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ON LOGICAL CHARACTERISATION OF HUMAN CONCEPT LEARNING BASED ON TERMINOLOGICAL SYSTEMS

Abstract. The central focus of this article is the epistemological assumption that knowledge could be generated based on human beings' experiences and over their conceptions of the world. Logical characterisation of human inductive learning over their produced conceptions within terminological systems and providing a logical background for theorising over the Human Concept Learning Problem (HCLP) in terminological systems are the main contributions of this research. In order to make a linkage between 'Logic' and 'Cognition', Description Logics (DLs) will be employed to provide a logical description and analysis of actual human inductive reasoning (and learning). This research connects with the topics 'logic & learning', 'cognitive modelling', and 'terminological knowledge representation'.

Keywords: concept; human concept learning; concept construction; terminological knowledge; terminological systems; logic and cognition

1. Introduction

The point of departure is my special focus on the conceptualisation of 'learning'. In this research, learning will be seen as the process of construction, and thus, learning will be assumed to be supported by an epistemology which argues that knowledge is constructed based on human beings' experiences, and over their conceptions of the world (see [25, 10, 6, 28, 26]). According to [3], concepts might be understood as representations of (aspects of) reality in human beings' minds. Frederic Bartlett — in his studies in experimental psychology — arrived at the phenomenon of 'concept' with his focus on memory analysis. In memory

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studies subjects recalled details of stories that were not actually there. Considering concepts as mental representations, any concept could be recognised to be equivalent to a psychological (and mental) entity (see [24, 32]).

Furthermore, concepts as mental entities could be studied by the representational theory of the mind (and the theory of mental representation); see [30, 18, 19]. In this research, the term 'concept' is suggested as following: "a concept could be said to be a linkage [and interconnection] between the mental representations of linguistic expressions and other mental images (e.g., aspects of representations of the world, and of inner experiences) that a human being has in her/his mind"; see [7]. Kindly observe that the ontological, existential and structural analysis of these linkages is beyond the scope of this paper, but it could be assumed that humans' conceptions are the outcomes and manifestations of their constructed concepts. Let me begin with an example; Suppose that one (say John) has a visualisation of 'Book' in his mind. Regarding his mental image, he describes (and defines) 'Book' by "Set of written Sheets". Note that I am not interpreting the truth/falsity of John's description (definition) of 'Book', but I am just analysing the logical structure of his description (definition). First, I shall mention that Description Logics (see [1]) and concept languages recognise a *definition* of a concept as a definition of a new [and/or more specified] concept in terms of other previously defined concepts. More precisely, a definition could be defined as an equation between a concept and its description (based on one's conceptions). Obviously, John's conception of the concept 'Book' is transformable into a hypothesis like "Book is a Set of written Sheets". The most important word of this hypothesis is the distinct entity 'Book'. Therefore, John has established the correspondence "Set of written Sheets" to the entity 'Book'. More particularly, John has created a mental assignment from the description "Set of written Sheets" to the entity 'Book'. In fact, John — by means of "is a" — has made a logical relation between the distinct entity 'Book' and the description "Set of written Sheets". More specifically, he has made two parallel relationships (i.e., a (i) hyperonym-hyponym or SuperClass-SubClass relationship and a (ii) hyponym-hyperonym or SubClass-SuperClass relationship) between a distinct entity and a description. Consequently, all characteristics, features and properties of 'Set of written Sheets' are assigned to 'Book', and vice-versa. From Johns point of view (that has supported him in producing his pre-conception of 'Book'), all applications of 'Book' are determined and supported by his primary mental expression and definition "Book is a Set of written Sheets".

This article will — by considering concepts as the amalgamations of mental images and linguistic expressions — focus on analysing the logical characteristics of "humans' inductive[1] reasoning (and learning) over their conceptions". In fact, it will focus on logical analysis of how terminological knowledge could reasonably be assumed to be built based on an individual's conceptions of the world. It attempts to offer an explanatory framework to draw a linkage between logic and cognition. It aims at providing a logical background for providing a terminological scheme and for theorising over the Human Concept Learning Problem (HCLP) in terminological systems.

2. On Human Concept Learning

In my opinion, one of the most fundamental characteristics of human concept learning is using background knowledge, which is formed and shaped over humans' pre-formed concepts, and, respectively, over their pre-conceptions. So this article relies on the idea that humans' pre-formed concepts form their background knowledge. In general — referring to [21] — the term 'background knowledge' represents the knowledge of the world in general, or of the life in the specific society, the understanding of which people can be assumed to share as a framework for talking with each other. It shall, therefore, be claimed that any background knowledge could represent an individual's universal knowledge of the world in general, or of any local knowledge of a specific part of the world in particular. Furthermore, in the learning sciences, the background knowledge can be defined as the knowledge that learners have (learned) both formally in the learning environments as well as informally through their life experiences (see [20, 5]). Therefore, the background knowledge of any individual can be constructed based on

(a) her/his descriptions of the world based on her/his pre-formed concepts (and pre-concept descriptions), and

[1] An inductive logic is a system of evidential support that extends deductive logic to less-than-certain inferences, see [8]. The premises of a strong inductive argument should be capable of providing some degree of support for the logical conclusion, where such support means that the truth of the premises indicates, with some degree of strength, that the conclusion is (could be) true.

(b) her/his own conceptions which are generated with regard to the structures of different phenomena/things in her/his mind.

However, there are strong dependencies between (a) pre-concept descriptions (and, thus, pre-conceptions) and (b) the structural conceptions of different phenomena/things. More specifically, any pre-concept description could be interpreted as an expression of a human's conception of a phenomenon's/things's structure. For example, pre-concept descriptions could be produced based on humans' qualitative and phenomenographic realisations of different phenomena/things. Furthermore, human beings — within processes of concept learning — are concerned with their explanatory, structural, existential and comprehensive conceptions of different phenomena/objects. According to the mentioned characteristics of background knowledge, human concept learning could be suggested as following.

2.1. Human concept learning

Concept learning is an inductive learning theory that is supported by humans' inductive reasoning processes. Concept learning is logically shaped over a system of evidential support, which is structured over less-than-certain inferences of human beings. This theory is supported by humans' reasoning processes based on their constructed concepts (and their produced conceptions). Concept learning could be generated based on humans' background knowledge (and over their pre-formed concepts and preconceptions) and with regard to their conceptualisations of the characteristics and properties of concepts and through experiencing (e.g., observing, hearing, touching, reading about) various groups of examples of those concepts. Accordingly, humans could focus on hypothesis generation. It shall be concluded that humans become concerned with specification of the conceptualisation of their constructed concepts within their concept learning.

It is worth mentioning that some approaches (in educational and social sciences) have focused on applications of the analysed notion in inductive teaching and inductive learning strategies (see [31, 22, 4, 27]). As mentioned, humans could focus on hypothesis generation within their concept learning. The next section will focus on logical analysis of hypothesis generation. In this research, the main references to logic of induction, inductive reasoning and inductive concept learning paradigm are [12, 13, 23, 14, 15, 16, 17]. Also, [1] is the main reference to Description Logics and Concept Languages. At this point I shall state that Description Logics (DLs) — under a plethora of names (among them terminological systems and Concept Languages) — attempt to provide descriptive knowledge representation formalisms based on formal semantics. They can support us in revealing some hidden conceptual assumptions that could support us in having a better understanding of 'concept learning' and in proposing more realisable logical descriptions for it in terminological systems. That's why I have employed DLs in this research.

3. A Terminological Basis

A series of inductive learning approaches make humans concerned with three kinds of non-monotonic processes:

1. The process of producing their descriptions of more specified concepts regarding their descriptions of more general concepts. For example, Mary could describe her own constructed concept (and conception of) 'Red Dog' regarding her descriptions of her constructed concepts (and conceptions of) 'Red' and 'Dog'. Also, Ann could describe her own constructed concept (and conception of) 'Big Brown Horse' regarding her descriptions of her constructed concepts (and conceptions of) 'Big', 'Brown' and 'Horse'.

2. The processes of reforming and re-organising their conceptions of the same concepts with insights based on their acquired knowledge and new experiences. For example, James could describe his constructed concept (and conception of) 'Spring' by the term "a Season; when all the Trees are Green", or equivalently: $\texttt{Season} \sqcap \forall \texttt{hasTree.Green}$. Later on, he may reform his conception of 'Spring' and may produce the description "a season; when some (and, in fact, not all) trees are green", or equivalently: $\texttt{Season} \sqcap \exists \texttt{hasTree.Green}$.

3. The process of producing their more specific descriptions of the same concepts with insights based on acquired knowledge within their interactions with new experiences. For example, James could describe the concept 'Spring' by "a season; when some trees are green", or equivalently: $\texttt{Season} \sqcap \exists \texttt{hasTree.Green}$. Later on, he may produce the more specific description "a Season of the Moderate Weather; when some Trees are Green", or equivalently: $\texttt{Season} \sqcap \exists \texttt{hasTree.Green} \sqcap \exists \texttt{hasWeather.Moderate}$.

Analysing the processes involved in human concept learning can be interpreted as a comprehensive study [of humans' minds] and as an explanatory and structural study [of humans' selves] that enable them to develop their own understanding of different phenomena's/things' realities within the world. It shall, therefore, be suggested that the phenomenon of 'human concept learning' has strong interrelationships with an existential and structural consciousness (that is related to a kind of ontology; see [29]). It might, then, be said that any human being actively deals with this ontology in her/his concept learning processes, and becomes concerned with its specification on various levels of her/his conceptualisation. Relying on this ontology, human beings focus on specification and categorisation of their conceptualisation, and accordingly, they induce [and learn] from the [more, and more] specific descriptions of their conceptions. It shall be stressed that this kind of ontology is shaped over the interrelationships between humans' mental images of the world and their linguistic expressions of the world. Here I focus on analysing terminological knowledge construction (and describing the world based on constructed terminological knowledge) over humans' conceptualisation. The analysis consists of two main categories:

I. The produced conceptions of human beings support them in modifying their terminologies. Suppose that Mary already knows about (and recognises) the concept 'Cyan'. The general concept 'Cyan' is the amalgamation of the mental word 'Cyan' and the mental image of Cyan. In this research, it's assumed that the word 'Cyan' belongs to Mary's terminologies, and, respectively, to her terminological knowledge. The concept 'Cyan' (i) is already experienced and known by Mary, (ii) is existing in her terminological knowledge and (iii) could be — terminologically — interpreted by her (in order to produce a meaningful comprehension for her). Then, it can provide a terminological principle in her mind. For example, she could identify (and interpret) 'Cyan' as a kind of 'Colour' (i.e., $\texttt{Cyan} \sqsubseteq \texttt{Colour}$) in her terminology[2]. Consequently, Mary can apply the terminological principle $\texttt{Cyan} \sqsubseteq \texttt{Colour}$ in creating the new more specified world descriptions over the concept 'Cyan'. For example, she may describe a 'Cyan Stone' by "a Stone that has Cyan Colour", formally: $\texttt{CyanStone} \doteq \texttt{Stone} \sqcap \exists \texttt{hasColour.Cyan}$. This concept description (in the form of a concept definition) is expanded over her/his conception of the concept 'Cyan'. In Description Logics, a

[2] It is worth mentioning that the concept inclusion $\texttt{Cyan} \sqsubseteq \texttt{Colour}$ presupposes so-called 'colour realism'.

concept definition (represented by $\doteq$) is a definition of a new (or more specified) concept in terms of other previously defined concepts.

II. The produced conceptions of human beings support their developmental processes of terminological construction. Suppose that Simon does not know about the concept 'Cyan'. Then he — as a basis for his reasoning (and inducing) processes — generates the general concept 'Cyan' and categorises it into his terminology T (let T be a set), i.e., $\texttt{Cyan} \in T$. In fact, he tries to provide a background for satisfying the concept 'Cyan' by his terminology. Relying on $\texttt{Cyan} \in T$, T could satisfy Cyan (i.e., $T \models \texttt{Cyan}$); It means that his developed terminology could provide a model for satisfying 'Cyan' based on his terminological interpretations. In particular, $\texttt{Cyan} \in T$ (as a set membership) could be developed by designing different world descriptions (in the form of assertive principles). For example, Simon may construct assertions like Cyan(c) and Colour(d), where the individuals c and d are the instances of the concepts 'Cyan' and 'Colour' respectively. Note that producing Cyan(c) and Colour(d) are the products of his conceptions and based on his experiences. Therefore, he — mentally — satisfies the assertions Cyan(c) and Colour(d) by semantic models like "Terminological Knowledge $\models$ Cyan(c)" and "Terminological Knowledge $\models$ Colour(d)". For example, according to the semantic model '$\models$ Cyan(c)', Simon (i) has experienced the individual c, (ii) has interpreted that c is an instance of his constructed concept Cyan, and (iii) has made a mental principle based on the assertion Cyan(c). Consequently, (iv) Simon's terminological knowledge could satisfy Cyan(c). Furthermore, he will be able to — inductively — subsume more specified concepts and their instances under his comprehension of 'Cyan'. For example, he could induce that the Cyan Chairs are all Cyan (i.e., $\texttt{CyanChair} \sqsubseteq \texttt{Cyan}$). As another example, Simon — by following his mental principle "$\texttt{Cyan} \sqsubseteq \texttt{Colour}$" and by considering the fact that "all chairs are some kinds of furnitures" — could induce that $(\texttt{CyanChair} \sqsubseteq \texttt{Colour})$ & $(\texttt{CyanChair} \sqsubseteq \texttt{Furniture})$. This logical term means that all Cyan Chairs are Colourful and Furnitures. Accordingly, he can induce that $\texttt{CyanChair} \doteq (\texttt{Chair} \sqcap \exists \texttt{hasColour.Cyan}) \equiv (\texttt{Furniture} \sqcap \exists \texttt{hasColour.Cyan})$.

3.1. Human concept learning and hypothesis generation

In concept learning, human beings — with regard to a set of experienced examples and over their background knowledge — focus on generating hypotheses. Subsequently, they focus on their generated hypotheses in order to adopt them in their world descriptions and for their reasoning processes. More specifically, they construct large numbers of hypotheses and record them in their minds. Accordingly, they search through the huge space of their hypotheses in order to find the most proper and useful ones and to describe and specify their constructed and developed knowledge based on them. Assessed by mathematical logic, the search spaces are capable of archiving the reflexive and transitive relations. In fact, these spaces must be expressed by (i) strictly-less-than (as well as less-than) and by (ii) proper-subset (as well as subset) relations. Recording the relations 'a is less than b (and b is greater than a)' and 'A is the subset of B (and B is the superset of A)' are expressible over *reflexive* and *transitive* relations. Focusing on these characteristics, the search spaces could be represented by the binary quasi order relation $(Q, \preccurlyeq)$, where Q is a set and $\preccurlyeq$ is a relation defined over Q. Any infinite sequence of the elements of Q could contain an increasing pair like (p_i, p_j), where $i \preccurlyeq j$; see [9, 11]. In my opinion, the most significant characteristic of a binary quasi order relation is that "if p and q are two elements of a binary quasi order relation, then they will be *comparable*, and, in fact, the relations $p \preccurlyeq q$ and $q \preccurlyeq p$ support the *comparability*".

According to the mentioned features of search spaces, the binary relation $(C, \preccurlyeq)$ over the set of a human's constructed concepts (represented by C) is quasi-order and it is reflexive and transitive. The main reason for applying quasi order relations is the fact that any concept learning relies on *induction* and on *comparability*, and inductive reasoning is expressible over 'less-than' and 'subset' relations. More specifically, a quasi-order, as a well-founded induction, can be applied to the set of humans' constructed concepts (or C) in order to express concept *subsumption* (or concept *inclusion*). It shall be emphasised that concept subsumption is the most fundamental feature of hierarchical structures in terminological knowledge. Accordingly, the terminological principles based on concept subsumption could be expressed.

Mechanism. In concept learning, human beings generate mental mappings (like L) from their primary constructed set of concepts (or C) into all its subsets (or 2^C) such that: $\forall p \in C, L(p) \subseteq \{q \in C \mid q \preccurlyeq p\}$. L is a proper mental mapping if, and only if, for all constructed concepts A and B, $B \in L(A) \Rightarrow A \not\equiv B$. Going back to the example of Cyan, for human beings, it is conceptualisable, and thus, understandable that there is a subsumption relation between `Cyan` and `CyanStone` like

CyanStone $\sqsubseteq$ Cyan. Furthermore, its conceptualisable and understandable that Cyan $\in$ {Cyan, Stone}. Obviously, Cyan and CyanStone are not equivalent (they are not the same based on all possible terminological interpretations). It shall be stressed that there are two kinds of mental mappings[3]:

1. *Complete Mapping*: Considering Cyan and CyanStone, humans could conceptualise, interpret, and, respectively, understand that CyanStone $\sqsubseteq$ Cyan (i.e., all Cyan Stones are Cyan). For example, Maria can reach the concept CyanDoor from Cyan (regarding CyanStone $\sqsubseteq$ Cyan) by means of a complete mental mapping. Then, she can induce: CyanDoor $\sqsubseteq$ Cyan.

2, *Weak Mapping*: Regarding isA(Cyan, Colour) $\sqsubseteq$ $\top$,[4] Michael could induce the world description isA(Cyan,Paint) based on his own conception of "the equivalence relation between the world descriptions isA(Cyan, Colour) and isA(Cyan, Paint)". In fact, the world description isA(Cyan, Paint) could be induced from a Tautology by means of a weak mental mapping.

4. The Human Concept Learning Problem (HCLP)

Suppose that the function $\mathcal{C}_{\mathcal{K}}(C)$ describes that a human being has constructed ($\mathcal{C}$) the concept C on a basis provided by her/his constructed knowledge ($\mathcal{K}$). I shall draw your attention to the following components of knowledge:

Component I (terminologies). $\mathcal{T}$ stands for humans' *terminological knowledge*, which is represented over constructed concepts (in humans' minds). The terminological component of knowledge is highly concerned with (A) concept subsumption (concept inclusion) and (B) concept equality.

(A) Human beings — by interpreting concept subsumptions (like $E \sqsubseteq F$) — produce their terminological models (like $\mathcal{I}$) in order to [mentally]

[3] I define a mental mapping as "a mapping (function) from a concept into another concept". In fact, this function is definable from a mental entity into another mental entity.

[4] isA(Cyan, Colour) is a role assertion (= a world description over a role in Description Logics). It expresses that "Cyan is a Colour". This world description is a tautology (it is true for all possible interpretations), and thus, it belongs to the top concept (or $\top$).

satisfy the concept subsumptions. Formally, they produce $\mathcal{I} \models (E \sqsubseteq F)$. This model is semantically valid (and it is logically meaningful) if and only if the interpretation of E is the subset of the interpretation of F, or formally: $E^{\mathcal{I}} \subseteq F^{\mathcal{I}}$. Accordingly, human beings by limiting their terminological interpretations to their meaningful understandings could — terminologically — understand[5] that E is the sub-concept of (sub-concept description of) F, or formally: $E \sqsubseteq F$. In fact, humans focus on producing semantic models (like *UND*) in order to satisfy $E \sqsubseteq F$; Formally: $UND \models (E \sqsubseteq F)$. Subsequently, $E^{UND} \subseteq F^{UND}$. For example, a terminological interpretation like $\mathcal{I}$ could be produced in order to provide a terminological model and to support $\texttt{Cyan}^{\mathcal{I}} \subseteq \texttt{Colour}^{\mathcal{I}}$ (regarding concept subsumption $\texttt{Cyan} \sqsubseteq \texttt{Colour}$). Subsequently: $\texttt{Cyan}^{UND} \subseteq \texttt{Colour}^{UND}$, and, in fact, it has been understood that 'Cyan is a Colour'.

(B) Human beings — by interpreting concept equalities (like $C \equiv D$) — produce their terminological models (like $\mathcal{I}$) in order to satisfy the concept equalities. Formally, $\mathcal{I} \models (C \equiv D)$. This model is semantically valid (and it is logically meaningful) if and only if the terminological interpretation of C is equal to the terminological interpretation of D, or formally: $C^{\mathcal{I}} = D^{\mathcal{I}}$. Subsequently, humans by limiting their terminological interpretations to their meaningful understandings could understand that C and D are equal ($C = D$). In fact, they produce understanding models (like *UND*) in order to satisfy $C \equiv D$. Formally: $UND \models (C \equiv D)$. Subsequently, $C^{UND} = D^{UND}$. For example, a terminological interpretation like $\mathcal{I}$ could be produced in order to provide a terminological model and to support $\texttt{Colour}^{\mathcal{I}} = \texttt{Paint}^{\mathcal{I}}$ (regarding $\texttt{Colour} \equiv \texttt{Paint}$). Subsequently, $\texttt{Colour}^{UND} = \texttt{Paint}^{UND}$, and, in fact, it has been understood that 'Colour and Paint are equivalent'.

Component II (world descriptions). The symbol $\mathcal{W}$ stands for humans' *World Descriptions* over their constructed concepts. This component of knowledge is concerned with (A) instance assertion (or identifying a phenomenon/thing as a member of a constructed concept) and with (B) relation assertion (or relating the instances of various concepts to each other).

(A) Humans — by interpreting that the individual a is an instance of the constructed concept C — produce their terminological interpretations (like $\mathcal{I}$) in order to satisfy the concept assertions $C(a)$. Formally,

[5] This 'understanding' is a limit of a 'terminological interpretation' based on own 'conceptualisation'. Therefore it is existentially terminological and conceptual.

they produce semantic models like $\mathcal{I} \models C(a)$. This model is logically meaningful if and only if the terminological interpretation interprets the individual a as an element of (and as a kind of) the concept C, or formally: $a^{\mathcal{I}} \in C^{\mathcal{I}}$. Humans by limiting their terminological interpretations to their meaningful understandings could understand that a is a kind of C. In fact, they produce understanding models (like UND) in order to satisfy $C(a)$ (ie. $UND \models C(a)$). Subsequently, $a^{UND} \in C^{UND}$. For example, Elizabeth can interpret, and, respectively, understand that her personal computer (represented by $\texttt{pcez}$) is a 'Machine'. Formally: $\mathcal{I} \models \texttt{Machine}(\texttt{pcez})$. So, $\texttt{pcez}^{\mathcal{I}} \in \texttt{Machine}^{\mathcal{I}}$. Therefore, $UND \models \texttt{Machine}(\texttt{pcez})$, and thus, $\texttt{pcez}^{UND} \in \texttt{Machine}^{UND}$, and, in fact, Elizabeth has understood that her personal computer is a machine.

(B) Humans — by interpreting that the relation (a, b) is an instance of (is a kind of) the relationship R — produce their interpretation models (like $\mathcal{I}$) in order to satisfy the role assertion $R(a, b)$. Formally, they produce semantic models like $\mathcal{I} \models R(a, b)$. This semantic model is logically meaningful if and only if the interpretation of the tuple (a, b) belongs to the interpretation of R, or formally: $(a, b)^{\mathcal{I}} \in R^{\mathcal{I}}$. Humans by limiting their terminological interpretations to their meaningful understandings could understand that the individuals a and b are related by a relation like $R(a, b)$. In fact, they produce understanding models (like UND) in order to understand the relation $R(a, b)$. Formally: $UND \models R(a, b)$. Subsequently, $(a, b)^{UND} \in R^{UND}$. For example, Bob could interpret, and, respectively, could understand that his magnet (represented by $\texttt{magb}$) attracts a pin (represented by $\texttt{pinb}$). Then: $\mathcal{I} \models \texttt{Attract}(\texttt{magb}, \texttt{pinb})$, and thus, $(\texttt{magb}, \texttt{pinb})^{\mathcal{I}} \in \texttt{Attract}^{\mathcal{I}}$. Accordingly, $UND \models \texttt{Attract}(\texttt{magb}, \texttt{pinb})$, and thus, $(\texttt{magb}, \texttt{pinb})^{UND} \in \texttt{Attract}^{UND}$, and, in fact, Bob has understood that his magnet attracts his pin.

Component III (rules). Suppose that the symbol $\mathcal{R}$ stands for *Rules*. First, I shall claim that any rule (in such a terminological system) is logically dependent on (and supported by) a logical implication. For example, considering 'Thirst' and 'Drinking' as two concepts, the logical term $\mathcal{R} \models (\texttt{Thirst} \Rightarrow \texttt{Drinking})$ denotes that if one (say John) has been interpreted [and, respectively, has been understood] to be thirsty and be an instance of (and described by) the concept 'Thirst', then John is, also, an instance of (and described by) the concept 'Drinking'. Formally: $[(\texttt{john} \in \texttt{Thirst}) \sqcap (\texttt{Thirst} \sqsubseteq \texttt{Drinking})] \Rightarrow \texttt{john} \in \texttt{Drinking}$. This logical term expresses that "John is thirsty, and the thirst will be followed by drinking, so, John is supposed to drink".

4.1. Knowledge

Regarding the components **I**, **II**, and **III** (terminologies, world descriptions and rules), the triple $\mathcal{K} = \langle \mathcal{T}, \mathcal{W}, \mathcal{R} \rangle$ could represent the humans' *constructed knowledge*. However, it could be believed that the third component ($\mathcal{R}$) is expressible based upon the components $\mathcal{T}$ and $\mathcal{W}$. Let me be more specific:

a. Considering the logical term '$\mathcal{R} \models (C \Rightarrow D)$', where $\mathcal{R}$ stands for a rule, and C and D stand for two concepts (concept descriptions), we can understand that there exists an individual (like c) for which, there is a logical implication between $C(c)$ and $D(c)$. Formally, $\exists c : \mathcal{R} \models (C(c) \Rightarrow D(c))$. Obviously, $\mathcal{R}$ satisfies an implication and indication from a concept assertion (as a world description) into another concept assertion (as a world description). Therefore, the rule $\mathcal{R} \models (C(c) \Rightarrow D(c))$ has been split into two world descriptions and over a terminology (i.e., it has been expressed in the form of two world descriptions and based on a terminology).

b. Considering the logical term '$\mathcal{R} \models (P \Rightarrow Q)$', where $\mathcal{R}$ stands for a rule, and P and Q stand for two roles (role descriptions), we can understand that there are two individuals (like a and b) for which a logical implication between $P(a, b)$ and $Q(a, b)$ is satisfied. Formally, $\exists a, b : \mathcal{R} \models (P(a, b) \Rightarrow Q(a, b))$. This formalism expresses that the rule $\mathcal{R}$ satisfies "P implies Q" (where P and Q are two relations between a and b). Again, this implication has been described from a role assertion (as a world description) into another role assertion (as a world description). Therefore, such a rule has been split into two world descriptions and over a terminology (i.e., it has been expressed in the form of two world descriptions and based on a terminology).

Taking **a** and **b** into consideration, it shall be concluded that the constructed knowledge could be expressed in the form of world descriptions and based on terminologies. Therefore, the component $\mathcal{R}$ could be eliminated, and, thus, it could be concluded that the tuple $\mathcal{K} = \langle \mathcal{T}, \mathcal{W} \rangle$ represents the humans' constructed knowledge.

4.2. Knowledge construction – mechanism

The mechanism of knowledge construction must be checked over concepts and roles.

A. Construction over Concepts. In concept learning, human beings become concerned with a set of experienced phenomena/things in order to form new building blocks of their knowledge construction. Expressing new concept descriptions is highly dependent on:

- a set of *Experienced Constructive* examples of *Concepts* (or Exp_c^+),
- a set of *Experienced Non-Constructive* examples of *Concepts* (or Exp_c^-)

over pre-formed concepts (and pre-conceptions). Accordingly, humans become concerned with a unifying set (like Exp_c) of their multiple constructive and non-constructive examples. Therefore:

$$Exp_c = \{Exp_c^+, Exp_c^-\},$$

where:

- $Exp_c^+ = \{a \in Exp_c(\mathcal{W}) \mid C(a) \in \mathcal{W}\} \subseteq \mathcal{C}_{\mathcal{K}}(C)$, and
- $Exp_c^- = \{b \in Exp_c(\mathcal{W}) \mid \neg C(b) \in \mathcal{W}\} \subseteq \mathcal{C}_{\mathcal{K}}(\neg C)$.

In fact, any Exp_c^+ consists of the individuals which could be described by humans' constructed concepts. Any member of Exp_c^+ can be supported by humans' world descriptions (based on concept assertions). Also, Exp_c^- consists of the individuals which could not be described by humans' constructed concepts and cannot be supported by their world descriptions (based on concept assertions). For example, considering Exp_{Dog}^+ as the set of Martin's experienced constructive examples of 'dogs', Martin's German Shepherd belongs to Exp_{Dog}^+. Also, considering Exp_{Dog}^- as the set of Martin's experienced non-constructive examples of 'dogs', a friend's rabbit belongs to Exp_{Dog}^-. Martin — by increasing the number of his experienced constructive and non-constructive examples of 'Dogs' — could develop his knowledge of dogs over his own construction, conception, interpretation and meaningful comprehension of 'Dogs'.

B. Construction over roles. Describing more specified roles is dependent on:

- a set of *Experienced Constructive* examples of *Roles* (or Exp_r^+), and
- a set of *Experienced Non-Constructive* examples of *Roles* (or Exp_r^-)

over pre-formed roles (as the relations between the instances of pre-formed concepts). Accordingly, humans become concerned with a unifying set of their multiple constructive and non-constructive examples with regard to their pre-constructions of roles. This unifying set is denoted by Exp_r. Then, $Exp_r = \{Exp_r^+, Exp_r^-\}$, where:

- $Exp_r^+ = \{p, q \in Exp_r(\mathcal{W}) \mid R(p, q) \in \mathcal{W}\} \subseteq \mathcal{C}_{\mathcal{K}}(R)$, and
- $Exp_r^- = \{p, s \in Exp_r(\mathcal{W}) \mid \neg R(p, s) \in \mathcal{W}\} \subseteq \mathcal{C}_{\mathcal{K}}(\neg R)$.

In fact, any Exp_r^+ consists of the individuals which could be described by humans' constructed concepts, and, respectively, by relating the instances of the constructed concepts. Accordingly, any member of Exp_r^+ can be supported by their world descriptions (based on role assertions). Additionally, Exp_r^- consists of the individuals which could not be described by humans' constructed concepts, and, respectively, cannot be described by relating the instances of the constructed concepts. Thus, they cannot be supported by humans' world descriptions (based on role assertions). For example, consider $Exp^+_{marriedTo}$ as the set of David's experienced constructive examples of 'people who are married to each other'. David knows that Ronald and Susan are married. So, the relation `(rolnad, susan)` belongs to David's $Exp^+_{marriedTo}$. Now let $Exp^-_{marriedTo}$ be the set of David's experienced non-constructive examples of 'people who are married to each other'. David knows that Peter and Rebeca are not married. So, the relation `(peter, rebeca)` does not belong to $Exp^-_{marriedTo}$. David — by increasing the number of his experienced constructive and non-constructive examples of 'people who are married to each other' (and by knowing more married and more non-married pairs of people) — could develop his knowledge.

Conclusion. According to $\mathcal{K} = \langle \mathcal{T}, \mathcal{W} \rangle$, human beings — by generating $\mathcal{T}$ and $\mathcal{W}$ — support their knowledge ($= \mathcal{K}$) construction. Consequently,

- if a concept assertion (like $D(a)$) is satisfied by the constructed knowledge, then: $\mathcal{K} \models D(a) \quad \forall a \in Exp_c^+(\mathcal{W})$,
- if a concept assertion (like $D(b)$) is not satisfied by the constructed knowledge, then: $\mathcal{K} \not\models D(b) \quad \forall b \in Exp_c^-(\mathcal{W})$,
- if a role assertion (like $R(a, b)$) is satisfied by the constructed knowledge, then: $\mathcal{K} \models R(a, b) \quad \forall a, b \in Exp_r^+(\mathcal{W})$, and
- if a role assertion (like $R(a, c)$) is not satisfied by the constructed knowledge, then: $\mathcal{K} \not\models R(a, c) \quad \forall a, c \in Exp_r^-(\mathcal{W})$.

4.3. From conceptions to predicates

According to my research in [2], there is a sort of reflector functions from humans' conceptions into predicates. Obviously, we could not directly move from 'concepts' to 'predicates' (and, respectively, to 'truth'), but, I shall stress that the central focus of this research is on a logical analysis of concept learning and hypothesis generation (and not on an ontological analysis of concepts). So, logics allow me to relate conceptions and predicates. My focus is on the logical fact that human beings can transform their conceptions [of their constructed concepts] into the [logically] equivalent ones in the form of predicates. Subsequently, the expressed predicates provide reasonable logical models that semantically can satisfy the collections of their generated hypotheses. Representing *Concepts*, *Predicates*, and *Hypotheses* by C, P and H, respectively, we formally have: $[\mathit{Reflection}\colon C \rightarrow P] \models H$.

4.3.1. The detailed examination I

A. Experienced concepts. The set of experienced concepts (or $\mathit{Exp}_{\mathit{Concept}}$) is equal to the union of the following sets:

Set 1. The set of experienced constructive examples of constructed concepts that could be represented by $\mathit{Exp}^{+}_{\mathit{Concept}} = \{\text{individual} \in \mathit{Exp}_{\mathit{Concept}}(\mathcal{W}) \mid \text{Concept(individual)} \in \mathcal{W}\}$. $\mathit{Exp}^{+}_{\mathit{Concept}}$ is subsumed under humans' constructed concepts. They could be generated over humans' constructed knowledge (or $\mathcal{C}_K(\mathit{Concept})$). Consequently, the humans' constructed knowledge satisfy their own world descriptions based on their concept assertions and over their experienced constructive examples.

Set 2. The set of experienced non-constructive examples of constructed concepts that could be represented by $\mathit{Exp}^{-}_{\mathit{Concept}} = \{\mathit{individual} \in \mathit{Exp}_{\mathit{Concept}}(\mathcal{W}) \mid \neg \mathit{Concept}(\mathit{individual}) \in \mathcal{W}\}$. $\mathit{Exp}^{-}_{\mathit{Concept}}$ is subsumed under humans' non-constructed concepts. They could not be generated over humans' constructed knowledge (or $\mathcal{C}_K(\neg \mathit{Concept})$). Consequently, the humans' constructed knowledge does not satisfy their own world descriptions based on their concept assertions and over their experienced non-constructive examples.

B. Experienced Roles. The set of experienced roles (or $\mathit{Exp}_{\mathit{Role}}$) is equal to the union of the following sets:

Set 1. The set of experienced constructive examples of constructed roles that could be represented by $\mathit{Exp}^{+}_{\mathit{Role}} = \{\mathit{individual}_1 \,\&\, \mathit{individual}_2 \in \mathit{Exp}_{\mathit{Role}}(\mathcal{W}) \mid \mathit{Role}(\mathit{individual}_1, \mathit{individual}_2) \in \mathcal{W}\}$. $\mathit{Exp}^{+}_{\mathit{Role}}$ is subsumed under humans' constructed roles, and, in fact, under humans' constructed concepts. They could be generated over humans' constructed knowledge (or $\mathcal{C}_K(Role)$). Consequently, the humans' constructed knowledge satisfy their own world descriptions based on their role assertions and over their experienced constructive examples.

Set 2. The set of experienced non-constructive examples of constructed roles that could be represented by $Exp^{-}_{Role} = \{individual_1 \; \& \; individual_2 \in Exp_{Role}(\mathcal{W}) \mid \neg Role(individual_1, individual_2) \in \mathcal{W}\}$. Exp^{-}_{Role} is subsumed under humans' non-constructed roles. They could not be generated over humans' constructed knowledge (or $\mathcal{C}_K(\neg Role)$). Consequently, the humans' constructed knowledge does not satisfy their own world descriptions based on their role assertions and over their experienced non-constructive examples.

4.3.2. The detailed examination II

Humans' conceptions [of their own constructed concepts] could be represented in the form of 'unary predicates' in order to be described and represented. Similarly, humans' conceptions [of their own constructed roles] could be represented in the form of 'binary (or any other n-ary) predicates' in order to be represented. A predicate could be interpreted to be an expression and assigner of concepts' different characteristics. Then, it could assign different characteristics to [and transmit them into] propositions (and statements) or even into truth-values. Consequently, regarding humans' constructed knowledge ($\mathcal{K}$) and reflecting (mirroring) the conceptions in predicates, the tuple $\langle \mathcal{T}, \mathcal{W} \rangle$ could be expressed and anlysed by predicate logic. Therefore, the terminological knowledge ($\mathcal{T}$) could be structured over (i) predicate symbols (e.g., unary, binary, ..., n-ary), (ii) variable symbols, (iii) constant symbols, and (iv) function symbols. It is worth mentioning that these four kinds of symbols are identified as non-logical symbols in Predicate Logic, because they — independently — do not cause any logical consequence in logical descriptions. Also, all world descriptions are shaped by utilising multiple descriptions over the provided terminologies. Subsequently, a predicate symbol denotes something which is a predication of the subject. A variable symbol is what a human asserts the predicate to it. The constant symbols could be asserted to any variable. Beside them, function symbols operate the variable symbols. Taking the translated terminological knowledge [, and, subsequently, the translated world descriptions] into consideration, we could express the components of the detailed examination **I** as follows:

A. Expressed unary predicates. The set of humans' experiences could be represented in the form of unary predicates ($Exp_{Predicate}$) and be acknowledged as the union of the following sets:

Set 1. The set of humans' experienced constructive examples of their described unary predicates. It could be represented by $Exp^{+}_{Predicate} = \{constant \in Exp_{Predicate}(\mathcal{W}) \mid Predicate(constant) \in (W)\}$ for expressing world descriptions based on unary predicates. The experienced constructive examples are expressed by humans' described predicates within their constructed knowledge (i.e., $\mathcal{C}_{\mathcal{K}}(Predicate)$). Consequently, $\mathcal{C}_{\mathcal{K}}$ satisfies the humans' world descriptions based on their concept assertions and over their experienced constructive examples. For example, consider the constant `palm` as an experienced constructive example of the concept `Tree`. Formally: `palm` $\sqsubseteq$ `Tree` within $\mathcal{C}_{\mathcal{K}}$. Note that this constructive example could be subsumed under the described binary predicate (relation) '*is a*' within constructed knowledge. In fact, `is a`(`palm`, `Tree`) supports `palm` $\sqsubseteq$ `Tree`.

Set 2. The set of humans' experienced non-constructive examples of their described unary predicates. It could be represented by $Exp^{-}_{Predicate} = \{constant \in Exp_{Predicate}(\mathcal{W}) \mid \neg Predicate(constant) \in \mathcal{W}\}$. The experienced non-constructive examples could not be expressed by described predicates within constructed knowledge (i.e., $\mathcal{C}_{\mathcal{K}}(\neg Predicate)$). Consequently, $\mathcal{C}_{\mathcal{K}}$ does not satisfy the world descriptions over experienced non-constructive examples. For example, consider the constant `rose` as an experienced non-constructive example of the concept `Tree`. Formally: `rose` $\not\sqsubseteq$ `Tree`. This non-constructive example is not expressible and is not satisfied by the binary predicate (relation) "is a" within $\mathcal{C}_{\mathcal{K}}$. In fact, $\mathcal{C}_{\mathcal{K}}$ does not satisfy the world description "rose is a tree". Therefore, we have "¬`isA(rose,tree)` over the experienced non-constructive example `rose` $\not\sqsubseteq$ `Tree`.

B. Expressed binary predicates. The set of humans' experiences could be represented in the form of binary predicates ($Exp_{Predicate}$) and be acknowledged as the union of the following sets:

Set 1. The set of humans' experienced constructive examples of their described binary predicates. It could be represented by $Exp^{+}_{Predicate} = \{constant_1 \ \& \ constant_2 \in Exp_{Predicate}(\mathcal{W}) \mid Predicate(constant_1, constant_2) \in (W)\}$ for expressing world descriptions over binary predicates. The experienced constructive examples are expressed by humans' described predicates within their constructed knowledge (i.e.,

$\mathcal{C}_{\mathcal{K}}(Predicate)$). Consequently, $\mathcal{C}_{\mathcal{K}}$ satisfies the humans' world descriptions based on their concept assertions and over their experienced constructive examples. For example, the constants `bob` and `mary` could be the experienced constructive examples of the role `fatherOf`. Formally: $(\texttt{bob}, \texttt{mary}) \sqsubseteq \texttt{fatherOf}$. This constructive example could be subsumed under a binary predicate within constructed knowledge. Equivalently, we have the world description $\texttt{fatherOf}(\texttt{bob}, \texttt{mary})$.

Set 2. The set of humans' experienced non-constructive examples of their described binary predicates. It could be described by $Exp^{-}_{Predicate} = \{constant_1 \ \& \ constant_2 \in Exp_{Predicate}(\mathcal{W}) \mid \neg Predicate(constant_1, constant_2) \in \mathcal{W}\}$. The experienced non-constructive examples could not be expressed by described predicates within constructed knowledge (i.e., $\mathcal{C}_{\mathcal{K}}(\neg\text{Predicate})$). Consequently, $\mathcal{C}_{\mathcal{K}}$ does not satisfy the world descriptions based on role assertions and over experienced non-constructive examples. For example, the constants `bob` and `silvia` could be the experienced non-constructive examples of the role `fatherOf`. Formally: $(\texttt{bob}, \texttt{silvia}) \not\sqsubseteq \texttt{fatherOf}$. We have the world description $\neg\texttt{fatherOf}(\texttt{bob}, \texttt{silvia})$, and in fact, $\mathcal{C}_{\mathcal{K}}$ does not satisfy the world description "Bob is the father of Silvia".

4.4. Summarisation

Let me go back to the example of Martin's conception of 'dogs'. In this example, Exp_{Dog} is the set of Martin's experienced examples of 'Dog' and it is equal to $Exp^{+}_{Dog} \cup Exp^{-}_{Dog}$. Relying on Exp_{Dog}, we have:

(a) $Exp^{+}_{Dog} = \{i \in Exp_{Dog}(\textit{Martin's description of dogs}) \mid \texttt{Dog}(i) \in \textit{Martin's description of dogs}\} \subseteq \textit{Martin's constructed knowledge of dogs}$. Consequently, Martin's knowledge of dogs satisfies the concept assertion $\texttt{Dog}(i)$, where the individual i belongs to his constructive examples of dogs within his own description of dogs.

(b) $Exp^{-}_{Dog} = \{j \in Exp_{Dog}(\textit{Martin's description of dogs}) \mid \neg\texttt{Dog}(j) \in \textit{Martin's description of dogs}\} \subseteq \textit{Martin's knowledge of NOT-dogs}$. Consequently, Martin's knowledge of dogs does not satisfy the concept assertion $\texttt{Dog}(j)$, where the individual j belongs to his non-constructive examples of dogs within his own description of dogs.

According to the logical term $[\textit{Reflection}: C \rightarrow P] \models H$, a reflection has been described as a function that mirrors the humans' conceptions [as the outcomes of their constructed concepts] in predicates. These predicates could be used in expressing humans' constructed concepts in terminological systems. A sort of reflector functions from human beings' *constructed concepts and conceptions* into *predications and described predicates* could formal-semantically satisfy the logical hypotheses. It shall be claimed that the transmission of concepts from the detailed examination **I** to the detailed examination **II**, is a kind of logical reflection that semantically forms new hypotheses. The generated hypotheses could be used to describe humans' grasps of the world in terminological systems (and terminological knowledge representation systems).

5. Conclusions

In this article, the phenomenon of 'learning' has been assumed to be supported by an epistemology which argues that knowledge could be constructed from an interaction between human beings' experiences and over their conceptions of what they have experienced (e.g., studied, seen, heard, felt, touched). Accordingly, in this research, the term 'concept learning' has been expressed as "the developmental process of concept construction and specification of the constructed concepts". Note that I have not focused on an ontological analysis of concepts, but I have — by considering the theoretical idea that concepts might be said to be a linkage between the mental representations of linguistic expressions and the other mental images that a human being has in her/his mind — focused on conceptual and logical analysis of how terminological knowledge could reasonably be assumed to be built based on an individual's conceptions of the world and over her/his constructed terminological basis. Consequently, a logical background for theorising over the Human Concept Learning Problem (HCLP) has been provided. It has been concluded that the problem of human concept learning could be expressed in the form of a function that expresses a human who constructs a concept on a basis provided by her/his constructed knowledge. Also, her/his knowledge is constructed based upon her/his terminological basis and over her/his descriptions of the world. Accordingly, humans are concerned with a set of experienced information in order to form new building blocks of their knowledge. Then, the union of the (i) Experienced Constructive Examples of their Constructed Concepts and (ii) Experienced Non-Constructive Examples of their Constructed Concepts have been considered as the set of Experienced Concepts that support the developmental processes of knowledge construction. Assessed by logics, a function (which I have called 'reflector') mirrors the humans' conceptions in predicates. The predicates could be used in expressing humans' constructed concepts within terminological systems. A sort of reflector functions from humans' "constructed concepts and conceptions" into "predications and described predicates" could semantically satisfy the collections of humans' hypotheses. The succession "Concept → Conception → Predication → Predicate" represents a logical flow that attempts to satisfy the collection of logical hypotheses. More specifically, the transmission of concepts from the detailed examination **I** to the detailed examination **II**, has expressed a kind of logical reflection that semantically forms new hypotheses in order to describe humans' grasps of the world in terminological systems. It could make sense in terminological knowledge representation systems and terminological knowledge bases.

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