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Application of Numerical Dispersion Compensation of the Yee-FDTD Algorithm on Elongated Domains

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***Abstract*—A postprocessing method to compensate for the numerical dispersion of the Yee-FDTD scheme is presented. The method makes use of frequency domain deconvolution of the erroneous phase shift from the obtained results and can be applied on certain specific conditions, such as for simulations on elongated computational domains. Validation of the method is performed by comparing to analytical solution in a simplified empty-space scenario. Application to simulation of an UWB deflection sensing system is demonstrated, with good match between numerical and measured results.**

1 INTRODUCTION

Numerical dispersion and anisotropy are two well-known obstacles in obtaining accurate results from the Yee finite-difference time-domain (FDTD) method [1,2]. For typical cell size of 1/10 of wavelength, the phase error reaches approx. 1 % which may be too much for simulations on large multi-wavelength domains and/or those running for extended period of time. What makes matters worse is that the error is anisotropic, ie. the phase velocity and the corresponding phase error varies with angle of propagation which in most instances does not allow for compensation of the error in post-processing.

Since the FDTD method became widely-used and the numerical dispersion problem acknowledged, additional numeric schemes other than the original Yee-FDTD [1] have been proposed to alleviate the dispersion problem, such as higher-order FDTD [3] or the multiresolution time-domain method [4]. However, the proposed methods typically come with certain tradeoffs that might not be acceptable in given situation. The dispersion error is indeed reduced, but on the cost of higher number of calculations per single FDTD cell. In addition, larger spatial stencil brings problems with accurate modeling of material interfaces.

In our recent research work, we were presented with a task to simulate propagation of ultrawideband signal (3.1–5.3 GHz) along a 58.7 m long wind turbine blade. The cell size was set to be 5 mm in this case, which is less that 1/10 of wavelength in free space, but only 1/5 in the fiberglass material that the blade is made of. The propagation distance was corresponding to 963 wavelengths in free space, consequently the resulting phase error was enormous and the obtained output signals were severely distorted.

In this paper we present the method and results of applying a dispersion compensation by means of deconvolution to the output signals of the wind turbine blade simulation. In most other situations, the calculated field would be a mixture of plane waves coming from all directions, which would prevent us from obtaining meaningful results by applying such compensation. In this case, however, we took the advantage of the given problem and exploited the fact that the wave is, for the most of its path, propagating in free space and close to the direction along the FDTD grid. Therefore, we were able to deconvolve (via frequency domain) the dispersion effects out from the output signals, assuming that the wave was propagating along the grid. Thus obtained corrected output signals contained only small residual errors from propagating in slightly other directions and through the fiberglass shell, and compared well with measurements on a real blade.

The paper is organized as follows. Section 2 introduces the original problem that suffered from excessive numerical dispersion error and necessitated the use of the dispersion compensation method, followed by explanation of the method itself. In Section 3, the simulation results are presented and compared first to theory and then to data obtained by measurement on a real wind turbine blade. Finally, Section 4 concludes the paper and outlines the conditions on which the presented compensation method can be used in other applications.

2 BACKGROUND

2.1 FDTD Simulation of Wind Turbine Blade

Our recent research work was to design a deflection sensing system for wind turbine blades based on UWB technology [5]. The system is composed of a TX antenna mounted near the tip and two RX antennas near the root of the blade (Fig. 1). The deflection (bending) of the blade under operational conditions is determined by triangulation based on precise timing of the UWB signals received at the root antennas.

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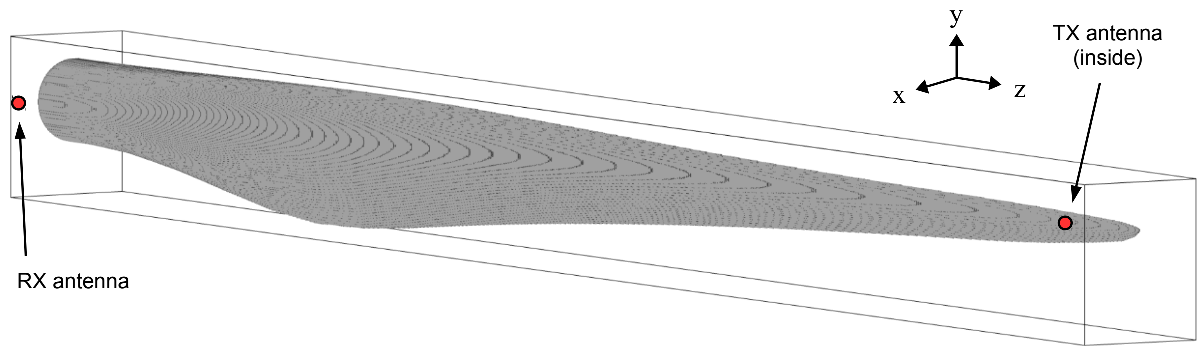

Figure 1: Wind turbine blade model simulated by Yee-FDTD on elongated computational domain.

Whereas the RX antennas at the root are placed 40 cm away from the blade, the tip TX antenna has to be mounted inside the blade due to aerodynamic reasons and to protect it against lightning strikes. Therefore, the signal traveling between the TX and RX antennas has to penetrate the blade shell made of fiberglass material and propagate close to the blade surface along almost the entire length of the blade.

In order to predict the available signal levels and distortion of the pulses due to multipath propagation, we simulated the UWB wireless link using the FDTD method. The excitation pulse was a Gauss-sine with the frequency range 3.1–5.3 GHz. The cell size was fixed to 5 mm in all directions (cubical cell) in the entire computational domain.

The simulation turned out to be a difficult task mainly for two reasons. First, due to the chosen cell size the computational domain dimensions were 764 × 945 × 11824 cells (Fig. 1), plus 50 cells of PML in order to suppress reflections at shallow angles, exceeding a total of 10 billion mesh cells. One simulation originally needed 33 hours to complete on our parallel cluster; this running time was eventually reduced by 7 hours after employing load balancing of the parallel processes [6].

The second problem was the numerical dispersion inherent in the FDTD method. The chosen cell size represents 1/10 of wavelength in free space, and only 1/5 in the fiberglass material that the blade is made of. This cell size would likely produce acceptable error in most small scenarios, but not in this case. As the analyzed wind turbine blade is 58.7 m long, the signal has to traverse 963 wavelengths in free space, resulting in enormous phase errors and distortion of the received pulse.

In order to cope with the pulse distortion, we have added dispersion compensation to the postprocessing of the results. The procedure is explained in the following section.

2.2 Numerical Dispersion Compensation

The fields calculated by the Yee-FDTD method are affected by numerical dispersion and anisotropy. If we assume numerical wavevector with arbitrary direction $\tilde{k} = (\tilde{k}_x, \tilde{k}_y, \tilde{k}_z)$, then the dispersion relation reads

$$\left[\frac{1}{c\Delta t}\sin\frac{\omega\Delta t}{2}\right]^2 = \sum_{\xi\in x,y,z}\left[\frac{1}{\Delta\xi}\sin\frac{\tilde{k}_\xi\Delta\xi}{2}\right]^2 \quad (1)$$

where $\Delta x, \Delta y, \Delta z$ are the dimensions of the FDTD mesh, Δt is the time step, c is speed of light in the background media, and $\omega = 2\pi f$ is the angular frequency.

Generally, FDTD results calculated in large computational domains are composed of waves coming from various directions and having a history of propagating through various media and various parts of the mesh (in case of non-homogeneous mesh). The phase error caused by the numerical dispersion is thus unknown, because, as follows from (1), $\tilde{k}$ is dependent on the mesh parameters, medium, and direction of propagation.

We have noted, however, that our simulation of a wind turbine blade as described in section 2.1 and Fig. 1 is a special case in which the dispersion error can be partially compensated for. In our case, the signal propagates along the blade in a direction close to parallel to the orientation of the mesh and mostly in air. If we approximate the numerical wavevector as having only the z (longitudinal) component, then its value can be expressed as

$$\tilde{k}_z = \frac{2}{\Delta z}\arcsin\left(\frac{\Delta z}{c\Delta t}\sin\frac{\omega\Delta t}{2}\right) \quad (2)$$

The E-fields at the root E can then be compensated for the phase errors by deconvolution in frequency domain

$$E^{(\mathrm{comp})} = E\cdot \mathrm{e}^{\mathrm{j}(\tilde{k}_z - k)r} \quad (3)$$

where $k = \omega/c$ is the true wavenumber and r is the known distance between the TX antenna and the point of interest. This approach can be applied not only to E- and H-fields, but also to any output signals captured by the RX antennas, and requires the received pulse to be transformed from time to frequency domain and back.

Described compensation procedure can be used on any elongated domain, where the waves are expected to be propagated in a single direction or close to that direction, and through single media with known properties. Paths leading through media having numerical propagation properties different from those determined by (2) then act as sources of additional errors.

In our case, the wave is mostly propagating inside or along the blade in longitudinal direction and through air. The distances traversed through the fiberglass blade shell are relatively short, hence the additional error is expected to be small.

3 RESULTS

The compensation method was first validated by comparing FDTD computed data to theoretical formula in a simplified scenario of an empty space. Then it was applied to the simulation of the 58.7 m long wind turbine blade, and the results were verified by measurements obtained using vector network analyzer (VNA).

3.1 Theory

In this scenario, the blade and both antennas were excluded from the simulation, leaving only the elongated computational domain with 50 cells thick PML layers on each wall. Hertzian dipole oriented in x direction was used in place of the TX antenna at the tip, and the E-field at the position of the root RX antenna was observed. The E_x field at large distance perpendicular to the x-oriented Hertzian dipole can be expressed analytically as

$$E_x \approx -\frac{\mathrm{j}k\eta_0 Il}{4\pi r}\mathrm{e}^{-\mathrm{j}kr} \tag{4}$$

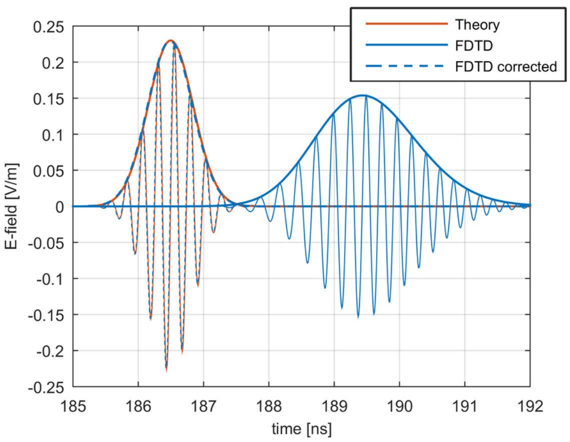

Figure 2: Comparison of theoretical (red), simulated (blue) and simulated with numerical dispersion compensation (dashed blue) E-field produced at a distance of 57 m across empty space.

where $\eta_0 = 120\pi\ \Omega$ is the free-space wave impedance, I is the dipole current with amplitude 1 A, and l is the dipole length. The distance r between the dipole and the observed E-field is 55.525 m.

As can be seen in Fig. 2, the FDTD simulated pulse (solid blue line) suffers heavily from numerical dispersion: its amplitude and shape are distorted, and

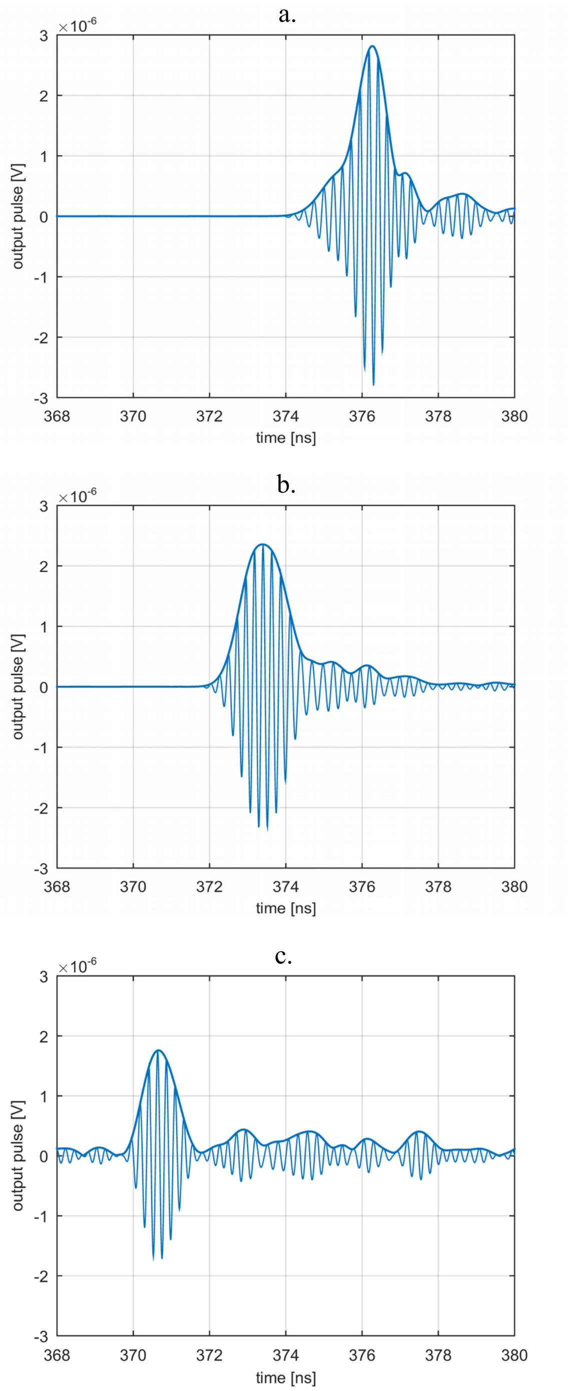

Figure 3: UWB pulses at the output of the complete deflection sensing system mounted on a 58.7 m long wind turbine blade: a. simulated, b. simulated with numerical dispersion compensation, c. measured.

the pulse comes late due to different group delay. After compensation (3) (dashed blue line) the pulse is almost identical to the result obtained analytically from (4) (red line).

3.2 Measurements

Comparison of the original and compensated FDTD results with measured data is shown in Fig. 3. The shape of the pulse after numerical dispersion compensation (Fig. 3b) matches well with the VNA-measured pulse (Fig. 3c). On the other hand, without compensation (Fig. 3a) the pulse shape is distorted. Because the correlator detector at the receiver locks on the rising edge, it is this particular part of the pulse that is the most important to the present study.

The amplitude of the simulated pulse is slightly overestimated even with compensation, but the error is within 3 dB. The delay of the measured pulse is adjusted by 20.53 ns so that it corresponds to the simulation setup (extra antenna cable), the remaining error in delay is then about 1.5 ns.

4 CONCLUSION

In this paper, the numerical dispersion compensation method for the Yee-FDTD method has been described. The method was demonstrated on simulation of a 58.7 m long wind turbine blade, where the shape of the received pulse compared well with the measured data. The FDTD method has thus been proved to be a viable tool to optimize positions of the RX antennas of the deflection sensing system and to predict behavior of the received UWB pulse under various deflections of the blade.

The compensation method should be applicable to other scenarios as well, provided that certain specific requirements are met. The received signal should travel through the numerical grid in known predominant direction, which is satisfied on elongated computational domains. It should also mostly travel through a single medium, as any portion of the propagation path spent inside other medium contributes to further error. Finally, the compensation improves the shape of the initial arrived pulse only, or generally any pulse for which the propagation distance is known. If these conditions are met, the residual error after the compensation is expected to be small.

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References

[1] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Transactions on Antennas and Propagation*, vol. 14, no. 3, pp. 302–307, May 1966.

[2] A. Taflove, and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, Artech House, 3rd ed., 2005.

[3] T. T. Zygiridis, and T. D. Tsiboukis, "Development of higher order FDTD schemes with controllable dispersion error," *IEEE Transactions on Antennas and Propagation*, vol. 53, no. 9, pp. 2952–2960, Sep. 2005.

[4] M. Krumpholz, and L. P. B. Katehi, "MRTD: New time-domain schemes based on multiresolution analysis," *IEEE Transactions on Microwave Theory and Techniques*, vol. 44, no. 4, pp. 555–571, Apr. 1996.

[5] O. Franek, S. Zhang, T.L. Jensen, P.C.F. Eggers, K. Olesen, C. Byskov, and G.F. Pedersen, "Wind turbine blade deflection sensing system based on UWB technology," in *Proc. IEEE-APS Topical Conference on Antennas and Propagation in Wireless Communications (APWC 2016)*, 2016, pp. 260–263.

[6] O. Franek, "A simple method for static load balancing of parallel FDTD codes," in *Proc. International Conference on Electromagnetics in Advanced Applications (ICEAA 2016)*, 2016, pp. 587–590.