p, q, r\}$,

$\widetilde{\Theta}_1 = \widehat{Q}_1 + \widetilde{S} + \widetilde{A}_1 P^{-1} + P^{-1} \widetilde{A}_1^T + \overline{\tau} \widetilde{X} + \widetilde{Y} + \widetilde{Y}^T$, the definition regarding $\widetilde{K}_p, \widetilde{K}_q, \widetilde{K}_r, \widetilde{K}_{pq}, \widetilde{K}_{pr}, \widetilde{K}_{qp}, \widetilde{K}_{rp}, \widetilde{S}, \widetilde{X}, \widetilde{Y}$ is similar to

remark 4.2.

C. Implementation of the Distributed Coordinated Control

The pth DER unit in DC MG can be controlled in four kinds of modes. The first mode is that the DER unit is controlled only by the local controller in the first-level unit agent. The second mode is that the DER unit is controlled by the distributed coordinated controller between both pth and qth DERs based on MAS. The third mode is by the distributed coordinated controller between both pth and rth DERs. The fourth mode is by the distributed coordinated controller among the pth, qth and rth DERs. The implementation flowchart of four kinds of control modes is given in Fig.5.

Fig.5 Implementation flowchart of four kinds of control modes based on the MAS

Fig.6 PCC control performance in Case 1

(a) Load currents in the DERp and DERq; (b) PCC voltage of the DERp in the local control mode 1 when τ=20ms; (c) PCC voltage of the DERq in the local control mode 1 when τ=20ms; (d) PCC voltage of the DERp in the distributed coordinated control mode 2 when τ=20ms; (e) PCC voltage of the DERq in the distributed coordinated control mode 2 when τ=20ms; (f) PCC voltage of the DERp in the distributed coordinated control mode 2 when τ=2s; (g) PCC voltage of the DERq in the distributed coordinated control mode 2 when τ=2s

V. EXPERIMENT STUDIES

To evaluate the performance of the distributed coordinated control in a radial DC MG, two kinds of cases are designed in order to test different scenarios. The two cases mainly focus on two aspects: the control performance in response to different load changes, as well as the control performance in different time delays. The simulation parameters are given in Table.1.

TABLE 1. SIMULATION PARAMETERS

Parameter	Symbol	Value
DC power supply	U_{DC}	1kV
Output capacitance	C_p	2.2mF
Output capacitance	C_q	2.0mF
Output capacitance	C_r	1.8mF
Converter inductance	L_{tp}	2.0mH
Converter inductance	L_{tq}	1.8mH
Converter inductance	L_{tr}	1.6mH
Converter resistance	R_{tp}	0.2Ω
Converter resistance	R_{tq}	0.2Ω
Converter resistance	R_{tr}	0.2Ω
Transmission line inductance	L_{pq}	2.0μH
Transmission line resistance	R_{pq}	0.05Ω
Transmission line inductance	L_{pr}	2.2μH
Transmission line resistance	R_{pr}	0.06Ω

A. Case 1: the load demand doubled in the DERj

The load demand in the DERq increases twice at t=3s instant as shown in Fig.6(a). The load current disturbance leads to a big bus voltage deviation between both pth and qth DER units, i.e. $\left| u_q - u_p \right| > \varepsilon_{pq}$ and $\left| u_r - u_p \right| \le \varepsilon_{pr}$. In this case, the secondary-level agent determines to execute the distributed coordinated control mode 2 in both pth and qth DER units after t=3s, while the rth DER unit is still controlled by local control mode 1. By solving LMI convex optimization problem in Remark 4.2, we find that the integrated system of both pth and qth DER units is robust stable for any time delays satisfying $0 \le \bar{\tau} \le 2.2848s$. For the comparison purpose, in Figs.6(b) and (c), we firstly give the PCC voltage performance of both pth and qth DER units still in local control mode 1 after t=3s. Then the PCC voltage performance in the proposed distributed coordinated control mode 2 is displayed in Figs.6(d)-(g) taking into account different time delays.
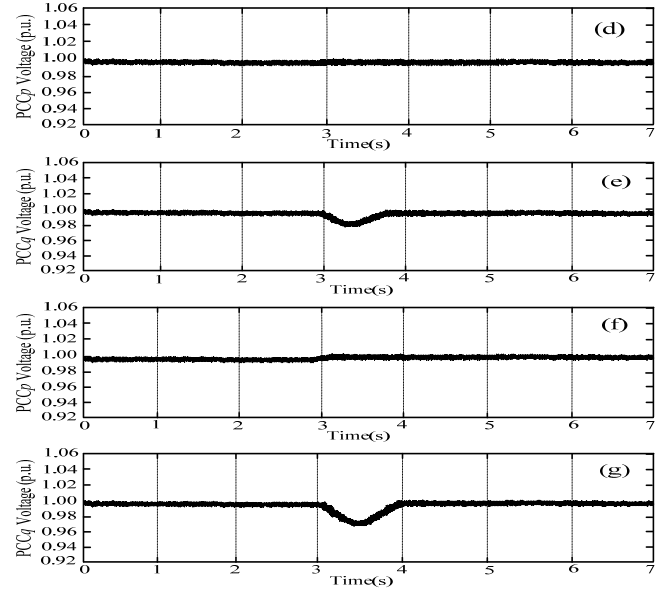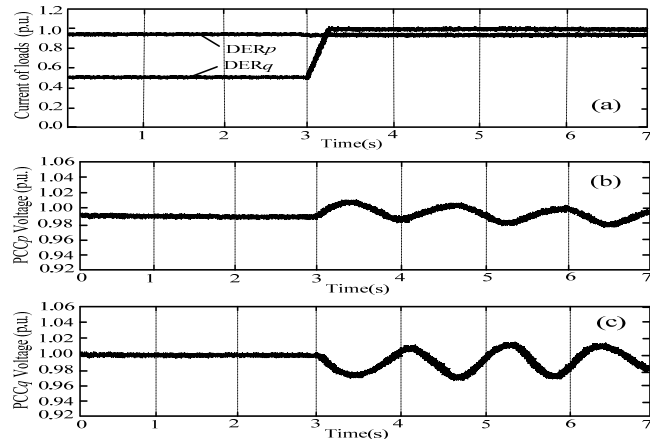

From Figs.6(b) and (c), it can be shown that if the two DER units are still controlled by means of the local control mode 1 after t=3s, the PCC bus voltages, especially the PCCq voltage, have larger fluctuation. It is because that, only by local controller in each unit control agent, no decoupling control signal eliminates the coupling effect between both pth and qth DER units, so that the bus voltage deviation can not be controlled effectively. From Figs.6(d)-(g), it can be implied that, by means of the proposed distributed coordinated control mode 2, the PCC voltages of both pth and qth DER units can be stabilized within the secure range with less bus voltage deviation. Moreover, although taking into account different time delays, the PCC voltage performance in each DER is almost same. It is because that, by solving LMI convex optimization problem in Remark 4.2, as long as $\tau_{pq} \in [0, \bar{\tau}]$, the designed distributed coordinated controller can guarantee the system optimal control performance. The above simulation results imply that the MAS based distributed coordinated control can improve bus voltage performance in response to the load change and different transmission time delays.

Also for comparison purpose, Fig.7 gives the voltage performance by means of the proposed method of reference [12] in Case 1.
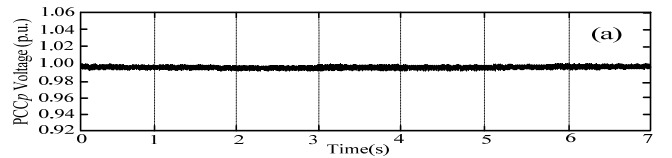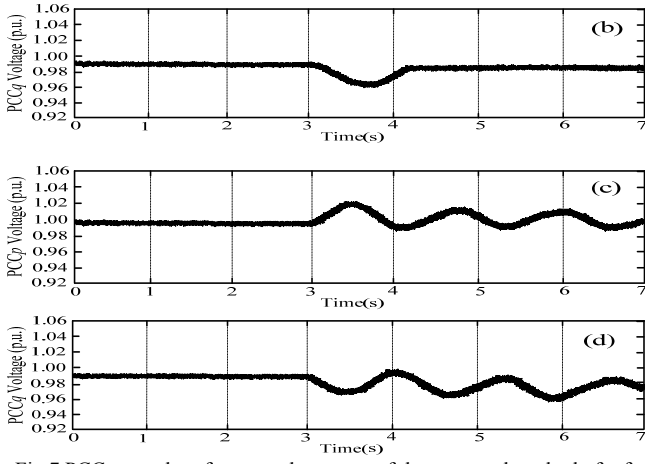
Fig.7 PCC control performance by means of the proposed method of reference [12] in Case 1

(a) PCC voltage of the DER*p* when τ =20ms; (b) PCC voltage of the DER*q* when τ =20ms; (c) PCC voltage of the DER*p* when τ =2s; (d) PCC voltage of the DER*q* when τ =2s

When τ =20ms, the voltage performance in Figs.7(a) and (b) by means of the proposed method of reference [12] is similar to that in Figs.6(d) and (e), only there is a bit larger deviation between both *p*th and *q*th bus voltages than in Figs.6(d) and (e). While when τ =2s, compared with Figs.6(f) and (g), there are larger voltage fluctuations in Figs.7(c) and (d) by means of the proposed method of reference [12] after *t*=3s. These voltage fluctuations can lead to much larger voltage deviation between both *p*th and *q*th DER units. The comparative results above imply that the proposed MAS based distributed coordinated control has better control performance especially in large time delays.

B. Case 2: the load changes in both DERp and DERq

The load demand in the DER*q* increases twice at *t*=3s instant, at the same time, the load demand in the DER*p* decreases half as shown in Fig.7(a). The load changes in the two DER units result in a larger bus voltage deviation among three adjacent DER units, i.e. $|u_q - u_p| > \varepsilon_{pq}$ and $|u_r - u_p| > \varepsilon_{pr}$. Therefore, the secondary-level agent determines to execute the distributed coordinated control mode 4 for the *p*th, *q*th and *r*th DER units after *t*=3s. By using LMI convex optimization technique in Remark 4.3, it can be found that the augmented system by integrating the *p*th, *q*th and *r*th DER units is asymptotically stable for any time delays satisfying $0 \le \bar{\tau} \le 2.1774s$. To evaluate the performance of the distributed coordinated control by comparison with the local control, the PCC voltage performance in the local control mode 1 is firstly given in Figs.7(b)-(d). Then taking into account different time delays, the PCC voltage performance in the distributed coordinated control mode 4 is displayed in Figs.7(e)-(j).
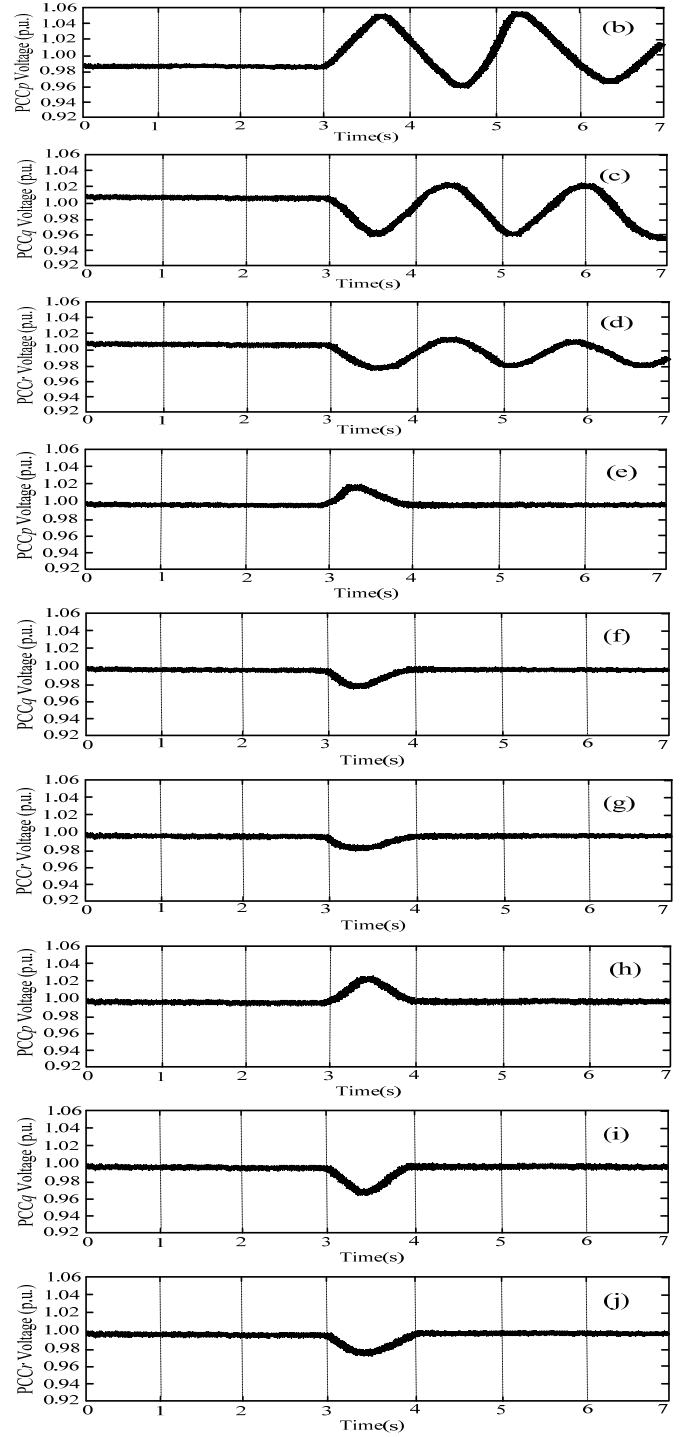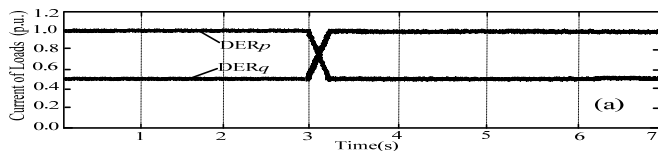

Fig.8 PCC voltage control performance in Case 2

(a) Load currents in the DER*p* and DER*q*; (b) PCC voltage of the DER*p* in the local control mode 1 when τ =200ms; (c) PCC voltage of the DER*q* in the local control mode 1 when τ =200ms; (d) PCC voltage of the DER*r* in the local control mode 1 when τ =200ms; (e) PCC voltage of the DER*p* in the distributed coordinated control mode 4 when τ =200ms; (f) PCC voltage of the DER*q* in the distributed coordinated control mode 4 when τ =200ms; (g) PCC voltage of the DER*r* in the distributed coordinated control mode 4 when τ =200ms; (h) PCC voltage of the DER*p* in the distributed coordinated control mode 4 when τ =2s; (i) PCC voltage of the DER*q* in the distributed coordinated control mode 4 when τ =2s; (i) PCC voltage of the DER*r* in the distributed coordinated control mode 4 when τ =2s

From Figs.8(b)-(d), it can seen that the local control is not

able to maintain the PCC bus voltage stabilization in Case 2. There are very large voltage deviations especially between both pth and qth DER units since there are larger load changes in the two units. From Figs.8(e)-(j), it can shown that by means of the proposed distributed coordinated control mode 4, at the t=3s instant, the larger load disturbances only lead to a bit bus voltage fluctuations in the three DER units, afterwards the PCC voltages are rapidly stabilized down almost without any deviation in different time delays. The simulation results above indicate that the proposed MAS based distributed coordinated control still can ensure the bus voltages performance better in the face of larger load disturbances.

In order to testify the performance regarding current sharing by comparison with the proposed method of reference [12], the current performance is given in Fig.9 by using two kinds of methods taking into account different time delays in Case 2.
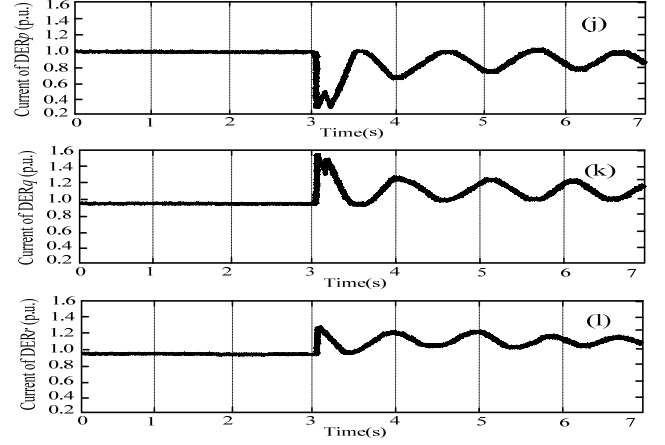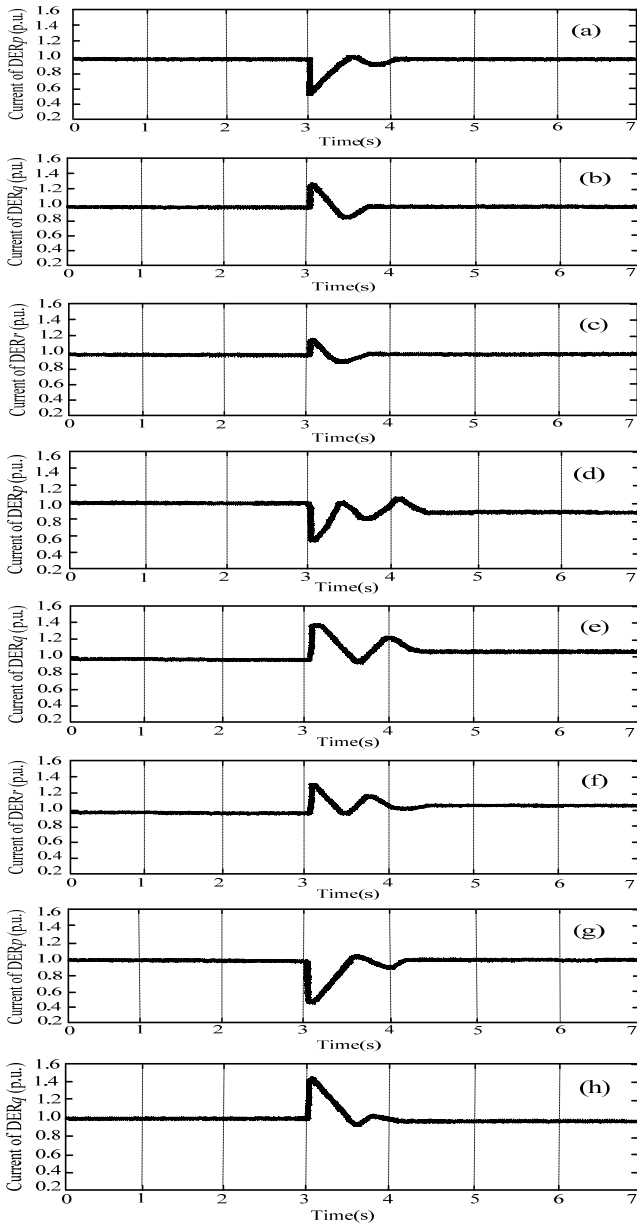

Fig.9 Current control performance in Case 2

(a) Current of the DERp in the proposed control mode 4 when τ=200ms; (b) Current of the DERq in the proposed control mode 4 when τ=200ms; (c) Current of the DERr in the proposed control mode 4 when τ=200ms; (d) Current of the DERp in the proposed method of reference [12] when τ=200ms; (e) Current of the DERq in the proposed method of reference [12] when τ=200ms; (f) Current of the DERr in the proposed method of reference [12] when τ=200ms; (g) Current of the DERp in the proposed control mode 4 when τ=2s; (h) Current of the DERq in the proposed control mode 4 when τ=2s; (i) Current of the DERr in the proposed control mode 4 when τ=2s; (j) Current of the DERp in the proposed method of reference [12] when τ=2s; (k) Current of the DERq in the proposed method of reference [12] when τ=2s; (l) Current of the DERr in the proposed method of reference [12] when τ=2s

From Figs.9(a)-(f), it can be shown that when τ=200ms, the current sharing in the three DER units by means of the proposed control mode 4 is a bit better than that in the proposed method of reference [12], since after t=4.5s there are a bit current deviation among the three DER units by means of the proposed method of reference [12]. When τ=2s, the current sharing is almost similar to that when τ=200ms in the each DER unit by means of the proposed control mode 4 as shown in Figs.(g)-(i). While by means of the proposed method of reference [12], when τ=2s, after t=3s there are very large current fluctuations in the three DER units, which leads to much larger current sharing deviation among the three DER units. The results above imply that when there is a large time delay, the proposed method in this paper can achieve better performance regarding current sharing.

In order to illustrate the robust performance of the proposed method, we assume that there is a failure of the communication link between both pth and qth secondary-level agents. Still in Case 2, the voltage performance of the three DER units is shown in Fig.10.
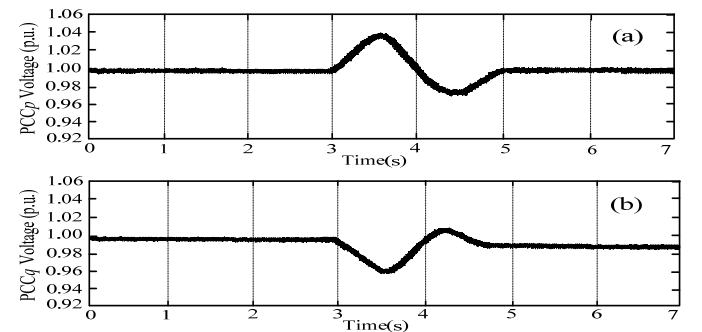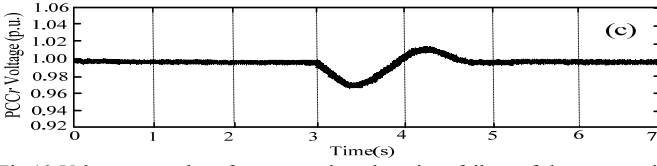
Fig.10 Voltage control performance when there is a failure of the communication in Case 2 when τ =200ms

(a) PCC voltage of the DERp; (b) PCC voltage of the DERq; (c) PCC voltage of the DERr

If there is no failure of the communication link between both pth and qth secondary-level agents, in case 2 the voltage performance in the three DER units should be the same as Figs.8(e)-(g). Therefore, compared with Figs.8(e)-(g), the PCC voltage fluctuations in the three DER units as shown in Figs.10(a)-(c) are a bit larger. However, the voltage fluctuations can damp down after t=4.5s. it is because that, though there is a failure of the communication link between both pth and qth secondary-level agents, but based on the MAS platform, through the interactions between first-level and secondary-level agents of pth DER unit and of qth DER units, as well as the interaction between both pth and qth first-level agents, the coordinated control laws between both pth and qth DER units can be still sent. Therefore, the proposed MAS distributed coordinated control method still can guarantee the PCC voltage stabilization. Only since it takes a larger time when the coordinated control laws are sent than that without failure, PCC voltage fluctuations are a bit larger.

VI. CONCLUSION

This paper develops a delay-dependent distributed coordinated control based on the two-level MAS, so that the DC MG can ensure voltage and current sharing performance better in response to large load disturbances and different time delays. Compared with the relevant research results, this paper contributes the following original works: The MAS based distributed coordinated control is proposed, where beside local-state-feedback control in unit control agent, the remote states that only come from adjacent DER based on low bandwidth communication are used to synthesize decoupling coordinated control law by means of the distributed coordinated control agent. Moreover, taking into account transmission time delays, the distributed coordinated control is designed by means of delay-dependent H$_\infty$ robust control method. Finally, the better control performance has been demonstrated by means of simulation results.

However, the MAS based distributed coordinated control scheme still a bit relies on communication technology. Based on communication technology, the scheme can not only be applied into DC MG, but also be feasible for controlling any kind of smart grids with multiple DERs by extending the control function of the agents and creating additional agents in the system. The future development of this study will focus on extending application.

Appendix A. Proof of Theorem 4.1:

Define a Lyapunov function for the tracking system (10) as

$$V_i(t) = \hat{x}_i^T(t) P_i \hat{x}_i(t), \text{ where } P_i = P_i^T > 0.$$

Then, it is easy to obtain

$$\int_0^{t_f} [(x_i(t) - x_{ri}(t))^T Q_i(x_i(t) - x_{ri}(t))]dt = \int_0^{t_f} \hat{x}_i^T(t)\hat{Q}_i\hat{x}_i(t)dt$$

$$= \hat{x}_i^T(0) P_i \hat{x}_i(0) - \hat{x}_i^T(t_f) P_i \hat{x}_i(t_f)$$

$$+ \int_0^{t_f} \{\hat{x}_i^T(t)\hat{Q}_i\hat{x}_i(t) + \frac{d}{dt}(\hat{x}_i^T(t)P_i\hat{x}_i(t))\}dt$$

$$\leq \hat{x}_i^T(0) P_i \hat{x}_i(0) + \int_0^{t_f} \{\hat{x}_i^T(t)\hat{Q}_i\hat{x}_i(t) + \dot{\hat{x}}_i^T(t)P_i\hat{x}_i(t) + \hat{x}_i^T(t)P_i\dot{\hat{x}}_i(t)\}dt$$

$$= \hat{x}_i^T(0) P_i \hat{x}_i(0) + \int_0^{t_f} \left\{ \begin{bmatrix} \hat{x}_i(t) \\ \hat{\omega}_i(t) \end{bmatrix}^T \begin{bmatrix} \hat{A}_i^T P_i + P_i\hat{A}_i + \hat{Q}_i & P_i\hat{D}_i \\ \hat{D}_i^T P_i & -\rho_i^2 I \end{bmatrix} \begin{bmatrix} \hat{x}_i(t) \\ \hat{\omega}_i(t) \end{bmatrix} + \rho_i^2 \hat{\omega}_i(t)^T \hat{\omega}_i(t)\}dt \right.$$

By the above inequality, it can be implied that if the inequality (13) is satisfied, the ith augmented DER unit system (10) is asymptotically stable with the guaranteed H$_\infty$ control performance in (12) for $\forall \hat{\omega}_i(t)$. The proof is completed.

Appendix B. Proof of Theorem 4.2

In order to prove the Theorem 4.2, firstly give a Lemma.
Lemma 1[21] : For all vectors **a** and **b**, matrices N、X、Y、Z with appropriate dimensions，if $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$, then

$$-2a^T Nb \leq \inf_{X,Y,Z} \left\{ \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\},$$

where, X and Z are symmetrical positive matrices.

Define a delay-dependent Lyapunov function for the system (18) as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{1B}$$

where $V_1(t) = \tilde{x}^T(t)P\tilde{x}(t)$, $V_2(t) = \int_{t-\bar{\tau}}^t \tilde{x}^T(\tau)S\tilde{x}(\tau)d\tau$,

$$V_3(t) = \int_{-\bar{\tau}}^0 \int_{t+\beta}^t \dot{\tilde{x}}^T(\alpha)Z\dot{\tilde{x}}(\alpha)d\alpha d\beta.$$

And P , S , Z are symmetric positive definite weighting matrices.

The derivative of $V_1(t)$ along the trajectory of system (18) satisfies that

$$\dot{V}_{i1}(t) = 2\tilde{x}^T(t)P\ (\tilde{A}+\hat{A})\ \bar{x}_i(t) - 2\tilde{x}^T(t)P\tilde{A}\int_{t-\bar{\tau}}^t \dot{\tilde{x}}(\alpha)d\alpha + 2\tilde{x}^T(t)P\tilde{D}\tilde{\omega}(t)$$

By using Lemma 3.1, it is very easy to obtain

$$-2\tilde{x}^T(t)P\hat{A}\dot{\tilde{x}}(\alpha) \leq \tilde{x}^T(t)X\tilde{x}(t)$$

$$+ \dot{\tilde{x}}^T(\alpha)[Y^T - (P\tilde{A})^T]\tilde{x}(t) + \tilde{x}^T(t)(Y - P\hat{A})\dot{\tilde{x}}(\alpha) + \dot{\tilde{x}}^T(\alpha)Z\dot{\tilde{x}}(\alpha)$$

Then $\dot{V}_1(t) \leq \{\tilde{x}^T(t)(P\tilde{A} + \tilde{A}^T P + \bar{\tau}X + Y + Y^T\}\tilde{x}(t)$

$$-2\tilde{x}^T(t)(Y - P\tilde{A})\tilde{x}(t-\bar{\tau}) + \int_{t-\bar{\tau}}^t \dot{\tilde{x}}^T(\alpha)Z\dot{\tilde{x}}(\alpha)d\alpha + 2\tilde{x}^T(t)P\tilde{D}\tilde{\omega}(t)\}. \tag{2B}$$

The derivative of other Lyapunov functions along the trajectory of system (18) satisfy that

$$\dot{V}_2(t) = \tilde{x}^T(t)S\tilde{x}(t) - \tilde{x}^T(t-\bar{\tau})S\tilde{x}(t-\bar{\tau}) \tag{3B}$$

$$\dot{V}_3(t) = \bar{\tau}\dot{\tilde{x}}^T(t)Z\dot{\tilde{x}}(t) - \int_{t-\bar{\tau}}^t \dot{\tilde{x}}^T(\alpha)Z\dot{\tilde{x}}(\alpha)d\alpha \tag{4B}$$

Then, it can be obtained

$$\int_0^{t_f} \tilde{x}^T(t)\tilde{Q}\tilde{x}(t)dt = V(0) - V(t_f) + \int_0^{t_f} \{\tilde{x}^T(t)\tilde{Q}\tilde{x}(t) + \dot{V}(t)\}dt$$

$$\leq V(0) + \int_0^{t_f} \{\tilde{x}^T(t)(\tilde{Q} + S + P\tilde{A} + \tilde{A}^T P + \bar{\tau}X + Y + Y^T)\tilde{x}(t)$$

$$-2\widetilde{\boldsymbol{x}}^T(t)(\boldsymbol{Y}-\boldsymbol{P}\widehat{\boldsymbol{A}})\widetilde{\boldsymbol{x}}(t-\overline{\tau})+2\widetilde{\boldsymbol{x}}^T(t)\boldsymbol{P}\widetilde{\boldsymbol{D}}\widetilde{\boldsymbol{\omega}}(t)-\widetilde{\boldsymbol{x}}^T(t-\overline{\tau})\boldsymbol{S}\widetilde{\boldsymbol{x}}(t-\overline{\tau})$$

$$+\overline{\tau}\dot{\widetilde{\boldsymbol{x}}}^T(t)\boldsymbol{Z}\dot{\widetilde{\boldsymbol{x}}}(t)+\rho^2\widetilde{\boldsymbol{\omega}}^T(t)\widetilde{\boldsymbol{\omega}}(t)-\rho^2\widetilde{\boldsymbol{\omega}}^T(t)\widetilde{\boldsymbol{\omega}}(t)\}dt$$

$$=\mathrm{V}(0)+\int_0^{t_f}\left\{\begin{bmatrix}\widetilde{\boldsymbol{x}}(t)\\\widetilde{\boldsymbol{x}}(t-\overline{\tau})\\\widetilde{\boldsymbol{\omega}}(t)\end{bmatrix}^T\begin{bmatrix}\Theta & \boldsymbol{P}\widehat{\boldsymbol{A}}-\boldsymbol{Y} & \boldsymbol{P}\widetilde{\boldsymbol{D}}\\ * & -\boldsymbol{S} & 0\\ * & * & -\rho^2\end{bmatrix}\right.$$

$$+\overline{\tau}\begin{bmatrix}\widetilde{\boldsymbol{A}}\\\widehat{\boldsymbol{A}}\\\widetilde{\boldsymbol{D}}\end{bmatrix}^T\boldsymbol{Z}\begin{bmatrix}\widetilde{\boldsymbol{A}}\\\widehat{\boldsymbol{A}}\\\widetilde{\boldsymbol{D}}\end{bmatrix}\begin{bmatrix}\widetilde{\boldsymbol{x}}(t)\\\widetilde{\boldsymbol{x}}(t-\overline{\tau})\\\widetilde{\boldsymbol{\omega}}(t)\end{bmatrix}+\rho^2\widetilde{\boldsymbol{\omega}}^T(t)\widetilde{\boldsymbol{\omega}}(t)\}dt$$

According to the above inequality, by using Schur complement, it is easy to obtain that, if the inequality (20) hold, the system (18) is asymptotically stable with the H$_\infty$ performance in (19). This completes the proof.

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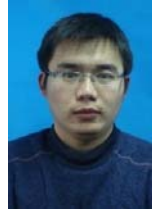
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