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# Variation in models for simple dynamic structure–soil–structure interaction problems

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**Abstract**

To account for dynamic cross-coupling of structures via the soil, a computational model must be accurate enough to provide the correct overall behaviour of the scattered wave field. However, simplicity is also important when a model should be used for design purposes, especially in the early design stages and feasibility studies. The paper addresses the accuracy of simple models in which an array of structures is simplified into blocks placed on the ground surface or embedded within the soil. Comparisons are made between models that account or do not account, in a proper manner, for the inertia and embedment of the structures. Especially, the limitations of simplified models are discussed regarding their capability to quantify the insertion loss accurately.


*Keywords:* Ground vibration; insertion loss; SSI; Green’s function; embedded structures.

## 1. Introduction

Dynamic coupling of structures and the ground vibration around them (termed environmental vibration) must be analysed in the assessment of ground vibration resulting from construction work, heavy traffic or factory machinery. Reducing or mitigating vibration in the urban environment is classified in two main categories, both aiming to protect sensitive buildings by creating a “shadow zone” within the propagation path beyond the “isolation” element. The main categories are: an isolating screen and a wave-impeding block (WIB), see Ref. [1]. The present study focuses on wave-impeding blocks or stiff plates and their location either on the surface or embedded partially into the ground.

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Single WIBs work in the following ways. Firstly, due to its large mass, a heavy rigid block imposes a constraint over the ground surface. This impedes the Rayleigh wave transmission but has limited efficiency due to other ground waves that may "diffract" around the rigid boundary. However, a second effect, introduced by the ground, is the resultant mass–stiffness system which has the ability to store and release energy similar to a one-dimensional single-degree-of-freedom system. Given certain site-specific characteristics, it is possible to control the isolation efficiency.

Many researchers have considered the dynamic behaviour of the ground surface and its influence on soil coupling to structures. One of the earliest contributions was by Warburton *et al.* [2] who considered the interaction of two rigid circular foundations using a mixed integral equation approach, and Krylov [3] studied the effect of single blocking masses such as concrete blocks placed on a homogenous ground. More recently Alic and Persson [4] proposed form finding for ground vibration mitigation systems, and Dijckmans *et al.* [5] investigated the ground vibration mitigation effectiveness of an array of heavy masses placed along a railway track. The principle of the latter approach was to modify the wave propagation regime of the ground by introducing an inertial mass near the load. A few experiments near a rail track were carried out on possible measures to attenuate wave propagation at low frequencies. The results of experiments such as these are notoriously difficult to analyse given the inherent variation of ground properties between different stretches of track and the differences in vibration levels generated by different trains.

As an alternative to placing an array of heavy masses along a road or track, Andersen [6] suggested to construct an array of periodically repeated WIBs or changes in the ground surface within the wave propagation path perpendicular to the road or track. Depending on periodicity, such a solution may lead to so-called band gaps, forming a range of frequencies in which no wave propagation occurs. Numerical studies and small-scale laboratory tests of similar periodic structures, but with only few repetitions of the WIB, were conducted by Andersen *et al.* [7].

The present paper follows the idea of placing an array of heavy masses within the propagation path between a source and a receiver point located on the ground surface. One aim is to quantify the overall mitigation that can be achieved by including increasingly more WIBs. The primary objective is however to quantify the uncertainty related to poor modelling of the mass distribution and position of the WIBs. In Section 2, the applied methodology is briefly outlined, and Section 3 presents a study of the insertion loss provided by an array of blocks placed on the ground surface or embedded within the soil. Subsection 3.1 examines the importance of accounting for rotational inertia, whereas Subsection 3.2 investigates the significance of modelling embedment. Finally, a short summary and conclusions are given in Section 4.

## 2. Methodology

Andersen and Clausen [8] provided a formulation of the impedance associated with rigid surface footings on layered ground. As an extension to this, a formulation is here given for multiple rigid blocks on the surface of the ground or embedded in the soil. The first step is to derive a relationship between the displacement $\mathbf{u}(x, y, z, t)$ at one point in space and time and the load $\mathbf{p}(x', y', z', t')$ applied at another position and an earlier time:

$$\mathbf{u}(x,y,z,t) = \int_{-\infty}^{t}\int_{-\infty}^{0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{g}(x-x', y-y', z, z', t-t') \cdot \mathbf{p}(x', y', z', t')\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'\,\mathrm{d}t' . \tag{1}$$

For a layered half-space with horizontal stratification, the Green's function $\mathbf{g}(x-x', y-y', z, z', t-t')$ cannot be found in closed form, but applying a triple Fourier transformation, the Green's function $\overline{\mathbf{G}}(k_x, k_y, z, z', \omega)$ in horizontal wavenumber–frequency domain can be determined analytically. The relationship

$$\overline{\mathbf{U}}(k_x, k_y, z, \omega) = \overline{\mathbf{G}}(k_x, k_y, z, z', \omega)\,\overline{\mathbf{P}}(k_x, k_y, z', \omega) \tag{2}$$

can then be evaluated for a number of discrete values of the wavenumbers $k_x$ and $k_y$, receiver depth $z$, source depth $z'$, and angular frequency $\omega$. As suggested by Andersen and Clausen [8], employing a distributed load with "bell shape" (a double Gaussian distribution with rotational symmetry around each source point) leads to a particularly fast and simple semi-analytical evaluation of the inverse Fourier transform regarding the wavenumbers.

As proposed by Andersen [9] and Bucinskas *et al.* [10], structure–soil–structure interaction for systems consisting of $N_F$ rigid bodies can be expressed via the impedance matrix $\mathbf{Z}_F(\omega)$ relating the displacement and rotation components of all bodies to the forces and moments acting on all bodies:

$$\mathbf{Z}_\mathrm{F}(\omega)\,\mathbf{D}_\mathrm{F}(\omega) = \mathbf{F}_\mathrm{F}(\omega)\,, \tag{3a}$$

$$\mathbf{D}_\mathrm{F}(\omega) = \left\{\mathbf{D}_{\mathrm{f},1}^T(\omega)\;\; \mathbf{D}_{\mathrm{f},2}^T(\omega)\;\; \cdots\;\; \mathbf{D}_{\mathrm{f},N_\mathrm{F}}^T(\omega)\right\}^T, \qquad \mathbf{D}_{\mathrm{f},n}^T = \left\{U_{x,n}\;\; U_{y,n}\;\; U_{z,n}\;\; \Theta_{x,n}\;\; \Theta_{y,n}\;\; \Theta_{z,n}\right\}^T, \tag{3b}$$

$$\mathbf{F}_\mathrm{F}(\omega) = \left\{\mathbf{F}_{\mathrm{f},1}^T(\omega)\;\; \mathbf{F}_{\mathrm{f},2}^T(\omega)\;\; \cdots\;\; \mathbf{F}_{\mathrm{f},N_\mathrm{F}}^T(\omega)\right\}^T, \qquad \mathbf{F}_{\mathrm{f},n}^T = \left\{P_{x,n}\;\; P_{y,n}\;\; P_{z,n}\;\; M_{x,n}\;\; M_{y,n}\;\; M_{z,n}\right\}^T. \tag{3c}$$

As indicated, each three-dimensional rigid body has three translational and three rotational degrees of freedom. Each individual rigid body is discretized into a number of points, and the Green's function is utilized to evaluate the influence from all points to all points in the model, eventually leading to a flexibility matrix the inverse of which multiplied by the displacement of all points associated with each individual mode of rigid-body motion provides $\mathbf{Z}_\mathrm{F}(\omega)$. The displacement response at any points, *e.g.* on the surface of the ground, can be found by post processing, using again the Green's function to obtain the influence from interaction forces provided by the rigid-body motion.

## 3. Analysis of insertion loss due to an array of blocks on the ground surface

The study concerns an array of blocks placed on the ground surface or embedded within the ground. The soil is either homogenous or layered with the properties listed in Tab. 1. The distance between the blocks, centre to centre, is 4.0 m, and the blocks are considered rigid with mass density 2400 kg/m$^3$. The blocks are 2.0 m long in the longitudinal direction (along the array), 4.0 m wide in the transverse direction and 2.0 m high. One to five blocks are present in the array, and no blocks are present in the reference case. The first block is placed at a distance of 4.0 m (centre to centre) from a rigid massless plate being subjected to harmonic excitation. Example results are shown in Fig. 1 for a case in which five blocks are embedded to a depth of 2.0 m within a homogeneous half-space.

To quantify the change in response at an observation point due to the presence of a number of blocks placed on the ground or embedded within the soil, the insertion loss $IL_\mathrm{dB}$ (in decibel) is calculated:

$$IL_\mathrm{dB} = 20\log_{10}\frac{|U_0|}{U_\mathrm{ref}} - 20\log_{10}\frac{|U_k|}{U_\mathrm{ref}} = 20\log_{10}\frac{|U_0|}{|U_k|}\,,\;\; U_\mathrm{ref} = 10\ \mathrm{pm}\,. \tag{4}$$

Here $|U_0|$ denotes the magnitude of the displacement response at an observation point in the case with no blocks present (*i.e.* the Green-field solution), and $|U_k|$ is the magnitude of displacement response at the same position when $k$ blocks are present. Results are presented for an observation point placed at the distance 28 m away from the centre of the loaded area (see Fig. 1). The frequency range [0 Hz; 80 Hz], relevant to whole-body vibration, will be considered.

Table 1. Properties of the soil.

| Soil | Shear modulus (MPa) | Poisson's ratio | Mass density (kg/m$^3$) | Loss factor |
|---|---|---|---|---|
| Homogeneous half-space | 80 | 1/3 | 2000 | 0.020 |
| Soft topsoil (top layer of 8 m over stiff half-space) | 20 | 1/3 | 2000 | 0.020 |
| Stiff subsoil (only present in layered half-space) | 2000 | 1/3 | 2000 | 0.020 |

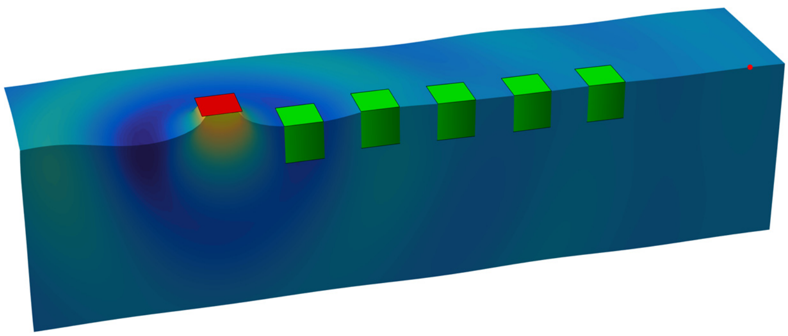

Fig. 1 Example results for homogeneous half-space subjected to a harmonic load applied at 20 Hz on a rigid (red) plate and with five rigid (green) blocks embedded in the soil. The ground surface and a vertical cut through the centre line of the array are illustrated. The red dot indicates the observation point used to evaluate the insertion loss. Dark blue and bright yellow shades indicate negative and positive vertical displacements.

### *3.1. Influence of mass and rotational inertia of blocks placed on the ground surface*

Fig. 2 shows the insertion loss (IL) at the observation point 28 m away from the centre of the loaded area for different numbers of blocks placed on the surface of the homogeneous half-space. A comparison is made between models accounting, or not accounting, for rotational inertia. The results at 15 Hz and 45 Hz clearly indicate an accumulated effect of inserting more blocks in the propagation path. At the frequency 15 Hz, the IL at positions in the range 20–30 m (*i.e.* 20–30 m from the centre of the loaded area) become increasingly more negative each time another block is added to the array. Contrarily, each added mass provides a positive increment in the IL at 45 Hz. The model without rotational inertia significantly underestimates the effect of the array.

Fig. 3 shows the IL at the observation point as function of the frequency. As already indicated by Fig. 2, negative IL occurs at low frequencies, whereas the array of blocks provides a positive IL at higher frequencies. For the homogeneous half-space, the cross-over point is around 17 Hz in the model accounting for rotational inertia and around 25 Hz for the case with no rotational inertia. The difference in frequency is a result of the difference in mass between the two models. The inability of the simplified model to estimate the IL is again observed. Differences of more than 10 dB are present in configurations with two or more blocks.

Compared to results for the homogeneous half-space, the results for the layered half-space do not have a clear trend. In general, the IL is lower than observed for the homogeneous half-space at most frequencies above 25 Hz. Especially, a clear dip in IL is present around 45 Hz. This dip is captured well by the model with no rotational inertia and is a result of resonance in the soft layer of topsoil. However, the simplified model provides inaccurate results for most frequencies. Differences compared to the model accounting for rotational inertia are within the range ±5 dB.

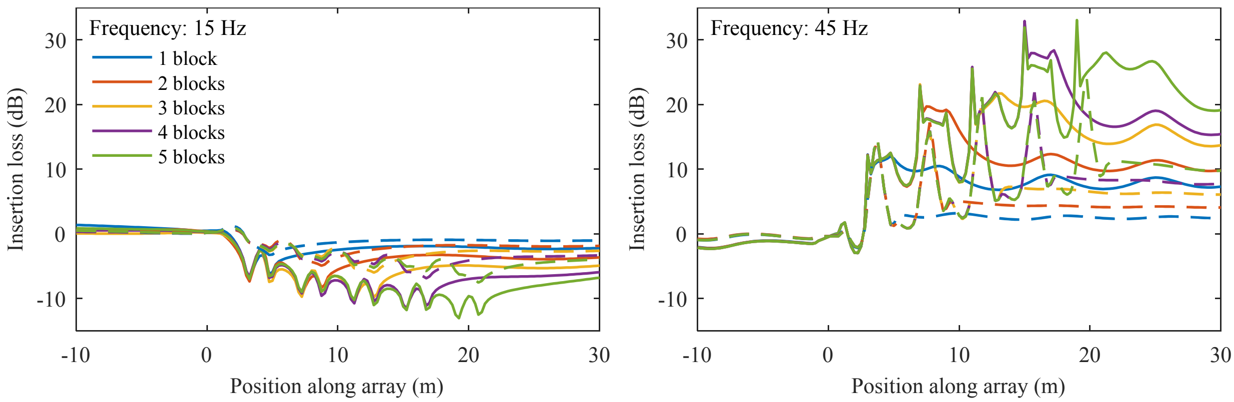


Fig. 2. Insertion loss as function of distance for different numbers of blocks on a homogeneous half-space at the frequencies 15 Hz and 45 Hz. Full lines (——) indicate results for blocks *with* rotational inertia, whereas dashed lines (— —) indicate results for blocks *without* rotational inertia.

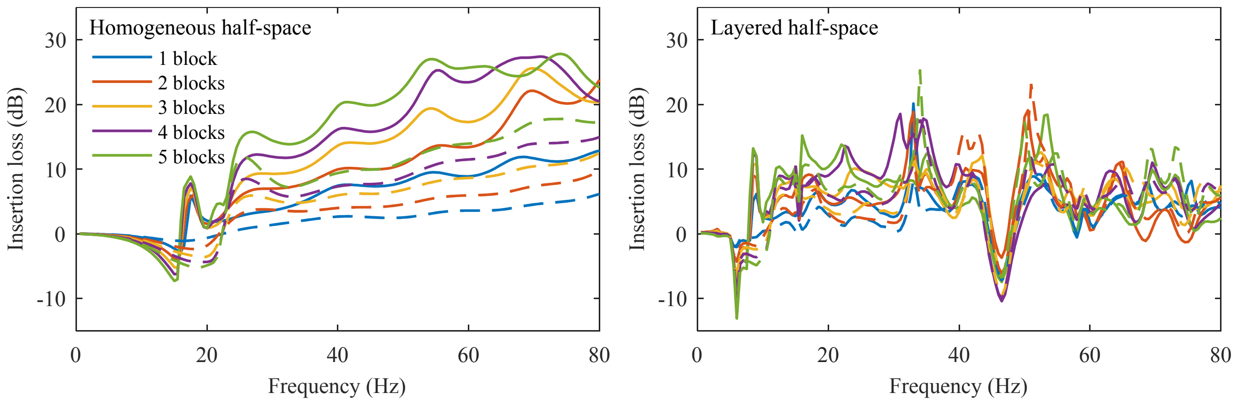


Fig. 3. Insertion loss as function of frequency for different numbers of blocks on a homogeneous half-space (left) or layered half-space (right). Full lines (——) indicate results for blocks *with* rotational inertia, whereas dashed lines (— —) indicate results for blocks *without* rotational inertia.

### *3.2. Influence of embedment and vertical extent of buried obstacles in the propagation path*

Embedding the solid blocks within the soil has two effects. Firstly, placing the blocks at a given depth below the ground surface implies a different response compared to blocks placed on the surface, since the sides and not only the bottom of the blocks are in contact with the soil. Secondly, embedment will lead to a screening effect similar to placing an array of vertical barriers. To study the importance of including these two effects in a model, comparisons are made between models that account for none, one or both of the effects. Thus, Fig. 4 shows the results obtained for models in which the blocks are either placed on the ground surface or embedded 1.0 or 2.0 m into the ground. Evidently, embedment has a significant impact on the insertion loss at the distance 28 m from the centre of the source, *i.e.* 8 m behind the centre of the backmost block when five blocks are present. For the homogeneous half-space, the embedment depth 1.0 m provides a 4 dB decrease of the IL in the frequency range from 25 Hz to about 60 Hz, relative to the IL obtained when the blocks placed on the surface; but at higher frequencies the embedment is generally favourable. Especially, at low frequencies (below 15 Hz) embedment leads to a positive IL, unlike the case with blocks placed on the ground surface. The larger embedment depth of 2.0 m is consistently leading to about 5 dB higher IL than the block placed on the surface. Finally, for the layered half-space no general trends can be observed. The dip in IL around 45 Hz is captured by all the models, but apart from this the models with or without embedment lead to differences in the IL of about 5 to 10 dB. It cannot be concluded that embedment to a depth of 1.0 or 2.0 m depth is favourable. However, it can be concluded that embedment must be modelled correctly.

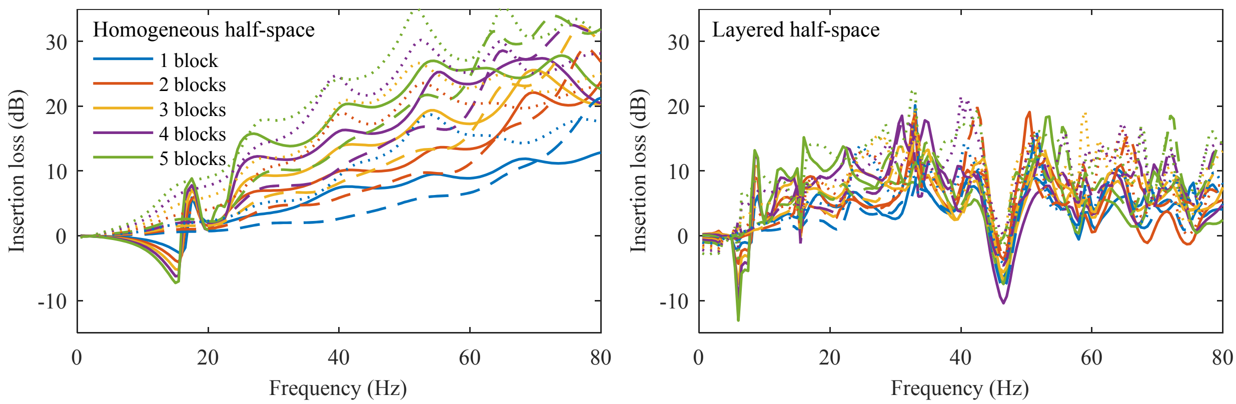


Fig. 4. Insertion loss as function of frequency for different numbers of blocks on a homogeneous half-space (left) or layered half-space (right). Full lines (——) indicate results for rigid solid blocks placed on top of the ground surface; dashed lines (— —) indicate results for rigid solid blocks embedded 1.0 m into the ground; dotted lines (······) indicate results for rigid solid blocks embedded 2.0 m into the ground.

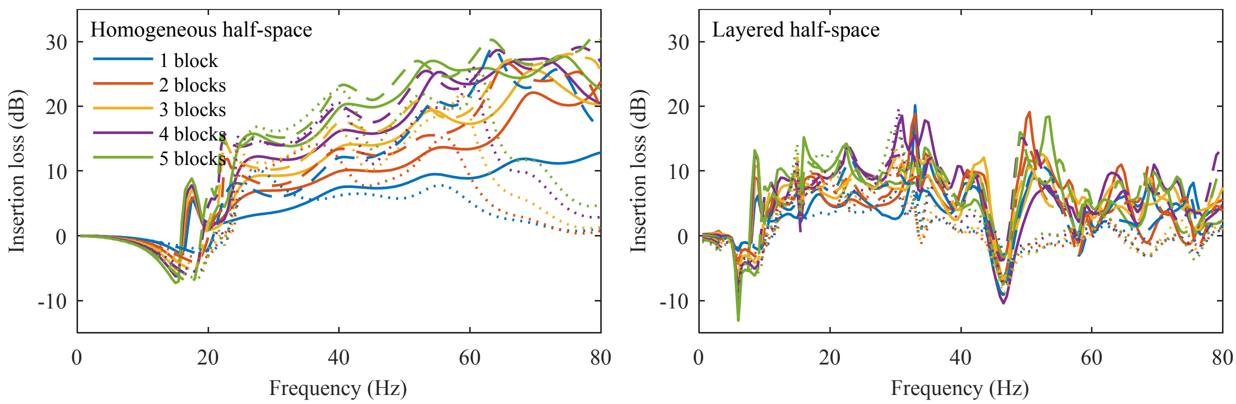


Fig. 5. Insertion loss as function of frequency for different numbers of blocks on a homogeneous half-space (left) or layered half-space (right). Full lines (——) indicate results for rigid solid blocks placed on top of the ground surface; dashed lines (— —) indicate results for rigid plates embedded 1.0 m into the ground; dotted lines (······) indicate results for rigid plates embedded 2.0 m into the ground.

Fig. 5 shows the IL obtained by embedding rigid plates 1.0 or 2.0 m into the ground. The plates have the same footprint as the blocks and are modelled with the same mass and rotational inertia; but they have no vertical extent. Clearly, this is not an adequate model of an embedded block, since the IL in the homogeneous half-space deviates by more than 10 dB at low frequencies compared to those obtained with the rigid blocks in Fig. 4. At higher frequencies the error is even larger for the plates embedded to a depth of 2.0 m. This can be due to resonance of the soil above the plates, which is not possible in the case with embedded blocks. For the layered half-space, similar deviations (more than 10 dB) exist between the results of the models with embedded blocks and plates.

## 4. Conclusions

In this paper, a semi-analytical model predicting ground vibration from rigid rectangular loads has been extended to estimate the impact of heavy masses or plates placed either on the ground surface or embedded into the soil. Generally, such a solution was found to provide a positive IL. The main conclusions can be listed as follows:

- A single heavy mass can provide a reduction in vibration levels above the mass–spring resonance frequency. Vibration reduction up to 10 dB can be expected for heavy masses over 10 tonnes. For layered media, similar reductions are observed but drawbacks could arise in the form of vibration amplification at higher frequencies.
- For stiff plates or rigid blocks embedded into the soil extra attenuation of vibration levels were observed. This can be attributed to the breaking of the Rayleigh-wave propagation path.
- For a periodic linear array of masses, reductions up to 30 dB were observed. Due to the waviness of the IL across the frequency range it was not possible to observe evidence of band-pass etc. familiar in periodic theory.
- Large variations in the solutions shown highlight the fact that care must be taken when drawing conclusions over small perturbations in this kind of modelling.

In future research, the impact of having flexible structures, representing buildings, and three-dimensional patterns of masses may be studied. It can also be of interest to relate the IL to the Rayleigh wavelengths of the soil.

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