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A repetitive control scheme aiming to compensate the 6k+1 harmonics for three-phase hybrid active filter

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Abstract: The traditional repetitive controller has relatively worse stability and poor transient performance due to the facts that it generates infinite gain at all the integer multiples of the fundamental frequency, and its control action is postponed by one fundamental period (T_0). To improve these disadvantages, many repetitive controllers with reduced delay time have been proposed, which can selectively compensate the odd harmonics or 6k±1 harmonics with delay time reduced to $T_0/2$ and $T_0/3$,repectively. To further study in this area, this paper proposes an improved repetitive scheme implemented in stationary reference frame, which only compensates the 6k+1 harmonics (e.g. -5, +7, -11, +13) in three-phase systems and reduces the time delay to $T_0/6$. So compared with the earlier reduced delay time repetitive controllers, the robustness and transient performance is further improved, the waste of control effort is reduced, and the possibility of amplifying and even injecting any harmonic noises into system is avoided to the greatest extent. Moreover, the proposed repetitive scheme is used in the control of a three-phase hybrid active power filter. The experimental results validate the effectiveness of the proposed repetitive control scheme.

Keywords: repetitive control; hybrid active power filter; power quality; harmonic compensation;

PACS: J0101

1. Introduction

Recently, due to the widespread applications of distributed generations, adjustable speed drives，uncontrolled AC/DC rectifiers, and other nonlinear loads, the harmonic pollution in power systems is getting more and more serious. The passive power filter (PPF) and active power filter (APF) are the two common solutions applied to mitigate these harmonics [1-2]. PPFs have the advantages of low-cost and high-efficiency. However, they also have some inherent drawbacks. Their compensation characteristics are strongly influenced by supply impedance and they are highly susceptible to series and parallel resonances with the supply and load impedance. The APFs，which are based on the power electronics, can overcome above drawbacks of PPFs [3-4]. Additionally, APFs are more flexible and efficient compared with PPFs. But, pure APFs usually require a PWM inverter with large kilovoltampere (KVA) rating. Thus, they do not constitute a cost-effective harmonic filtering solution for nonlinear loads above 500-1000 kW[5-6]. To address this issue, hybrid active power filter (HAPF) have been developed, which are composed of small rated APF and PPF in different configurations. Among the various viable hybrid active filter topologies, parallel hybrid active filters present a cost-effective solution for harmonic filtering and reactive power compensation of high power nonlinear industrial loads, due to small rating of the active filter—2%–3% of load KVA rating [6]. Thus, they have attracted increasing attention [7-11].

Among various control strategies, the repetitive control, as a kind of control method based on internal model principle, can accurately track the periodic signal or reject periodic interference. Hence it is widely used in harmonic compensation scheme for active filters [12-15]. The traditional repetitive control technique can generate infinite gains at all the integer multiples of the fundamental frequency, including the odd, even harmonics and dc component. However, in most cases, the introduction of high gain for all frequencies is not necessary, as it could waste control effort and reduce the system robustness without improving system performance, even amplify irrelevant signal and reinject distortions to systems [16]. Moreover, the control action of traditional repetitive controller is postponed by one fundamental period (T_0), hence the transient performance is poorer. To improve the drawbacks above, and considering the fact that even harmonic components do not regularly appear in power systems, literatures [16-17] propose a repetitive control scheme aiming at compensating only the odd harmonics. It uses a negative feedback array instead of the usual positive feedback in the traditional repetitive controller. Meanwhile, the delay time of control action is reduced to $T_0/2$. As a sequence, the control performance is improved and the convergence rate is enhanced. Furthermore, among the odd harmonics, the group of $6k \pm 1\ (k = 0, \pm 1, \pm 2 \cdots)$ harmonic components in electric industry are dominated due to the wide use of uncontrolled rectifiers and six-pulse converters. Thus, many improved repetitive control schemes aiming at compensating $6k \pm 1$ harmonics have been developed [18-20]. For instance, in [18] a repetitive control scheme based on the feedback array of two delay lines plus a feedforward path is presented, which can only compensate $6k \pm 1$ harmonics and reduce delay time to $T_0/3$; In [19], the authors propose a $6k \pm 1$ repetitive control scheme in there-phase synchronous reference frame (SRF). It has an advantage of $T_0/6$ delay time. However, it needs complex coordinate transformation and much more calculation in both positive-rotating and negative-rotating SRFs.

Considering in three-phase power systems, harmonic of the same frequency can be decomposed into positive sequence, negative sequence and zero sequence. Generally speaking, a normal balanced three-phase system mainly contains 6k+1 harmonics (such as -5, +7, -11, +13), and rarely contains 6k-1 harmonics (such as +5, -7, +11, -13). For this reason, this paper proposes a repetitive control scheme aiming at compensating the 6k+1 harmonics implemented in three-phase stationary reference frame with $T_0/6$ delay time. So that the transient performance is further improved. The 6k+1 repetitive controller is expressed with complex-vector notation, so that the dual-input/dual-output control system (in the $\alpha\beta$ reference frame) can be simplified into one single-input/single-output system. Meanwhile, the general design method of Lk+M repetitive controller is also introduced, with which a repetitive controller aiming at compensating Lk+M harmonics can be easily deduced. Moreover, taking the transformerless parallel hybrid active filter as controlled object, a harmonic compensating control system based on the proposed $6k+1$ repetitive control scheme is presented. Finally, the experimental results validate the effectiveness of the $6k+1$ repetitive control scheme.

2. System structure and mathematical modeling of HAPF

2.1. Topological structure analysis

The topology of the transformerless parallel hybrid active filter is shown in Figure 1. It consists of a LC passive filter and a three-phase voltage source inverter (VSI). The purposes of installing the LC filter are: 1) to provide reactive power compensating and absorb some harmonics; 2) to sustain fundamental voltage at the point of common coupling (PCC). And the active filter (VSI) is responsible for improving the filtering characteristics of passive filter and avoiding the undesirable resonances with the grid. To minimize its own KVA rating, VSI doesn't participate in reactive compensation, and the grid voltage is almost fully dropped on the capacitor in LC filter. Thus the fundamental voltage sustained by VSI is small. So that the dc bus voltage rating of VSI can be set very low, the KVA rating and power losses are reduced greatly. Due to the presence of VSI, LC filter is not necessary to be accurately tuned at a certain harmonic frequency. The design objective of LC filter is to offer a lowest possible impedance path for injecting harmonic currents, on the premise of ensuring reactive power compensating.

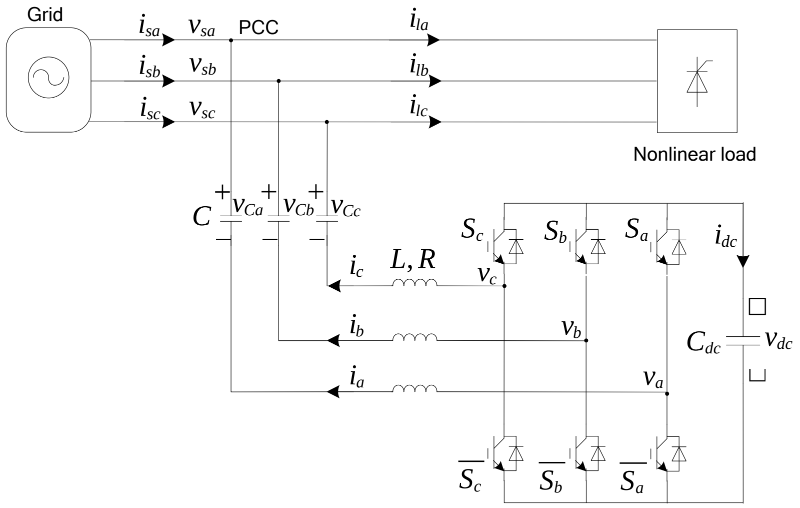


Figure 1. Topology of parallel hybrid active power filter

2.2. Mathematical modeling

According to Figure 1, the mathematical model in state-space representation for the system is formulated as

$$\begin{cases} L\dfrac{di_a}{dt} = -v_{sa} + v_{Ca} - Ri_a + v_a \\ L\dfrac{di_b}{dt} = -v_{sb} + v_{Cb} - Ri_b + v_b \\ L\dfrac{di_c}{dt} = -v_{sc} + v_{Cc} - Ri_c + v_c \end{cases} \tag{1}$$

$$\begin{cases} C\dfrac{dv_{Ca}}{dt} = -i_a \\ C\dfrac{dv_{Cb}}{dt} = -i_b \\ C\dfrac{dv_{Cc}}{dt} = -i_c \end{cases} \tag{2}$$

$$C_{dc}\frac{dv_{dc}}{dt} = i_{dc} = -(S_a i_a + S_b i_b + S_a i_a) \tag{3}$$

where S_a, S_b and S_c are switching functions defined by

$$S_x = \begin{cases} 1, \ (when\ S_x \text{ on}, \ \overline{S_x}\ off) \\ 0, \ (when\ S_x \text{ off}, \ \overline{S_x}\ on) \end{cases} \quad (x = a, b, c) \tag{4}$$

2. System control

According to (1-2), it can be inferred that the state equation of output current i_x $(x = a, b, c)$ is a second-order differential equation. If the output current control is implemented in dq synchronous reference frame, it needs to sample and feed back the AC capacitor voltage v_{Cx} $(x = a, b, c)$ to achieve decoupling control between d-axis and q-axis. Therefore, in this paper, the output current control is implemented in $\alpha\beta$ stationary frame, which has the advantages of no need of complex decoupling control and AC capacitor voltage sampling. The overall system control diagram is shown in Figure 2, it is mainly composed of dc-link voltage control and harmonic current tacking control. In this figure, *Bpf*(*s*) is a band-pass filter to extract the fundamental frequency component of input signal, and its expression is given by

$$Bpf(s) = \frac{\gamma\omega_0 s}{s^2 + \gamma\omega_0 s + \omega_0^2} \tag{5}$$

where ω_0 is the grid frequency; γ is the control coefficient of passband width and $\gamma > 0$.

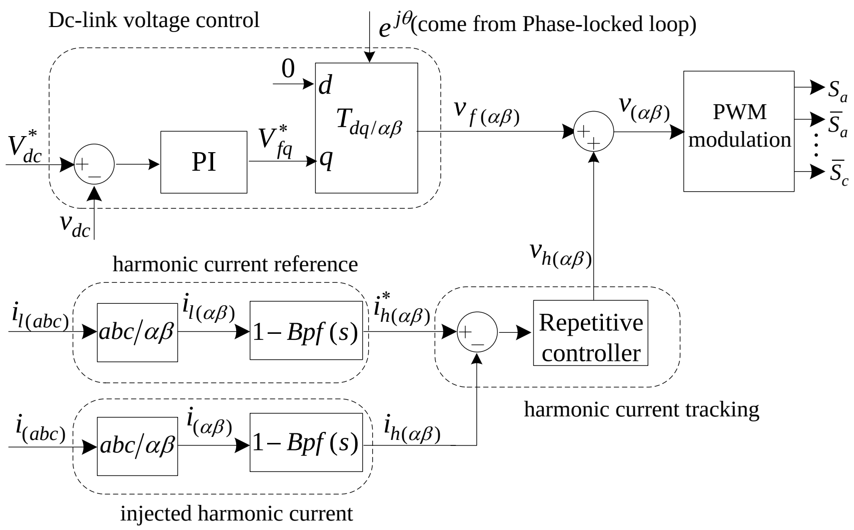


Figure 2. Overall block diagram of system control

2.1. DC-link voltage stabilisation method

Assuming that the VSI doesn't provide reactive power compensation for the load and only absorbs active power from grid to maintain its power loss. According to the power conservation principle, there is

$$P_{in} = C_{dc} v_{dc} \frac{dv_{dc}}{dt} + P_{loss} \tag{6}$$

where P_{in} is the active power absorbed from gird, $C_{dc}v_{dc}\frac{dv_{dc}}{dt}$ is the power of the dc-link capacitor and P_{loss} is the power loss of inverter. It can be inferred from (6) that if P_{loss} is regarded as a disturbance, v_{dc} could be controlled by adjusting P_{in}. In the dq synchronous reference frame (grid voltage orientation), the active power P_{in} and reactive power Q_{in} absorbed by VSI can be given by

$$\begin{cases} P_{in} = -\dfrac{V_{sd}V_{fq}}{X} \\ Q_{in} = \dfrac{V_{fd}^2 - V_{sd}V_{fd} + V_{fq}^2}{X} \end{cases} \tag{7}$$

where X denotes the fundamental frequency impedance of LC filter; V_{sd} is the d-axis component of grid voltage ($V_{sq}=0$); V_{fq}, V_{fq} are the d-axis and q-axis component of VSI output fundamental voltage, respectively.

It can be inferred from (7) that P_{in} is only regulated by V_{fq}. Generally, V_{fq} is small. Thus, $Q_{in} \approx 0$ could be achieved when V_{fq} is set to 0. Thus, the dc-link voltage regulator can be designed as (8), and the corresponding bode diagram is shown in Figure 2.

$$\begin{cases} V_{fq}^* = (k_p + k_i/s)(V_{dc}^* - v_{dc}) \\ V_{fd}^* = 0 \end{cases} \tag{8}$$

where V_{dc}^* is the rated value of dc-link voltage; k_p, k_i are the parameters of PI controller.

2.2. Harmonic current tracking control

Harmonic current tracking control is the important part of system control, which contributes directly to the performance of harmonic compensating. The block diagram of current control is shown in Figure 2. Considering the case that the three-phase load current mainly contains 6k+1 harmonics, this paper presents a 6k+1 repetitive control scheme to compensate these harmonics. The detail theoretical derivation, analysis and design of proposed 6k+1 repetitive controller is given in the next section.

3. 6k+1 repetitive control scheme

3.1. internal model of 6k+1 repetitive controller

Firstly, the internal model of the wellknown traditional repetitive controller is given by

$$RC_t(s) = \frac{e^{-sT_0}}{1-e^{-sT_0}} \tag{9}$$

where e^{-sT_0} is the periodic time delay unit, and T_0 is the fundamental period, i.e., $T_0 = 2\pi/\omega_0$.

By setting the denominator $1-e^{-sT_0}$ in (9) equal to zero, it can be obtained that

$$s_{pk} = jk\omega_0 \quad (k = 0, \pm 1, \pm 2, \ldots, \pm\infty) \tag{10}$$

where s_{pk} is the pole of (9). Seen from (10), it is clear that the traditional repetitive controller has an infinite number of poles located at $jk\omega_0$, which is the reason traditional repetitive controller has resonant peaks at every integral multiple of fundermental frequency ω_0.

In order to make repetitive controller with poles only located at a selected group of harmonic frequencies, a new internal model is need to be structured. Assume the order *h* of these harmonics meets the rule:

$$h = Lk + M \tag{11}$$

where *L*, *M* are intergers, and *L* is not equal to zero.

Then, the poles of the new internal model should be located at

$$s'_{pk} = j(Lk + M)\omega_0 \quad (k = 0, \pm1, \pm2, \ldots, \pm\infty) \tag{12}$$

Moreover, to enhance the frequency selectivity, an infinite number of zeros of the new internal model that located in the midpoints between two consecutive poles are introduced as

$$s'_{zk} = j(Lk + M + 0.5L)\omega_0 \quad (k = 0, \pm1, \pm2, \ldots, \pm\infty) \tag{13}$$

These zeros bring another benefit that allowing bigger gains with improved performance.

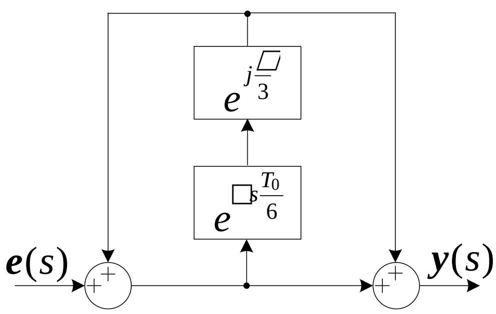
In order to satisfy (12-13) , the general internal model for $Lk + M$ harmonics can be structured as

$$RC_g(s) = \frac{1 - e^{-\frac{(s - j\omega_0(M + 0.5L))}{L}T_0}}{1 - e^{-\frac{(s - j\omega_0 M)}{L}T_0}} \tag{14}$$

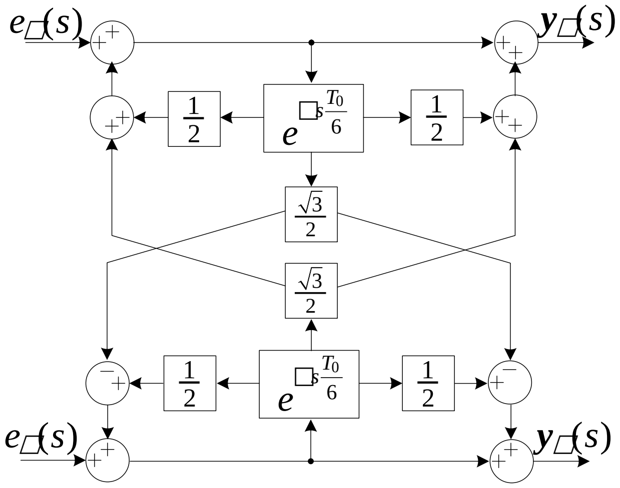
After the substitution of *L*=6 and *M*=1 in (14), the 6k+1 internal model is given by

$$RC(s) = \frac{1 + e^{j\frac{\pi}{3}} e^{-s\frac{T_0}{6}}}{1 - e^{j\frac{\pi}{3}} e^{-s\frac{T_0}{6}}} \tag{15}$$

Comparing (15) with the traditional internal model given by (9), it can be found that the delay time of 6k+1 internal model is reduced to $T_0/6$, which means a much faster dynamic response. what's more, it should be noted that the 6k+1 internal model is expressed using the complex-vector notation, as it contains the complex coefficient $e^{j\frac{\pi}{3}}$. As a consequence, the input signal of RC (s) is required to be a complex vector. The block of the proposed 6k+1 internal model is shown in Figure 3.



(a)



(b)

Figure 3. Block diagram of the 6k+1 repetitive controller internal model: (a) complex-vector notation; (b) scalar notation

Assuming that the fundermental frequcny f_0 = 50 Hz, i.e., T_0 = 0.02 s, the bode plot of the 6k+1 repetitive controller internal model is shown in Figure 4. As expected, the amplitude-frequency response curve shows that 6k+1 internal model has resonant peaks that located at frequency multiples 6k+1 of 50 Hz (50, -250, 350, -550, 650 Hz ...), and has notches that located at frequency multiples 6k+4 of 50 Hz (-100, 200, -400, 500 Hz ...). The phase-frequency response curve shows the phase shift is bounded between 90 and - 90 degree, and zero at the peaks and notches.

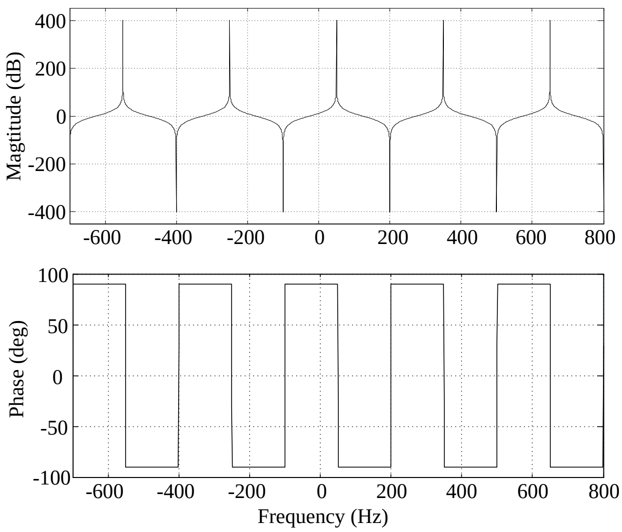


Figure 4. Bode plot of the proposed repetitive controller internal model.

3.2. Fractional delay compensation

In a practical application, the implementation of repetitive control scheme is usually performed in the digital form. Using the transformation $z = e^{sT_s}$, (15) can be discretized and its expression in discrete time domain is given by

$$RC(z)\Big|_{z=e^{sT_s}} = \frac{1+e^{j\frac{\pi}{3}}z^{-\frac{N_0}{6}}}{1-e^{j\frac{\pi}{3}}z^{-\frac{N_0}{6}}} \qquad (16)$$

where T_s is the sampling period, and $N_0 = T_0/T_s$ (the number of samples per fundamental period).

In most cases, the sampling frequncy f_s ($f_s = 1/T_s$) is a fixed rate(e.g. 10 kHz，12.8 kHz, 20 kHz), and the grid frequecny detected by PLL is variabe in a certain range (e.g. 49~51Hz). Thus, $N_0/6$ is usually non-interger.

Let

$$z^{-\frac{N_0}{6}} = z^{-(D+d)} = z^{-D} \cdot z^{-d} \tag{17}$$

where D and d are the integral and fractional parts of $N_0/6$, repectively.

In a common implementation, $z^{-N_0/6}$ is approximately treated as z^{-D} and performed by reserving D memory locations, with the fractional order part z^{-d} neglected. But, this will cause the resonant peaks to deviate from the harmonic frequencies. As a consequence, the harmonic comperseation performance could be degraded.

To address this problem, fractional delay (FD) filters have been usd as approximations of z^{-d}. The magnitude-frequency and phase-frequency characteristics of z^{-d} can be given by

$$\begin{cases} \left|z^{-d}\right| = 1 \\ \angle z^{-d} = -d\omega T_s \end{cases} \tag{18}$$

Thus, it requires that FD filters should have a unit gain and linear phase in the low-middle frequencies, and acheive a high attenuation rate in the high frequencies to enhance the system stablity.

In the condition of $\left|z^{-1} - 1\right| < 1$ (i.e. $-\pi/(3T_s) < \omega < \pi/(3T_s)$), with the use of the Taylor expansion, z^{-d} can be expressed as

$$z^{-d} = (1 + z^{-1} - 1)^d = 1 + d(z^{-1} - 1) + \ldots + \frac{d(d-1)\ldots(d-n+1)}{n!}(z^{-1} - 1)^n \tag{19}$$

Specifically, choose the first-order Taylor expansion of z^{-d} as a FD filter, that is

$$Fd(z) = 1 - d + dz^{-1} \tag{20}$$

Figure 5 shows the bode plot of $Fd(z)$, with $T_s = 78.125$ μs, d =0.2, 0.5 and 0.8, respectively. It can be seen that $Fd(z)$ has the low-pass filter nature. In low frequecnies, $Fd(z)$ has a well linear phase approximated to the ideal value. However, the main disadvantages of $Fd(z)$ are that the cutoff frequecny is too high (greater than 3000 Hz), and it changes with the value of d. Only when d=0.5, $Fd(z)$ achieves the lowest cutoff frequecny and best linear phase.

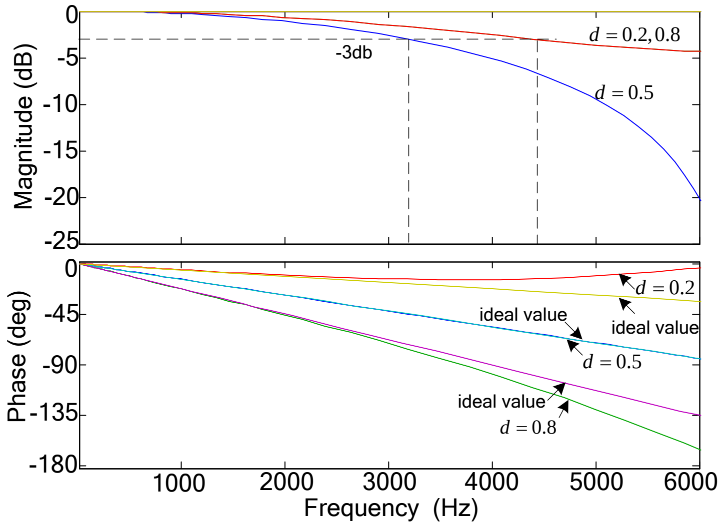


Figure 5. Bode plot of Fd(z)

To overcome above issue, this paper presents a FD filter $Q(z)$ by cascading Fd(z) with a zero-phase digital low-pass filter, i.e.,

$$Q(z) = Fd(z)M(z) \tag{21}$$

where $M(z)$ is the zero-phase digital low-pass filter used to low the cut-off frequency and Increase the attenuation rate in high frequecnies. Its expression is given as

$$M(z) = (a_1 z + a_0 + a_1 z^{-1})^n \tag{22}$$

where $a_0, a_1 > 0$ and $a_0 + 2a_1 = 1$; n is the order of filter.

Although $Q(z)$ is noncausal, the time delay term z^{-D} makes it applicable. After the fractional delay compseation, (16) should be revised as

$$RC(z) = \frac{1 + e^{j\frac{\pi}{3}} Q(z) z^{-D}}{1 - e^{j\frac{\pi}{3}} Q(z) z^{-D}} \tag{23}$$

3.3. Design of 6k+1 repetitive controller

Figure 6 shows the block diagram of the harmonic current tracking control. This paper adopts a plug-in repetive controller structure in the control loop，where the PI controller is used to enhance the stability and improve dynamic response, and the repetive controller is used to eliminate the steady-state error.

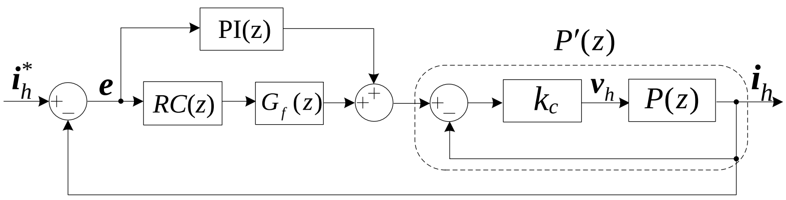


Figure 6. Block diagram of harmonic current tracking control

In Figure 6, P(z) is the plant of current control. According to (1-2), its expression in continuous domain can be obtained as

$$P(s) = \frac{1}{Ls + R + \frac{1}{Cs}} \tag{24}$$

Obviously, P(s) is a second-order system. To modify the characteristic of P(s), a method of output current status feedback is used. Accoding to Figure 6, the modified plant expression is given by

$$P'(s) = \frac{k_c P(s)}{1 + k_c P(s)} = \frac{1}{\frac{L}{k_c}s + \frac{(k_c + R)}{k_c} + \frac{1}{k_c C s}} \tag{25}$$

Equation (25) reveals that $P'(s)$ can be viewed as the R becomes to 1 ($k_c \gg R$), while L, C become $1/k_c$ and k_c times of its original values in P(s), respectively. The bode plots of $P(s)$ and $P'(s)$ are shown in Figure 7, with $L = 3\text{ mH}$, $C = 90\ \mu\text{F}$, $R = 0.1\ \Omega$. As seen, $P'(s)$ has cancelled the resonant peak appeared in $P(s)$, and presents the characteristics of a band-pass filter. The passban*d widt*h depends on the value of k_c. A larger k_c leads to bigger passband width and smaller phase lead/lag.

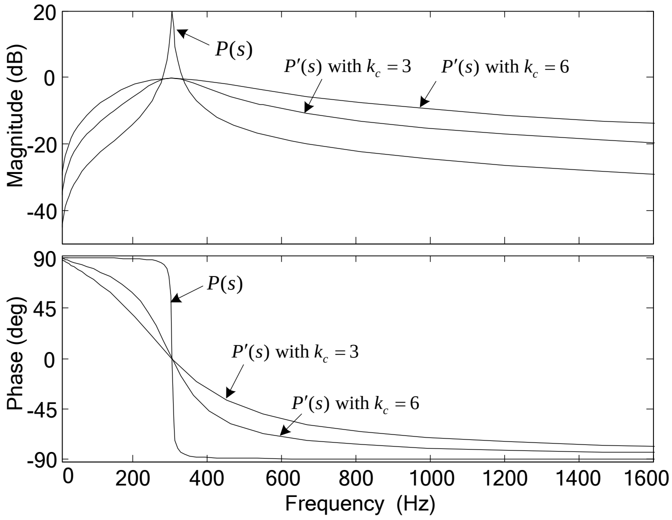


Figure 7. Bode plots of P(s) and $P'(s)$

In Figure 7, without the repetitive controller, the tracking error $\boldsymbol{e}$ between the reference $\boldsymbol{i}_h^*$ and output $\boldsymbol{i}_h$ is

$$\boldsymbol{e}_0(z) = \frac{1}{1 + PI(z)P'(z)} \boldsymbol{i}_h^* \tag{26}$$

where $PI(z)$ should be designed to guarantee the stability of $\boldsymbol{e}_0(z)$.

With the proposed 6k+1 repetitive controller, the tracking error $\boldsymbol{e}$ can be written as

$$\begin{aligned}\boldsymbol{e}(z) &= \boldsymbol{e}_0(z)\frac{1}{1+H(z)G_f(z)R(z)} \\ &= \frac{1}{2}\boldsymbol{e}_0(z)\frac{(1-e^{j\frac{\pi}{3}}Q(z)z^{-D})}{1-(1\text{-}H(z)G_f(z))(1+e^{j\frac{\pi}{3}}Q(z)z^{-D})/2}\end{aligned} \tag{27}$$

where $G_f(z)$ is the compensation function, and

$$H(z) = \frac{P'(z)}{1+PI(z)P'(z)} \tag{28}$$

By the small gain theorem, the sufficient condition for ensuring (27) stable can be given as

$$\left|(1\text{-}H(z)G_f(z))(1+e^{j\frac{\pi}{3}}Q(z)z^{-D})/2\right| < 1 \tag{29}$$

Clearly, $\left|(1+e^{j\frac{\pi}{3}}Q(z)z^{-D})/2\right| \le 1$ is true. To make (29) true, it only needs $\left|1\text{-}H(z)G_f(z)\right| < 1$ being satisfied. Thus, $G_f(z)$ can be chosen as

$$G_f(s) = \frac{1}{H(s)} \cdot \frac{1}{\tau s+1} \tag{30}$$

where $G_f(s)$ and $H(s)$ are the functions of $G_f(z)$ and $H(z)$ in Laplace domain, respectively; $1/(\tau s+1)$ is a low-pass filter.

Moreover, on the premise of system stability, it can be derived that the numerator of (27) has such a steady-state relationship:

$$1-e^{j\frac{\pi}{3}}Q(e^{j(6k+1)\omega_0 T_s})e^{-j(6k+1)\omega_0 DT_s} = 0 \tag{31}$$

Equation (31) indicates that the 6k+1 repetitive control scheme can eliminate the steady-state error of 6k+1 harmonics tracking in D+d T_s (i.e., $T_0/6$), which means the proposed repetitive control scheme could has a much faster transient state response than the traditional one.

4. Experimental results

To validate the correctness and effectiveness of the proposed 6k+1 repetitive control scheme, a prototype of three-phase parallel hybrid APF is built in lab, which is shown in Figure 8. The control system is realized by a combination of digital signal processor TMS320F28335 and field programmable gate array FPGA EP2C8T144C8N. The power switches use three Infineon IGBT modules and the drive circuit uses M57962L driver chips. The non-linear load used in the experiments is a three-phase diode rectifier bridge with resistive load. The overall experimental parameters are given in Table 1.

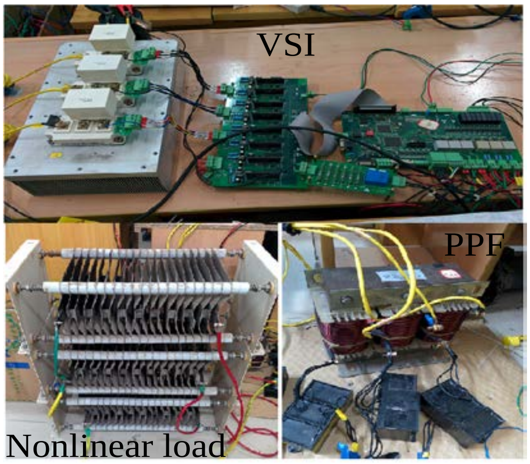


Figure 8. Photograph of the prototype

Table 1. Experimental parameters

Parameters	Symbol	Value	Unit
grid phase voltage	v_s	60	V (rms)
grid frequency	f	50	Hz
Inductance	L	3	mH
capacitor	C	90	μF
dc bus voltage	V_{dc}	80	V
load resistance	R_L	6.6	Ω
switching period	T_s	78.125	μs

4.1. Controller parameters

In the implementation of experiments, the parameters of controllers are given as follows.

1) Dc-link voltage PI controller: $k_{p1} = 2$, $k_{i1} = 5$.

2) In the harmonic current tacking loop:

a. The zero-phase low-pass filter $M(z)$ is given as $M(z) = (0.25z + 0.5 + 0.25z^{-1})^2$;

b. The number of delay sample is 42, and the FD filter is given as

$$Fd(z) = 1 - d + dz^{-1} = 0.333 + 0.667z^{-1} \tag{32}$$

c. Output current state feedback gain $k_c = 3$;

d. PI controller in the plug-in repetitive controller: $k_{p2} = 1, k_{i2} = 1$.

e. The compensation function $G_f(z)$ is given as

$$G_f(z) = \frac{5 - 9.303z^{-1} + 4.397z^{-2}}{1 - 1.677z^{-1} + 0.6766z^{-2}} \tag{33}$$

4.2. LC filter parmaters

As the non-linear load used in this paper is a three-phase diode rectifier bridge with resistive load，the 5th, 7th, 11th, 13th harmonic currents are dominated in load current. Assume that the load harmonic currents are fully compensated by the hybrid APF, the voltage drop across the LC filter by the injected compensating harmonic current is

$$\left|v_h\right| = \left|\sum_{m>1} i_{lh}^m (\omega_m L - \frac{1}{\omega_m C})\right| \le \sum_{m>1} (I_{lh}^m \left|\omega_m L - \frac{1}{\omega_m C}\right|) \tag{34}$$

where i_{lh}^m is the m-th order harmonic component of load current, and I_{lh}^m is the amplitude of i_{lh}^m.

For the hybrid APF, the design objective of LC filter is to offer a lowest possible impedance path for injecting harmonic currents, in other words, to minimize the voltage drop $\left|v_h\right|$. Thus, the dc-link voltage rating of VSI can be minimized.

Then, an optimization function can be given as

$$f_{\min} = \sum_{m=5,7,11,13} (I_h^m \left|\omega_m L - \frac{1}{\omega_m C}\right|) = I_f \cdot \sum_{m=5,7,11,13} (HD_m \left|\omega_m L - \frac{1}{\omega_m C}\right|) \tag{35}$$

where HD_m is the individual m-th order harmonic distortion rate, and I_{lf} is the amplitude of fundamental component in load current.

The capacitor C in LC filter can be chose by the rule as follow:

$$Q_C = 3\omega_1 C V_s^2 \tag{36}$$

where Q_C is the reactive power demanded by load, ω_1 is the grid frequency, V_s is the grid voltage amplitude.

Assume the capacitor C has been determined, such as *C*=90uf. According to Fig.11(b), it can obtained that $HD_5 = 22.4\%$, $HD_7 = 8\%$, $HD_{11} = 5.7\%$ and $HD_{13} = 2.6\%$. Substituting the above parameters into (32), the optimal inductor *L* can be obtained as *L*=2.8 mH. So we choose *L*=3 mH for the hybrid APF experimental prototype without loss of much performance, and the resonant frequency of LC filter is 306 Hz.

4.3. Experimental results

Figure 9 shows the dynamic behaviour of dc-link capacitor voltage in start-up process. To avoid the inrush current caused by capacitors, the series-resistance soft-start mode is used in experiments. Specifically, when $v_{dc} \le V_{set} = 60$ V, the IGBTs are turned off, the capacitors are charged up with small current due to the series-resistance ; When $v_{dc} > V_{set}$, the series-resistance is bypassed and then the PWM pulses will be activated. v_{dc} reaches the setting value 80 V in the steady state, which verifies the correctness of the dc voltage control strategy.

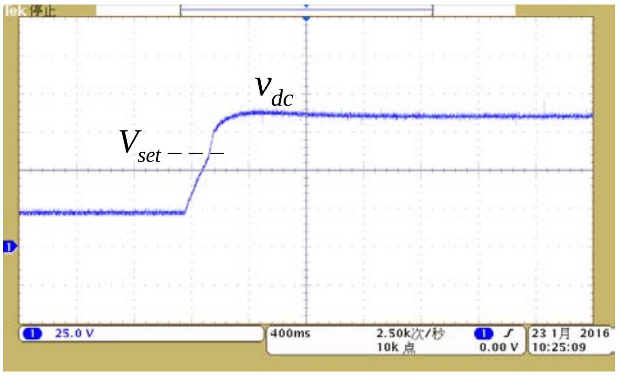


Figure 9. Start-up process of dc-link voltage

To validate the static and dynamic performances of the proposed 6k+1 repetitive control scheme, the related experimental results are shown in Figure 10-13. For the sake of simplicity, only the a-phase waveforms are displayed.

Figure 10 shows the harmonic compensation results when nonlinear load is disconnected ($i_{la} = 0$). As seen, $i_{sa} = i_a$, and i_{sa} is almost the reactive power current provided by LC filter. The waveform of i_a is sinusoidal with less distortion, which indicates that the proposed repetitive control scheme can well suppress the undesired harmonic components.

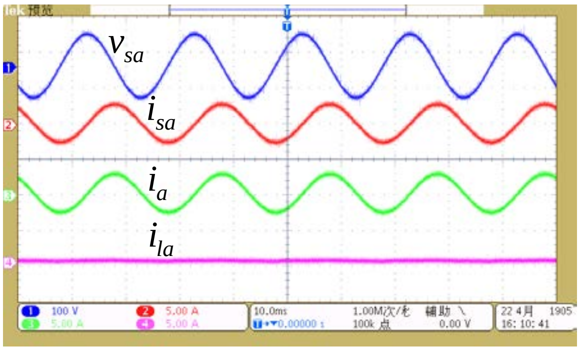
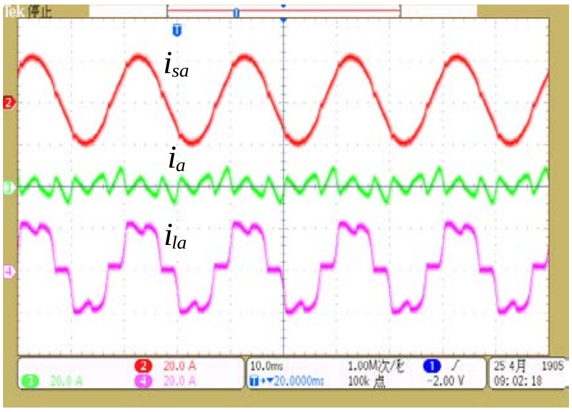
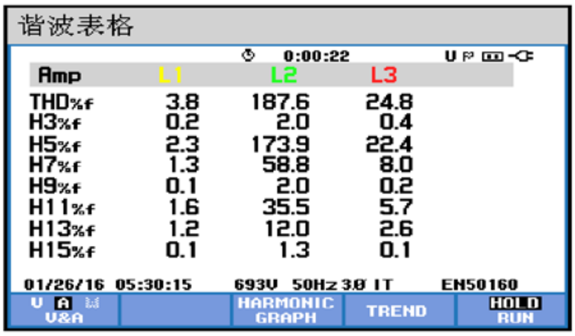


Figure 10. Harmonic compensation results with nonlinear load disconnected

Figure 11-12 show the steady-state harmonic compensation results with and without fractional delay compensation when the nonlinear load is connected, respectively. In Figure 11, the total harmonic distortion (THD) of the source current i_{sa} is reduced to 3.8% from 24.8% (THD of the load current), and the distortion ratio of 5th, 7th, 11th and 13th harmonics in i_{sa} are reduced to 2.3%, 1.3%, 1.6% and 1.2%, respectively. As a contrast, the THD of i_{sa} is 4.9% in Figure 12. These comparison experiment results demonstrate the good static performance of 6k+1 repetitive controller and effectiveness of the fractional delay compensation.



(**a**)



谐波表格

0:00:22

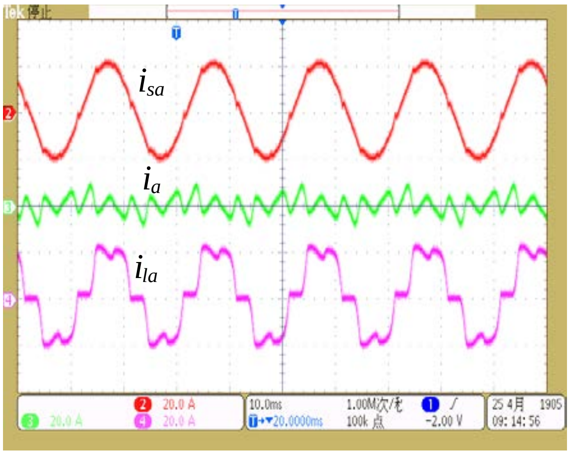
Amp	L1	L2	L3
THD%f	3.8	187.6	24.8
H3%f	0.2	2.0	0.4
H5%f	2.3	173.9	22.4
H7%f	1.3	58.8	8.0
H9%f	0.1	2.0	0.2
H11%f	1.6	35.5	5.7
H13%f	1.2	12.0	2.6
H15%f	0.1	1.3	0.1

01/26/16 05:30:15 693V 50Hz 3Ø IT EN50160

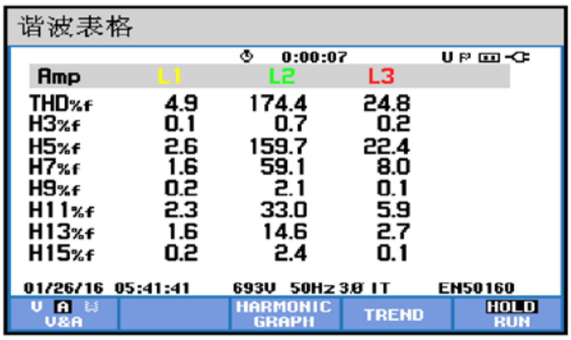
V&A HARMONIC GRAPH TREND HOLD RUN

(**b**)

Figure 11. Harmonic compensation results with FD compensation: (**a**) source (L1), compensating (L2), load (L3) current waveforms ; (**b**) harmonic distortion rate graph.



(**a**)



谐波表格

0:00:07

Amp	L1	L2	L3
THD%f	4.9	174.4	24.8
H3%f	0.1	0.7	0.2
H5%f	2.6	159.7	22.4
H7%f	1.6	59.1	8.0
H9%f	0.2	2.1	0.1
H11%f	2.3	33.0	5.9
H13%f	1.6	14.6	2.7
H15%f	0.2	2.4	0.1

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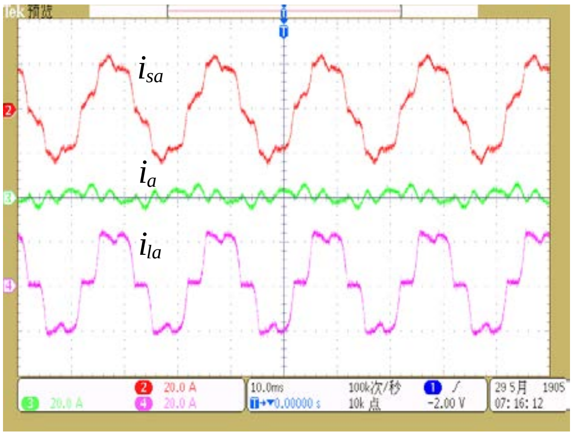
V&A HARMONIC GRAPH TREND HOLD RUN

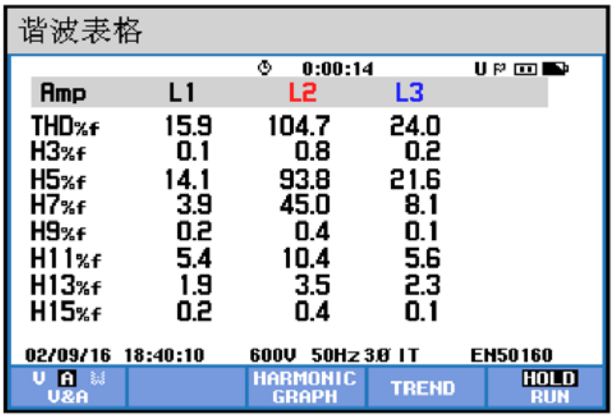
(**b**)

Figure 12. Harmonic compensation results without FD compensation: (**a**) source (L1), compensating (L2), load (L3) current waveforms ; (**b**) harmonic distortion rate graph.

Also, to highlight the effectiveness of the 6k+1 repetitive control scheme, the harmonic compensation results by only the LC filter is shown in Figure 13. As seen, the source current is still highly distorted after the compensation of the LC filter, with a THD of 15.9%. The main reasons are that the resonant frequency of LC filter is not precisely tuned at a domain harmonic frequency, and the performance of LC filter seriously depends on the internal resistance of grid source.



(**a**)



谐波表格

Amp	L1	L2	L3
THD%f	15.9	104.7	24.0
H3%f	0.1	0.8	0.2
H5%f	14.1	93.8	21.6
H7%f	3.9	45.0	8.1
H9%f	0.2	0.4	0.1
H11%f	5.4	10.4	5.6
H13%f	1.9	3.5	2.3
H15%f	0.2	0.4	0.1

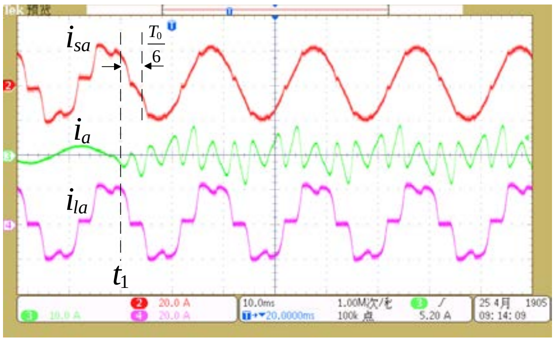
02/09/16 18:40:10 600V 50Hz 3Ø IT EN50160

(**b**)

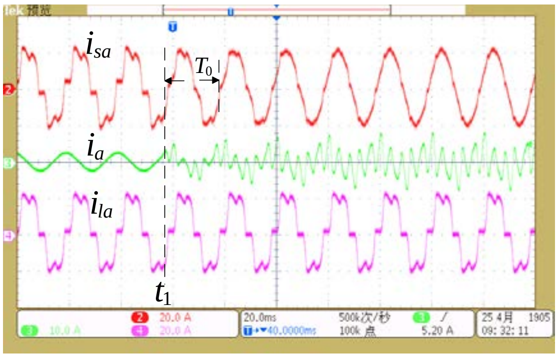
Figure 13. Harmonic compensation results by only the LC filter: (**a**) source (L1), compensating (L2), load (L3) current waveforms ; (**b**) harmonic distortion rate graph.

To verify the dynamic performance of the 6k+1 repetitive control scheme, Figure 14 shows the comparison experimental results of the proposed and traditional repetitive control schemes in transient process. As seen, before the time t_1, the harmonic compensation function is not enabled, i_a is only the reactive power current provided by LC filter with sinusoidal waveform, and i_{sa} is distorted by the load harmonics. At the time t_1, the harmonic compensation function is enabled.

The 6k+1 repetitive control scheme can take effect after $T_0/6$ time, and eliminate the steady-state error of harmonic tracking quickly. As a contrast, the traditional repetitive control scheme takes effect after T_0 time, and needs several T_0 periods to eliminate the steady-state error. The experimental results demonstrates that the 6k+1 repetitive control scheme has a much better dynamic performance than the traditional repetitive control scheme, which is consistent with theoretical analysis.



(**a**)



(**b**)

Figure 14. Dynamic performance comparison: (**a**) 6k+1 repetitive control scheme; (**b**) traditional repetitive control scheme

5. Conclusions

In this paper, a 6k+1 repetitive control scheme for HAPF is proposed, which aims at compensating the 6k+1 harmonics in three-phase power systems. The internal model of the 6k+1 repetitive controller is constructed by the general mathematical principles of traditional repetitive controller, and expressed using the complex-vector notation. A FD compensating method for 6k+1 repetitive controller is also presented. Through theoretical analysis and experiments, it is demonstrated that the 6k+1 repetitive control scheme can achieve a fast transient response with delay time of $T_0/6$, and good performance for compensating or suppressing the 6k+1 harmonics. Furthermore, due to the above features, the 6k+1 repetitive control scheme is also suitable to used in the current or voltage control for other three-phase grid-connected inverters.

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