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## CHAPTER 10

### PROBABILISTIC SIMULATION OF OFFSHORE COLLISIONS<sup>1</sup>

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#### Abstract

Collisions between tankers and offshore structures may result in serious problems such as oil pollution. It is therefore of great interest to estimate the probability of a collision. In this paper this is done by means of a simulation model using the simulation programme GASP IV. The new method presented here is based on a statistical prediction of the ship path after some kind of critical failure affecting the maneuverability of the tanker has taken place.

#### 1. INTRODUCTION

Due to the risk of a serious catastrophe involving losses of human life, oil out slip etc. it is of very great interest to be able to evaluate the probability of collision between a tanker and an off- shore platform. If a reasonably accurate method to do this can be developed, it may be possible to optimize the relative arrangement of platforms and buoys in an oil field.

At Det Norske Veritas (DTN) a research project on this matter is in progress and a number of progress reports has been published, Amdahl, Hysing & Furnes [1]. The author has worked as consultant on this project.

Risk of collision exists in several situations, but only tankers for offshore loading in an oilfield will be considered here. Even with such a limitation it is of course an almost impossible task to evaluate the total collision probability taking into account all kinds of human and technical failures. However to get some information on the probability of collisions it is reasonable to consider initially some idealized situations. When some experience has been gained from the idealized model one can then proceed with a more sophisticated model later.

In this chapter the mathematical modelling of the physical system will be described in some details and less effort will be put on the numerical simulations.

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<sup>1</sup> Appl.Math. Modelling, Vol.4, 1980, pp. 166-170.

In section 2 an extremely simple model is discussed as an introduction to the subject. The physical system and its mathematical modelling are presented in sections 3 and 4, respectively. The numerical simulation is described in section 5, and an example of its use in section 6. In section 7 some concluding remarks concerning improvements are given.

## 2. A SIMPLE MODEL

The first attempt to use simulation as described in this chapter was outlined by Thoft-Christensen [5] in July 1977. In [5] an extremely simple model is treated with the purpose to test whether the main Idea behind the method is promising. If so, more extensive models can be formulated.

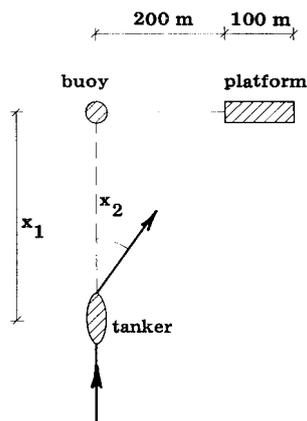


Figure 1. Test example for the simple model.

A tanker is assumed to move straight toward a loading buoy as shown in figure 1. The buoy is placed 200 m from the platform and the moving direction of the tanker is perpendicular to the line connecting the buoy and the platform. Further it is assumed that steering control of the tanker is lost in a distance  $x_1$  (m) from the buoy in such a way that the tanker then follows a straight path under the angle  $x_2$  (radians) with the original direction. In this model only two random variables  $X_1$  and  $X_2$  are involved. It depends now on the values  $x_1$  and  $x_2$  whether the tanker will hit the platform or not.

In this case it is easy to do a simulation study when probability density functions for  $X_1$  and  $X_2$  are known. The simulation involves only generation of values of  $x_1$  and  $x_2$  according to the chosen distributions. By simply counting the number of times such a set  $(x_1, x_2)$  results in collision, the probability of collision can be estimated.

Such a simulation has been performed on an HP67 pocket calculator on the basis of the following two density functions:

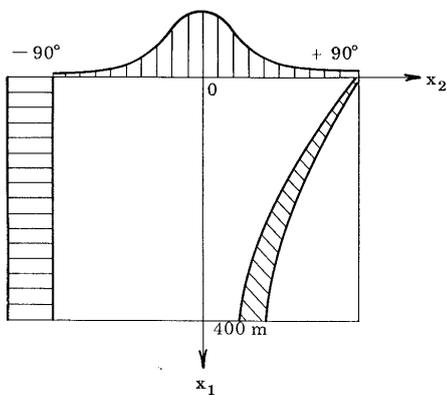


Figure 2. Probabilistic simulation.

$$p_{X_1}(x_1) = \begin{cases} \frac{1}{400} & \text{for } x_1 \in [0; 400 \text{ m}] \\ 0 & \text{for } x_1 \notin [0; 400 \text{ m}] \end{cases} \quad (1)$$

$$p_{X_2}(x_2) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{x_2}{\sigma}\right)^2\right) \quad (2)$$

where  $\sigma = 1/2$  radian. This can be illustrated as shown in figure 2, where the shaded area indicates the set of  $(x_1, x_2)$  resulting in collision. By using a random number generator a total number of 3256 simulations has been performed and 95 of them were in the shaded area. According to this the following probability of collision can be estimated

$$P(\text{collision}) = 95/3256 = 2.9 \% .$$

In this case numerical integration can of course be used as well.

### 3. THE PHYSICAL SYSTEM

It is obvious that the model in chapter 2 is much too simple to reflect the behaviour of the real physical system satisfactorily. On the other hand, the physical system is extremely complicated so that rather extensive limitations have to be made to get a usable model. In this chapter some of the more important aspects of the physical system will be discussed from a modelling point of view.

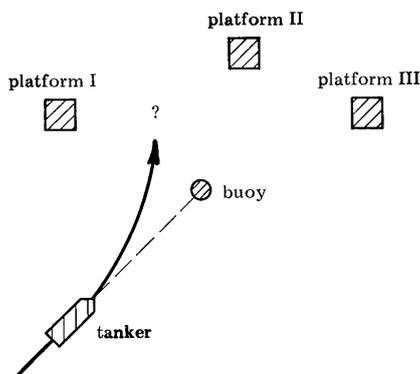


Figure 3. Outline of the physical problem.

The general outline of the physical problem is shown in figure 3 where a tanker is shown heading towards a buoy. Forces from waves, current and wind influence the movement of the tanker. Under normal conditions the tanker will approach the buoy with decreasing speed and after loading leave the buoy again. But it may be so that the normal procedure cannot be followed due to some kind of failure. Such a failure may be that propulsion is lost; the rudder is locked in a given position, etc. In this paper it is assumed that some kind of critical failure takes place some time during the heading of the buoy and that no effort is made from the crew to avoid

collision, i.e. it is assumed that the tanker after failure has taken place will continue in a path which will depend on a great number of factors. By overlooking the possibility of the crew to intervene it is reasonable to expect that the calculated probabilities of collision are too high. As mentioned in chapter 7 it is possible to extend the simulation model in such a way that intervention from the crew is taken into account.

It is clear that several factors will influence the path of the tanker after failure has taken place and they will thus be of importance for the estimate of the collision probability between the tanker and a given platform. First of all the speed of the tanker, its position and orientation at the time of failure are significant. But also very important is the rudder angle at failure. Rudder failure is assumed to occur in one of two different ways. One possibility is that the rudder locks in the instantaneous position. Another possibility is that it locks in the maximum rudder angle.

Also of great importance for the path of the tanker are of course the external forces acting on the ship. These external forces are hydrodynamic still water forces, wave forces, and wind forces.

The above-mentioned factors affecting the path of tankers can be described by 13 stochastic variables, namely:

- a. Position and orientation of ship (3 variables)
- b. Velocity components and yaw rate (3 variables)
- c. Rudder angle (1 variable)
- d. Current velocity and direction (2 variables)
- e. Wave height and direction (2 variables)
- f. Wind speed and direction (2 variables)

Some of these variables are correlated so that the number of independent variables can be strongly reduced. The wave height can be expressed by the wind speed and the wave direction can be put equal to the wind direction. Further it is assumed that the tanker moves towards the loading buoy into the wind within a deviation of  $\pm 15$  degrees, uniformly distributed. Next, only the distance to the platform is considered important and the velocity of the ship is assumed to depend only on the distance to the loading buoy. The current speed is assumed to be the mean tidal current speed.

In this way the number of uncorrelated stochastic variables has been reduced from 13 to 6, namely

- $X^1$  Distance between tanker and platform at time of failure,
- $X_2$  Direction of wind,
- $X_3$  Speed of wind
- $X_4$  Speed of current
- $X_5$  Rudder angle at failure
- $X_6$  Direction of current.

#### 4. MATHEMATICAL MODELLING

In the last chapter it is argued that the physical system can be described by 6 independent stochastic variables. In this chapter the necessary formula for estimating the probability of collision will be derived. Let  $F$  be the event that a critical failure takes place and let  $C$  be the probability of collision is

$$P(C) = P(C|F) P(F) \quad (3)$$

$P(F)$  is the probability that the critical failure occurs when the tanker is approaching the buoy. The magnitude of  $P(F)$  must be determined on the basis of experience. The order

of magnitude might be something like  $10^{-4}$ .

$P(C|F)$  is the conditional probability of collision. It is the purpose of this paper by simulation to estimate  $P(C|F)$ , which will be called the relative probability of collision.

By estimating  $P(C|F)$  for different arrangements of platforms relative to the buoy it is possible to choose the "best" arrangement if  $P(F)$  can be considered invariant in this respect. Only the six stochastic variables  $X_1, \dots, X_6$  are taken into account in the estimate of  $P(C|F)$ .

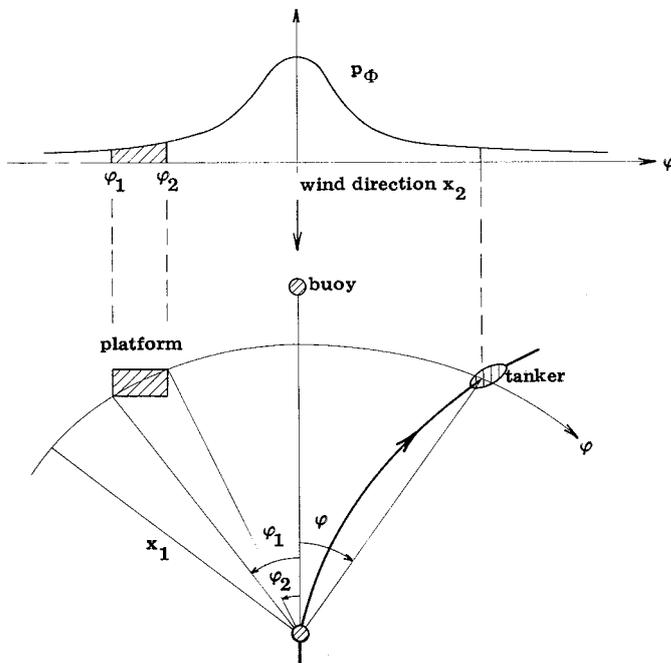


Figure 4. Definition of  $p_\phi$ .

In figure 4 the situation for a given wind direction  $x_2$  is that a critical failure has occurred at a distance  $x_1$  of an offshore platform. By simulation and numerical

integration the subsequent path of the tanker can then be determined and its intersection with the circle through the platform can be calculated. Let this intersection be determined by the angle  $\varphi$  as shown in figure 4, and let the density function for the corresponding stochastic variable  $\Phi$  be  $p_\Phi(\varphi)$ . For simulated values of the wind speed  $x_3$ , the speed of the current  $x_4$ , the rudder angle at failure  $x_5$ , the direction of the current  $x_6$ , the density function  $p_\Phi$  will depend of given values of  $x_1$  and  $x_2$ . Therefore, we will use the symbol  $p_\Phi(\varphi | x_1, x_2)$  for the density function.

For given values of the stochastic variables  $X_1$  and  $X_2$  the relative probability of collision is

$$P(C | F | x_1, x_2) = \int_{\varphi_1}^{\varphi_2} p_\Phi(\varphi | x_1, x_2) d\varphi. \quad (4)$$

Finally

$$P(C | F) = \int_{x_2=0}^{2\pi} \int_{x_1}^{\varphi_2} p_\Phi(\varphi | x_1, x_2) p_{X_1}(x_1 | x_2) p_{X_2}(x_2) d\varphi dx_1 dx_2 \quad (5)$$

In (5)  $p_{X_2}(x_2)$  is the density function for the wind direction  $X_2$ , and  $p_{X_1}(x_1 | x_2)$  is the density function for the failure distance  $X_1$  given the wind direction. Integration with respect to  $x_1$  is over a chosen interval.

The density function  $p_{X_2}(x_2)$  is supposed to be known for the site of the offshore platform. The density function  $p_{X_1}(x_1 | x_2)$  can e.g. be derived from the assumption that critical failure is uniformly distributed over time if the velocity profile of the tanker is known. The velocity of the tanker at the time of critical failure is determined by dividing the last part of the approach activity in equal time intervals and assigns to each interval a mean velocity (and a mean distance of the buoy) according to the chosen velocity profile.

To sum up for a given offshore installation the density functions  $p_{X_1}$  and  $p_{X_2}$  are known and  $p_\Phi(\varphi | x_1, x_2)$  is determined by simulation. Then the relative probability of collision can be calculated by numerical integration of the multi-integral in (5).

## 5. NUMERICAL SIMULATION

It should be clear from the previous chapters that the main problem in estimating the relative probability of collision is the determination of  $p_\Phi$  by simulation. The simulation study is concerned with the path of the tanker after critical failure has occurred. It is outside the purpose to go into details with the system of differential equations determining the path of the tanker (see e.g. Norrbin [3] and the reports [1]). But the quality of the simulation is of course closely connected with the precision of the prediction of the path of the tanker. Fortunately experience seems to indicate that such a prediction can be made with good confidence. Of great importance are of course the initial values of the state variables mentioned above, i.e. the values of the stochastic variables just after failure has taken place. These values must be generated randomly according to their respective distributions.

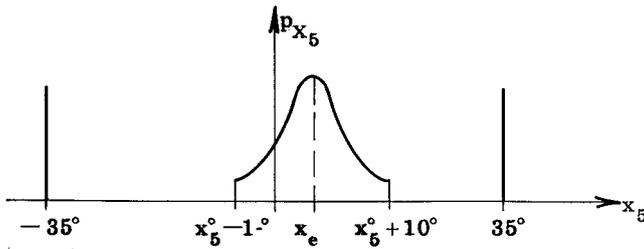


Figure 5. Density function for the rudder angle.

The stochastic variable  $X_5$  describing the rudder angle at the time of failure has been mentioned earlier in chapter 3. The density function  $p_{X_5}$  is shown in figure 5. It is of the mixed continuous-discrete type. In the interval  $[x_e - 10^\circ; x_e + 10^\circ]$ , where  $x_e$  is the equilibrium rudder angle the

density function follows a normal distribution. In some failure situations the rudder returns to the maximum rudder angle (e.g.  $\pm 35^\circ$ ), so that non-zero probability must be given to these values.

The speed of wind  $X_3$  and the direction of current  $X_6$  must be defined in accordance with measurements on the site of the offshore platform.

When a set of the above-mentioned parameters has been generated the ship path can be calculated by numerical integration of the equations of motion. For this purpose a general simulation programme such as GASP IV can be applied. In the applications shown in chapter 6 the GASP IV programme developed by Prietsker [4] was used. From the histograms obtained by the simulation it is possible by curve-fitting to determine density functions for the crossing angle  $\Phi$  for a number of values of  $X_1$ , that is for the crossing of circles with different radii, say  $r_1, \dots, r_m$  (see figure 6).

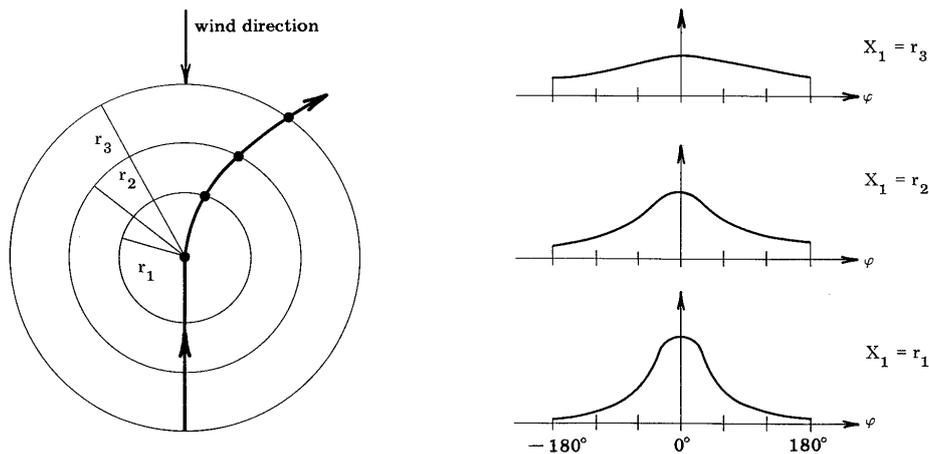


Figure 6. Determination of density functions  $p_{\Phi}$ .

For a number of initial values  $u_1, u_2, \dots, u_n$  for the velocity of the tanker at the time of critical failure a set of density functions  $p_{\Phi}$  can be produced as shown in figure 6. Combined with known distributions for the remaining variables they form the basic information needed for the computation of the relative probability of collision. Notice that each simulated ship path contributes to several but not all density functions. For immediate values of speed  $u$  and radius  $r$  density functions can be determined by interpolation.

A set of density functions connected with an initial value of the speed will be called the *density function rosette*. The idea of a density function rosette is very

illustrative because one can imagine that it follows the tanker continuously when the tanker is approaching the loading buoy.

Instead of considering only distinct values of  $X_1$  it is of course possible to divide the area around the tanker in some sub areas and by simulation connect some probability to each area. In this case the density function rosette is like a two-dimensional density function.

The idea of a density function rosette is of course not only usable in connection with an estimate of the probability of collision between a tanker and an offshore platform. It seems to be a useful concept also in investigations of the probability of grounding of e.g. tankers in narrow straits.

## 6. APPLICATIONS

The simulation technique outlined above has been used on a number of special situations. More detailed description of the results is given in the reports [1]. Only some results reported in a recent paper by Furnes & Amdahl [2] will be briefly presented here.

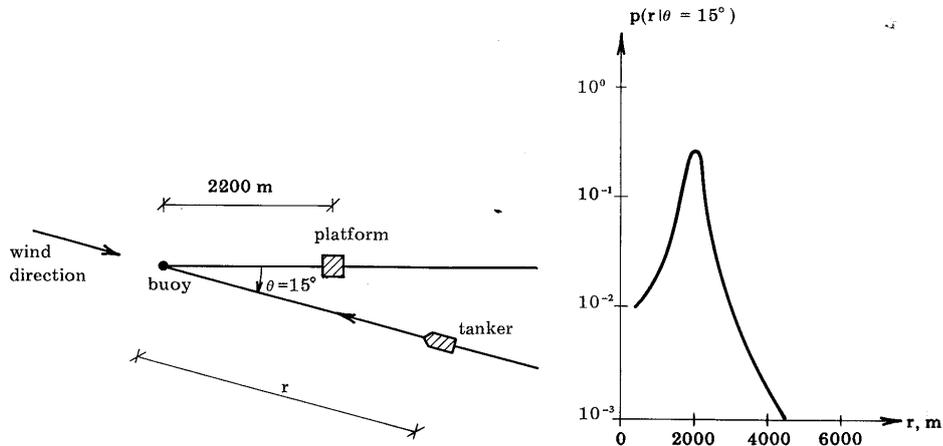


Figure 7. Collision probability for a given wind direction.

Consider the situation shown in figure 7, where a tanker is approaching a buoy on a course very close to a platform ( $\theta = 15^\circ$ ). In the figure the probability of collision  $p$  related to critical failure in a distance  $r$  from the buoy is shown. As one could expect the collision probability has a peak when the ship passes the platform. The curve is based on calculations carried out at a number of points each representative of a 10 minute interval.

For any wind direction  $\theta$  a similar curve can be determined. Based on such a set of curves it is possible simply by integration to compare the probability of collision  $p(\theta)$  for different directions. For the field geometry shown in figure 7 one gets e.g. the following values:

$\theta$	$p(\theta)$
$15^\circ$	$1.7 \times 10^{-2}$
$105^\circ$	$4.4 \times 10^{-4}$
$195^\circ$	$\approx 10^{-5}$

The tendency of these figures is as one can expect namely that a course of the tanker close to the platform is more dangerous than a course farther away. When figures as given in (6) are known for all wind directions  $\theta \in [0; 360^\circ]$  and if the long-term distribution of the wind is known for the site of the platform then the relative probability of collision  $P(C|F)$  can be computed. In the case presented above one gets (see [2])

$$P(C|F) = 6 \times 10^{-3} \quad (7)$$

if the so-called ‘‘Lerwick’’ statistics of the wind is used.

Although only the relative probability of collision can be determined by the method above the simulation technique used here seems to have some potentials. It is e.g. possible to find the most favorable position of a buoy relative to the platforms. It is to expect that the buoy should be placed on the leeward side of the platform. This expectation has been confirmed by the method as emphasized in Furnes & Amdahl [2].

## 7. DISCUSSIONS AND CONCLUSION

It is obvious that the simulation study presented in this paper can be improved in several ways. Improvements can be made in relation to the determination of the path of a tanker after critical failure has occurred, that is a better (more accurate) mathematical model for the movement of the ship could be of importance. But also from a probabilistic point of view improvements are possible. The main idea, namely the use of a density function rosette, does not limit the method to the technique used here. It has quite general potentials.

The object of the initial study has been to try to make as simple a model as possible reflecting the main characteristics of the problem. A more sophisticated model might give a more accurate result but also a more complicated model to use. An important improvement will be to make it possible for the crew to intervene after failure has taken place. Such an intervention (interactive simulation) is possible in connection with the simulation programme GASP IV.

The main advantages of a simple model as used here is that it is easy in all steps of the simulation to verify the calculations so that unreasonable results can be corrected before it is too late.

The applications made until now seem to indicate that the method have some advantageous potentials. Unfortunately it has not been possible to compare the method with similar methods because apparently no simulation study of collision between a tanker and a platform has been published earlier.

Compared with more traditional methods of estimating collision probabilities this method can also give information with regard to e.g. the velocity of the tanker at the moment of collision. This is because the course of the ship is followed in all details. The simulation method might therefore be of some help in defining loads on offshore platforms and thus improve the basis for a rational design.

## ACKNOWLEDGEMENT

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