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Thoft-Christensen, Palle

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CHAPTER 6

APPLICATION OF OPTIMIZATION TECHNIQUES FROM THE USER'S POINT OF VIEW¹

P. Thoft-Christensen, Aalborg University, Denmark

Extended abstract

The existence of effective optimization methods is well known, but it is also a fact that such methods can be very difficult to use for scientists or engineers, who are not experts on optimization theory. Limitations of the following types can be difficult to handle in choosing a suitable method:

- a. The algorithms are usually only tested on unimodal functions.
- b. Convergence considerations may only be valid for quadratic functions.
- c. Only test functions with a few variables are used.
- d. Often the algorithms are only tested on unconstrained problems.

The user may conclude that it is more convenient to use a primitive method and not care about computer-time. It is much more important that the quality of the solution is high.

A recent stochastic strategy by Thoft-Christensen and Hartmann [1], [2] based on a combination of local and global steps is believed to have some advantages with regard to the problems mentioned above.

In this presentation a very brief discussion of this situation will be given. Let us therefore try to look at optimization methods in the same way as a user, who for the first time is considering the use of optimization techniques. So we assume that we are scientists or engineers having some kind of a project leading to an optimization situation. How can we attack such a problem?

It is first of all important to remember that optimization is not the main problem, but some kind of tool to solve a perhaps minor part of the problem. To make the situation a little more explicit we can assume that we want to find minimum for a function $f: R^n \rightarrow R$ with a number of equality and inequality constraints. The mathematical formulation will then be:

¹ Symposium on Applied Mathematics. Gothenburg, Sweden, April 1976.

Minimize $f(\bar{x})$, $\bar{x} \in R^n$

with the constraints

$$h_j(\bar{x}) = 0, \quad j = 1, 2, \dots, m$$

$$g_j(\bar{x}) \leq 0, \quad j = 1, 2, \dots, p$$

Let us further assume that we have a rather good knowledge of mathematics. With such a background we will probably go to the library and get some books and papers on optimization methods. Having looked closer at one of the recent methods we will be very pleased to see that a considerable amount of mathematics is used. According to the introduction in the chosen paper the method can be used on our problem, and in the conclusion we are told:

“The method is clearly superior to other methods, it has good convergence properties and the use of computer-time is reasonable”.

Now we are very careful persons, so we read the paper critically and then a lot of new problems and questions arise.

a. *“The algorithm is only tested on unimodal functions”.*

Is my function unimodal? Will the algorithm only disclose a local optimum and not the global optimum?

b. *“Convergence considerations are only valid for quadratic function”.*

Is my function quadratic? How good is it to approximate my function with a quadratic function?

c. *“Only test functions with 1, 2, or 3 variables are investigated”.*

Is it possible to use the same algorithm on problems with more than three variables? Is it possible to extend the conclusions to functions with more than three variables?

d. *“The algorithm is only tested on unconstrained problems”.*

Is it possible to modify the algorithm so that it can be used on constrained problems?

As a result of such questions it may be very difficult for an engineer who is not an expert on optimization methods to choose a good method. Further he will often be faced with one more serious problem namely that his computer centre does not have the right method in its programming library.

After these critical remarks it is fair to stress now that actually several very good optimization methods exist, but they are usually not very easy to use. Further you will most certainly prefer to use a method you can understand in details and feel familiar with. So you may conclude that you will try to find the minimum by using a less sophisticated technique. In the method described in the literature much work is used to reduce computer time resulting in limitations of the same type as shown above. But the steadily increasing speed of the digital computers makes it perhaps not so important to think of computer-time. If you use a programme only once a year or so, you can afford to be very large with regard to computer-time, it is much more important for you to be sure that the quality of the solution is high.

It is outside the scope of this presentation to give particulars of optimization methods. Only a recent strategy based on random search will be briefly presented. The

strategy is very simple to understand and easy to programme, so it may be of interest for a user in the situation mentioned above.

The search for optimization is in many respects equivalent to search for new scientific discoveries. The famous mathematician Hadamard has illustrated a scientific method used by several scientists in the following way:

“It is well known that good hunting cartridges are those which have a proper scattering. If this scattering is too wide, it is useless to aim, but if it is too narrow, you have too many chances to miss your game by a line”.

The same thing happens in optimization. You need a method, which at the same time has a wide and a narrow scattering. A wide scattering is essential to secure the global minimum, and a narrow scattering is essential to get a good convergence.

The strategy by Thoft-Christensen & Hartmann [1], [2] is based on a combination of so-called local and global steps. The intention with the local steps is to obtain a high degree of convergence to the nearest minimum not depending on whether it is a global or a local minimum. So the step-sizes for the local steps are rather small. On the contrary the global steps are not limited in size as their only purpose is to escape from a local minimum by making sufficiently large steps.

First a starting point is chosen at random in the feasible region. Then some trial steps are taken with different step-sizes chosen in an appropriate way and with a directional adaptation so that the direction of the last successful step is given a preference. When a prescribed number of successful steps are carried out, a global trial step follows. The intention with the global steps is to be able to reach any point in the feasible region, so no information on any “good” direction for global steps can be given in general. The step-size is chosen at random in a pre-determined interval.

In this way a prescribed number of global steps are made until an improvement may happen. In such a case the new point is then used as the starting point for local steps. Otherwise further local trial steps are made from the original point, but now with new directions and sizes. The procedure described above continues until a prescribed number of successful steps have occurred or until no further improvements by local steps can be found.

This method has been used successfully on multi modal functions with up to 15 variables. Further details about the experimental results are given in the papers [1], [2], and [3]. Based on the main idea behind the strategy, namely the local and global steps, a lot of different methods can be developed by changing the way in which the local and global steps are chosen.

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