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# Effect of placement of droop based generators in distribution network on small signal stability margin and network loss

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## Abstract

Optimal location of distributed generators (DGs) in a utility-connected system is well described in literature. For a utility-connected system, issues related to small signal stability with DGs are insignificant due to the presence of a very strong grid. Optimally placed sources in utility connected microgrid system may not be optimal/stable in islanded condition. Among others issues, small signal stability margin is on the fore. The present research studied the effect of location of droop-controlled DGs on small signal stability margin and network loss on an IEEE 33-bus distribution system and a practical 22-bus radial distribution network. A complete dynamic model of an islanded microgrid was developed. From stability analysis, the study reports that both location of DGs and choice of droop coefficient have a significant effect on small signal stability and transient response of the system. For multi-objective optimization of the DG network, Pareto fronts were identified and the non-dominated solutions found with two and three generators. Results were validated by time domain simulations using MATLAB.

*Keywords:* Islanded microgrid, droop control, small signal stability margin.

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## 1. Introduction

Growing environmental concerns competitive energy policies has led to the decentralization of power generation. Installations of distributed generators (DGs) photovoltaic, wind, etc.) are expected to increase worldwide in the next decade [1]. Due to their location being close to consumers, DGs provide better power in terms of quality and reliability [2]. Controllable DGs along with controllable loads present themselves to the upstream network as microgrid. Microgrids when operating in grid-connected mode provide/draw power

14 based on supply/demand within. In islanded mode (when not connected to  
15 the main grid), microgrids operate as an independent power system [2].

16 The optimality in placement of a DG is decided by the owner based on the  
17 availability of primary resource, site, and climatic conditions. Thus, choosing  
18 an inappropriate location may result in losses and fall in power quality. Lit-  
19 erature has widely addressed optimal placement of DGs in a network based  
20 on objective functions of energy/power loss minimization, cost minimization,  
21 voltage deviation minimization, profit maximization, loadability maximiza-  
22 tion, etc [3]. Different approaches, methods, and optimization techniques for  
23 DG siting and sizing are presented in [3]-[9].

24 DG siting and sizing is a multi-objective optimization problem classifiable  
25 into two groups. The first group focuses on economics of the system [9]-[17].  
26 With respect to islanded microgrids, minimization of total annual energy  
27 losses and cost of energy for distributed generation is an area of much interest  
28 to investors [10]. One study [9] presented a multi-objective optimization  
29 problem of minimization of photovoltaic, wind generator and energy storage  
30 investment cost, expectation of energy not supplied, and line loss. Economic  
31 and environmental restrictions for a microgrid are outlined in [11]. Operation  
32 cost (local generation cost and grid energy cost) minimization is presented  
33 in [12]. An optimization problem considering operation cost and emission  
34 minimization is presented in [13]. Economic dispatch problem in a hybrid,  
35 droop-based microgrid is presented in [14].

36 The second group focuses on the optimal design of a microgrid based on  
37 technical parameters such as network losses, maximum loadability, voltage  
38 profile, reactive power, power quality, and droop setting. The assessment of  
39 maximum loadability for a droop-based islanded microgrid is presented in  
40 [18]-[20] considering reactive power requirements and various load types. A  
41 decision-making program for load procurement in a distribution network is  
42 presented in [21] based on uncertainty parameters like electricity demand,  
43 local power investors, and electricity price. Optimal setting of droop to  
44 minimize the cost of wind generator is presented in [22]. One wind-generation  
45 study combined economics and stability issues due to uncertainty (volatility)  
46 and its effect on small signal stability [23]-[24]. This study of small signal  
47 stability in droop-based islanded microgrids is thus worthy in the context of  
48 potential benefits of optimal DG placement to grid managers.

49 A microgrid may present as much complexities as a conventional power  
50 system. When connected to a grid, these optimally placed and sized DGs  
51 (inverter-based) operate in current control mode, feeding maximum power to

the network. When a grid is not available, these DGs shift to droop control mode for effective power sharing.

Two important aspects of an islanded microgridload sharing and stability are widely addressed in literature. A higher droop in these DGs is desired for better power sharing and transient response [25]-[28]. Higher droop and stability margin improves the transient response of the system and hence power sharing among the sources [28]]. Inappropriate settings of droop value may cause a power controller to operate at low frequency mode and fall into an unstable region[29]-[31]. Stability of islanded microgrids [25]-[27] is a growing operational challenge. A grid-connected system optimized for DG sizing and siting may be vulnerable to small signal stability when islanded.

The impact of optimal DG placement on enhancement of small signal stability margin and loss minimization is investigated on a standard IEEE 33-bus distribution system and a practical 22-bus radial distribution network of a local utility. The rest of the paper is organized as follows: Section 2 presents a description of the system considered and the mathematical model designed for stability studies. Eigen value analysis and identified Pareto fronts are presented in Section 3. Validation of Eigen value analysis by time domain simulation is presented in Section 4, followed by conclusions of the study in Section 5.

## 2. System Description and Mathematical Modeling

Microgrids integrated with renewable energy sources through voltage source inverters (VSIs), together with loads and interconnecting lines, were considered for the present study. An IEEE 33-bus radial distribution system (Fig. 1) and a 22-bus practical radial distribution network of Andhra Pradesh Eastern Power Distribution Company Limited (APEPDCL) (Fig. 2) were considered.

### 2.1. System State Space Equation

The modeling of VSIs, line, and load in  $d$ - $q$  axis reference frame for small signal stability is defined in [32]. Equation (1) is the overall state space (matrix) equation for the total system under consideration. For the IEEE 33-bus system, the size of matrix  $A_{MG}$  with two generators is  $152 \times 152$ , which includes 26 states of DGs, 62 states of lines, and 64 states of loads. With three generators, the size of  $A_{MG}$  is  $165 \times 165$  (39 states of DGs, 62 states of lines, and 64 states of loads). Similarly, for the 22-bus practical radial

87 distribution network of APEPDCL, the size of  $A_{MG}$  with three generators is  
 88  $121 \times 121$  (39 states of DGs, 40 states of lines, and 42 states of loads).

$$\begin{bmatrix} \Delta \dot{X}_{DG} \\ \Delta I_{DQ_{Line}} \\ \Delta I_{DQ_{Load}} \end{bmatrix} = A_{MG} \begin{bmatrix} \Delta X_{DG} \\ \Delta I_{DQ_{Line}} \\ \Delta I_{DQ_{Load}} \end{bmatrix} \quad (1)$$

### 89 2.2. Loss calculation

90 Consider a line of impedance  $(R + jX) \Omega$  connected between two nodes  
 91 through which current  $I_i$  is flowing. This current ( $I_i$ ) can be expressed as:

$$I_i = I_d \pm jI_q \quad (2)$$

92 Real power loss in the line can be calculated using :

$$P_{loss,i} = I_i^2 \times R \quad (3)$$

93 where,  $I_i^2 = I_d^2 + I_q^2$ . Total real power loss of the network containing  $n$  lines  
 94 is the sum of individual line loss which is

$$P_{loss} = \sum_{i=1}^n P_{loss,i} \quad (4)$$

### 95 2.3. Small Signal Stability Margin and Constraint

96 In this study, small signal stability margin is related to droop parameters.  
 97 Higher droop is desired for better power sharing and transient response. The  
 98 system is said to be stable if the real part of all Eigen values (other than 0)  
 99 is negative. Small signal stability constraint is thus defined as::

$$R[\lambda_i] < 0, \forall \text{ eigenvalues except } 0 \quad (5)$$

100 where,  $\lambda_i$  is the  $i^{th}$  Eigenvalue of the system and  $R[\lambda_i]$  is the real part of  
 101 that Eigenvalue. Small signal stability limit can be obtained by varying the  
 102 stability constraints. In this study, droop parameters ( $m_p$  and  $n_q$ ) are taken  
 103 as system variables. The droop constants are designed using (6) and (7). For  
 104 the present work, initial values of  $m_p$  and  $n_q$  are taken as  $1.0 \times 10^{-6} \text{ rpm/W}$   
 105 and  $1.0 \times 10^{-5} \text{ V/VAR}$ , respectively.

$$m_{p1} \times P_1 = m_{p2} \times P_2 = \dots = m_{pn} \times P_n \quad (6)$$

$$n_{q1} \times Q_1 = n_{q2} \times Q_2 = \dots = n_{qn} \times Q_n \quad (7)$$

To perform Eigen value analysis, draw the root locus plot and calculate the losses, we obtain the operating condition/point using time domain simulation or from load flow analysis. Literature on load flow analysis for islanded systems is scarce [33]. The present study preferred time domain simulation using MATLAB/SIMULINK to obtain the operating point. The time domain simulation is also used to validate the Eigen value analysis.

The optimal location of DGs for an IEEE 33-bus radial distributed system presented in [34] is taken as base case for this study. The line and load data for a standard IEEE 33-bus network is available in [35]. Description of the 22-bus practical radial distribution network of APEPDCL is available in [36]-[37].

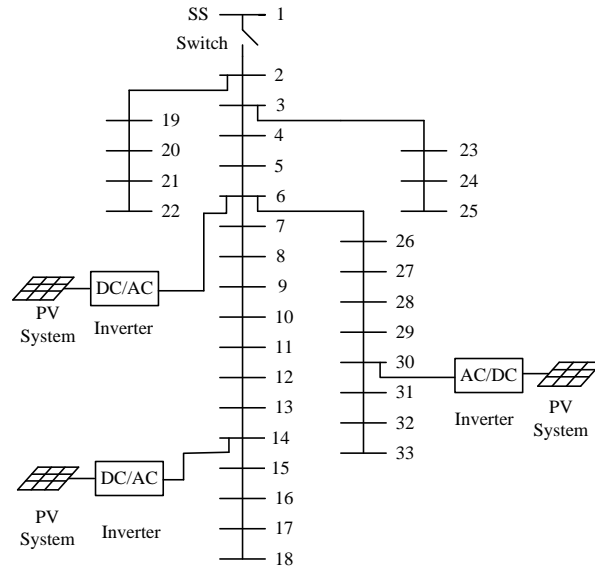


Figure 1: IEEE 33-bus radial distribution system

### 3. Eigen Value Analysis and Pareto Front Identification

#### 3.1. IEEE 33-bus system with two DGs

The optimal locations of two generators (in a grid-connected system) based on loss minimization proposed in [34] are at nodes 6 and 30. When islanded, these two generators operate in droop control mode (for size in

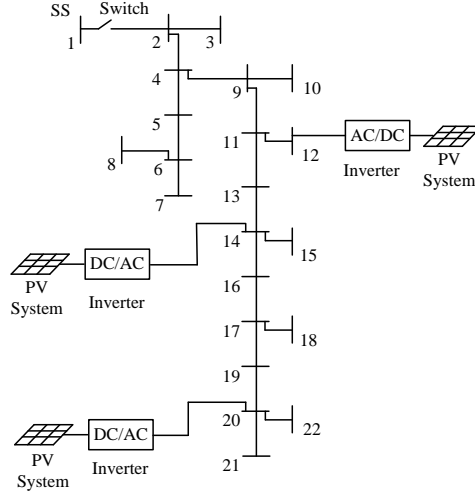


Figure 2: Practical radial distribution (22 bus) network APEPDCL

122 proportion of 1:0.50) for load sharing. From the droop law, we know that  
 123 system frequency takes a new steady state value till secondary control acts.  
 124 System simulation (time domain) is performed with these two generators  
 125 at various locations (cases) in a standard IEEE 33-bus radial distribution  
 126 network. From the operating points, state space matrix is obtained using  
 127 (1). Root locus analysis is performed for these cases by varying the droop  
 128 constants to identify the stability limit. The values of  $m_{p,max}$  and  $n_{q,max}$   
 129 are noted when the system reaches an unstable region. Losses in the system,  
 130 minimum voltage value in the total network,  $m_{p,max}$ ,  $n_{q,max}$ , and minimum  
 131 distance between the DGs for all these cases are presented in Table 1. It  
 132 is clear that the maximum values of  $m_{p,max}$  and  $n_{q,max}$  are not the best  
 133 for case 1. This is true since the decision for placement of generators in this  
 134 location in [34] was made with separate conditions (grid-connected, exporting  
 135 power, etc.). However, in systems where grid reliability is poor (true in many  
 136 developing countries), such location may not be optimum. From network loss,  
 137 stability, and voltage perspectives, case 1, case 6, and case 13 are preferred  
 138 options, respectively.

139 Figure 3 shows the plot between  $m_{p,max}$  and  $Z$ , while Fig. 4 shows the  
 140 plot between  $n_{q,max}$  and  $Z$  for the cases tabulated in Table 1. Electrical  
 141 distance (in terms of impedance) between generators is an important param-  
 142 eter contributing to small signal stability margin. From Figs. 3 and 4, it  
 143 is observed that higher electrical distance between sources results in better



Table 1: Various case study results for two DGs placement for IEEE 33-bus radial network

Case	DG-1 Node	DG-2 Node	$P_{loss}$ (kW)	$V_{min}$ (p.u.)	$m_{p,max}$ ( $10^{-5}$ )	$n_{q,max}$ ( $10^{-4}$ )	Z ( $\Omega$ )
1	6	30	65.05	0.9469	1.24	1.34	3.5709
2	24	30	74.27	0.9303	2.30	2.21	7.1671
3	18	24	120.48	0.9193	4.90	5.92	16.8053
4	13	30	264.07	0.9206	3.43	2.84	11.1844
5	18	25	143.45	0.9068	5.33	6.10	17.9422
6	18	22	207.91	0.8855	5.55	6.31	19.6787
7	22	33	185.24	0.9003	3.39	3.84	12.4616
8	22	25	175.09	0.8906	2.08	2.72	7.3835
9	25	33	106.39	0.9131	3.16	3.48	10.7276
10	18	33	386.46	0.8833	5.39	4.58	19.2281
11	6	14	83.96	0.9528	2.44	3.12	8.4827
12	6	18	120.38	0.9524	3.60	5.08	0.9524
13	6	10	72.29	0.9532	1.53	1.97	5.1831
14	3	5	97.04	0.9335	0.88	0.37	0.8118
15	6	26	84.97	0.9487	0.82	0.23	0.2278
16	3	4	103.26	0.9273	0.80	0.27	0.4107
17	9	10	238.42	0.8823	0.73	0.59	1.2764
18	32	33	291.85	0.8507	0.43	0.47	0.6304
19	17	18	525.83	0.7425	0.65	0.50	0.9302
20	24	25	182.31	0.8890	0.66	0.58	1.1377

144 stability margin. Root locus plot and time domain simulation further prove  
145 this point. Case 1 (base case), case 6 (highest stability margin), and case 18  
146 (least stability margin) are considered for detailed analysis.

147 Figure. 5 shows the root locus plot of the system for case -1, case -6, and  
148 case -18.  $\lambda_{12}$  indicates the interaction of low-frequency modes between two  
149 sources. From the three sets of Eigen traces, it s clear that the system is  
150 going into an unstable region after a certain value of  $m_P$ . In Fig. 5,  $\lambda_{12}$  for  
151 case -1 starts from  $-15.066 \pm j 16.60$  and reaches the imaginary axis at  $0 \pm j$   
152  $74.40$ , while for case -6 and case -18 the starting points for  $\lambda_{12}$  are at  $-15.346$   
153  $\pm j 1.1835$  and  $-12.971 \pm j 28.278$  and they reach the imaginary axis at  $0 \pm$   
154  $j 87.05$  and  $0 \pm j 58.84$ , respectively. From these root locus plots, the effect  
155 of impedance between sources on stability margin is observed, and it is clear  
156 that, distance between sources influences the stability of the system.

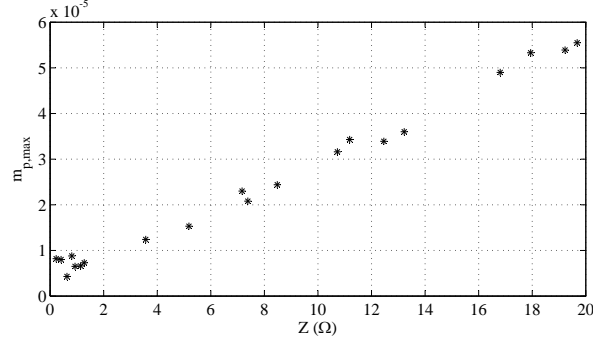


Figure 3: Impedance vs.  $m_{p,max}$  plot

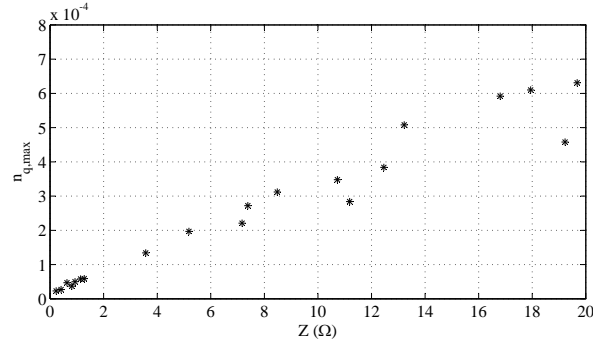


Figure 4: Impedance vs.  $n_{q,max}$  plot

### 157 3.2. IEEE 33-bus system with three DGs

158 Optimal locations of three generators (in grid connected system) based  
 159 on loss minimization, proposed in [34], are at nodes 6, 14, and 30. When is-  
 160 landed, these three generators operate in droop control mode for load sharing.  
 161 System simulation (time domain) is performed with these three generators  
 162 at various locations (cases) in a standard IEEE 33 bus radial distribution  
 163 network. From the operating points, state space matrix is obtained using  
 164 (1). Root locus analysis is performed for these cases by varying droop con-  
 165 stants to identify the stability limit. The values of  $m_{p,max}$  and  $n_{q,max}$  are  
 166 noted when the system reaches an unstable region. Losses in the system,  
 167 minimum voltage value in the total network,  $m_{p,max}$ ,  $n_{q,max}$  and minimum  
 168 distance between the DGs for all these cases are presented in Table. 2.

169 It is clear that the maximum values of  $m_{p,max}$ ,  $n_{q,max}$  are not the highest

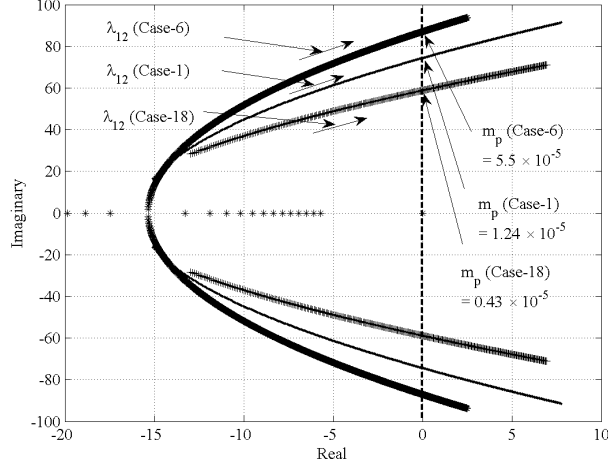


Figure 5: Table 1, cases-1, 6, 18 : Rootlocus plot with variation in droop gain  $m_p$

for case-1. This is true since the decision for this location for placement of generators in this location in [34] was done with separate conditions (grid-connected, exporting power, etc). From network loss, stability, and voltage perspectives, case -37, case -3 and case -33 are preferred options.

Figure. 6 shows the eigenvalues plot for case -1 (base case). Out of 165 eigenvalues 92 eigenvalues are shown in figure (rest of the Eigenvalues are highly damped). For dynamic stability, low-frequency mode Eigenvalues, which are sensitive to the droop gains of the system, are of interest. These low-frequency modes correspond to the power controller mode of the VSI. Case -1 (base case), case -3 (highest stability margin), and case -41 (least stability margin) are considered for detailed analysis. Two complex conjugate low-frequency mode trajectories sensitive to real power droop gain for these cases are shown in Fig. 7, Fig. 8 and Fig. 9, respectively.  $\lambda_{12}$  shows the interaction of low frequency modes between VSIs 1 and 2 while  $\lambda_{13}$  shows the interaction of low frequency modes between VSIs 1 and 3. This trajectory shows that  $\lambda_{12}$  goes into an unstable mode at a lower value of  $m_p$  than  $\lambda_{13}$ .

In Fig. 7,  $\lambda_{12}$  starts at  $-15.7 \pm j 6.2054$  and reaches the imaginary axis at  $0 \pm j 81.265$ . In Figs. 8 and 9,  $\lambda_{12}$  starts from  $-15.24 \pm j 8.065$  and  $-11.213 \pm j 33.373$  and reaches to imaginary axis at  $0 \pm j 86.75$  and  $0 \pm j 60.118$  respectively. From these root locus plots, the impact of minimum distance between sources on stability margin is clearly observed, and it is understood that sources separated with higher impedance have relatively higher stability

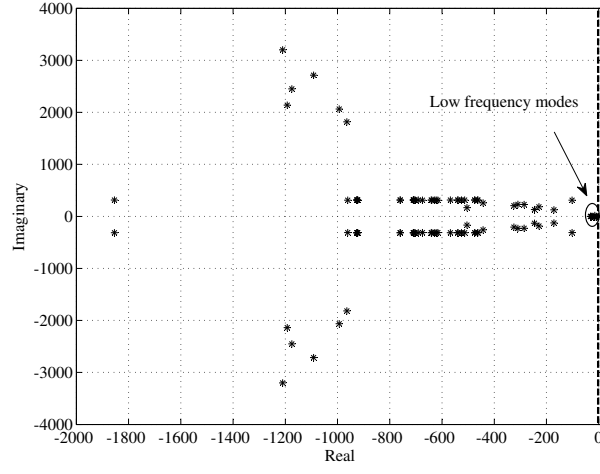


Figure 6: Eigenvalue plot of the microgrid

192 margin.

Table 2: Various case study results for three DGs placement for IEEE 33-bus radial network

Case	DG-1 Node	DG-2 Node	DG-3 Node	$P_{loss}$ (kW)	$V_{min}$ (p.u.)	$m_{p,max}$ ( $10^{-5}$ )	$n_{q,max}$ ( $10^{-4}$ )	$Z_{min}$ ( $\Omega$ )
1	6	30	14	60.03	0.9581	1.81	1.31	3.5709
2	25	33	18	67.98	0.9635	2.91	4.12	10.7274
3	22	33	18	86.76	0.9441	2.94	4.73	12.4616
4	24	30	8	32.36	0.9694	0.92	1.80	6.1455
5	24	30	18	44.86	0.9751	2.38	2.62	7.1671
6	6	30	18	79.30	0.9577	1.78	1.38	3.4992
7	24	30	6	45.94	0.9530	0.53	0.76	3.5965
8	24	30	22	52.07	0.9364	1.35	2.05	6.2483
9	24	6	18	84.58	0.9613	1.22	1.29	3.5965
10	10	30	15	126.06	0.9347	1.19	1.31	4.0902
11	10	24	15	153.56	0.9360	1.18	1.30	4.0902
12	10	22	15	151.49	0.9167	1.19	1.35	4.0902
13	24	30	20	45.87	0.9370	1.08	1.50	4.4788
14	24	20	18	95.61	0.9321	1.80	1.84	4.4788
15	24	30	3	46.11	0.9471	0.54	0.57	1.6905
16	24	3	18	75.06	0.9422	0.44	0.16	1.6905
17	24	21	3	135.0	0.9235	0.43	0.53	1.6905
18	24	22	18	114.78	0.9299	2.23	2.62	6.2483
19	6	11	18	212.26	0.9554	0.94	1.71	5.3783
20	2	6	18	65.20	0.9617	1.07	0.97	2.456
21	24	21	2	139.08	0.9181	0.40	0.59	2.2352
22	2	6	30	36.66	0.9528	0.61	0.79	2.456
23	24	21	6	96.28	0.9509	0.82	1.04	3.5965
24	8	14	18	362.75	0.9062	0.62	1.46	4.7548
25	2	4	6	76.63	0.9514	0.41	0.42	0.964
26	24	21	11	91.14	0.9409	1.63	2.01	5.0983
27	7	26	30	64.41	0.9514	0.64	0.26	0.8209
28	10	14	18	386.80	0.8604	0.60	1.16	3.2999
29	3	6	11	41.92	0.9627	0.72	0.77	2.3629
30	3	6	30	34.80	0.9532	0.60	0.72	2.3629
31	24	21	14	85.60	0.9363	1.83	2.08	5.0983
32	24	30	11	25.81	0.9770	1.48	2.68	7.1429
33	23	30	18	51.56	0.9779	2.31	2.25	6.0099
34	23	33	18	77.61	0.9746	2.84	3.22	15.6619
35	23	19	3	112.85	0.9244	0.37	0.25	0.5472
36	6	12	18	231.84	0.9552	0.82	1.70	5.7461
37	24	30	14	25.69	0.9759	1.94	2.64	7.1671
38	23	3	4	100.50	0.9301	0.43	0.25	0.4170
39	19	2	3	130.38	0.9224	0.41	0.27	0.2267
40	5	6	26	67.69	0.9532	0.43	0.21	0.2278
41	29	30	31	193.0	0.8980	0.33	0.34	0.6214
42	24	23	3	119.09	0.9236	0.37	0.31	0.5472
43	21	20	19	246.34	0.9051	0.42	0.45	0.6297
44	4	6	8	52.80	0.9630	0.44	0.64	1.5007
45	28	30	32	175.37	0.9153	0.35	0.61	1.6249
46	10	11	12	295.94	0.8632	0.50	0.22	0.2071

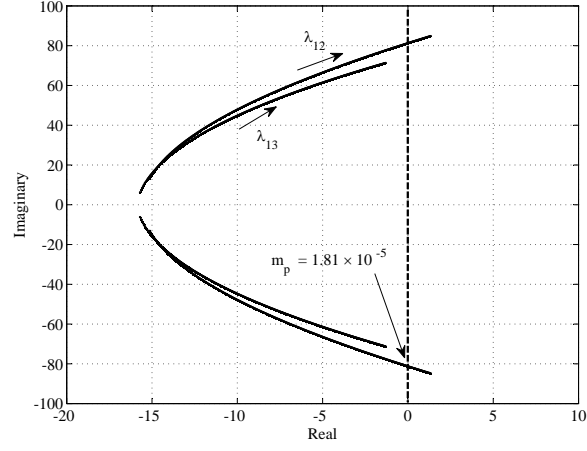


Figure 7: Table 2, case-1 : Rootlocus plot with variation in droop gain  $m_p$

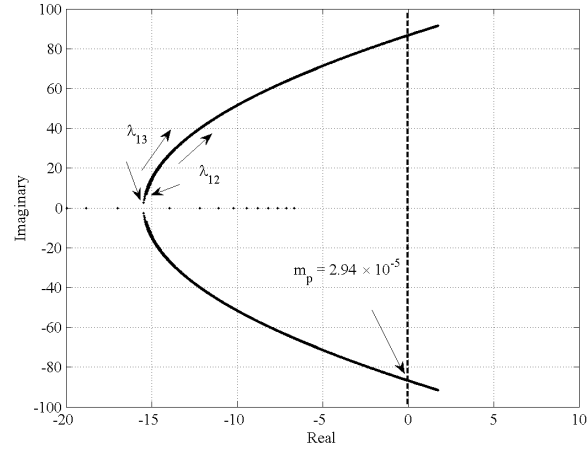


Figure 8: Table 2, case-3 : Rootlocus plot with variation in droop gain  $m_p$

### 193 3.3. 22-bus APEPDCL Distribution Network

194 The optimal locations of three generators (in a grid-connected system)  
 195 based on loss minimization, proposed in [36], are at nodes 12, 14, and 20.  
 196 System simulation (time domain) is performed with these three generators  
 197 at various locations (cases) in the 22-bus APEPDCL distribution network.  
 198 From the operating points, state space matrix is obtained using (1). Root

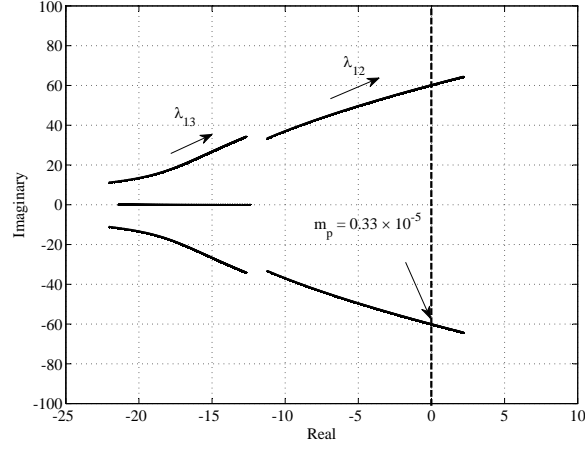


Figure 9: Table 2, case-41 : Rootlocus plot with variation in droop gain  $m_p$

locus analysis is performed for these cases by varying droop constants to identify the stability limit. The values of  $m_{p,max}$  and  $n_{q,max}$  are noted when the system reaches an unstable region. Losses in the system, minimum voltage value in the total network,  $m_{p,max}$ ,  $n_{q,max}$ , and minimum distance between the DGs for all these cases are presented in Table. 3.

It is clear that the maximum values of  $m_{p,max}$ ,  $n_{q,max}$  are not the highest for case 1. This is true since the decision for placement of generators in this location was made with separate conditions (grid-connected, exporting power, etc.). From network loss, stability, and voltage perspectives, case 8, case 6, and case 8 are preferred options. Case 1 (base case), case 6 (highest stability margin) and case 20 (least stability margin) are considered for detailed analysis.

Table 3: Various case study results for three DGs placement for APEPDCL 22-bus practical radial network

Case	DG-1 Node	DG-2 Node	DG-3 Node	$P_{loss}$ (kW)	$V_{min}$ (p.u.)	$m_{p,max}$ ( $10^{-6}$ )	$n_{q,max}$ ( $10^{-5}$ )	$Z_{min}$ ( $\Omega$ )
1	12	14	20	0.740	0.9952	7.23	4.48	1.2137
2	3	14	20	0.752	0.9967	7.29	4.90	1.2137
3	8	12	22	3.154	0.9951	12.06	8.49	3.6752
4	8	13	22	2.627	0.9958	11.01	8.04	3.0911
5	4	15	22	0.612	0.9971	8.41	6.10	1.8402
6	8	10	22	4.4459	0.9942	13.01	8.16	2.9157
7	3	15	22	0.953	0.9965	8.45	6.25	1.8402
8	4	14	20	0.367	0.9972	7.26	4.85	1.1897
9	9	15	22	0.732	0.9968	8.33	5.76	1.8402
10	8	9	17	5.078	0.9965	10.32	7.10	2.8026
11	3	10	17	3.675	0.9965	10.04	5.60	1.5681
12	8	11	17	3.586	0.9967	8.95	6.49	2.0428
13	8	10	18	5.041	0.9961	10.66	7.47	2.9157
14	12	15	18	0.943	0.9953	5.91	3.49	0.5567
15	15	18	22	2.712	0.9903	7.60	3.50	0.5567
16	10	12	15	2.050	0.9954	8.06	4.11	0.8826
17	13	14	15	1.514	0.9945	6.02	2.13	0.0249
18	20	21	22	6.410	0.9840	6.43	2.17	0.0980
19	9	10	11	5.281	0.9879	6.61	2.16	0.0615
20	6	7	8	19.336	0.9683	5.83	2.15	0.0673

Plots of  $m_{p,max}$  vs.  $Z_{min}$  (minimum impedance among sources) and  $n_{q,max}$  vs.  $Z_{min}$  are shown in Figs. 10 and 11, respectively.

Figures. 12, 13 and 14 show root locus plot for cases 6, 8 and 20, respectively.  $\lambda_{12}$  shows the interaction of low-frequency modes between VSIs 1 and 2 while  $\lambda_{13}$  shows the interaction of low frequency modes between VSIs 1 and 3. This trajectory shows that  $\lambda_{12}$  goes into an unstable mode at a lower value of  $m_p$  than  $\lambda_{13}$ . In Fig. 12  $\lambda_{12}$  starts from an approximate value of  $-15.55 \pm j 21.27$  and reaches the imaginary axis at an approximate value of  $0 \pm j 63.3$ . In Figs. 13 and 14,  $\lambda_{12}$  approximately starts from  $-15.27 \pm j 15.275$  and  $-16.19 \pm j 24.70$  and reaches the imaginary axis approximately at  $0 \pm j 71.1$  and  $0 \pm j 59.7$  respectively. The following are some critical observations from the case studies:

- The system configuration (generator location) with low losses in grid



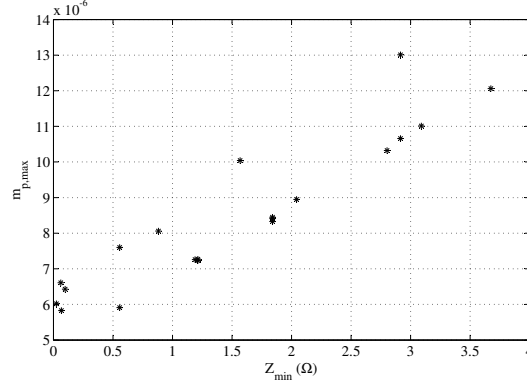


Figure 10: Plot between  $m_{p,max}$  vs.  $Z_{min}$

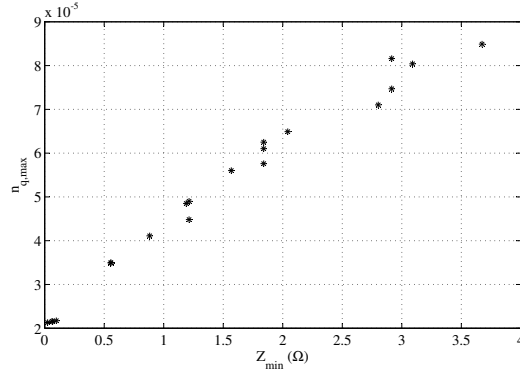


Figure 11: Plot between  $n_{q,max}$  vs.  $Z_{min}$

connected mode may suffer from stability issues when islanded. This can be a serious problem when the reliability of the main grid is poor.

- The interaction of low-frequency modes between various DGs is different and the location of some inverters is critical (inverter 2 in this case) with respect to the stability.
- Stability margin (gain of droop constant) is a function of minimum distance between the generators in an islanded network.
- It is important to choose an optimal location for these generators by considering stability and network losses.

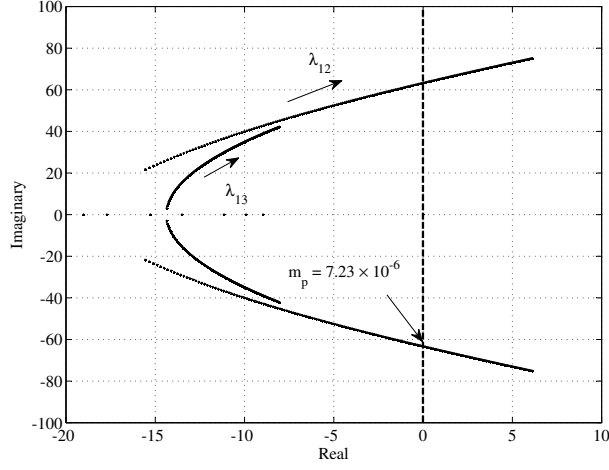


Figure 12: Table 3, case-1 : Rootlocus plot with variation in droop gain  $m_p$

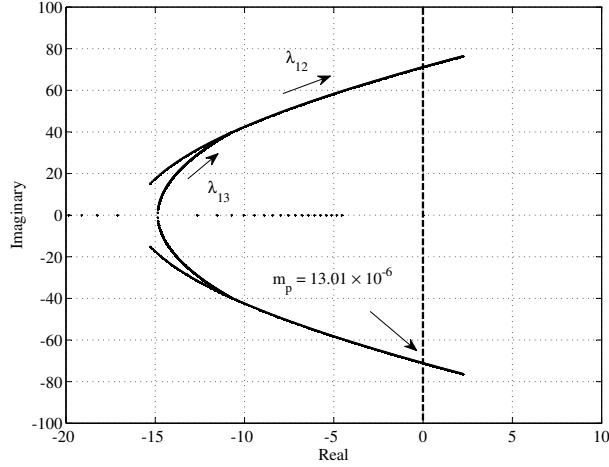


Figure 13: Table 3, case-6 : Rootlocus plot with variation in droop gain  $m_p$

### 233 3.4. Determination of Pareto Front in an Islanded Microgrid

234 The locations of generators should depend on network losses and overall  
 235 stability of the system. For multi-objective optimization of the DG network,  
 236 Pareto optimal front should be identified. Data in Tables 2 and 3 are plotted  
 237 and Pareto fronts (set of non dominated solutions) obtained between  $m_{p,max}$   
 238 vs. real power loss and  $n_{q,max}$  vs. reactive power loss (Figs. 15, 16 and 17,  
 239 18 respectively).

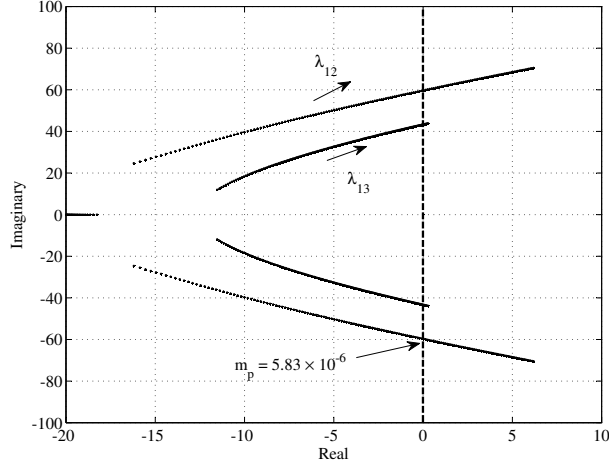


Figure 14: Table 3, case-20 : Rootlocus plot with variation in droop gain  $m_p$

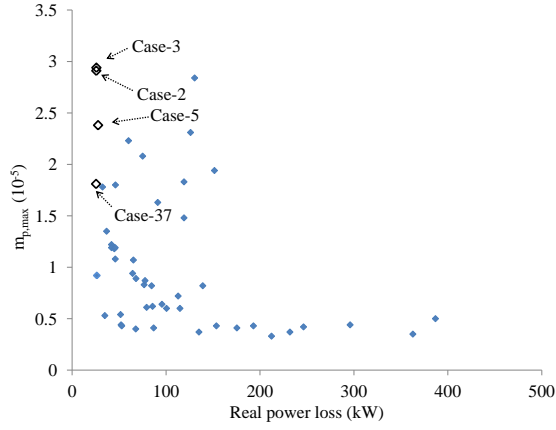


Figure 15: Real power loss vs.  $m_{p,max}$  for IEEE 33 bus system with three DGs - Pareto front shown in open boxes

- 240 Critical observations from Pareto fronts (for 33-bus system) are:
- 241 • Cases corresponding to Pareto fronts (shown in open box) obtained in
- 242 Fig. 15 are 2, 3, 5 and 37.
- 243 • Cases corresponding to Pareto fronts (shown in open box) obtained in
- 244 Fig. 16 are 2, 3 and 32.

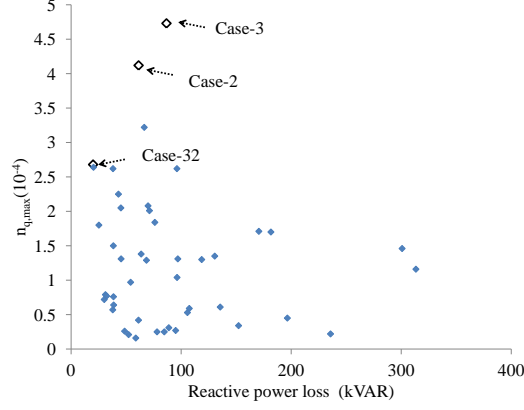


Figure 16: Reactive power loss vs.  $n_{q,max}$  for IEEE 33 bus system with three DGs - Pareto front shown in open boxes

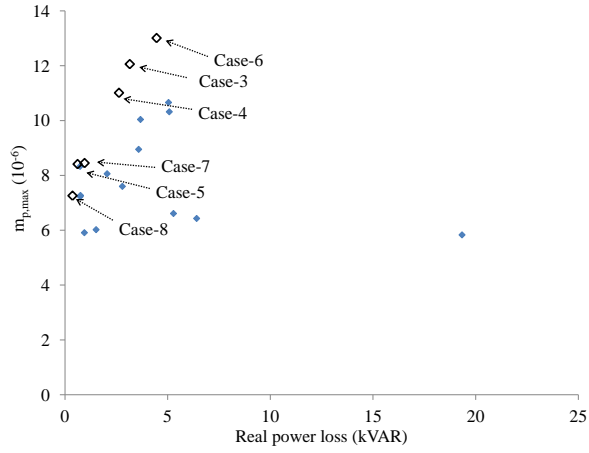


Figure 17: Real power loss vs.  $m_{p,max}$  for 22 bus APEPDCL network with three DGs - Pareto front shown in open boxes

- Case-1 which represents optimal location of sources in a grid-connected system, does not lie on the Pareto front. This clearly indicates that the optimal placement of sources in a grid-connected microgrid is not optimal during islanding.

Critical observations from Pareto fronts (for 22 bus practical system) are:

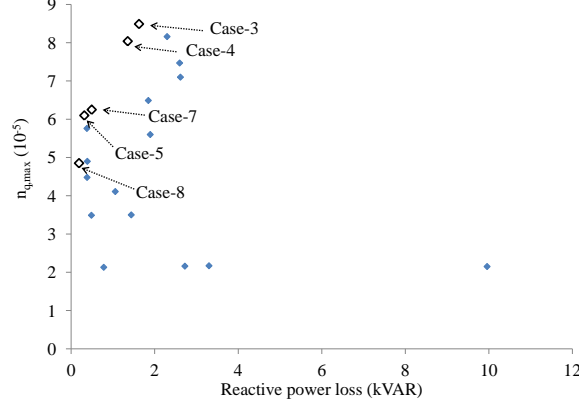


Figure 18: Reactive power loss vs.  $n_{q,max}$  for 22 bus APEPDCL network with three DGs  
- Pareto front shown in open boxes

- 250 • Cases corresponding to Pareto fronts (shown in open box) obtained in  
251 Fig. 17 are 3, 4, 5, 6, 7 and 8.
- 252 • Cases corresponding to Pareto fronts (shown in open box) obtained in  
253 Fig. 18 are 3, 4, 5, 7 and 8.
- 254 • Similar to the previous example, case -1 does not lie on the Pareto  
255 front.
- 256 • Cases 3, 4, and 6 have high stability margin and higher losses, while  
257 cases 5, 7, and 8 have low stability margin and low losses.

#### 258 4. Simulation - Time Domain Validation

259 Time domain simulation is performed on both the networks for validation  
260 of stability analysis. Simulation results for the three DG system (case -1 of  
261 Table 2) and for the practical network (case -1 of Table. 3) are shown in Fig.  
262 19 and Fig. 20, respectively.

263 The system is stable and sharing power as per the droop law. The effect  
264 of higher value of droop parameter is investigated by changing the droop  
265 value (beyond  $m_{p,max}$ ). At time  $t = 2s$  for a higher value of  $m_p (> m_{p,max})$ ,  
266 power output of DGs is oscillating with increasing amplitude as shown in  
267 Fig. 19, which indicates that the system is now unstable.

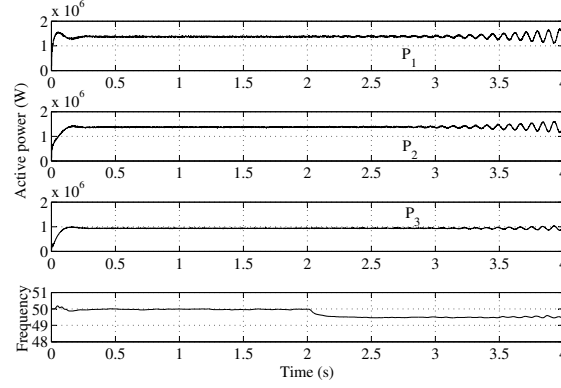


Figure 19: Real power output of DGs and system frequency in Std. IEEE 33 network

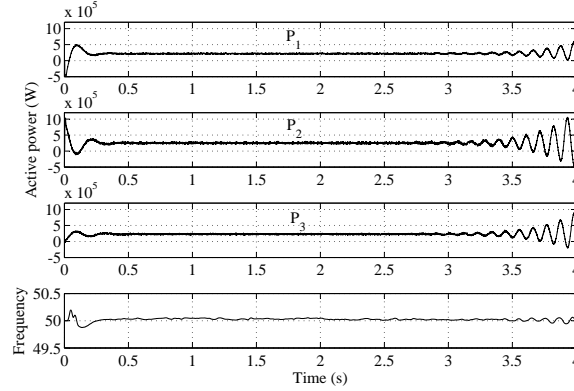


Figure 20: Real power output of DGs and system frequency in practical 22 bus distribution network

## 5. Conclusion

The effect of location of droop-based sources on small signal stability, transient response, and network losses in an islanded network is investigated. A standard IEEE 33-bus network and a 22-bus practical distribution network are chosen. A microgrid model is developed for both the networks with droop-based sources, network components, and loads for stability analysis. Higher droop in DGs is desired for better power sharing and transient response. Small signal stability is studied for various locations of DGs (two/three) by varying the droop constant. From the stability study, it is found that a system optimized for losses in grid-connected mode may suffer from small signal

278 stability issues and poor transient response when in islanded configuration.  
 279 The minimum distance between generators in the network also has an im-  
 280 pact on small signal stability. For multi-objective optimization of the DG  
 281 network, Pareto optimal front is identified. Results of small signal stability  
 282 analysis are verified using time domain simulation in MATLAB for both the  
 283 networks.

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