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Abstract— In this paper, a mathematical model for calculating the energy yields of offshore wind farm with mixed types of wind turbines is proposed. The Jensen model is selected as the base and developed to a three dimension wake model to estimate the energy yields. Since the wind turbines are with different hub heights, the wind shear effect is also taken into consideration. The results show that the proposed wake model is effective in calculating the wind speed deficit. The calculation framework is applicable for energy yields calculation in offshore wind farms.

Index Terms—different hub heights; shear effect; calculation framework; energy yields; wake model.

Nomenclature

C_{opt}	Power coefficient at λ_{opt}
d_{ij}	Distance from O_i to O_j
E_{tol}	Total energy yields
h_{ij}	Length of diagonal line in blue quadrangle
$Hub(i, j)$	Hub height matrix
k	Decay constant
L_{ij}	Distance from the center of upstream WT to downstream WT's center
$M(i, j)$	Element of matrix M at row I, column j
N_col	Total number of WTs in a column
N_row	Total number of WTs in a row
O_i	Center of the downstream WT
O_j	Center of the wake that developed from the upstream WT
$P_{m,ij}$	Mechanical power generated by WT at row I, column j
$P_{tol,t}$	Wind farm power production within the corresponding sample time interval t
R	Rotor radius
R_i	Radius of the downstream WT's rotor
R_j	Radius of the wake that generated from the upstream WT rotor
R_0	Radius of the upstream WT's rotor
$R(x)$	Generated wake radius at x distance along the wind direction
S_i	Fan shaped area of the sweeping area that in downstream WT rotor
S_j	Fan shaped area of the wake area
S_0	Sweeping area of WT's rotor with radius R_0
S_{ol}	Blue area in Figure 3(b) which shows the wake effect region of downstream WT
S_q	Blue quadrangle area in Figure 3
V_{ij}	Wind speed deficit generated by the WT at i^{th} row, j^{th} column of wind farm
$V_{ij}(\alpha, V_{0,ij})$	Wind speed of the upstream wind turbine (WT) when the inflow wind direction angle is α and velocity is $V_{0,ij}$
V_{nm}	Wind velocity at WT at row n, column m.
$V_{nm}(\alpha, V_{0,ij})$	Wind speed of the upstream WT when free wind direction angle is α and velocity is $V_{0,ij}$
$V_{0,ij}$	Wind velocity at the blade of WT at i^{th} row, j^{th} column of wind farm
x_i, y_i	Position of the downstream WT in coordinate system
z_0	Surface roughness
z_{ref}	Reference height for the measured wind speed
z_{ij}	Hub height of WT at row i, column j
λ_{opt}	Optimal tip speed ratio for the pitch angle β', at which the power coefficient will be maximum
β'	Pitch angle
γ	Chord angle corresponding to S_j
μ	Chord angle corresponding to S_i
ρ	Air density, $1.225 kg/m^3$ in standard condition

I. INTRODUCTION

The WTs extracts the power from the wind which incurs the wind speed reduction and turbulence increase at downstream WT. The physic change of speed and turbulence for the wind is called wake effect [1]. With the development of the capacity of the wind farm, the wake losses estimation becomes particularly evident. Because the overestimation of energy yields means a higher voltage level selection of electrical equipment and higher capacity of cables are required, this will induce the waste of investment on components' redundancy. In addition, the wind farm control strategy and operating reserve will be influenced as well [2], [3]. Presently, there are three wake models that are widely used in solving the Wind Farm Optimization Problem

(WFOP) as: Jensen model, Ainslie model and G.C. Larsen model [4]. Based on momentum conservation theory, Jensen proposed a wake model which assumes a linear expansion of the wake after the upstream WT in 1983. After that, several wake models are proposed for the wake calculation [5]. In [6], a method to calculate the wake losses by Jensen model is proposed while the Larsen eddy viscosity model is specified in [7]. Besides using analytical model to predict the energy yields of the wind farm, some works have been done on the wake effect simulation by using CFD (Computational Fluid Dynamics) which is a more precise way to estimate the wake losses [8]. The authors tried to accurately describe the wake effect by solving differential equations, however, the calculation time is quite long so that it is not an expected way for energy yields calculation in WFOP.

In this paper, a 3D wake model which considers the wake losses within an offshore wind farm with different hub height WTs is proposed. The proposed model is used for the energy yields calculation of two reference wind farm and the results show that the proposed method is an effective and efficient way for regular and irregular wind farm energy yields calculation.

The analytical equations for the wake model are specified in section II, the calculation framework is presented in Section III. The FINO3 reference wind farm is chosen as the study case to demonstrate the proposed method in Section IV. Finally, conclusions and future work are given In Section V.

II. WIND FARM MODEL

In this section, the Jensen wake model is firstly introduced. Based on which, the wake model which concerns the total wake effects from different height WTs are proposed. The energy model is presented at last.

A. Jensen Model

In this simulation, the Jensen wake model is adopted as the basic wake model to analyze the wake effect for its simplicity. The formula for Jensen single wake model is [9]:

$$V_{ij} = V_0 - V_0 \left(1 - \sqrt{1 - C_{t,ij}}\right) \left(\frac{R_i}{R_j(x)}\right)^2 \quad (1)$$

$$R_j(x) = R_i + kx \quad (2)$$

The recommended value of k is 0.04 for offshore environment which is suitable for a free wind condition (turbulence-free, that is to say not affected by any upstream turbine) [10].

B. Multiple Wakes

Within the wind farm, there is a probability that one downstream WT would be in the affected region of wake that generated by several upstream WTs. The problem has been solved by sum of squares of velocity deficits method [4]. As a consequence, the wind velocity at WT in row n, column m can be expressed as:

$$V_{nm} = V_0 \left[1 - \sqrt{\sum_{i=1}^{N\_row} \sum_{j=1}^{N\_col} \left[\left[1 - \left(\frac{V_{ij}}{V_0}\right)\right]\right]^2}\right] \quad (3)$$

C. Shear Effect

When the height is above 1 km, atmosphere is hardly influenced by the friction against the ground. However, in the lower layers, wind speed increases as the height of air goes up. This is called wind shear effect [10]. So if the height of some WTs is different, this effect should be also incorporated. Then, the wind speed can be rewritten as:

$$V_{0,ij} = V_{ref} \frac{\ln\left(\frac{z_{ij}}{h_0}\right)}{\ln\left(\frac{z_{ref}}{z_0}\right)} \quad (4)$$

D. Wake Model for Mixed WT Offshore Wind Farm

In this work, it is assumed that there are two types of WTs with different heights exist in one offshore wind farm. The wake model for this wind farm should take shear effect into consideration. The effected wake area that contributes to the wind speed deficit when two WTs are in a line is illustrated in Figure 1.
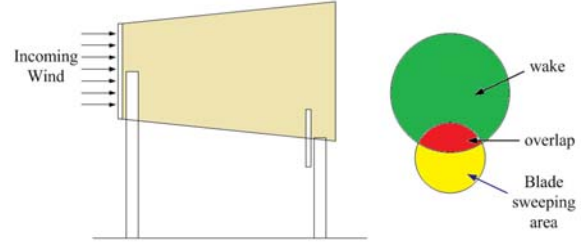

Figure 1. Illustration of wake overlpping with two wind turbines in a line.

As can be seen in Figure 1, the downstream WT are partial within the wake that generated by the upstream WT due to the hub height difference. If two WTs are not in a line, then four conditions should be considered as shown in Figure 2.
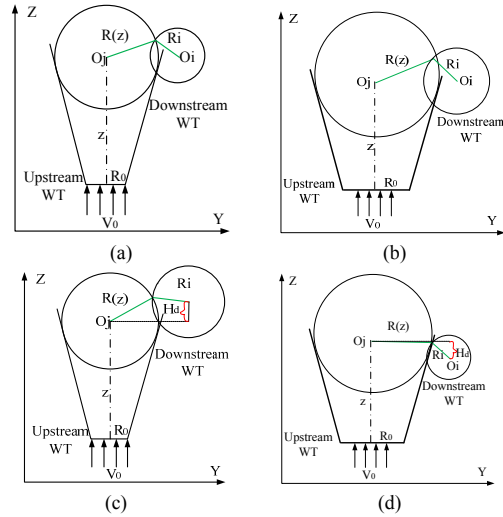

(a) (b)

(c) (d)

Figure 2. Wake model with mixed WT wind farm in y-z coordinate. (a) Upstream and downstream WTs are both WT type 1. (b) Upstream WT is WT type 1 and downstream WT is WT type 2. (c) Upstream WT is WT type 1 and downstream WT is WT type 2. (d) Upstream and downstream WTs are both WT type 2.

The affected wake area in four conditions is shown in y-z coordinate in Figure 2 (a) and (d) are the cases when upstream and downstream WT are in the same type. So the circle centers are in the same height. If the WTs are with different height, then the affected wake area will be reduced because of the height difference, H_d, as shown in Figure 2. (b) and (c). In this model, the wind is considered to be existed in 4 quadrants. In each quadrant two cases are required to be specified as shown in Figure 3. In which, the red line is the distance from the center of the upstream WT to downstream WT. The green area, denoted as S_{ol} is the overlapped area. The blue quadrangle area is denoted as S_q. If all the WTs are with same height, then a 2 dimension (2D) wake model will be used to wake losses estimation while the 3D wake model is the updating version by taking the hub height difference's impact on the wake affected area into consideration.
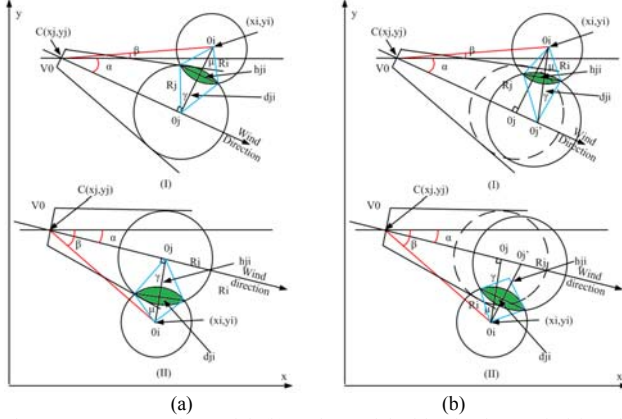

Figure 3. (a) 2 D wake model. (b) Wake model with mixed WT wind farm in x-y coordinate.

The 2D wake model is shown in Figure. 3 (a) while (b) indicates the proposed model. A series of analytical equations for wake velocity calculation of 3D wake model in case (I) can be derived as:

$$L_{ij} = \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2} \quad (5)$$

$$L_d = \overline{O_j O_j'} = H_b - H_s \quad (6)$$

$$d_{ij} = \sqrt{\left(L_{ij}\left|\sin(\alpha+\beta)\right|\right)^2 + \left(L_d\right)^2} \quad (7)$$

$$R_j = R_i + k L_{ij}\left|\cos(\alpha+\beta)\right| \quad (8)$$

$$\gamma = 2\cos^{-1}\frac{R_j^2 + d_{ij}^2 - R_i^2}{2R_j d_{ij}} \quad (9)$$

$$\mu = 2\cos^{-1}\frac{R_i^2 + d_{ij}^2 - R_j^2}{2R_i d_{ij}} \quad (10)$$

$$h_{ij} = 2R_i\left|\sin(\mu/2)\right| \quad (11)$$

$$S_0 = \pi R_i^2 \quad (12)$$

$$S_j = \frac{\gamma(R_j)^2}{2} \quad (13)$$

$$S_i = \frac{\mu R_i^2}{2} \quad (14)$$

$$S_q = 0.5 h_{ij} d_{ij} \quad (15)$$

$$S_{ol} = S_j + S_i - S_q \quad (16)$$

Then the wind velocity at j^{th} WT can be rewritten as:

$$V_{ij}(\alpha, V_{0,ij}) = V_{0,ij}[1 - \left(1 - \sqrt{1 - C_{t,ij}}\right)\left(\frac{R_0}{R_j}\right)^2 \left(\frac{S_{ol}}{S_0}\right)] \quad (17)$$

In case (II), the analytical equations is merely modified by changing all (6) and (7) into (β-α) while keeping all other terms the same.

The model proposed above is valid when the wake and the rotor sweeping area are intersected. The general principle of intersection judgement is shown in Figure 4.
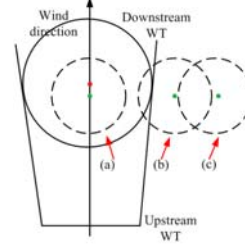

Figure 4. Three cases in wake losses calculation

The dotted circle represents the location of the downwind WT. The red dot and green dot show the circle centers for the generated wake at downstream WT and downstream WT itself. The wake effect will be receded gradually if the downwind WT is moving from position (a) to (c). The specifications of three cases are summarized in Table I as follow:

TABLE I.   JUDGEMENT SPECIFICATIONS

Case	Category	Condition	Analytical equations
(a)	full wake effect	$0 \leq d_{ji} \leq R_j - R_i$	(1) - (4)
(b)	partial wake effect	$R_j - R_i \leq d_{ji} \leq R_j + R_i$	(5) - (17)
(c)	non-wake effect	$d_{ji} \geq R_j + R_i$	$V_j = V_0$

E. Energy model

The power produced by WT can be calculated using the following equations [11], [12]:

$$P_{m,ij} = 0.5 C_{p,opt}(\beta', \lambda_{opt})\rho\pi R^2 V_{ij}^3 \quad (18)$$

In the simulation, the power production of each WT is found by assuming a maximum power point tracking (MPPT) control strategy [13]. Hence, the total power production that generated by the WTs at row i, column j can be written as:

$$P_{tol} = \sum_{j=1}^{N\_col}\sum_{i=1}^{N\_row} P_{m,爆} \quad (19)$$

III. BINARY MATRIX METHOD FOR IRREGULAR WIND FARM ENERGY CALCULATION

The energy yields calculation for irregular wind farm is difficult since there is no explicit rule to define the distance between the WTs. The problem is solved by introducing a binary matrix, M(i, j) as shown in Figure 5.
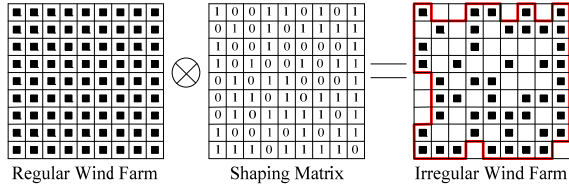
Figure 5.   Binary matrix method

Regular Wind Farm   Shaping Matrix   Irregular Wind Farm

The black solid square in Figure 5 represents the WT. Number 1 means there is a WT in this position while 0 means the position is empty. By using the binary matrix, the original full occupied wind farm can be shaped into an irregular wind farm. Then, the wake speed as well as the energy yields of wind farm can be calculated as follow:

$$V_{nm}(\alpha, V_{0,ij}) = V_{0,ij}[1-$$
$$\sqrt{\sum_{i=1}^{N\_row} \sum_{j=1}^{m} [1-M(i,爆)\left(\frac{V_{ij}}{V_{0,ij}}\right)\left(\frac{S_{ol,ij}}{S_0}\right)^2]} \quad (20)$$

$$E_{tol} = \sum_{t=1}^{TE} P_{tol,t} T \quad (21)$$

In order to evaluate the wake interaction between WTs with different hub height, a hub height matrix, $Hub(i, j)$, is defined according to $M(i, j)$. The positions which the type 2 WTs are in will be indicated as number '10' instead of '1' in $M(i, j)$. Then the original binary matrix will be changed into a '0-1-10' matrix while 0 means no WT in this position, 1 means there is a type 1 WT in this position and 10 means there is a type 2 WT in this position.

A. Calculation framework

The energy yields calculation for irregular wind farm should be easier solved with the binary matrix method as mentioned above. The calculation framework can be seen in Figure 6.
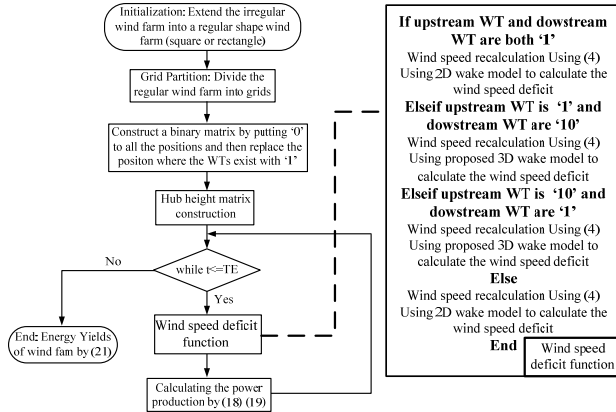

Figure 6.   Calculation framework

Firstly, the binary matrix will be used to help identify the distance between each pair of WT by partitioning the wind farm into small grids. Each grid could have one WT or not. Then the hub height matrix will help decide which condition (as shown in Figure 2) should be considered at this moment. If both turbines are with same type then 2D wake model will be used to estimate the wake losses otherwise the energy yields of the wind farm will be calculated base on 3D wake model.

IV.   CASE STUDY

The simulation is implemented on the platform of Matlab R2013a. Two study cases are adopted to verify the feasibility of the proposed method.

A. Case I

FINO3 reference wind farm is sited 80km west of German island of Sylt. In the first case, the wind farm layout is assumed to be as shown in Figure 7 [14].
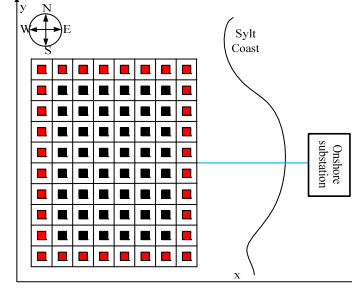

Figure 7.   Case I Wind Farm Layout

Two types of WTs (Vestas V90-2.0 [15] and DTU 10MW reference WT [16]) are considered as the reference WTs in this paper. The distance between each pair of WT is 630m (7 rotor diameter of 2MW WT, 7D). As shown in Figure 7, the red squares show the positions of 10 MW WT while black squares indicate the 2 MW WT.

The input wind velocity and direction distribution for the simulation are illustrated in Figure 8. The input time series wind speed for the calculation is obtained from the Norwegian Meteorological Institute [14]. Following the calculation framework, the energy yields for this case are obtained as in Table II.
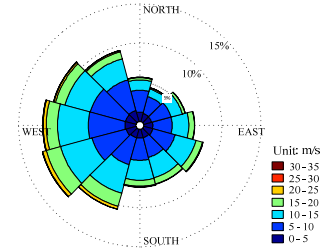

Figure 8.   Wind Rose of FINO3 [17]

TABLE II.   SIMULATION RESULTS

Name	Mixed WT farm	10MW WT farm	2MW WT farm
Wind farm capacity	416 MW	800 MW	160 MW
Enegy yields	1813.4 GWh	3393.29 GWh	704.97 GWh
Capacity Factor	49.76%	48.42%	50.30%
Energy yields without wake effect	2220.3 GWh	4386.12 GWh	776.38 GWh
Capacity Factor (no wakes)	60.93%	62.59%	55.39%
Wake losses	18.33%	22.64%	9.20%
Simulation time	964seconds	959seconds	960seconds

The program is performed on a computer which is an Intel(R) Core(TM) i7-4800MQ CPU @ 2.70 GHz processor with 8 GB RAM. The wake losses take up to 18.33% of total energy extracted from the wind. The energy yields of this wind farm with mixed WT (Mixed WT farm) was compared with the results obtained from 10MW WT farm (wind farm composed by 10MW WTs) and 2MW WT farm (composed by 2MW WTs). It can be seen that the wake losses in Mixed WT farm and 10MW WT farm is relatively higher than 2MW WT farm. This is because that the designed reference wind farm is with a smaller separation between each pair of WT, which makes the wake effect more obvious when the bigger size WTs are adopted.

B. Case II

In this case, the reference wind farm layout is assumed to be elliptic as shown in Figure 9. The red stars show the 10MW WT locations while the blue stars shows the locations of 2MW WT locations.
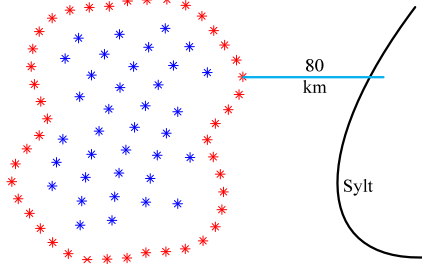

Figure 9.   Case II Wind Farm Layout

Based on the same wind input as illustrated in Figure 8, the energy yields considering either wake effect or not are listed in Table III. In this case, the wake effect reduced the energy production to 15.95%.

TABLE III.   SIMULATION RESULTS

Name	Mixed WT farm	10MW WT farm	2MW WT farm
Wind farm capacity	512 MW	800	160
Enegy yields	2321.3 GWh	3870.68 GWh	684.57 GWh
Capacity Factor	51.76%	55.23%	48.84%
Energy yields without wake effect	2761.7 GWh	4386.12 GWh	776.38 GWh
Capacity Factor (no wakes)	61.58%	62.59%	55.39%
Wake losses	15.95%	11.75%	11.83%
Simulation time	1782s	1773	1765

V.   CONCLUSIONS AND FUTURE WORK

In this paper, a new wake model which is applicable of calculating the wind speed in the wakes generated by WTs with different hub heights is proposed. The shear effect is considered to estimate the wind speed difference in different height and incorporated into the Jensen model so that a 3D wake model can be generated to evaluate the wind speed deficit in a wind farm with different hub height WTs. The studied cases demonstrate that it is an effective way to calculate any shape wind farm energy yields considering wake effect. In the future, the proposed model may be used for layout optimization work of the wind farm with different hub height and power curve WTs to build up a more cost-effective wind farm.

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