Cost Based Droop Schemes for Economic Dispatch in Islanded Microgrids

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Abstract—In this paper, cost based droop schemes are proposed, to minimize the total active power generation cost in an islanded microgrid (MG), while the simplicity and decentralized nature of the droop control are retained. In cost based droop schemes, the incremental costs of distributed generators (DGs) are embedded into the droop schemes, where the incremental cost is a derivative of the DG cost function with respect to output power. In the steady state, DGs share a single common frequency, and cost based droop schemes equate incremental costs of DGs, thus minimizing the total active power generation cost, in terms of the equal incremental cost principle. Finally, simulation results in an islanded MG with high penetration of intermittent renewable energy sources are presented, to demonstrate the effectiveness, as well as plug and play capability of the cost based droop schemes.

Index Terms—Microgrid (MG), incremental cost, generation cost, cost based droop schemes, decentralized control.

I. INTRODUCTION

Facing the challenges arising from fossil fuel exploitation and associated pollution, people now turn their attention to the renewable energy resources [1]. Note that the microgrid (MG) is considered as a solution for flexible and reliable integration of renewable energy resources into the main grid. An MG is a cluster of distributed generators (DGs), storage systems, and loads, which is connected to the main grid at the point of common coupling, and it operates in either islanded mode or grid-connected mode, and it is able to perform seamless transitions between these two modes. In the grid-connected mode, the power shortfall can be compensated by the main grid, and the excess power in the MG can be absorbed by the main grid. When islanded, the power generation of DGs ought to be in balance with the total load demand in the MG [2].

Conventionally, the centralized control is implemented for islanded MGs, where the control decisions are performed based on global information, requiring an extensive communication network between the MG central controller (MGCC) and DGs. On the contrary, in distributed control, the decision making is performed based on local information, and it requires no MGCC and extensive communication network. Moreover, it assumes that the interactions among DGs are negligible in decentralized control, and the requirement for communication network is obviated. Therefore, the distributed and decentralized control methods are more practical for a large sized MG than centralized control, due to their flexibilities in plug and play, as well as sparse communication channels, etc [3]–[5].

Note that the distributed techniques for the secondary control [6]–[11] and proportional power sharing [12]–[17] have been extensively studied. However, the stability of voltage and frequency, as well as proportional power sharing might not be comprehensive enough for the MG, where both the generation costs and emission penalties are different for different types of DGs. Therefore, the economic dispatch is desired, which aims at distributing generation responsibility among DGs in an economical manner, while satisfying both equality and inequality constraints in the MG [18]. In the last few years, the efforts toward the distributed economic dispatch have been performed. Based on JADE platform and multi-agent system (MAS), Zhang et al. [19] proposed a distributed gradient based algorithm, to minimize the total active power generation cost online. Later, further work was carried out to integrate the economic dispatch and demand response management together [20]. Additionally, considering the generation and ramping rate limits of DGs, an MAS based dynamic programming algorithm was developed, to minimize the total power generation cost [21].

On the other hand, the consensus algorithms have found their applications in economic dispatch. First, it is worth noting that the total generation cost of the MG is minimized so long as the incremental costs of DGs are equal, i.e., the equal incremental cost principle, where the incremental cost is a derivative of the DG cost function with respect to output power [22], [23]. Therefore, the incremental cost was chosen as the consensus variable, and a consensus algorithm was developed to drive incremental costs of DGs to a common value [24]. In [25], a consensus algorithm for equal incremental costs was formulated, which required a strongly connected communication for information exchanges. Additionally, the incremental welfare consensus algorithm with the consideration of responsive loads was proposed [26]. Further, a network consisting of cooperative dynamic agents
was formulated to solve the cost minimization problem in a distributed manner [27]. Considering the power losses and generation limits, a distributed solution for the cost minimization problem was provided in [23]. Moreover, the consensus algorithm with exponential convergence rate was explored in [28]. And Binetti et al. [29] focused on an economic dispatch model incorporating the transmission losses, power generation constraints and prohibited operating zones. Considering the impacts of intermittent DGs, a fully distributed consensus algorithm was proposed for social welfare maximization of both the users and energy providers [30]. Similar work was carried out in [31], where both the conventional thermal DGs and wind turbines were taken into consideration. Moreover, a fully distributed strategy integrating the frequency recovery and economic dispatch in one process was developed in [32].

As discussed above, the economic dispatch problem was effectively solved by consensus algorithms in distributed manners, and the consensus algorithms precluded the need for a centralized coordinator so that they were fully distributed. On the other hand, the consensus algorithms were implemented with asynchronous or synchronous communications. In other words, the units in the communication network updated and exchanged information with other units synchronously or asynchronously, which may degrade the convergence speed [21]. Moreover, the consensus algorithms were highly dependent on the communication network for information exchange, and the malfunction of communication links may possibly result in the invalid algorithms.

Therefore, the vulnerabilities of the consensus algorithms prompted the interest in cost based droop schemes, which required neither communication network nor interactions among DGs. Note that Xin et al. [33], [34] proposed a fully distributed control strategy for frequency-power and power-frequency droop type DGs, respectively, which allowed all DGs share loads according to their generation costs, however, the capability of plug and play was not demonstrated. With the consideration of generation costs, efficiencies, and emission penalties of DGs, Nutkani et al. [18], [35], [36] formulated nonlinear droop schemes to allow the least costly DG produce more power. Further work was carried out in [37], where the costlier DGs were turned OFF when the load was light, to achieve a reduction in no-load cost. Considering the lower accuracy and increased complexity of the nonlinear droop control, a linear droop scheme was proposed to achieve a reduction in generation cost [38]. Later, the constraints such as online power reserve, frequency, voltage, and power limits were taken into consideration [39].

By embedding the generation costs into the droop control, the cost based droop schemes discussed in [18], [35]–[39] allowed autonomous identification of the appropriate DGs for generation, in terms of the generation cost. Therefore, compared to traditional droop control, the cost based droop schemes demonstrated significant effectiveness in the reduction of the total generation cost of the MG. On the other hand, the cost based droop control schemes were incapable of keeping the total generation cost to a minimum, because the power outputs of the DGs were tuned according to their respective generation costs rather than the total generation cost of the MG.

To address these concerns, the alternative cost based droop schemes are developed in this paper, which are implemented by embedding the incremental costs into the droop control. The cost based droop schemes have the functionality for driving the incremental costs of DGs to a consensus value, therefore, the total active power generation cost of the MG is minimized, in terms of the equal incremental cost principle. Furthermore, the simplicity, the capability of plug and play, as well as the decentralized nature of the droop control are preserved. In practical applications, the plug and play capability allows the plug out operation of the faulted DG, while maintaining the optimal operation states of remaining DGs. Moreover, when the DG is plugged back to the MG after the repair and maintenance, the control schemes with plug and play capability are able quickly to drive the incremental costs of DGs to a consensus value, to minimize the total active power generation cost of the MG.

Compared to the existing methods, the salient features of the proposed cost based droop schemes are (i) they require no complicated mathematical modeling, and they are simple enough to be implemented, therefore making them more suitable for practical applications; (ii) they are fully decentralized and require no communication network among DGs, therefore, they offer increased reliability; (iii) they have the capability of plug and play, therefore, the plug out and plug in behaviors of DGs do not affect the performance of the control schemes. Finally, five cases are performed, and simulation results show that the total active power generation cost of the MG is minimized, while satisfying given constraints, in the presence of intermittent DGs and highly variable load demands.

The rest of the paper is organized as follows. The economic dispatch problem and the equal incremental cost principle are introduced in Section II. And in Section III, the structure and parameters of the islanded MG under study are introduced and listed, further, the cost based droop schemes are developed. Later, in Section IV, five cases are carried out to evaluate the performance of the cost based droop schemes, and then the simulation results are analyzed and discussed. Finally, Section V concludes the paper.

II. PROBLEM FORMULATION

This section introduces the economic dispatch problem for islanded MGs, which aims at minimizing the total active power generation cost. Further, the economic dispatch problem is solved by the Lagrangian method, and then the equal incremental cost principle is presented.

A. Economic Dispatch Problem

For the case that there are a total of \( n \) DGs in an islanded MG, the \( i \)th DG has a cost function \( C_i(P_i) \). And it is assumed that the photovoltaic (PV) and wind generation, as well as reactive power generation are free. Note that the transmission loss is small in the MG, which is about 3–5\% of the total load, therefore, the transmission loss \( P_{\text{loss}} \) can be approximately
represented by multiplying the total load demand \( P_D \) by 5\% [20], [30], [40].

Thereafter, with the aim of minimizing the total active power generation cost, while satisfying power balance and power generation constraints, the economic dispatch problem being considered in this paper can be expressed as

\[
\min \sum_{i=1}^{n} C_i(P_i),
\]

\[
\sum_{i=1}^{n} P_i = P_D + P_{\text{loss}} = (1 + 5\%) \cdot P_D,
\]

\[
P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}},
\]

where \( P_i \) is the active power output of DG, \( C_i(P_i) \) is the cost function of DG, \( P_D \) is the total active load demand in the MG, \( P_{\text{loss}} \) is the transmission loss, and \( P_i^{\text{min}}, P_i^{\text{max}} \) are lower and upper limits of active power generation of DG, respectively.

B. The Equal Incremental Cost Principle

The economic dispatch problem formulated in (1) can be solved by the Lagrangian method, and the corresponding Lagrangian function takes the following forms [22], [23],

\[
L(P_1, P_2, \cdots, P_n) = \sum_{i=1}^{n} C_i(P_i) + \lambda \cdot (P_D + P_{\text{loss}} - \sum_{i=1}^{n} P_i) + u_1 \cdot (P_i - P_i^{\text{max}}) + u_2 \cdot (P_i^{\text{min}} - P_i),
\]

where \( \lambda, u_1, \) and \( u_2 \) are Lagrange multipliers.

In terms of first order optimality conditions, we have

\[
\frac{\partial L}{\partial P_i} = \frac{\partial C_i(P_i)}{\partial P_i} - \lambda + u_1 - u_2 = 0,
\]

\[
\frac{\partial L}{\partial u_1} = P_D + P_{\text{loss}} - \sum_{i=1}^{n} P_i = 0,
\]

\[
\frac{\partial L}{\partial u_2} = P_i - P_i^{\text{max}} \leq 0,
\]

\[
\frac{\partial C_i(P_i)}{\partial P_i} \bigg|_{P_i = P_i^{\text{max}}} = \lambda_i,
\]

If \( P_i^{\text{min}} < P_i < P_i^{\text{max}} \), according to (9), (10) and (11), it yields

\[
\lambda = \frac{\partial C_i(P_i)}{\partial P_i} = \lambda_i,
\]

where \( \frac{\partial C_i(P_i)}{\partial P_i} = \lambda_i \) is the incremental cost of DG, and \( \lambda^* \) is the optimal incremental cost of DG.

Similarly, if \( P_i = P_i^{\text{max}} \), according to (5), (9), (10) and (11), it yields

\[
\begin{align*}
    u_1 & = u_2 = 0.
\end{align*}
\]

Applying conditions (12) to (5), we have

\[
\lambda = \frac{\partial C_i(P_i)}{\partial P_i} \bigg|_{P_i = P_i^{\text{max}}} = \lambda_i,
\]

And if \( P_i = P_i^{\text{min}} \), similar relationship between \( \lambda^* \) and \( P_i \) can be obtained as follows,

\[
\begin{align*}
    u_1 & = 0, \\
    u_2 & \geq 0, \\
    \lambda^* = \frac{\partial C_i(P_i)}{\partial P_i} \bigg|_{P_i = P_i^{\text{min}}} + u_1 \geq \frac{\partial C_i(P_i)}{\partial P_i} \bigg|_{P_i = P_i^{\text{max}}} = \lambda_i.
\end{align*}
\]

Finally, the solution to (1) can be obtained, which is the equal incremental cost principle, and it takes the following forms

\[
\begin{align*}
    \lambda_i & = \frac{\partial C_i(P_i)}{\partial P_i} \bigg|_{P_i = P_i^{\text{min}}} < P_i < P_i^{\text{max}}, \\
    \lambda_i & \geq \frac{\partial C_i(P_i)}{\partial P_i} \bigg|_{P_i = P_i^{\text{max}}},
\end{align*}
\]

The equal incremental cost principle in (16) denotes that the total active power generation cost of the MG is minimized, if only the equal incremental cost principle is satisfied. Moreover, in this case, the DGs that operate within power generation limits have the equal incremental cost of \( \frac{\partial C_i(P_i)}{\partial P_i} \bigg|_{P_i = P_i^{\text{min}}} \) and DGs have the incremental costs of \( \frac{\partial C_i(P_i)}{\partial P_i} \bigg|_{P_i = P_i^{\text{max}}} \) or \( \frac{\partial C_i(P_i)}{\partial P_i} \bigg|_{P_i = P_i^{\text{max}}} \), when operating on their upper or lower generation limits, respectively.

III. INCREMENTAL COST BASED DROOP SCHEMES

In this section, the structure and parameters of the islanded MG for test is introduced first. Next, how the nonlinear and linear cost based droop schemes are derived from the exponential and quadratic cost functions is investigated, respectively.

A. The Structure and Parameters of The Islanded MG

In this paper, an islanded MG shown in Fig. 1 is considered, which is established in MATLAB/Simulink. The specifications of loads and power ratings of intermittent DGs are listed in Fig. 1. Here, DG1 and DG3 are PVs, while DG6, DG8 and DG9 are permanent magnet synchronous generator (PMSG) based wind turbines, all of which operate in maximum power point tracking (MPPT) control mode. Additionally, dc-links of controllable DGs, DG2, DG4, DG5, DG7 and DG10 are modeled as constant dc voltage sources \( V_{dc} \), and we assume that the voltage variations of dc-links are well regulated. And controllable DGs operate in terms of the cost based droop schemes. Moreover, the line impedance of low-voltage transmission lines in the MG is set at 0.642 + j0.083 \( \Omega/km \).

B. Traditional Linear Droop Scheme

In an islanded droop-controlled MG, the active power output of a DG is controlled by adjusting frequency of the voltage reference [41]. Further, the \( P-\omega \) droop control is implemented as follows,

\[
\begin{align*}
    f_i = f_{\text{max}} - \frac{f_{\text{max}} - f_{\text{min}}}{P_{\text{max}}} \cdot P_i,
\end{align*}
\]

where \( f_i \) is the frequency of DG, and \( f_{\text{max}} = 51 \text{ Hz}, f_{\text{min}} = 49 \text{ Hz} \), which are maximum and minimum frequencies allowed by the MG, respectively [42].
C. Nonlinear Cost Based Droop Scheme

In order to minimize the total active power generation cost, the *nonlinear cost based droop scheme* is developed in this subsection. First, it is assumed that the \( i \)th DG has an exponential cost function \( C_i(P_i) \) including the maintenance cost, fuel cost, emission penalty and no-load cost, which takes the following forms [18], [34],

\[
C_i(P_i) = \alpha_i \cdot P_i^2 + \beta_i \cdot \exp(\gamma_i \cdot P_i) + \delta_i \cdot P_i + \epsilon_i,
\]

where \( \alpha_i, \beta_i, \gamma_i, \delta_i, \epsilon_i \) are cost coefficients.

Therefore, for \( i \)th DG, the incremental cost can be obtained

\[
\lambda_i = \frac{\partial C_i(P_i)}{\partial P_i} = 2\alpha_i \cdot P_i + \beta_i \cdot \gamma_i \cdot \exp(\gamma_i \cdot P_i) + \delta_i
\]

where \( 2\alpha_i = a_i, \beta_i = b_i, \gamma_i = c_i, \delta_i = d_i \).

**TABLE I**

<table>
<thead>
<tr>
<th>DG</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
<th>( P_{\text{min}} )</th>
<th>( P_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DG2</td>
<td>0.50552</td>
<td>0.08290</td>
<td>3.33</td>
<td>-0.42094</td>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>DG4</td>
<td>0.65912</td>
<td>0.06259</td>
<td>2.857</td>
<td>0.10762</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>DG5</td>
<td>0.47936</td>
<td>0.04552</td>
<td>2.857</td>
<td>0.07827</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>DG7</td>
<td>0.35952</td>
<td>0.03414</td>
<td>2.857</td>
<td>0.05870</td>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>DG10</td>
<td>0.26964</td>
<td>0.02561</td>
<td>2.857</td>
<td>0.04430</td>
<td>0</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 3. Incremental cost curves with exponential cost functions.

Furthermore, the cost coefficients and power generation limits of controllable DGs are summarized in Table I [18], [34], and the incremental cost curves are plotted in Fig. 3 correspondingly. Note that the equal incremental cost principle discussed in Section II is equivalent to draw horizontal lines in Fig. 3. Therefore, different levels of active power outputs (p.u.) can be read from their respective points, and it clearly denotes that the load demands are not proportionally shared among controllable DGs, and DG10 always has the highest level of output power, when DGs operate at the equal incremental cost. Moreover, it can be found in Fig. 3 that the incremental costs of DG2, DG4, DG5, DG7, DG10 remain constant at 2.8, 1.85, 1.35, 1, 0.76, when power outputs reach their respective upper limits, respectively.
droop curves. Following this idea, a parameter $k$ of the MG, which can be achieved by tuning the gradients of
regulated within the permissible range, to ensure the stability
operate at their respective upper limits.

Moreover, there exists a frequency permitted by the MG, when $DG_2$, $DG_4$, and $DG_5$ operate at their respective upper limits.

On the other hand, it is desired that the frequency is regulated within the permissible range, to ensure the stability
of the MG, which can be achieved by tuning the gradients of
droop curves. Following this idea, a parameter $k$ is added to
the droop equation in (20) to arrive at the proposed cost based
droop scheme in (21)

$$
\left\{
\begin{align*}
    f_i &= f_{\text{max}} - (f_{\text{max}} - f_{\text{min}}) \cdot k \cdot \frac{\partial C_i(P_i)}{\partial P_i}, \\
    \min(f_2, f_4, f_5, f_7, f_{10}) &> f_{\text{min}} = 49, \\
    \max(f_2, f_4, f_5, f_7, f_{10}) &< f_{\text{max}} = 51.
\end{align*}
\right
$$

In terms of (21), a number of solutions for $k$ can be obtained, and $k$ is taken as 0.357 in this paper. Note that the parameter $k$
is properly chosen before the implementation of the cost based
droop scheme, while requiring no communication network
for information exchanges among DGs, when the system is
running.

Finally, in terms of (21), the cost based droop curves are
plotted in Fig. 4(b), where the droop curves are nonlinear, and
the frequency of the MG stays within the allowable range.
Moreover, in the steady state, DGs share a single common
frequency, according to (21), we have

$$
\frac{\partial C_2(P_2)}{\partial P_2} = \frac{\partial C_4(P_4)}{\partial P_4} = \frac{\partial C_5(P_5)}{\partial P_5} = \frac{\partial C_7(P_7)}{\partial P_7} = \frac{\partial C_{10}(P_{10})}{\partial P_{10}},
$$

which denote that the nonlinear cost based droop scheme
equates the incremental costs of controllable DGs. Therefore,
in terms of the equal incremental cost principle, the total active
power generation cost is minimized.

**D. Linear Cost Based Droop Scheme**

On the other hand, considering the fuel cost, the generation
cost of $i$th DG can also be approximated with a quadratic cost
function, which takes the following forms [19], [23],

$$
C_i(P_i) = d_i \cdot P_i^2 + e_i \cdot P_i + f_i,
$$

where $d_i$, $e_i$, $f_i$ are cost coefficients.

<table>
<thead>
<tr>
<th>Table II</th>
<th>Coefficients of quadratic cost functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>DG</td>
<td>$d$</td>
</tr>
<tr>
<td>DG2</td>
<td>0.094</td>
</tr>
<tr>
<td>DG4</td>
<td>0.078</td>
</tr>
<tr>
<td>DG5</td>
<td>0.105</td>
</tr>
<tr>
<td>DG7</td>
<td>0.082</td>
</tr>
<tr>
<td>DG10</td>
<td>0.074</td>
</tr>
</tbody>
</table>

Moreover, the cost coefficients are listed in Table II [28],
and for $i$th DG, the incremental cost can be derived,

$$
\lambda_i = \frac{\partial C_i(P_i)}{\partial P_i} = 2d_i \cdot P_i + e_i.
$$

Furthermore, similar to the derivation process of the
nonlinear cost based droop scheme, the cost based droop
scheme derived from the quadratic cost function takes the
following forms,

$$
\begin{align*}
    f_i &= f_{\text{max}} - (f_{\text{max}} - f_{\text{min}}) \cdot h \cdot \frac{\partial C_i(P_i)}{\partial P_i}, \\
    \min(f_2, f_4, f_5, f_7, f_{10}) &> f_{\text{min}} = 49, \\
    \max(f_2, f_4, f_5, f_7, f_{10}) &< f_{\text{max}} = 51.
\end{align*}
$$

where $h = 0.1$.

Thereafter, the incremental cost curves and droop curves
are plotted in Fig. 5 and Fig 6, respectively. It can be seen
in Fig. 5 and Fig 6 that both of the incremental cost curves
Fig. 5. Incremental cost curves with quadratic cost functions.

Fig. 6. Linear cost based droop scheme.

IV. RESULTS

In this section, the performance of the cost based droop schemes are tested on the islanded MG shown in Fig. 1, and five simulation cases are presented and discussed in detail. In case 1, the nonlinear cost based droop scheme derived from the exponential cost function is utilized, and case 1 focuses on responses of the nonlinear cost based droop scheme to the fluctuation of intermittent DGs. For case 2, the large changes in load demands are considered, where the nonlinear cost based droop scheme is implemented. Further, case 3 illustrates the plug and play capability of the nonlinear cost based droop scheme, when both the active power outputs of intermittent DGs and load demands fluctuate at the same time. Later, the performance of the linear cost based droop scheme is investigated in case 4, which is derived from the quadratic cost function. For case 5, the performance of the linear cost based droop scheme with flat incremental cost and droop curves and droop curves are linear, termed here the **linear cost based droop scheme**. Similarly, in the steady state, DGs share a single common frequency, therefore, the equations in (22) are fulfilled, and the minimum generation cost is achieved, in terms of the equal incremental cost principle.
is investigated. Finally, simulation results are discussed and explained in detail.

In this paper, the distributed secondary voltage control strategy based on local communication among DGs [6], and the decentralized secondary frequency control strategy requiring no communication network are adopted [34], to restore the voltage and frequency to their nominal values, respectively. Note that all simulations are performed in MATLAB/Simulink, where the MG and equations are modeled and solved. In MATLAB/Simulink, it offers blocks for mathematical operation. Based on these blocks, the cost based droop schemes for controllable DGs are modeled, which provides a manner to avoid transmitting signals in or out of MATLAB/Simulink. Moreover, the cost based droop schemes simply require the local information of frequency and power output, while requiring no communication network for information exchanges among DGs. Therefore, the cost based droop schemes can be implemented in an effective manner.

A. Case 1: Fluctuating Power Outputs of Intermittent DGs

In this case, the nonlinear cost based droop scheme derived from the exponential cost function is implemented. Moreover, the total active load demand remains constant at 155 kW throughout simulations. On the contrary, the active power outputs of intermittent DGs, DG$_1$, DG$_3$, DG$_6$, DG$_8$ and DG$_9$ change between 10 kW and 30 kW, because the illumination intensity and wind speed are constantly varying. From Fig. 7(a), it can be seen that the active power outputs of DG$_3$, DG$_6$ and DG$_8$ decrease gradually, while those of DG$_1$ and DG$_9$ remain constant, which follow the changes in environmental conditions at the first three seconds.

Accordingly, the active power outputs of droop-controlled DGs are constantly increasing during this period, so that the generation-load balance is maintained, as illustrated in Fig. 7(b). Moreover, the incremental costs increase with rising active power outputs of controllable DGs, as shown in Fig. 7(c), because the incremental cost is an increasing function of output power. Similarly, from $t = 3$ s to $t = 5$ s, the lower the outputs of the controllable DGs, the smaller are the incremental costs.

Furthermore, from Fig. 7(c), it can be found that despite of fluctuations of power outputs of intermittent DGs, the equality of incremental costs among controllable DGs is assured by the nonlinear cost based droop scheme, for equations in (22) are always satisfied. In other words, the equal incremental cost principle is satisfied, and the minimum active power generation cost of the MG is achieved.

B. Case 2: Time-varying Load Demands

In this case, the controllable DGs are regulated in terms of the nonlinear cost based droop scheme. And it is assumed that active power outputs of all intermittent DGs are always at 50% of their respective power ratings. On the contrary, large changes in active load demands are scheduled as follows,

- $t = 3$ s: all active power loads decrease by 15%;
- $t = 6$ s: all active power loads decrease by 15%;

![Fig. 8. Case 2: simulation results under the constant active power outputs of intermittent DGs and fluctuating load demands, when the nonlinear cost based droop scheme is implemented.](image-url)
• $t = 8$ s: all active power loads increase by 25%, where the fluctuations of the total active load demand are illustrated in Fig. 8(a).

Under these settings, the total load demand is decreased significantly at $t = 3$ s by 23.25 kW from 155 kW to 131.75 kW, therefore, the active power outputs of all controllable DGs decrease simultaneously, fulfilling the generation-demand equality constraint in the MG, as shown in Fig. 8(b). Moreover, the incremental costs of controllable DGs drop from 0.47 to 0.43 together, due to lower power outputs, as illustrated in Fig. 8(c). Furthermore, a horizontal line corresponding to the incremental cost at 0.43 can be drawn in Fig. 3. Therefore, the levels of active power outputs (p.u.) of controllable DGs can be read from their respective points, which are 0.36, 0.28, 0.42, 0.56, 0.7 for DG2, DG4, DG5, DG7, DG10, respectively. According to the analysis in Section III, DG10 is supposed to has the highest level of output power, when controllable DGs operate at the equal incremental cost, and the simulation results are consistent with the discussion. Accordingly, the active power outputs of DG2, DG4, DG5, DG7, DG10 are 16.2 kW, 8.4 kW, 16.8 kW, 25.2 kW, 14 kW, respectively, as shown in Fig. 8(b). Moreover, the analyses are applicable to active power outputs and incremental costs of controllable DGs after $t = 3$ s.

On the other hand, it is clear from Fig. 8(b) and Fig. 8(c) that our method has a very short settling time about 0.2 s, which makes sense as our method does not require complicated mathematical modeling and computation, while ensuring the rapid convergence of incremental costs. Furthermore, from Fig. 8(c), we know that regardless of large changes in load demands, the incremental costs reach a common value quickly, satisfying the equal incremental cost principle, because equations in (22) are always satisfied. Therefore, the total active power generation cost is minimized.

C. Case 3: Plug and Play Capability of Nonlinear Cost Based Droop Scheme

In this case, the nonlinear cost based droop scheme is implemented, when both the active power outputs of intermittent DGs and active load demands change at the same time. In addition, five controllable DGs have already reached the optimal states before the plug out of controllable DG4 at $t = 4$ s. Considering the power mismatch under the new situations, the remaining controllable DGs have to produce more power to compensate for the amount of power previously generated by DG4, as shown in Fig. 9(a). Therefore, there are increases in incremental costs corresponding with rising power outputs of remaining controllable DGs. While the incremental cost of DG3 drops to 0.17201 during the plug out, for the incremental cost of DG4 at no-load is 0.17201, as illustrated in Fig. 9(b). Furthermore, at $t = 6.5$ s, the synchronization strategy is activated and the seamless plug in of DG4 into the MG is achieved at $t = 7.3$ s, and the other four controllable DGs reduce their power outputs to accommodate the plug in of DG4, as shown in Fig. 9(a).

Moreover, it can be found in Fig. 9(b) that the plug out and plug in operations of DG4 do not degrade the convergence of
the incremental costs, for the equations in (22) are satisfied regardless of behaviors of DG. Therefore, according to the equal incremental cost principle, the nonlinear cost based droop scheme keeps the total active power generation cost to a minimum. On the other hand, the capability of the droop scheme to meet the requirement for plug and play operation is verified.

Furthermore, Fig. 9(c) shows that the MG frequency stays around the nominal value, namely 50 Hz, in all situations. Also the line voltages of loads are still in a normal range, even if fluctuations occur at $t = 3$ s, $6$ s and $8$ s, which satisfy the IEEE Standard 1547 requirements [43].

D. Case 4: Performance of Linear Cost Based Droop Scheme

In this case, the linear cost based droop scheme derived from the quadratic cost function is adopted, and both the active power outputs of intermittent DGs and active load demands change at the same time. Moreover, the plug and play functionality is tested by disconnecting DG at $t = 4$ s and reconnecting it at $t = 7.2$ s. The results obtained under the linear cost based droop scheme are shown in Fig. 10.

From Fig. 10(a), it can be seen that DG always has the lowest level of output power, because it is the most costly DG in the MG and it has the lowest priority generating active power. On the other hand, in terms of (22), even if DG is unplugged, the incremental costs of remaining DGs stay at the same level, i.e., the equal incremental cost principle is satisfied. Therefore, it can be seen in Fig. 10(b) that the proposed method maintains the consensus of the incremental costs of remaining DGs, after DG is unplugged at $t = 4$ s. While the incremental cost of DG drops to 3.3 during the plug out, for the incremental cost of DG at no-load is 3.3.

Similarly, despite the plug in operation of DG at $t = 7.2$ s, the equations in (22) are fulfilled, and the linear cost based droop scheme quickly drives the incremental costs to a consensus value, as illustrated in Fig. 10(b). Accordingly, plug and play functionality can be realized with the linear cost based droop scheme. Finally, it can be found from Fig. 10(c) that the voltages and frequency of the MG are regulated to be within the IEEE Standard limits.

E. Case 5: Performance of Linear Cost Based Droop Scheme with Flat Incremental Cost Curve

In this case, the linear cost based droop scheme derived from the quadratic cost function is adopted, and DG with the power rating of 40 kW and flat incremental cost curve, i.e., constant incremental cost of 4.8 is considered. The incremental cost curves and corresponding cost based droop curves are illustrated in Fig. 11 and Fig. 12, respectively. Moreover, in this case, other settings for the control model follow those in Case 4, and the simulation results are shown in Fig. 13.

In the steady state, DGs share a single common frequency, and we have

$$\frac{\partial C_2(P_2)}{\partial P_2} = \frac{\partial C_4(P_4)}{\partial P_4} = \frac{\partial C_5(P_5)}{\partial P_5} = \frac{\partial C_7(P_7)}{\partial P_7} = \lambda_{10} = 0.48.$$  

(26)
In terms of (24) and (26), the power outputs of \( DG_2 \), \( DG_4 \), \( DG_5 \) and \( DG_7 \) remain constant at 19 kW, 8.9 kW, 10.8 kW and 4.8 kW, respectively, when DGs are plugged in to the MG, as shown in Fig. 13(a). On the other hand, the power output of \( DG_{10} \) is constantly varying to maintain the generation-load balance in the MG, and \( DG_{10} \) is turned off at very low-load condition, and it turns on when the total active load demand of the MG increases significantly. Therefore, it can be found in Fig. 13(a) that \( DG_{10} \) is turned off from \( t = 6.8 \text{ s} \) to \( t = 8 \text{ s} \) when the MG is at very low-load condition.

Furthermore, the incremental costs among DGs is maintained around 4.8, as shown in Fig. 13(b), and the decentralized secondary frequency control results in the slight deviation between the incremental cost of \( DG_{10} \) and those of other DGs. Therefore, the total active power generation cost of the MG is minimized.

V. CONCLUSION

In this paper, the linear and nonlinear cost based droop schemes are developed, to minimize the total active power generation cost of the islanded MG in a decentralized manner. By embedding the incremental costs into the droop schemes, the proposed cost based droop schemes are able to drive the incremental costs of DGs to a consensus value, minimizing the total active power generation cost of the islanded MG, in terms of the equal incremental cost principle. Finally, the expected minimization of the total active power generation cost, and the capability of plug and play are verified by numerical simulations, in the presence of a high penetration of intermittent renewable energy sources, and large changes in load demands.

On the other hand, the following observations are also worth noting about our method. Firstly, in this paper, we restrict ourselves in an islanded MG, where the MG has already reached the optimal state before the plug out and plug in of a DG. However, when a DG is newly installed in the MG, the cost based droop schemes can be tuned in terms of the given method, which is easy to implement in practice. Secondary, our method is based on droop control, while the intermittent renewable energy sources usually operate in the manner of MPPT rather than droop control, therefore, our method is not applicable to intermittent DGs.

For future work, the proposed method can be refined...
by considering the generation cost of intermittent renewable energy sources, the vigorous stability analysis of the control model, and the verification of our method on real time experimental platform.

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**References**


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