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Published in:
International Journal of Electrical Power & Energy Systems

DOI (link to publication from Publisher):
[10.1016/j.ijepes.2016.12.014](https://doi.org/10.1016/j.ijepes.2016.12.014)

Publication date:
2017

Document Version
Early version, also known as pre-print

[Link to publication from Aalborg University](#)

Citation for published version (APA):
Dheer, D. K., Doolla, S., Bandyopadhyay, S., & Guerrero, J. M. (2017). Effect of placement of droop based generators in distribution network on small signal stability margin and network loss. *International Journal of Electrical Power & Energy Systems*, 88, 108–118. <https://doi.org/10.1016/j.ijepes.2016.12.014>

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1 Effect of placement of droop based generators in
2 distribution network on small signal stability margin
3 and network loss

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4 **Abstract**

Optimal location of distributed generators (DGs) in a utility-connected system is well described in literature. For a utility-connected system, issues related to small signal stability with DGs are insignificant due to the presence of a very strong grid. Optimally placed sources in utility connected microgrid system may not be optimal/stable in islanded condition. Among others issues, small signal stability margin is on the fore. The present research studied the effect of location of droop-controlled DGs on small signal stability margin and network loss on an IEEE 33-bus distribution system and a practical 22-bus radial distribution network. A complete dynamic model of an islanded microgrid was developed. From stability analysis, the study reports that both location of DGs and choice of droop coefficient have a significant effect on small signal stability and transient response of the system. For multi-objective optimization of the DG network, Pareto fronts were identified and the non-dominated solutions found with two and three generators. Results were validated by time domain simulations using MATLAB.

5 *Keywords:* Islanded microgrid, droop control, small signal stability margin.

6 **1. Introduction**

7 Growing environmental concerns competitive energy policies has led to
8 the decentralization of power generation. Installations of distributed genera-
9 tors (DGsphotovoltaic, wind, etc.) are expected to increase worldwide in the
10 next decade [1]. Due to their location being close to consumers, DGs provide
11 better power in terms of quality and reliability [2]. Controllable DGs along
12 with controllable loads present themselves to the upstream network as micro-
13 grid. Microgrids when operating in grid-connected mode provide/draw power

14 based on supply/demand within. In islanded mode (when not connected to
15 the main grid), microgrids operate as an independent power system [2].

16 The optimality in placement of a DG is decided by the owner based on the
17 availability of primary resource, site, and climatic conditions. Thus, choosing
18 an inappropriate location may result in losses and fall in power quality. Lit-
19 erature has widely addressed optimal placement of DGs in a network based
20 on objective functions of energy/power loss minimization, cost minimization,
21 voltage deviation minimization, profit maximization, loadability maximiza-
22 tion, etc [3]. Different approaches, methods, and optimization techniques for
23 DG siting and sizing are presented in [3]-[9].

24 DG siting and sizing is a multi-objective optimization problem classifiable
25 into two groups. The first group focuses on economics of the system [9]-[17].
26 With respect to islanded microgrids, minimization of total annual energy
27 losses and cost of energy for distributed generation is an area of much interest
28 to investors [10]. One study [9] presented a multi-objective optimization
29 problem of minimization of photovoltaic, wind generator and energy storage
30 investment cost, expectation of energy not supplied, and line loss. Economic
31 and environmental restrictions for a microgrid are outlined in [11]. Operation
32 cost (local generation cost and grid energy cost) minimization is presented
33 in [12]. An optimization problem considering operation cost and emission
34 minimization is presented in [13]. Economic dispatch problem in a hybrid,
35 droop-based microgrid is presented in [14].

36 The second group focuses on the optimal design of a microgrid based on
37 technical parameters such as network losses, maximum loadability, voltage
38 profile, reactive power, power quality, and droop setting. The assessment of
39 maximum loadability for a droop-based islanded microgrid is presented in
40 [18]-[20] considering reactive power requirements and various load types. A
41 decision-making program for load procurement in a distribution network is
42 presented in [21] based on uncertainty parameters like electricity demand,
43 local power investors, and electricity price. Optimal setting of droop to
44 minimize the cost of wind generator is presented in [22]. One wind-generation
45 study combined economics and stability issues due to uncertainty (volatility)
46 and its effect on small signal stability [23]-[24]. This study of small signal
47 stability in droop-based islanded microgrids is thus worthy in the context of
48 potential benefits of optimal DG placement to grid managers.

49 A microgrid may present as much complexities as a conventional power
50 system. When connected to a grid, these optimally placed and sized DGs
51 (inverter-based) operate in current control mode, feeding maximum power to

52 the network. When a grid is not available, these DGs shift to droop control
53 mode for effective power sharing.

54 Two important aspects of an islanded microgridload sharing and stabil-
55 ityare widely addressed in literature. A higher droop in these DGs is desired
56 for better power sharing and transient response [25]-[28]. Higher droop and
57 stability margin improves the transient response of the system and hence
58 power sharing among the sources [28]]. Inappropriate settings of droop value
59 may cause a power controller to operate at low frequency mode and fall
60 into an unstable region[29]-[31]. Stability of islanded microgrids [25]-[27] is
61 a growing operational challenge. A grid-connected system optimized for DG
62 sizing and siting may be vulnerable to small signal stability when islanded.

63 The impact of optimal DG placement on enhancement of small signal
64 stability margin and loss minimization is investigated on a standard IEEE
65 33-bus distribution system and a practical 22-bus radial distribution network
66 of a local utility. The rest of the paper is organized as follows: Section 2
67 presents a description of the system considered and the mathematical model
68 designed for stability studies. Eigen value analysis and identified Pareto
69 fronts are presented in Section 3. Validation of Eigen value analysis by time
70 domain simulation is presented in Section 4, followed by conclusions of the
71 study in Section 5.

72 2. System Description and Mathematical Modeling

73 Microgrids integrated with renewable energy sources through voltage source
74 inverters (VSIs), together with loads and interconnecting lines, were consid-
75 ered for the present study. An IEEE 33-bus radial distribution system (Fig.
76 1) and a 22-bus practical radial distribution network of Andhra Pradesh
77 Eastern Power Distribution Company Limited (APEPDCL) (Fig. 2) were
78 considered.

79 2.1. System State Space Equation

80 The modeling of VSIs, line, and load in d - q axis reference frame for small
81 signal stability is defined in [32]. Equation (1) is the overall state space
82 (matrix) equation for the total system under consideration. For the IEEE
83 33-bus system, the size of matrix A_{MG} with two generators is 152×152 , which
84 includes 26 states of DGs, 62 states of lines, and 64 states of loads. With
85 three generators, the size of A_{MG} is 165×165 (39 states of DGs, 62 states
86 of lines, and 64 states of loads). Similarly, for the 22-bus practical radial

87 distribution network of APEPDCL, the size of A_{MG} with three generators is
 88 121×121 (39 states of DGs, 40 states of lines, and 42 states of loads).

$$\begin{bmatrix} \Delta \dot{X}_{DG} \\ \Delta I_{DQLine} \\ \Delta I_{DQLoad} \end{bmatrix} = A_{MG} \begin{bmatrix} \Delta X_{DG} \\ \Delta I_{DQLine} \\ \Delta I_{DQLoad} \end{bmatrix} \quad (1)$$

89 2.2. Loss calculation

90 Consider a line of impedance $(R + jX) \Omega$ connected between two nodes
 91 through which current I_i is flowing. This current (I_i) can be expressed as:

$$I_i = I_d \pm jI_q \quad (2)$$

92 Real power loss in the line can be calculated using :

$$P_{loss,i} = I_i^2 \times R \quad (3)$$

93 where, $I_i^2 = I_d^2 + I_q^2$. Total real power loss of the network containing n lines
 94 is the sum of individual line loss which is

$$P_{loss} = \sum_{i=1}^n P_{loss,i} \quad (4)$$

95 2.3. Small Signal Stability Margin and Constraint

96 In this study, small signal stability margin is related to droop parameters.
 97 Higher droop is desired for better power sharing and transient response. The
 98 system is said to be stable if the real part of all Eigen values (other than 0)
 99 is negative. Small signal stability constraint is thus defined as::

$$R[\lambda_i] < 0, \forall \text{ eigenvalues except } 0 \quad (5)$$

100 where, λ_i is the i^{th} Eigenvalue of the system and $R[\lambda_i]$ is the real part of
 101 that Eigenvalue. Small signal stability limit can be obtained by varying the
 102 stability constraints. In this study, droop parameters (m_p and n_q) are taken
 103 as system variables. The droop constants are designed using (6) and (7). For
 104 the present work, initial values of m_p and n_q are taken as $1.0 \times 10^{-6} \text{ rpm/W}$
 105 and $1.0 \times 10^{-5} \text{ V/VAR}$, respectively.

$$m_{p1} \times P_1 = m_{p2} \times P_2 = \dots = m_{pn} \times P_n \quad (6)$$

$$n_{q1} \times Q_1 = n_{q2} \times Q_2 = \dots = n_{qn} \times Q_n \quad (7)$$

106 To perform Eigen value analysis, draw the root locus plot and calcu-
 107 late the losses, we obtain the operating condition/point using time domain
 108 simulation or from load flow analysis. Literature on load flow analysis for
 109 islanded systems is scarce [33]. The present study preferred time domain
 110 simulation using MATLAB/SIMULINK to obtain the operating point. The
 111 time domain simulation is also used to validate the Eigen value analysis.

112 The optimal location of DGs for an IEEE 33-bus radial distributed system
 113 presented in [34] is taken as base case for this study. The line and load data
 114 for a standard IEEE 33-bus network is available in [35]. Description of the
 115 22-bus practical radial distribution network of APEPDCL is available in [36]-
 116 [37].

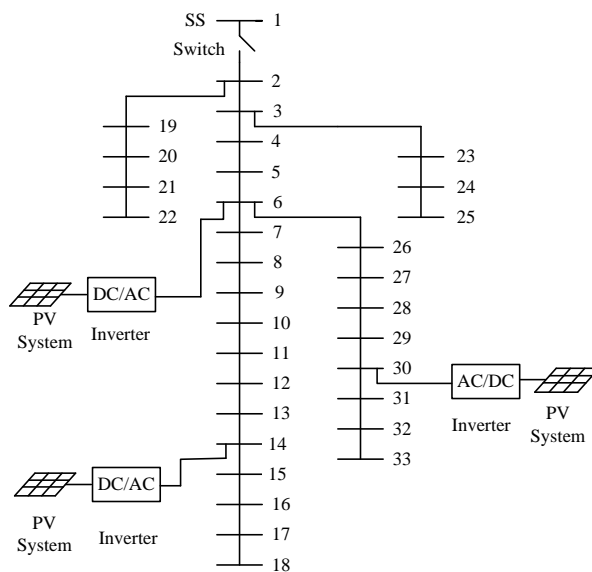


Figure 1: IEEE 33-bus radial distribution system

117 3. Eigen Value Analysis and Pareto Front Identification

118 3.1. IEEE 33-bus system with two DGs

119 The optimal locations of two generators (in a grid-connected system)
 120 based on loss minimization proposed in [34] are at nodes 6 and 30. When
 121 islanded, these two generators operate in droop control mode (for size in

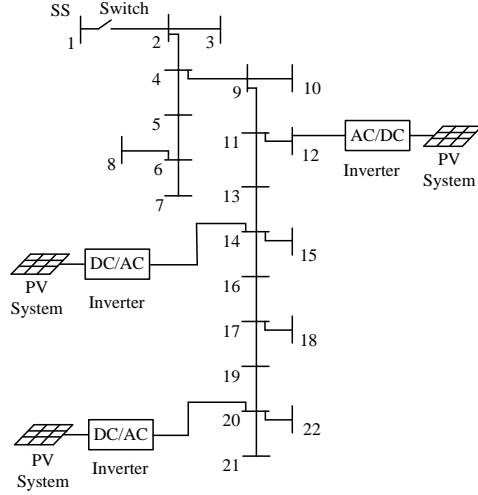


Figure 2: Practical radial distribution (22 bus) network APEPDCL

122 proportion of 1:0.50) for load sharing. From the droop law, we know that
 123 system frequency takes a new steady state value till secondary control acts.
 124 System simulation (time domain) is performed with these two generators
 125 at various locations (cases) in a standard IEEE 33-bus radial distribution
 126 network. From the operating points, state space matrix is obtained using
 127 (1). Root locus analysis is performed for these cases by varying the droop
 128 constants to identify the stability limit. The values of $m_{p,max}$ and $n_{q,max}$
 129 are noted when the system reaches an unstable region. Losses in the system,
 130 minimum voltage value in the total network, $m_{p,max}$, $n_{q,max}$, and minimum
 131 distance between the DGs for all these cases are presented in Table 1. It
 132 is clear that the maximum values of $m_{p,max}$ and $n_{q,max}$ are not the best
 133 for case 1. This is true since the decision for placement of generators in this
 134 location in [34] was made with separate conditions (grid-connected, exporting
 135 power, etc.). However, in systems where grid reliability is poor (true in many
 136 developing countries), such location may not be optimum. From network loss,
 137 stability, and voltage perspectives, case 1, case 6, and case 13 are preferred
 138 options, respectively.

139 Figure 3 shows the plot between $m_{p,max}$ and Z , while Fig. 4 shows the
 140 plot between $n_{q,max}$ and Z for the cases tabulated in Table 1. Electrical
 141 distance (in terms of impedance) between generators is an important param-
 142 eter contributing to small signal stability margin. From Figs. 3 and 4, it
 143 is observed that higher electrical distance between sources results in better

Table 1: Various case study results for two DGs placement for IEEE 33-bus radial network

Case	DG-1 Node	DG-2 Node	P_{loss} (kW)	V_{min} (p.u.)	$m_{p,max}$ (10^{-5})	$n_{q,max}$ (10^{-4})	Z (Ω)
1	6	30	65.05	0.9469	1.24	1.34	3.5709
2	24	30	74.27	0.9303	2.30	2.21	7.1671
3	18	24	120.48	0.9193	4.90	5.92	16.8053
4	13	30	264.07	0.9206	3.43	2.84	11.1844
5	18	25	143.45	0.9068	5.33	6.10	17.9422
6	18	22	207.91	0.8855	5.55	6.31	19.6787
7	22	33	185.24	0.9003	3.39	3.84	12.4616
8	22	25	175.09	0.8906	2.08	2.72	7.3835
9	25	33	106.39	0.9131	3.16	3.48	10.7276
10	18	33	386.46	0.8833	5.39	4.58	19.2281
11	6	14	83.96	0.9528	2.44	3.12	8.4827
12	6	18	120.38	0.9524	3.60	5.08	0.9524
13	6	10	72.29	0.9532	1.53	1.97	5.1831
14	3	5	97.04	0.9335	0.88	0.37	0.8118
15	6	26	84.97	0.9487	0.82	0.23	0.2278
16	3	4	103.26	0.9273	0.80	0.27	0.4107
17	9	10	238.42	0.8823	0.73	0.59	1.2764
18	32	33	291.85	0.8507	0.43	0.47	0.6304
19	17	18	525.83	0.7425	0.65	0.50	0.9302
20	24	25	182.31	0.8890	0.66	0.58	1.1377

144 stability margin. Root locus plot and time domain simulation further prove
 145 this point. Case 1 (base case), case 6 (highest stability margin), and case 18
 146 (least stability margin) are considered for detailed analysis.

147 Figure. 5 shows the root locus plot of the system for case -1, case -6, and
 148 case -18. λ_{12} indicates the interaction of low-frequency modes between two
 149 sources. From the three sets of Eigen traces, it s clear that the system is
 150 going into an unstable region after a certain value of m_P . In Fig. 5, λ_{12} for
 151 case -1 starts from $-15.066 \pm j 16.60$ and reaches the imaginary axis at $0 \pm j$
 152 74.40 , while for case -6 and case -18 the starting points for λ_{12} are at -15.346
 153 $\pm j 1.1835$ and $-12.971 \pm j 28.278$ and they reach the imaginary axis at $0 \pm j$
 154 87.05 and $0 \pm j 58.84$, respectively. From these root locus plots, the effect
 155 of impedance between sources on stability margin is observed, and it is clear
 156 that, distance between sources influences the stability of the system.

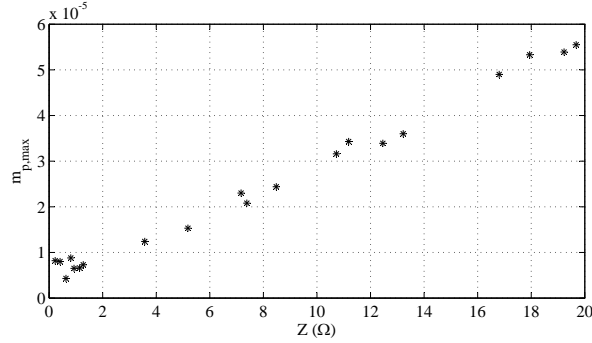


Figure 3: Impedance vs. $m_{p,max}$ plot

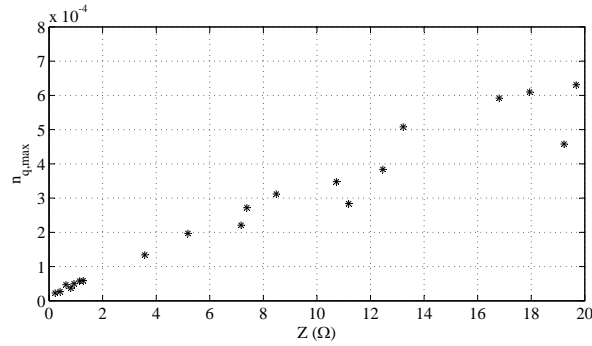


Figure 4: Impedance vs. $n_{q,max}$ plot

157 3.2. IEEE 33-bus system with three DGs

158 Optimal locations of three generators (in grid connected system) based
 159 on loss minimization, proposed in [34], are at nodes 6, 14, and 30. When is-
 160 lated, these three generators operate in droop control mode for load sharing.
 161 System simulation (time domain) is performed with these three generators
 162 at various locations (cases) in a standard IEEE 33 bus radial distribution
 163 network. From the operating points, state space matrix is obtained using
 164 (1). Root locus analysis is performed for these cases by varying droop con-
 165 stants to identify the stability limit. The values of $m_{p,max}$ and $n_{q,max}$ are
 166 noted when the system reaches an unstable region. Losses in the system,
 167 minimum voltage value in the total network, $m_{p,max}$, $n_{q,max}$ and minimum
 168 distance between the DGs for all these cases are presented in Table. 2.

169 It is clear that the maximum values of $m_{p,max}$, $n_{q,max}$ are not the highest

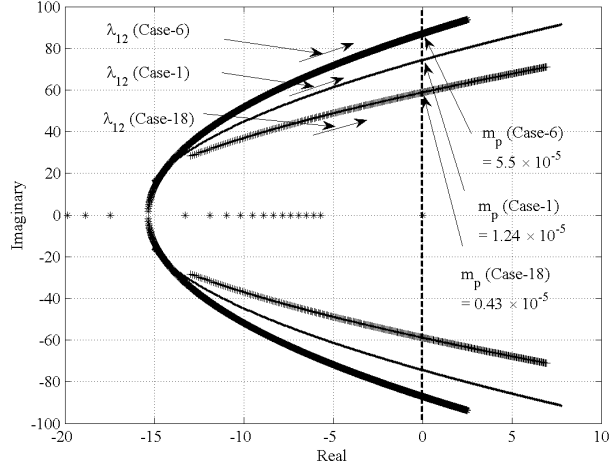


Figure 5: Table 1, cases-1, 6, 18 : Rootlocus plot with variation in droop gain m_p

170 for case-1. This is true since the decision for this location for placement of
 171 generators in this location in [34] was done with separate conditions (grid-
 172 connected, exporting power, etc). From network loss, stability, and voltage
 173 perspectives, case -37, case -3 and case -33 are preferred options.

174 Figure. 6 shows the eigenvalues plot for case -1 (base case). Out of 165
 175 eigenvalues 92 eigenvalues are shown in figure (rest of the Eigenvalues are
 176 highly damped). For dynamic stability, low-frequency mode Eigenvalues,
 177 which are sensitive to the droop gains of the system, are of interest. These
 178 low-frequency modes correspond to the power controller mode of the VSI.
 179 Case -1 (base case), case -3 (highest stability margin), and case -41 (least
 180 stability margin) are considered for detailed analysis. Two complex conjugate
 181 low-frequency mode trajectories sensitive to real power droop gain for these
 182 cases are shown in Fig. 7, Fig. 8 and Fig. 9, respectively. λ_{12} shows the
 183 interaction of low frequency modes between VSIs 1 and 2 while λ_{13} shows the
 184 interaction of low frequency modes between VSIs 1 and 3. This trajectory
 185 shows that λ_{12} goes into an unstable mode at a lower value of m_p than λ_{13} .

186 In Fig. 7, λ_{12} starts at $-15.7 \pm j 6.2054$ and reaches the imaginary axis
 187 at $0 \pm j 81.265$. In Figs. 8 and 9, λ_{12} starts from $-15.24 \pm j 8.065$ and -11.213
 188 $\pm j 33.373$ and reaches to imaginary axis at $0 \pm j 86.75$ and $0 \pm j 60.118$
 189 respectively. From these root locus plots, the impact of minimum distance
 190 between sources on stability margin is clearly observed, and it is understood
 191 that sources separated with higher impedance have relatively higher stability

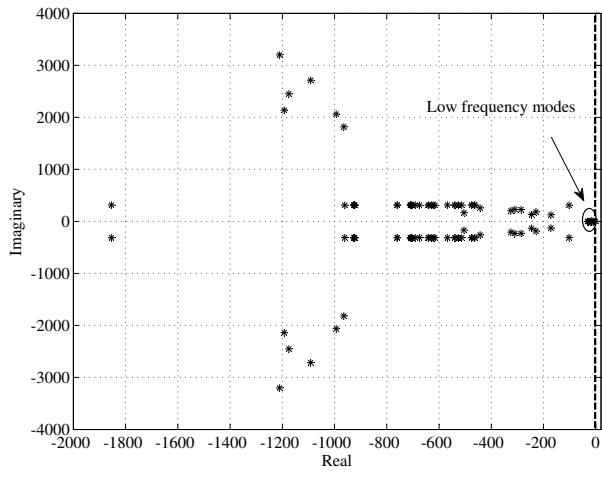


Figure 6: Eigenvalue plot of the microgrid

192 margin.

Table 2: Various case study results for three DGs placement for IEEE 33-bus radial network

Case	DG-1 Node	DG-2 Node	DG-3 Node	P_{loss} (kW)	V_{min} (p.u.)	$m_{p,max}$ (10^{-5})	$n_{q,max}$ (10^{-4})	Z_{min} (Ω)
1	6	30	14	60.03	0.9581	1.81	1.31	3.5709
2	25	33	18	67.98	0.9635	2.91	4.12	10.7274
3	22	33	18	86.76	0.9441	2.94	4.73	12.4616
4	24	30	8	32.36	0.9694	0.92	1.80	6.1455
5	24	30	18	44.86	0.9751	2.38	2.62	7.1671
6	6	30	18	79.30	0.9577	1.78	1.38	3.4992
7	24	30	6	45.94	0.9530	0.53	0.76	3.5965
8	24	30	22	52.07	0.9364	1.35	2.05	6.2483
9	24	6	18	84.58	0.9613	1.22	1.29	3.5965
10	10	30	15	126.06	0.9347	1.19	1.31	4.0902
11	10	24	15	153.56	0.9360	1.18	1.30	4.0902
12	10	22	15	151.49	0.9167	1.19	1.35	4.0902
13	24	30	20	45.87	0.9370	1.08	1.50	4.4788
14	24	20	18	95.61	0.9321	1.80	1.84	4.4788
15	24	30	3	46.11	0.9471	0.54	0.57	1.6905
16	24	3	18	75.06	0.9422	0.44	0.16	1.6905
17	24	21	3	135.0	0.9235	0.43	0.53	1.6905
18	24	22	18	114.78	0.9299	2.23	2.62	6.2483
19	6	11	18	212.26	0.9554	0.94	1.71	5.3783
20	2	6	18	65.20	0.9617	1.07	0.97	2.456
21	24	21	2	139.08	0.9181	0.40	0.59	2.2352
22	2	6	30	36.66	0.9528	0.61	0.79	2.456
23	24	21	6	96.28	0.9509	0.82	1.04	3.5965
24	8	14	18	362.75	0.9062	0.62	1.46	4.7548
25	2	4	6	76.63	0.9514	0.41	0.42	0.964
26	24	21	11	91.14	0.9409	1.63	2.01	5.0983
27	7	26	30	64.41	0.9514	0.64	0.26	0.8209
28	10	14	18	386.80	0.8604	0.60	1.16	3.2999
29	3	6	11	41.92	0.9627	0.72	0.77	2.3629
30	3	6	30	34.80	0.9532	0.60	0.72	2.3629
31	24	21	14	85.60	0.9363	1.83	2.08	5.0983
32	24	30	11	25.81	0.9770	1.48	2.68	7.1429
33	23	30	18	51.56	0.9779	2.31	2.25	6.0099
34	23	33	18	77.61	0.9746	2.84	3.22	15.6619
35	23	19	3	112.85	0.9244	0.37	0.25	0.5472
36	6	12	18	231.84	0.9552	0.82	1.70	5.7461
37	24	30	14	25.69	0.9759	1.94	2.64	7.1671
38	23	3	4	100.50	0.9301	0.43	0.25	0.4170
39	19	2	3	130.38	0.9224	0.41	0.27	0.2267
40	5	6	26	67.69	0.9532	0.43	0.21	0.2278
41	29	30	31	193.0	0.8980	0.33	0.34	0.6214
42	24	23	3	119.09	0.9236	0.37	0.31	0.5472
43	21	20	19	246.34	0.9051	0.42	0.45	0.6297
44	4	6	8	52.80	0.9630	0.44	0.64	1.5007
45	28	30	32	175.37	0.9153	0.35	0.61	1.6249
46	10	11	12	295.94	0.8632	0.50	0.22	0.2071

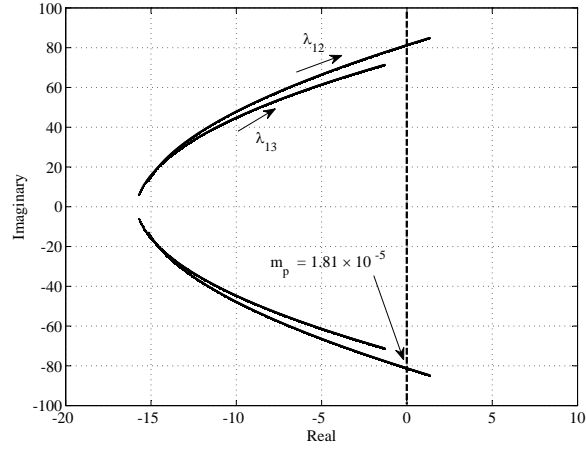


Figure 7: Table 2, case-1 : Rootlocus plot with variation in droop gain m_p

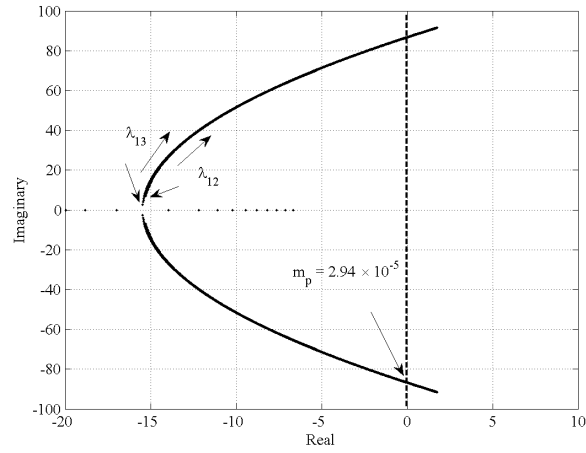


Figure 8: Table 2, case-3 : Rootlocus plot with variation in droop gain m_p

193 *3.3. 22-bus APEPDCL Distribution Network*

194 The optimal locations of three generators (in a grid-connected system)
 195 based on loss minimization, proposed in [36], are at nodes 12, 14, and 20.
 196 System simulation (time domain) is performed with these three generators
 197 at various locations (cases) in the 22-bus APEPDCL distribution network.
 198 From the operating points, state space matrix is obtained using (1). Root

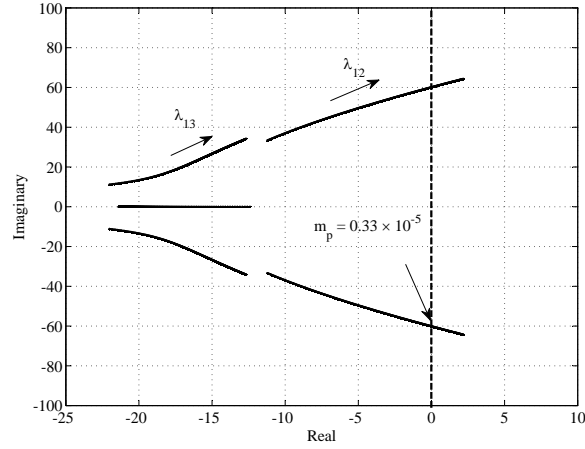


Figure 9: Table 2, case-41 : Rootlocus plot with variation in droop gain m_p

199 locus analysis is performed for these cases by varying droop constants to identify the stability limit. The values of $m_{p,max}$ and $n_{q,max}$ are noted when the system reaches an unstable region. Losses in the system, minimum voltage value in the total network, $m_{p,max}$, $n_{q,max}$, and minimum distance between the DGs for all these cases are presented in Table. 3.

204 It is clear that the maximum values of $m_{p,max}$, $n_{q,max}$ are not the highest for case 1. This is true since the decision for placement of generators in this location was made with separate conditions (grid-connected, exporting power, etc.). From network loss, stability, and voltage perspectives, case 8, case 6, and case 8 are preferred options. Case 1 (base case), case 6 (highest stability margin) and case 20 (least stability margin) are considered for detailed analysis.

Table 3: Various case study results for three DGs placement for APEPDCL 22-bus practical radial network

Case	DG-1 Node	DG-2 Node	DG-3 Node	P_{loss} (kW)	V_{min} (p.u.)	$m_{p,max}$ (10^{-6})	$n_{q,max}$ (10^{-5})	Z_{min} (Ω)
1	12	14	20	0.740	0.9952	7.23	4.48	1.2137
2	3	14	20	0.752	0.9967	7.29	4.90	1.2137
3	8	12	22	3.154	0.9951	12.06	8.49	3.6752
4	8	13	22	2.627	0.9958	11.01	8.04	3.0911
5	4	15	22	0.612	0.9971	8.41	6.10	1.8402
6	8	10	22	4.4459	0.9942	13.01	8.16	2.9157
7	3	15	22	0.953	0.9965	8.45	6.25	1.8402
8	4	14	20	0.367	0.9972	7.26	4.85	1.1897
9	9	15	22	0.732	0.9968	8.33	5.76	1.8402
10	8	9	17	5.078	0.9965	10.32	7.10	2.8026
11	3	10	17	3.675	0.9965	10.04	5.60	1.5681
12	8	11	17	3.586	0.9967	8.95	6.49	2.0428
13	8	10	18	5.041	0.9961	10.66	7.47	2.9157
14	12	15	18	0.943	0.9953	5.91	3.49	0.5567
15	15	18	22	2.712	0.9903	7.60	3.50	0.5567
16	10	12	15	2.050	0.9954	8.06	4.11	0.8826
17	13	14	15	1.514	0.9945	6.02	2.13	0.0249
18	20	21	22	6.410	0.9840	6.43	2.17	0.0980
19	9	10	11	5.281	0.9879	6.61	2.16	0.0615
20	6	7	8	19.336	0.9683	5.83	2.15	0.0673

211 Plots of $m_{p,max}$ vs. Z_{min} (minimum impedance among sources) and $n_{q,max}$
212 vs. Z_{min} are shown in Figs. 10 and 11, respectively.

213 Figures. 12, 13 and 14 show root locus plot for cases 6, 8 and 20, re-
214 spectively. λ_{12} shows the interaction of low-frequency modes between VSIs 1
215 and 2 while λ_{13} shows the interaction of low frequency modes between VSIs
216 1 and 3. This trajectory shows that λ_{12} goes into an unstable mode at a
217 lower value of m_p than λ_{13} . In Fig. 12 λ_{12} starts from an approximate value
218 of $-15.55 \pm j 21.27$ and reaches the imaginary axis at an approximate value
219 of $0 \pm j 63.3$. In Figs. 13 and 14, λ_{12} approximately starts from $-15.27 \pm j$
220 15.275 and $-16.19 \pm j 24.70$ and reaches the imaginary axis approximately
221 at $0 \pm j 71.1$ and $0 \pm j 59.7$ respectively. The following are some critical
222 observations from the case studies:

- 223 • The system configuration (generator location) with low losses in grid

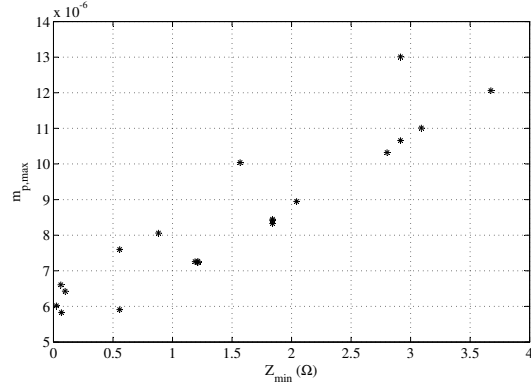


Figure 10: Plot between $m_{p,max}$ vs. Z_{min}

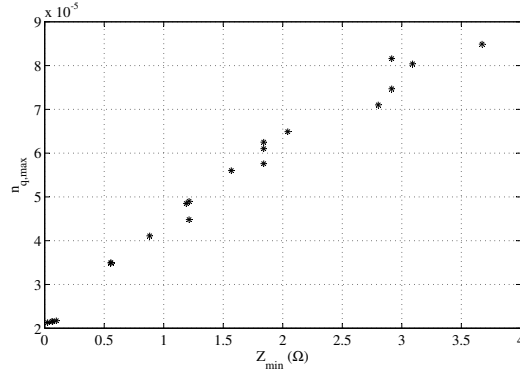


Figure 11: Plot between $n_{q,max}$ vs. Z_{min}

224 connected mode may suffer from stability issues when islanded. This
 225 can be a serious problem when the reliability of the main grid is poor.

226 • The interaction of low-frequency modes between various DGs is differ-
 227 ent and the location of some inverters is critical (inverter 2 in this case)
 228 with respect to the stability.

229 • Stability margin (gain of droop constant) is a function of minimum
 230 distance between the generators in an islanded network.

231 • It is important to choose an optimal location for these generators by
 232 considering stability and network losses.

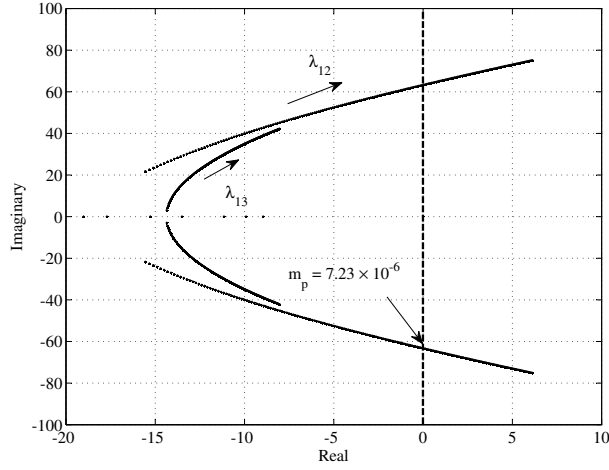


Figure 12: Table 3, case-1 : Rootlocus plot with variation in droop gain m_p

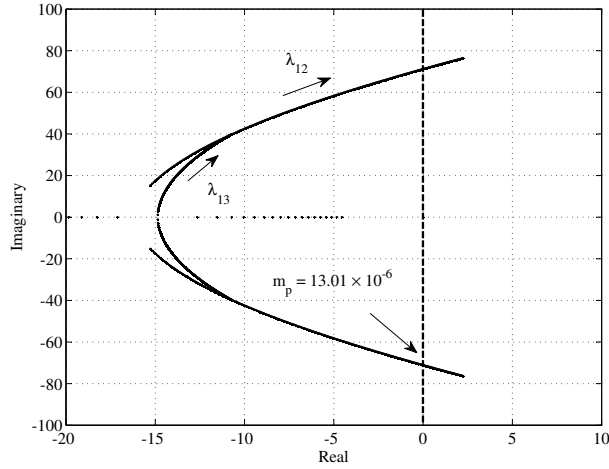


Figure 13: Table 3, case-6 : Rootlocus plot with variation in droop gain m_p

233 *3.4. Determination of Pareto Front in an Islanded Microgrid*

234 The locations of generators should depend on network losses and overall
 235 stability of the system. For multi-objective optimization of the DG network,
 236 Pareto optimal front should be identified. Data in Tables 2 and 3 are plotted
 237 and Pareto fronts (set of non dominated solutions) obtained between $m_{p,max}$
 238 vs. real power loss and $n_{q,max}$ vs. reactive power loss (Figs. 15, 16 and 17,
 239 18 respectively).

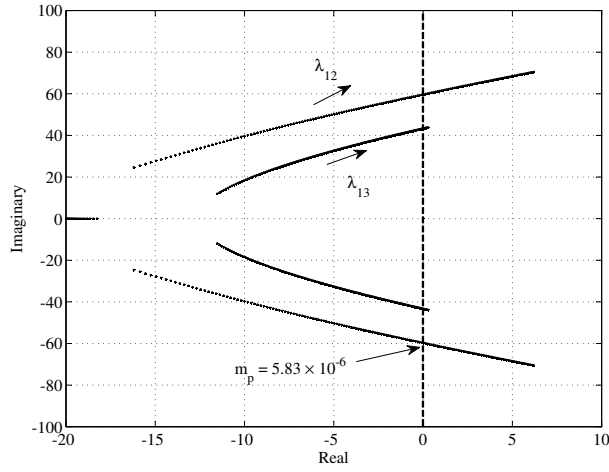


Figure 14: Table 3, case-20 : Rootlocus plot with variation in droop gain m_p

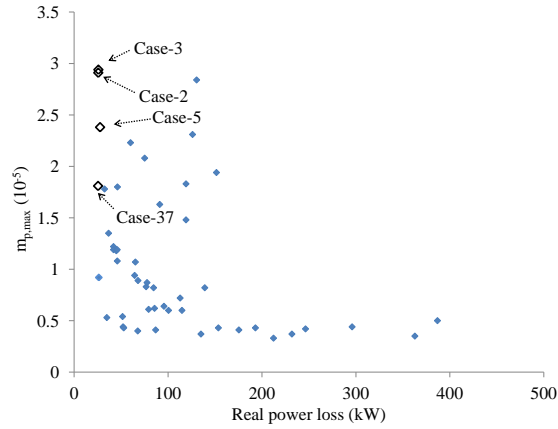


Figure 15: Real power loss vs. $m_{p,max}$ for IEEE 33 bus system with three DGs - Pareto front shown in open boxes

240

Critical observations from Pareto fronts (for 33-bus system) are:

241

- Cases corresponding to Pareto fronts (shown in open box) obtained in Fig. 15 are 2, 3, 5 and 37.

242

243

- Cases corresponding to Pareto fronts (shown in open box) obtained in Fig. 16 are 2, 3 and 32.

244

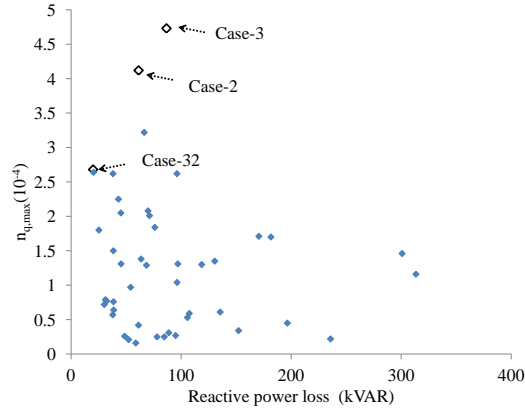


Figure 16: Reactive power loss vs. $n_{q,max}$ for IEEE 33 bus system with three DGs - Pareto front shown in open boxes

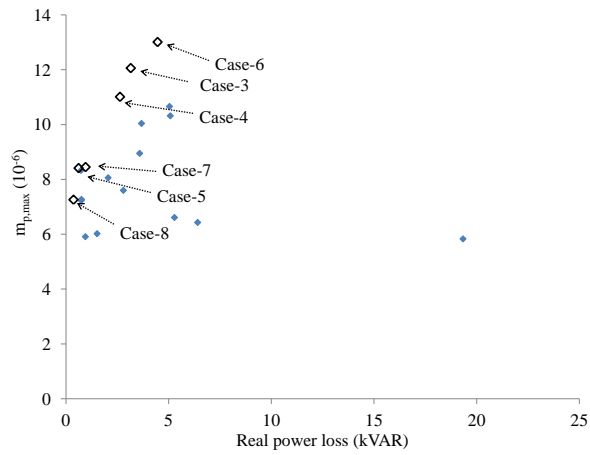


Figure 17: Real power loss vs. $m_{p,max}$ for 22 bus APEPDCL network with three DGs - Pareto front shown in open boxes

- 245 • Case-1 which represents optimal location of sources in a grid-connected
- 246 system, does not lie on the Pareto front. This clearly indicates that
- 247 the optimal placement of sources in a grid-connected microgrid is not
- 248 optimal during islanding.

249 Critical observations from Pareto fronts (for 22 bus practical system) are:

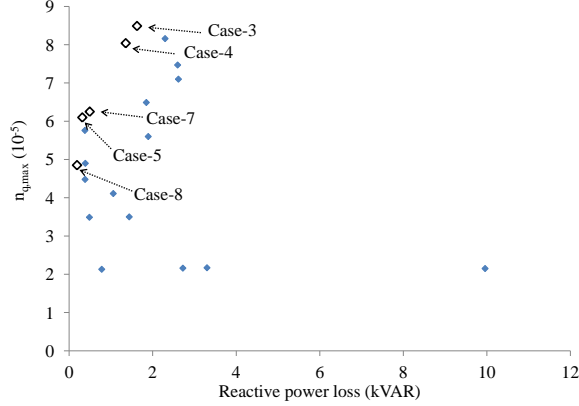


Figure 18: Reactive power loss vs. $n_{q,max}$ for 22 bus APEPDCL network with three DGs
- Pareto front shown in open boxes

- 250 • Cases corresponding to Pareto fronts (shown in open box) obtained in
251 Fig. 17 are 3, 4, 5, 6, 7 and 8.
- 252 • Cases corresponding to Pareto fronts (shown in open box) obtained in
253 Fig. 18 are 3, 4, 5, 7 and 8.
- 254 • Similar to the previous example, case -1 does not lie on the Pareto
255 front.
- 256 • Cases 3, 4, and 6 have high stability margin and higher losses, while
257 cases 5, 7, and 8 have low stability margin and low losses.

258 4. Simulation - Time Domain Validation

259 Time domain simulation is performed on both the networks for validation
260 of stability analysis. Simulation results for the three DG system (case -1 of
261 Table 2) and for the practical network (case -1 of Table. 3) are shown in Fig.
262 19 and Fig. 20, respectively.

263 The system is stable and sharing power as per the droop law. The effect
264 of higher value of droop parameter is investigated by changing the droop
265 value (beyond $m_{p,max}$). At time $t = 2s$ for a higher value of $m_p (> m_{p,max})$,
266 power output of DGs is oscillating with increasing amplitude as shown in
267 Fig. 19, which indicates that the system is now unstable.

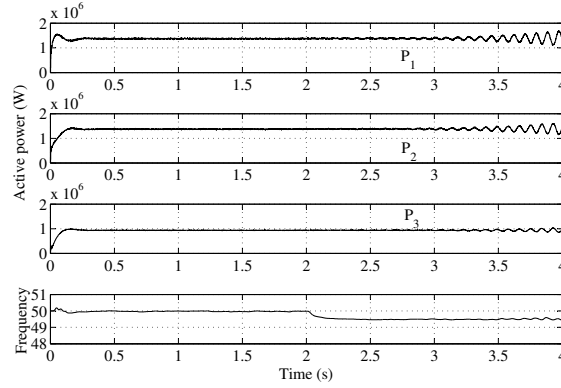


Figure 19: Real power output of DGs and system frequency in Std. IEEE 33 network

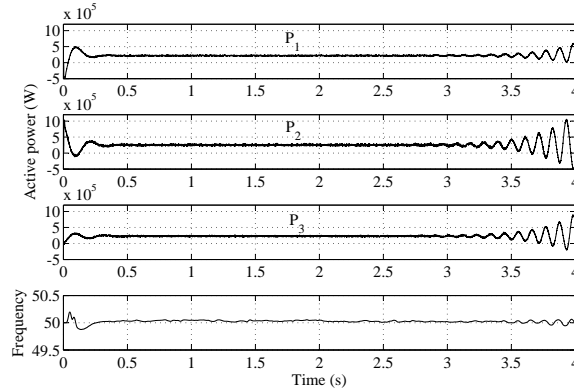


Figure 20: Real power output of DGs and system frequency in practical 22 bus distribution network

268 5. Conclusion

269 The effect of location of droop-based sources on small signal stability,
 270 transient response, and network losses in an islanded network is investigated.
 271 A standard IEEE 33-bus network and a 22-bus practical distribution network
 272 are chosen. A microgrid model is developed for both the networks with droop-
 273 based sources, network components, and loads for stability analysis. Higher
 274 droop in DGs is desired for better power sharing and transient response.
 275 Small signal stability is studied for various locations of DGs (two/three) by
 276 varying the droop constant. From the stability study, it is found that a sys-
 277 tem optimized for losses in grid-connected mode may suffer from small signal

278 stability issues and poor transient response when in islanded configuration.
279 The minimum distance between generators in the network also has an im-
280 pact on small signal stability. For multi-objective optimization of the DG
281 network, Pareto optimal front is identified. Results of small signal stability
282 analysis are verified using time domain simulation in MATLAB for both the
283 networks.

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