Adaptive Fuzzy Output Regulation for Formation Control of Unmanned Surface Vehicles

Li, Shaobao; Er, Meng Joo ; Wang, Ning ; Chien , Chiang-Ju ; Yang, Zhenyu

Published in:
Procedings of 2017 International Conference on Fuzzy Theory and Its Applications (iFUZZY 2017)

DOI (link to publication from Publisher):
10.1109/iFUZZY.2017.8311796

Publication date:
2017

Document Version
Accepted author manuscript, peer reviewed version

Adaptive Fuzzy Output Regulation for Formation Control of Unmanned Surface Vehicles

Shaobao Li
Department of Energy Technology, Aalborg University
Esbjerg, Denmark
shl@et.aau.dk

Meng Joo Er
School of Electrical and Electronic Engineering Nanyang Technological University
Singapore
emjer@ntu.edu.sg

Ning Wang
Marine Engineering College Dalian Maritime University
Dalian, China
n.wang.dmu.cn@gmail.com

Chiang-Ju Chien
Department of Electronic Engineering Huafan University
New Taipei City, Taiwan
cjc@cc.hfu.edu.tw

Zhenyu Yang
Department of Energy Technology, Aalborg University
Esbjerg, Denmark
yang@et.aau.dk

***Abstract*—In this paper, the formation control problem of unmanned surface vehicles (USVs) is investigated. Unlike the classical formation control problem where the reference signal is required to be second-order differentiable with respect to time, we consider a more general autonomous dynamic system as the reference system. A novel adaptive fuzzy output regulation approach is presented to solve the formation control problem, where a set of regulator equations using only approximation information and a distributed observer are constructed to obtain the feedforward information of the reference system. Based upon this, a distributed adaptive fuzzy control law is designed by using the backstepping technique. It is shown that USVs can effectively achieve the desired formation with the tracking error being adjusted as small as possible. Simulation studies demonstrate that the proposed formation control law is effective and efficient.**

***Index Terms*—formation control, output regulation, adaptive fuzzy control, unmanned surface vehicle**

I. Introduction

Benefiting from their prominent characteristics such as large-scale distribution, multi-task parallelism, economy, etc., unmanned surface vehicles (USVs) have attracted much attention in recent years. Formation control is one of the important research topics of USVs, and it can be potentially applied in ocean environment surveillance, dynamic networking of marine vessels and seabed mapping [1].

Recently, many researchers have devoted much effort to investigate the formation control problem of USVs. For example, in [2], a formation control algorithm based on terminal sliding mode observer was proposed, where the dynamics of USVs are required to be known precisely. In [3], a guidance system was needed to supply desired reference signals, based upon which a universal consensus control law for all formation reference points was proposed to make USVs achieve desired formation. In [4], the USVs were subject to uncertain kinematics and unknown dynamics induced by model uncertainties and ocean disturbances, and a modular adaptive control method was proposed to design the formation control algorithm. In [5], neural network was applied to approximate the unknown dynamics of USVs, and a line-of-sight formation algorithm was proposed. In [6], the formation control problem of fully-actuated USVs was studied, and a consensus-based adaptive control law under bidirectional communication topology is proposed based on potential function.

To the best of our knowledge, the existing formation control approaches are dependent on a given reference signal which must be second-order differentiable with respect to time. However, in many practical systems, e.g., motion target tracking without the trajectory known in advance, the existing approaches cannot be easily extended to cope with the formation control problem of USVs. To be more general, this paper considers the case where the reference trajectory for USVs is generated by an autonomous system. To make the problem more practical, we also assume that the dynamics of USVs are subject to unknown nonlinearity and disturbances. A novel adaptive fuzzy output regulation approach is then presented to solve the formation control problem under the aforementioned circumstances. It is shown that the proposed formation approach can not only be used for tracking an autonomous dynamic system by USVs, but also be applied to solve classical tracking problems with second-order differentiable reference signals as in [3], [5], [7].

Throughout this paper, $\mathcal{R}^p$ denotes the space of real p-vectors. $\|\cdot\|$ denotes the 2-norm, and $\lambda_{max}(\cdot)$ and $\lambda_{min}(\cdot)$ are the maximum and minimum eigenvalues of a matrix, respectively. Define $\mathrm{col}(x_1, \cdots, x_n) = [x_1^T, \cdots, x_n^T]^T$, and let $A = \mathrm{diag}(A_1, \cdots, A_n)$ denote a diagonal matrix with

The authors would like to acknowledge the funding support from the Ministry of Education, Singapore (Tier 1 AcRF, RG29/15), the National Natural Science Foundation of P. R. China under Grant 61403334, the Natural Science Foundation of Hebei Province under Grant F2017203109, the Postdoctoral Science Foundation of China under Grant 2015M581318, and Ministry of Science and Technology, R.O.C., under Grant MOST106-2221-E-211-001.

diagonal elements being $A_1, \cdots, A_n$. The symbol $=:$ is used to define an equation.

II. Preliminaries and Problem Formulation

A. Review of Graph Theory

It is convenient to describe the communication network of a multi-USV system by a graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$, where the vertex set $\mathcal{V} = \{1, \cdots, n\}$ describes the indexes of USVs, the edge set $\mathcal{E} \subseteq \{(i,j) \mid i, j \in \mathcal{V}, i \neq j\}$ denotes the communication links between USVs, and $\mathcal{A} = [a_{ij}]$ is the adjacency matrix with $a_{ij} > 0$ if USV i can receive information from USV j and $a_{ij} = 0$ otherwise. The set of neighbors of USV i is defined as $\mathcal{N}_i = \{j \in \mathcal{V} \mid (i,j) \in \mathcal{E}\}$. In this paper, without loss of generality, the communications between USVs are assumed to be unidirectional, and thus graph $\mathcal{G}$ is a directed graph. A path is a finite sequence of distinct edges in the form of $(i_1, i_2), (i_2, i_3), \cdots, (i_{k-1}, i_k)$, and vertex i_k is said to be reachable from vertex i_1. A graph contains a spanning tree if there exists a vertex (which is called the root) that can reach every vertex of the graph.

The in-degree of vertex i is denoted as $\deg_i = \sum_{j=1}^{n} a_{ij}$, and the matrix $\mathcal{D} = \mathrm{diag}\{\deg_1, \cdots, \deg_n\}$ is called the degree matrix of graph $\mathcal{G}$. The Laplacian matrix of $\mathcal{G}$ is defined to be $\mathcal{L}_{\mathcal{G}} = \mathcal{D} - \mathcal{A}$. Let $\lambda_1, \cdots, \lambda_n$ denote the eigenvalues of $\mathcal{L}_{\mathcal{G}}$. It is well known that $\mathcal{L}_{\mathcal{G}}$ has exactly one zero eigenvalue denoted by $\lambda_1 = 0$ with $\mathbf{1}_n$ being the corresponding eigenvector, if and only if graph $\mathcal{G}$ contains at least one spanning tree [8]. Furthermore, all the other eigenvalues $\lambda_2, \cdots, \lambda_n$ of $\mathcal{L}_{\mathcal{G}}$ have positive real parts.

B. Problem Formulation

Consider a multi-USV system consisting of n 3-DOF USVs. The dynamics of ith USV for $i = 1, \cdots, n$ is given as follows:

$$\dot{\eta}_i = J(\psi_i)\nu_i \tag{1}$$

$$M_i\dot{\nu}_i + C_i(\nu_i)\nu_i + D_i(\nu_i)\nu_i = \tau_i + d_i \tag{2}$$

where $\eta_i = [x_i, y_i, \psi_i]^T \in \mathcal{R}^3$ is the position state vector of coordinates (x_i, y_i) and heading angle (ψ_i) of USV i in the earth-fixed inertial frame, $\nu_i = [u_i, v_i, r_i]^T \in \mathcal{R}^3$ is the corresponding surge, sway and yaw velocities in the body-fixed frame of USV i, $\tau_i \in \mathcal{R}^3$ is the control input, $d_i \in \mathcal{R}^3$ is the unknown time-varying external disturbances due to wind, waves and ocean currents, and matrix $J(\psi_i)$ is the 3-DOF rotation matrix defined as follows:

$$J(\psi_i) = \begin{bmatrix} \cos(\psi_i) & -\sin(\psi_i) & 0 \\ \sin(\psi_i) & \cos(\psi_i) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

Matrices M_i, $C_i(\nu_i)$ and $D_i(\nu_i)$ are the inertia matrix, Coriolis and centripetal matrix and damping matrix, respectively, where M_i and $C_i(\nu_i)$ can be measurable but $D_i(\nu_i)$ is hard to be identified. Therefore, the dynamics (1)-(2) of USVs are considered to be unknown in this paper.

The objective of this paper is to make the USVs achieve a desired formation while tracking a reference trajectory. The reference trajectory is governed by the following dynamics:

$$\begin{cases} \dot{x}_d = Sx_d \\ y_d = q(x_d) \end{cases} \tag{4}$$

where $x_d \in \mathcal{R}^m$ is the state and $y_d \in \mathcal{R}^3$ is the output with $q(x_d)$ being a known sufficiently smooth nonlinear function. η_i, $i = 1, \cdots, n$, are desired to track the trajectory y_d while keeping a desired relative distance δ_i with y_d. Also, we can further define the desired relative position between neighboring USVs as $\delta_{ij} =: \lim_{t\to\infty}(\eta_i - \eta_j) = \delta_i - \delta_j$, $i \in \mathcal{V}$, $j \in \mathcal{N}_i$. Then we can define the formation control problem of the multi-USV system as follows:

Definition 1: Given the multi-USV system (1)-(2) and its underlying communication topology $\mathcal{G}$, the formation control problem is solved if we can find a distributed control law τ_i, $i = 1, \cdots, n$, such that the following two properties are satisfied:

Property 1: All signals in the closed-loop system are globally uniformly ultimately bounded (GUUB);

Property 2: For any initial conditions $\eta(t_0) = \mathrm{col}(\eta_1(t_0), \cdots, \eta_n(t_0))$ and $\nu(t_0) = \mathrm{col}(\nu_1(t_0), \cdots, \nu_n(t_0))^T$, there exist a sufficiently small constant $\varepsilon > 0$ and a finite time $T(\eta(t_0), \nu_i(t_0), \varepsilon)$ such that the tracking error $e_i = \eta_i - y_d - \delta_i$ satisfies $\|e_i\| < \varepsilon$, $\forall t > T(\eta(t_0), \nu_i(t_0), \varepsilon)$.

Remark 1: The reference system (4) can be an autonomous vehicle and viewed as a leader of the multi-USV system. Due to the limitation of sensing and communication ranges in practical USV systems, it is meaningful to assume that only the USVs closed enough to the reference system can get access to its information, which arises a problem that how to obtain the reference information for those USVs that cannot get access to the reference system directly. Another challenge problem significantly different from existing studies is how to design a distributed control law to achieve a desired formation while tracking the reference system in the form of (4) without precise USV dynamics.

Two standard assumptions are needed.

Assumption 1: The underlying communication topology $\mathcal{G}$, which consists of the reference system and the n USVs, contains a spanning tree with the root being the reference system.

Assumption 2: The disturbances d_i, $i = 1, \cdots, n$, are bounded by a constant d, i.e., $\|d_i\| \leq d$, $i = 1, \cdots, n$.

C. Fuzzy Logic System

A fuzzy logic system consists of fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier. The fuzzy rule base contains a number of fuzzy IF-THEN rules in the form as

R^l: IF x_1 is F_1^l and $\cdots$ and x_r is F_r^l, THEN y is G^l,

where $x = [x_1, \cdots, x_r]^T \in \mathcal{R}^r$ is the input and y is the output of the fuzzy logic system. $F_j^l, j = 1, \cdots, r$ and G^l are labels of fuzzy sets characterized by appropriate membership functions $\mu_{F_j^l}(x_j)$ and $\mu_{G^l}(y)$. $l \in \{1, \cdots, m\}$ describes the rule sequence.

We will use the same fuzzy logic system proposed as in [9], which can be expressed as

$$y(x) = \frac{\sum_{l=1}^{m} \bar{y}_l \prod_{j=1}^{r} \mu_{F_j^l}(x_j)}{\sum_{k=1}^{m} \prod_{j=1}^{r} \mu_{F_j^k}(x_j)}, \tag{5}$$

where $\bar{y}_l = \max \mu_{G^l}(y)$. Define the fuzzy basis functions as

$$\varphi_l(x) = \frac{\prod_{j=1}^{r} \mu_{F_j^l}(x_j)}{\sum_{k=1}^{m} \prod_{j=1}^{r} \mu_{F_j^k}(x_j)}. \tag{6}$$

Let $\theta = [\bar{y}_1, \cdots, \bar{y}_m]^T = [\theta_1, \cdots, \theta_m]^T$ and $\varphi(x) = [\varphi_1(x), \cdots, \varphi_m(x)]^T$, and then the fuzzy logic system (5) can be rewritten in the form as

$$y(x) = \theta^T \varphi(x). \tag{7}$$

This fuzzy logic system has the following property:

Lemma 1 ([9]): Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists a fuzzy logic system (3) such that

$$\sup_{x \in \Omega} |f(x) - \theta^T \varphi(x)| \leq \varepsilon. \tag{8}$$

III. ADAPTIVE FUZZY OUTPUT REGULATION DESIGN

In this section, the formation control problem of multi-USV systems will first be transformed into an output regulation problem, and then a novel distributed adaptive fuzzy control law will be presented to solve the output regulation problem.

A. Output Regulation Design

The reference system (4) can also be called as the exosystem of the multi-USV system (1)-(2). According to the output regulation theory [10], the formation control problem can be conveniently transformed to solving an output regulation problem.

Solving the formation control problem is equivalent to making the states η_i, $i = 1, \cdots, n$, track the reference signal $(y_d + \delta_i)$, $i = 1, \cdots, n$. Hence, we can define $\zeta_{di} = \mathrm{col}(x_d, \delta_i)$ and construct the following exosystem for ith USV:

$$\dot{\zeta}_{di} = \begin{bmatrix} S & O_{m\times 3} \\ O_{3\times m} & O_{3\times 3} \end{bmatrix} \zeta_{di} = \bar{S}\zeta_{di}. \tag{9}$$

Then the tracking error can be rewritten as $e_i = \eta_i - \bar{q}(\zeta_{di}) = \eta_i - y_d - \delta_i$.

Let $\xi_i = J(\psi_i)\nu_i$. The multi-USV system (1)-(2) can be rewritten as follows:

$$\begin{cases} \dot{\eta}_i &= \xi_i \\ \dot{\xi}_i &= \bar{C}_i(\eta_i, \xi_i)\xi_i + \bar{D}_i(\eta_i, \xi_i)\xi_i + \bar{\tau}_i + \bar{d}_i \\ e_i &= \eta_i - \bar{q}(\zeta_{di}) \end{cases} \tag{10}$$

where $\bar{C}_i(\eta_i, \xi_i) = -J(\psi_i)M_i^{-1}C_i(\nu_i)J^{-1}(\psi_i)$, $\bar{D}_i(\eta_i, \xi_i) = \dot{J}(\psi_i)J^{-1}(\psi_i) - J(\psi_i)M_i^{-1}D_i(\nu_i)J^{-1}(\psi_i)$, $\bar{\tau}_i = J(\psi_i)M_i^{-1}\tau_i$ and $\bar{d}_i = J(\psi_i)M_i^{-1}d_i$. Thus, the formation control problem is transformed to design a distributed control law $\bar{\tau}_i$ such that the tracking error e_i, $i = 1, \cdots, n$, can be regulated into a sufficiently small neighborhood of origin, which thus becomes an output regulation problem.

Let $f_i(\eta_i, \xi_i) = \bar{C}_i(\eta_i, \xi_i)\xi_i + \bar{D}_i(\eta_i, \xi_i)\xi_i$. Obviously, $f_i(\eta_i, \xi_i)$ is an unknown nonlinear function. Here, we will apply a fuzzy logic system to approximate $f_i(\eta_i, \xi_i)$ as follows:

$$\hat{f}_i(\eta_i, \xi_i \mid \hat{\theta}_i) = \hat{\theta}_i^T \varphi_i(\eta_i, \xi_i) \tag{11}$$

where $\hat{\theta}_i$ is the estimate of θ_i^*, which is the optimal parameter vector defined as

$$\theta_i^* = \arg \lim_{\hat{\theta}_i} \left(\sup_{\eta_i, \xi_i} \| \hat{f}_i(\eta_i, \xi_i \mid \hat{\theta}_i) - f_i(\eta_i, \xi_i) \| \right). \tag{12}$$

The approximation errors of the fuzzy logic system are defined as follows:

$$\epsilon_i^* = f_i(\eta_i, \xi_i \mid \theta^*) - \hat{f}_i(\eta_i, \xi_i), \tag{13}$$

$$\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i \tag{14}$$

To solve the output regulation problem (10), the following assumptions are further needed.

Assumption 3: There exists a positive constant ϵ such that $\|\epsilon_i^*\| \leq \epsilon$, $i = 1, \cdots, n$.

Assumption 4: There exist sufficiently smooth functions $\mathbf{X}_i(\zeta_{di})$, $\mathbf{Z}_i(\zeta_{di})$ and $\mathbf{U}_i(\zeta_{di})$ with $\mathbf{X}_i(0) = 0$, $\mathbf{Z}_i(0) = 0$ and $\mathbf{U}_i(0) = 0$ such that for any ζ_{di} the following regulator equations hold:

$$\begin{cases} \dfrac{\partial \mathbf{X}_i}{\partial \zeta_{di}} \bar{S}\zeta_{di} &= \mathbf{Z}_i \\ \dfrac{\partial \mathbf{Z}_i}{\partial \zeta_{di}} \bar{S}\zeta_{di} &= \hat{\theta}_i^T \varphi(\mathbf{X}_i, \mathbf{Z}_i) + \mathbf{U}_i \\ 0 &= \mathbf{X}_i - \bar{q}(\zeta_{di}) \end{cases} \tag{15}$$

Remark 2: Assumption 4 is quite standard in output regulation problem of nonlinear systems [10]. The regulator equations (15) define a center manifold $\mathcal{M} = \{(\eta_i, \xi_i, \zeta_{di}) \mid \eta_i = \mathbf{X}_i, \xi_i = \mathbf{Z}_i, i = 1, \cdots, n\}$ of the multi-USV system under the control $\mathrm{col}(\mathbf{U}_1, \cdots, \mathbf{U}_n)$. In classical output regulation problem, e.g., [11]–[13], system dynamics are usually assumed to be known such that the accurate regulator equations can be constructed and precise feedforward information dependent on exosystem states can be obtained. However, the dynamics of multi-USV systems in this paper have unknown nonlinearity and disturbances, and thus it is impossible to construct accurate regulator equations. In what follows, we will present a novel adaptive fuzzy control scheme to solve the formation control problem using only the inaccurate regulator equations (15). It is worth mentioning that Assumption 4 can be easily satisfied if $\bar{q}(\zeta_{di})$ is second-order differentiable along ζ_{di}.

B. Distributed Observer Design

It is noted that state x_d cannot be used for those USVs that cannot get access to the reference system directly. As a result, the solution of the regulator equations (15) cannot be directly used for distributed control law design. Towards this end, a distributed observer is necessary to be designed to estimate x_d.

The distributed observer can be given as follows:

$$\dot{\hat{x}}_{di} = S\hat{x}_{di} + k\left(\sum_{j=1}^{n} a_{ij}(\hat{x}_{dj} - \hat{x}_{di}) + a_{i0}(x_d - \hat{x}_{di})\right) \quad (16)$$

where $\hat{x}_{di}$ is the estimate of x_d, and $a_{i0} > 0$ if USV i can get access to the reference system and $a_{i0} = 0$ otherwise.

Then we can obtain the following theorem.

Theorem 1: Assume that the communication topology of the multi-USV system (1)-(2) satisfies Assumption 1. Let $k > \frac{\lambda_{\max}(S)}{\lambda_{\min}(\mathcal{L}_{\mathcal{G}}+\Lambda)}$, where $\Lambda = \mathrm{diag}(a_{10}, \cdots, a_{n0})$. Then $\hat{x}_{di}$, $i = 1, \cdots, n$, will achieve consensus to x_d asymptotically.

The consensus of linear multi-agent systems like (16) has been widely studied. The similar results can be found in [14], [15], and thus the proof is omitted here.

Designing the distributed observer aims at constructing a set of new regulator equations that only use local information. Let $\bar{\zeta}_{di} = [\hat{x}_{di}^T, \delta_i^T]^T$, and we have

$$\begin{cases} \dfrac{\partial \bar{\mathbf{X}}_i(\bar{\zeta}_{di})}{\partial \bar{\zeta}_{di}} \bar{S}\bar{\zeta}_{di} = \bar{\mathbf{Z}}_i(\bar{\zeta}_{di}) \\ \dfrac{\partial \bar{\mathbf{Z}}_i(\bar{\zeta}_{di})}{\partial \bar{\zeta}_{di}} \bar{S}\bar{\zeta}_{di} = \hat{\theta}_i^T \varphi(\bar{\mathbf{X}}_i, \bar{\mathbf{Z}}_i) + \bar{\mathbf{U}}_i(\bar{\zeta}_{di}) \\ \dfrac{\partial \hat{\mathbf{X}}_{di}(x_d)}{\partial x_d} S x_d = S\hat{\mathbf{X}}_{di} + k\Big(\sum_{j=1}^{n} a_{ij}(\hat{\mathbf{X}}_{dj} - \hat{\mathbf{X}}_{di}) \\ \qquad\qquad + a_{i0}(x_d - \hat{\mathbf{X}}_{di})\Big) \\ 0 = \bar{\mathbf{X}}_i(\bar{\zeta}_{di}) - \bar{q}(\bar{\zeta}_{di}) \end{cases} \quad (17)$$

One can easily observe that the new regulator equations (17) have solution $\bar{\mathbf{X}}_i(\bar{\zeta}_{di})$, $\bar{\mathbf{Z}}_i(\bar{\zeta}_{di})$, $\bar{\mathbf{U}}_i(\bar{\zeta}_{di})$ and $\hat{\mathbf{X}}_{di} = x_d$, $i = 1, \cdots, n$ if Assumption 4 holds.

C. Distributed Adaptive Fuzzy Control

Now we can use $\bar{\mathbf{X}}_i(\bar{\zeta}_{di})$, $\bar{\mathbf{Z}}_i(\bar{\zeta}_{di})$, $\bar{\mathbf{U}}_i(\bar{\zeta}_{di})$ instead of $\mathbf{X}_i(\zeta_{di})$, $\mathbf{Z}_i(\zeta_{di})$ and $\mathbf{U}_i(\zeta_{di})$ to design the distributed control law. Backstepping technique will be applied for the adaptive fuzzy control design. Note that we can directly obtain $\bar{\mathbf{X}}_i = \bar{q}(\bar{\zeta}_{di})$ from the last equation of (17), and thus the formation control problem will be solved if η_i can converge to $\bar{q}(\bar{\zeta}_{di})$. Therefore, we can define two auxiliary variables $s_{1i} = \eta_i - \bar{\mathbf{X}}_i$ and $s_{2i} = \xi_i - \alpha_i$, where $\alpha_i \in \mathcal{R}^3$ is a virtual controller.

Step 1: Note that the first equation of (17) implies that $\dot{\bar{\mathbf{X}}}_i = \bar{\mathbf{Z}}_i$. The derivative of s_{1i} along time t yields:

$$\begin{aligned} \dot{s}_{1i} &= \xi_i - \bar{\mathbf{Z}}_i \\ &= s_{2i} + \alpha_i - \bar{\mathbf{Z}}_i \end{aligned} \quad (18)$$

We can choose

$$\alpha_i = -c_i s_{1i} + \bar{\mathbf{Z}}_i. \quad (19)$$

where c_i is a positive constant to be determined later.

Substituting (19) into (18) yields

$$\dot{s}_{1i} = -c_i s_{1i} + s_{2i}. \quad (20)$$

Choose a Lyapunov function candidate as follows:

$$V_{1i} = \frac{1}{2} s_{1i}^T s_{1i}. \quad (21)$$

We can obtain the derivative of V_{1i} along time t as

$$\dot{V}_{1i} = -c_i s_{1i}^T s_{1i} + s_{1i}^T s_{2i}. \quad (22)$$

The first term in the right of (22) is negative and the second term will be handled in the next step.

Step 2: The derivative of s_{2i} along time t gives

$$\begin{aligned} \dot{s}_{2i} &= f_i(\eta_i, \xi_i) + \bar{\tau}_i + \bar{d}_i - \dot{\alpha}_i \\ &= f_i(\eta_i, \xi_i) + \bar{\tau}_i + \bar{d}_i + c_i \dot{s}_{1i} - \dot{\bar{\mathbf{Z}}}_i \\ &= f_i(\eta_i, \xi_i) + \bar{\tau}_i + \bar{d}_i + c_i s_{2i} - c_i^2 s_{1i} - \hat{\theta}_i^T \varphi(\bar{\mathbf{X}}_i, \bar{\mathbf{Z}}_i) - \bar{\mathbf{U}}_i \end{aligned} \quad (23)$$

Substitute $f_i(\eta_i, \xi_i) = \theta_i^{*T} \varphi(\eta_i, \xi_i) + \epsilon_i^*$ into (23) and let $\tilde{\tau}_i = \bar{\tau}_i - \bar{\mathbf{U}}_i$ and $\Delta_i = \bar{d}_i + \epsilon_i^*$. Then we have

$$\dot{s}_{2i} = \theta_i^{*T} \varphi(\eta_i, \xi_i) - \hat{\theta}_i^T \varphi(\bar{\mathbf{X}}_i, \bar{\mathbf{Z}}_i) + \tilde{\tau}_i + c_i s_{2i} - c_i^2 s_{1i} + \Delta_i, \quad (24)$$

Since d_i and ϵ_i^* are bounded, and hence Δ_i is also bounded. For simplicity, we assume that there exists a constant Δ_M such that $\|\Delta_i\| \leq \Delta_M$.

Now we are ready to give the distributed adaptive fuzzy control law for solving the formation control problem as follows:

$$\tilde{\tau}_i = -K_i s_{2i} - \hat{\theta}_i^T \left(\varphi(\eta_i, \xi_i) - \varphi(\bar{\mathbf{X}}_i, \bar{\mathbf{Z}}_i)\right) \quad (25)$$

$$\dot{\hat{\theta}}_i = \gamma_i \varphi(\eta_i, \xi_i) s_{2i}^T - \mu_i \hat{\theta}_i \quad (26)$$

where K_i, γ_i and μ_i are positive constants to be determined later.

Remark 3: $\tilde{\tau}_i$ in (25) is an intermediate controller. The real controller τ_i is given as follows:

$$\tau_i = J(\psi_i)^{-1} M_i (\bar{\mathbf{U}}_i + \tilde{\tau}_i). \quad (27)$$

Choose the Lyapunov function candidate as follows:

$$V_{2i} = V_{1i} + \frac{1}{2} s_{2i}^T s_{2i} + \frac{1}{2\gamma_i} \tilde{\theta}_i^T \tilde{\theta}_i. \quad (28)$$

The derivative of (28) gives

$$\begin{aligned} \dot{V}_{2i} &= -c_i s_{1i}^T s_{1i} + s_{1i}^T s_{2i} + s_{2i}^T \tilde{\theta}_i^T \varphi(\eta_i, \xi_i) + c_i s_{2i}^T s_{2i} \\ &\quad - c_i^2 s_{2i}^T s_{1i} + s_{2i}^T \Delta_i - K_i s_{2i}^T s_{2i} - \tfrac{1}{\gamma_i} \tilde{\theta}_i^T \dot{\hat{\theta}}_i \\ &= -c_i s_{1i}^T s_{1i} - (K_i - c_i) s_{2i}^T s_{2i} + (1 - c_i^2) s_{1i}^T s_{2_i} \\ &\quad + s_{2i}^T \Delta_i + s_{2i}^T \tilde{\theta}_i^T \varphi(\eta_i, \xi_i) - \tfrac{1}{\gamma_i} \tilde{\theta}_i^T \dot{\hat{\theta}}_i \end{aligned} \quad (29)$$

By applying Young's inequality $x^T y \leq \frac{a^p}{p}\|x\|^p + \frac{1}{q a^q}\|y\|^q$, $(a > 0, p > 1, q > 1, (p-1)(q-1) = 1)$, we have

$$s_{1i}^T s_{2i} \leq \frac{\pi_{1i}}{2} s_{1i}^T s_{1i} + \frac{1}{2\pi_{1i}} s_{2i}^T s_{2i} \quad (30)$$

$$\begin{aligned} s_{2i}^T \Delta_i &\leq \tfrac{\pi_{2i}}{2} \|\Delta_i\|^2 + \tfrac{1}{2\pi_{2i}} s_{2i}^T s_{2i} \\ &\leq \tfrac{\pi_{2i}}{2} \Delta_M^2 + \tfrac{1}{2\pi_{2i}} s_{2i}^T s_{2i} \end{aligned} \quad (31)$$

$$\begin{aligned} -\tfrac{1}{\gamma_i} \tilde{\theta}_i^T \dot{\hat{\theta}}_i &= -\tilde{\theta}_i^T \varphi(\eta_i, \xi_i) s_{2i}^T + \tfrac{\mu_i}{\gamma_i} \tilde{\theta}_i^T \hat{\theta}_i \\ &= -\tilde{\theta}_i^T \varphi(\eta_i, \xi_i) s_{2i}^T + \tfrac{\mu_i}{\gamma_i} \tilde{\theta}_i^T (\theta_i^* - \tilde{\theta}_i) \\ &= -\tilde{\theta}_i^T \varphi(\eta_i, \xi_i) s_{2i}^T + \tfrac{\mu_i}{2\gamma_i} \tilde{\theta}_i^{*T} \tilde{\theta}_i^* - \tfrac{\mu_i}{2\gamma_i} \tilde{\theta}_i^T \tilde{\theta}_i \end{aligned} \quad (32)$$

where π_{1i} and π_{2i} can be any positive constants.

Substituting (30)-(32) into (29), we have

$$\begin{aligned}\dot{V}_{2i} = &-\left(c_i - \tfrac{(1-c_i^2)\pi_{1i}}{2}\right) s_{1i}^T s_{1i} \\ &-\left(K_i - c_i - \tfrac{1-c_i^2}{2\pi_{1i}} - \tfrac{1}{2\pi_{2i}}\right) s_{2i}^T s_{2i} - \tfrac{\mu_i}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i \\ &+\tfrac{\pi_{2i}}{2}\Delta_M^2 + \tfrac{\mu_i}{2\gamma_i}\tilde{\theta}_i^{*T}\tilde{\theta}_i^*\end{aligned} \tag{33}$$

We can choose appropriate c_i and K_i such that

$$\beta_i =: c_i - \frac{(1-c_i^2)\pi_{1i}}{2} > 0, \tag{34}$$

$$\chi_i =: K_i - c_i - \frac{1-c_i^2}{2\pi_{1i}} - \frac{1}{2\pi_{2i}} > 0. \tag{35}$$

Let

$$\kappa_i = \min(2\beta_i, 2\chi_i, \mu_i), \tag{36}$$

$$\rho_i = \frac{\pi_{2i}}{2}\Delta_M^2 + \frac{\mu_i}{2\gamma_i}\|\tilde{\theta}_i^*\|^2. \tag{37}$$

Then (33) can be written as

$$\dot{V}_{2i} \le -\kappa_i V_{2i} + \rho_i, \tag{38}$$

and hence we have

$$V_{2i}(t) \le V_{2i}(0)e^{-\kappa_i t} + \frac{\rho_i}{\kappa_i},\ \forall t > 0, \tag{39}$$

which implies that $V_{2i}(t)$ is eventually bounded by $\frac{\rho_i}{\kappa_i}$. Hence, s_{1i}, s_{2i} and $\tilde{\theta}_i$ are GUUB. Thus, one can see that Property 1 of Definition 1 can be satisfied.

Next, consider the tracking error as follows:

$$\begin{aligned}\|\eta_i - y_d - \delta_i\| &= \|s_{1i} + \bar{q}(\bar{\zeta}_{di}) - \bar{q}(\zeta_{di})\| \\ &\le \|s_{1i}\| + \|\bar{q}(\bar{\zeta}_{di}) - \bar{q}(\zeta_{di})\|\end{aligned} \tag{40}$$

According to Theorem 1, $\hat{x}_{di}$ will converge to x_d asymptotically, which implies $\bar{q}(\bar{\zeta}_{di})$ will approach $\bar{q}(\zeta_{di})$ asymptotically. Therefore, there always exist a sufficiently small constant $\varepsilon > \sqrt{\frac{\rho_i}{\kappa_i}}$ and a finite time $T(\eta_i(t_0), \nu_i(t_0), \varepsilon)$ such that

$$\|\eta_i - y_d - \delta_i\| \le \varepsilon,\ \forall t > T(\eta_i(t_0), \nu_i(t_0), \varepsilon). \tag{41}$$

Moreover, one can observe that ε can be as small as possible by choosing c_i, K_i and γ_i as large as possible while μ_i as small as possible.

In summary, we can conclude the main results of this paper as follows:

Theorem 2: Consider the multi-USV system (1)-(2) satisfying Assumptions 1-4. Under the distributed observer (16) and distributed adaptive fuzzy control law (25)-(27), if $k > \frac{\lambda_{max}(S)}{\lambda_{min}(\mathcal{L}_{\mathcal{G}}+\Lambda)}$ and $K_i, \gamma_i, \mu_i, i = 1, \cdots, n$, are chosen to satisfy (34)-(35), then the formation control problem of the multi-USV system can be solved and the tracking error ε can be adjusted as small as possible.

Remark 4: It is noted that the reference system (4) can also be used to describe the typical nonlinear reference signals studied in existing works, e.g., [7], [16]. In other words, the proposed approach in this paper can be also directly applied to solve the existing classical trajectory tracking problem. Moreover, it is not difficult to see that the proposed formation control approach can be easily extended to more general cases that the reference system is nonlinear and can be well observed by designing a distributed observer.

IV. Simulation Results and Discussions

This section provides an example of a multi-USV system with 4 USVs. The system topology is shown in Fig. 1, where node 0 denotes the reference system and the other nodes denote the USVs. The system model of USVs are chosen the same as that in [7]. Parameters are set as follows:

$$M_i = \begin{bmatrix} 25.8000 & 0 & 0 \\ 0 & 33.8000 & 1.0948 \\ 0 & 1.0948 & 2.7600 \end{bmatrix}$$

$$C_i = \begin{bmatrix} 0 & 0 & c_{13} \\ 0 & 0 & c_{23} \\ -c_{13} & -c_{23} & 0 \end{bmatrix}$$

$$D_i = \begin{bmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix}$$

for $i = 1, \cdots, 4$, where $c_{13} = -25.8v - 1.0948r$, $c_{23} = -25.8u$, $d_{11} = 0.7225 + 13274|u| + 5.8664u^2$, $d_{22} = 0.8612 + 36.2823|v|$, $d_{23} = -0.1079$, $d_{32} = -0.1052 - 5.0437|v|$ and $d_{33} = 0$. The disturbances d_i, $i = 1, \cdots, 4$ are described by a Gaussian white noise.

The state matrix S of the reference system is

$$S = \begin{bmatrix} 0 & 0.5 & 0 \\ -0.5 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and

$$q(x_d) = \begin{bmatrix} 6 - 5\cos(\frac{\pi}{180}x_{d1}) \\ -0.5 + 2\sin(\frac{\pi}{180}x_{d2}) \\ \frac{\pi}{2}\cos(\frac{\pi}{2}x_{d3}) \end{bmatrix}.$$

The initial values are

$$\eta_1 = \begin{bmatrix} 0 & 0.5 & 0.1 \end{bmatrix}^T, \eta_2 = \begin{bmatrix} -0.5 & 0.3 & 0.4 \end{bmatrix}^T,$$
$$\eta_1 = \begin{bmatrix} 0.7 & 0.2 & 0.3 \end{bmatrix}^T, \eta_2 = \begin{bmatrix} -0.3 & -0.2 & 0.6 \end{bmatrix}^T,$$
$$\nu_{1,\cdots,4} = \begin{bmatrix} 1 & 1 & 0.1 \end{bmatrix}^T, x_d = \begin{bmatrix} 2 & 2 & \frac{\pi}{36} \end{bmatrix}^T.$$

The desired relative positions are

$$\delta_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T, \delta_2 = \begin{bmatrix} -1 & 0 & 0 \end{bmatrix}^T,$$
$$\delta_3 = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T, \delta_4 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T.$$

Control parameters are chosen as $k = 2$, $K_i = 10$, $\mu_i = 0.1$, $\gamma_i = 20$ and $c_i = 10$, $i = 1, \cdots, 4$.

The simulation results are shown in Fig. 2 and Fig. 3. Fig. 2 shows that all USVs (blue dots) track the reference trajectory (red square) and achieve the desired formation. Fig. 3 shows the tracking errors of USVs corresponding to their desired positions. One can observe that all tracking errors in surge, sway and yaw are bounded within a small neighborhood of origin. The simulation results demonstrate that the proposed control approach is effective and efficient.

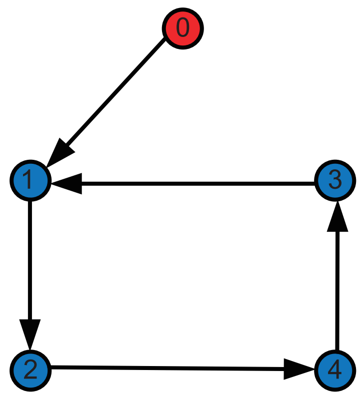

Fig. 1. Topology of the multi-USV system

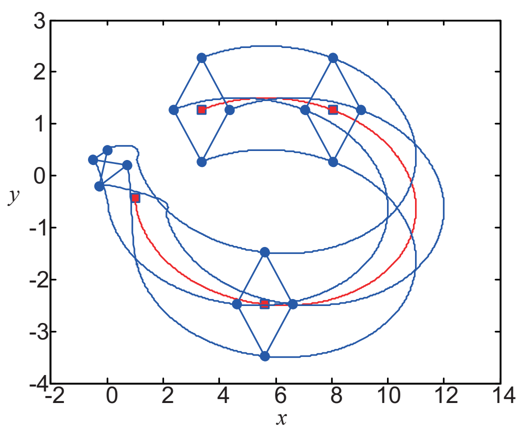

Fig. 2. Formation of the multi-USV system

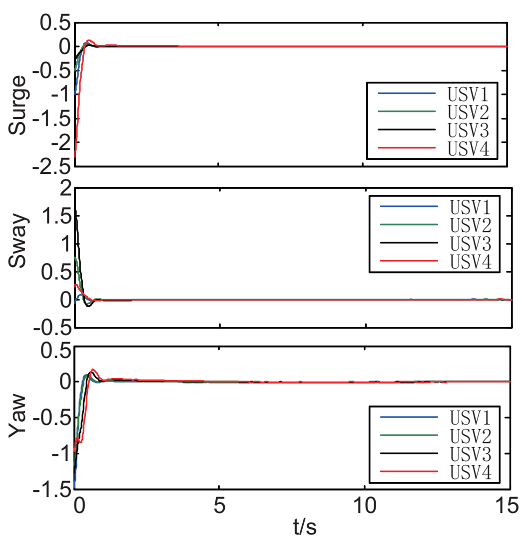

Fig. 3. Tracking errors in surge, sway and yaw

V. Conclusions

In this paper, the formation control problem of USVs with unknown nonlinearity and disturbances is investigated. The reference system is considered to be an autonomous system, and a novel adaptive fuzzy output regulation scheme is proposed. It has shown that the formation control problem can be conveniently transformed into an output regulation problem. By comprehensively using fuzzy approximation, output regulation and backstepping methods, the USVs can achieve desired formation under the proposed control law while tracking the reference system with the tracking errors being able to be adjusted as small as possible. It has shown that the proposed formation control scheme can also be applied to solve the classical formation control problem and is thus more general.

References

[1] T. I. Fossen, M. Breivik, and R. Skjetne, "Line-of-sight path following of underactuated marine craft," *IFAC Proc. Vol.*, vol. 36, no. 21, pp. 211–216, 2003.

[2] B. Xiao, X. Yang, and X. Huo, "A novel disturbance estimation scheme for formation control of ocean surface vessels," *IEEE Trans. Industr. Electron.*, vol. 64, no. 6, pp. 4994–5003, 2017.

[3] S. Yin, H. Yang, and O. Kaynak, "Coordination task triggered formation control algorithm for multiple marine vessels," *IEEE Trans. Industr. Electron.*, vol. 64, no. 6, pp. 4984–4993, 2017.

[4] L. Liu, D. Wang, Z. Peng, and T. Li, "Modular adaptive control for los-based cooperative path maneuvering of multiple underactuated autonomous surface vehicles," *IEEE Trans. Syst., Man, and Cyber.: Syst.*, vol. 47, no. 7, pp. 1613–1624, 2017.

[5] Z. Peng, D. Wang, Z. Chen, X. Hu, and W. Lan, "Adaptive dynamic surface control for formations of autonomous surface vehicles with uncertain dynamics," *IEEE Trans. Contr. Syst. Tech.*, vol. 21, no. 2, pp. 513–520, 2013.

[6] Y.-Y. Chen and Y.-P. Tian, "Coordinated adaptive control for formation tracking surface vessels with a time-invariant orbital velocity," in *Chinese Contr. Conf. (CCC)*. IEEE, 2013, pp. 7445–7450.

[7] N. Wang, J.-C. Sun, M. J. Er, and Y.-C. Liu, "A novel extreme learning control framework of unmanned surface vehicles," *IEEE trans. cyber.*, vol. 46, no. 5, pp. 1106–1117, 2016.

[8] C. Godsil and G. F. Royle, *Algebraic graph theory*. Springer Sci. & Business Media, 2013, vol. 207.

[9] L.-X. Wang, "Adaptive fuzzy systems and control: design and stability analysis," *Englewood Cliffs*, 1994.

[10] J. Huang, *Nonlinear output regulation: theory and applications*. SIAM, 2004.

[11] Y. Su and J. Huang, "Cooperative output regulation of linear multi-agent systems by output feedback," *Syst. & Contr. Lett.*, vol. 61, no. 12, pp. 1248–1253, 2012.

[12] ——, "Cooperative output regulation of linear multi-agent systems," *IEEE Trans. Automat. Contr.*, vol. 57, no. 4, pp. 1062–1066, 2012.

[13] H. Kim, H. Shim, and J. H. Seo, "Output consensus of heterogeneous uncertain linear multi-agent systems," *IEEE Trans. Automat. Contr.*, vol. 56, no. 1, pp. 200–206, 2011.

[14] W. He and J. Cao, "Consensus control for high-order multi-agent systems," *IET Contr. Theory & Appl.*, vol. 5, no. 1, pp. 231–238, 2011.

[15] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[16] J. Shin, D. J. Kwak, and Y.-i. Lee, "Adaptive path following control for an unmanned surface vessel using an identified dynamic model," *IEEE/ASME Trans. Mech.*, vol. 22, no. 3, pp. 1143–1153, 2017.