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A study of dynamic behaviour performance of DC/DC boost converter used in the photovoltaic system

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Abstract—As the continuously fluctuation of the operating point due to the permanent research of maximum power point which the photovoltaic source can produce, the dynamic behaviour of the used DC/DC converter -adopted to realize the maximum power point tracking- will be disturbed at all time. For the aim to design an efficient photovoltaic system, the dynamic behaviour of the converter is studied based on the damping factor and the natural frequency of the system. As an example, the system under study is composed with a photovoltaic module connected to an equivalent resistive load through a DC/DC boost converter, assuming that this last one has been controlled with a direct method of maximum power point tracking. The results of analytical studies allow having an overview about the dynamic behaviour performance of the used DC/DC converter for the aim to implement a prompt maximum power point tracking algorithm.

Keywords—Photovoltaic (PV) cell; DC/DC boost converter; maximum power point (MPP); maximum power point tracker (MPPT); damping factor ($\xi$); natural frequency ($\omega_0$).

I. INTRODUCTION

Solar photovoltaic is envisaged to be a popular source of renewable energy due to several advantages, notably low operational cost, almost maintenance free and environmentally friendly [1].

A PV module under uniform irradiation exhibits an $I-V$ characteristic with a unique point, called the maximum power point (MPP), where the module produces maximum output power [2]. The amount of the produced power depends on the operating point, which is the intersection between the $I-V$ characteristics of both PV module and the connected load. It is known that the efficiency of the PV cells is low (14 to 16% for the monocrystalline Si), therefore, it is necessary to operate at the MPP at all time whatever the climatic conditions and the connected load are.

The DC/DC converter is usually used for realizing the MPPT function through the injection on its principal switching power component, the right pulse width control signal ($d$) using a MPPT algorithm.

In cloudy days, the solar irradiation can be random and very volatile (which can have a rate of 100 W/m²/s [3]). Consequently most of the MPPT algorithms, among them the perturb and observe (P&O) which can be confused due to the fact that it is not able to distinguish the variations of the PV module output power caused by the duty cycle modulation from those ones caused by the irradiance variation [2], hence a judicious optimization of an efficient MPPT algorithm must take place. Among the optimized parameters, the sampling time ($T_s$) which is strongly depends on the dynamic behavior of the used DC/DC converter.

In the case where there is no effect of the connected load on the output PV voltage ($v_{pv}$), the knowledge of the transfer function of the control to $v_{pv}$ ($F_{v_{pv},d}$) is enough for designing a stable and robust system to face different fluctuations of MPP.

This work is focusing on the variation of both $\xi$ and $\omega_0$ of the system (see fig. 1) which their formulas are extracted from ($F_{v_{pv},d}$) of an approximated second order system. The interval variation of both $\xi$ and $\omega_0$ as well as the system operating point changes, allows us having an overview about the dynamic behaviour performance of the used DC/DC converter for the aim to implement a prompt maximum power point tracking algorithm.

The paper is organized as follows. The PV cell based on the two diode model is presented in section II. The transfer function ($F_{v_{pv},d}$) and its simplification to a second order are developed in section III. Criteria of the transient response are given in section IV. The results of analytical studies and discussions are shown in section V. Finally section VI is devoted to conclusion.

![Fig. 1. The considered system under study.](image-url)
II. MODELING OF THE PV CELL

The development of PV system has demanded the need of simulation tools capable of handling mathematical simulations [4]. Due to its good representation of the output I-V characteristic, under different values of irradiation and temperature, the model of two diode is taken, it is defined as indicated in fig. 2, by an electrical source associated to two diodes (D1, D2) and two resistances (Rs, Rsh). The diode (D1) represent the semi-conductor behaviour, which constitute the PV cells, and the other diode (D2) is a compliment of the first one that is used for more accurate current and the leakage current to the ground through the PV cell respectively.

![Two diode model of the PV cell](image)

Fig. 2. The two diode model of the PV cell.

Fig. 3. shows the obtained electrical characteristics of STP080S-12/Bb module from SUNTCH Company under some solar irradiation and temperature combinations.

![Output characteristics of the used PV module under some combinations of the solar irradiation and temperature](image)

Fig. 3. Output characteristics of the used PV module under some combinations of the solar irradiation and temperature; a) I-V curves b) P-V curves.

III. MODELING OF BOOST CONVERTER

A. Small signal model

The DC/DC boost topology presents a good efficiency in the PV application especially its continuous input current, which is useful for extracting all the amount of power as the PV module can product. Many DC/DC topologies have been used in this type of application [5][6]. The used equivalent circuit of the DC/DC boost converter (dotted line in fig. 4) is constituted with a passive components (L, Cpv and Co), which are sized away to have a continuous conduction mode (CCM). The formulas used are found in references [7][8]. The switches S and D are in general a transistor (IGBT) and a diode respectively.

The input and output of the converter are connected respectively to PV module -which is modelled with a dynamic resistance (rPV) that define the ratio between the instantaneous voltage and current of the PV module- and to an equivalent resistive load (Ro), when the input and the output voltages are the instantaneous values of the PV voltage (vPV(t)) and the capacitor voltage (vC(t)) respectively.

![Equivalent circuit of the converter](image)

Fig. 4. The equivalent circuit of the considered system

In power electronic, the switches’ number (N) determines the number of the elementary configurations as 2N. In the pulse width modulation (PWM) control, and for the CCM, there are two stable configurations which appear every switching period (Tsw) i.e. when S is switched-on, and D is switched-off and vice versa (when S is switched-off and D is switched-on in the interval time dTsw ≤ toff ≤ Tsw where d is the duty cycle (d = toff/Tsw).

The state variables according to these configurations are:

\[
\begin{align*}
\frac{dv_{PV}}{dt} &= -\frac{v_{PV}}{C_{PV}} - \frac{IL}{C_{PV}} \\
\frac{dv_{C}}{dt} &= \frac{v_{PV}}{L} - (1 - s) \frac{v_{C}}{L} \\
\frac{dL}{dt} &= -\frac{v_{C}}{CoR_{o}} + (1 - s) \frac{IL}{Co}
\end{align*}
\]  

(1)

Where s is time-dependent switching variable, defined as:

\[
s(t) = \begin{cases} 
1 & \text{For S is switched on} \\
0 & \text{For S is switched off}
\end{cases}
\]  

(2)
Based on (1) the small signal averaged state-space model is obtained as shown in (3) at the bottom of this page, where the variables with a hat are small AC variations about the equilibrium operating point, \( V_{pvE} \) is the equilibrium value of the output PV voltage and \( r_{pv} \) is a dynamic resistance that define the ratio \( \bar{v}_{pv}/V_{pv} \). Other symbols in (3) refer to Fig. 4.

**B. Transfer function and the obtained second order system**

By applying the Laplace transform to (3), the small signal control to module voltage transfer function \( F_{pv, \text{d}}(s) \) is obtained as shown in (4) at the bottom of this page, where \( s \) here is the Laplace variable.

According to control theory of linear systems, the transient response and dynamic behavior of (4) strongly depends on the nature of the poles’ denominator \( (D(s)) \). We note that this last one is a third degree polynomial.

To facilitate the study of the converters’ dynamic behavior, the idea is to get a comparable system to a second order in the general form:

\[
D(s) = s^2 + 2\omega_0\xi s + \omega_0^2
\]

(5)

Where the natural frequency \( \omega_0 \) characterizes the time response of the system (i.e., the good variations of \( \omega_0 \) provide a fast transient response) since the damping factor \( \xi \) characterizes the oscillation during the transient response (i.e., the good variations of \( \xi \) provide a well damped system).

The pair of complex conjugate poles then characterize the system of second order are:

\[
s_{1,2} = -\omega_0\xi \pm j\omega_0\sqrt{1-\xi^2} \quad (\text{If} \quad 0 \leq \xi < 1)
\]

(6)

The characteristic polynomial wanted, take the following form:

\[
D(s) = (s^2 + 2\xi \omega_0 s + \omega_0^2)(s - s_r)
\]

(7)

Equation (7) accepts three poles, at least one is real \( (s_r) \), the other two poles \( (s_{1,2}) \) are either real (if the discriminate \( \Delta_D(s) > 0 \)) or complex conjugate (if \( \Delta_D(s) < 0 \)). Therefore, to obtain a second order system, we must put \( s_r \) as far as possible in the left half complex plane and place the two complex poles \( (s_{1,2}) \) the closest possible to the imaginary axis (see Fig. 5). In this case \( s_r \) has a negligible influence on the transient response.

Therefore, we can write the second condition:

\[
|\text{Re}(s_r)| \gg |\text{Re}(s_{1,2})| \quad (8)
\]

The denominator \( D(s) \) of (4) is a third degree polynomial of the general form:

\[
D(s) = d_3 s^3 + d_2 s^2 + d_1 s + d_0
\]

(9)

Applying (8) on (7, 9), we obtain requirements imposed on the coefficients \( d_0, d_1, d_2, d_3 \) of (9) for getting a second order system dominate. We obtained the following system of equations:

\[
\begin{align*}
  s_r &= -\frac{d_2}{d_3} \\
  \omega_0 &= \frac{d_0}{\sqrt{d_2}} \\
  \xi &= \frac{d_1 d_2 - d_0}{\sqrt{4 d_0 d_2}}
\end{align*}
\]

(10)

Based on (10), the values of \( L, C_{pv}, \) and \( C_o \) will be chose (we respect the CCM). The dynamic resistance \( r_{pv} \) defines the operating point on the characteristic \( I-V \) and it depends on the temperature and solar irradiance. This resistance takes big values if the solar irradiation takes small values and vice versa.

\[
\begin{align*}
  \frac{d}{dt} [\begin{array}{c} v_{pv} \\ v_o \\
  \end{array}] &= \begin{bmatrix} -1/(C_{pv} r_{pv}) & -1/C_{pv} & 0 \\ 1/L & 0 & -(1-d)/L \\
  0 & (1-d)/C_o & -1/(C_o R_o) \\
  \end{bmatrix} \begin{bmatrix} v_{pv} \\ v_L \\
  \end{bmatrix} + \begin{bmatrix} \frac{V_{pvE}}{L (1-d)} \\ -\frac{V_{pvE}}{C_o r_{pv}} \\
  \end{bmatrix} (d) \\
  F_{pv, \text{d}}(s) &= \frac{-V_{pvE}}{L C_{pv} (1-d)^3} - \frac{2 V_{pvE} (1-d)}{L C_{pv} C_o r_{pv}} \\
  &\quad + \frac{1}{C_{pv} r_{pv} (1-d)^2 + 1/L C_{pv} J} s^3 + \frac{1}{C_o r_{pv} (1-d)^2 + 1/L C_{pv} J} s^2 + \frac{1}{C_{pv} r_{pv} (1-d)^2 + 1/L C_{pv} J} s + \frac{2 (1-d)^2}{L C_{pv} C_o r_{pv}} \\
\end{align*}
\]
The second parameter \((d)\) defines the transformation ratio between the input and the output of the converter, it gives the level voltage of the load connected.

The first step for having a second order system is to have discriminate \(\Delta_{D(s)} < 0\) whatever the variations in irradiation and temperature (the interval variation of \(r_{pv}\)) and whatever the variation in load (The interval of duty cycle \(0 \leq d < 1\)). This condition decides the first interval dimensioning the elements (\(L, C_{pv}, \) and \(C_o\)).

The following figure shows the variations of the discriminant \(\Delta_{D(s)}\) in the predefined intervals of \(r_{pv}\) and \(d\) for our converter topology.

![Image of variations of discriminant-boost topology](image)

Fig. 6. Variations of discriminant-boost topology.

Obviously, \(\Delta_{D(s)}\) is negative whatever the values of \(r_{pv}\) and \(d\) used in the design of the elements of storage. So the first condition is verified.

The second condition \((8)\) which is to having a very real pole away from the imaginary axis. This condition determines the sub-range of \(L, C_{pv}, \) and \(C_o\). Solving \((10)\) gives a relationship between \(L, C_{pv}, \) and \(C_o\) for having a real pole non-dominant.

Fig. 7 shows the placement of poles and zeros of the transfer function \(F_{pv, D(s)}\). For example if \(r_{pv} = 20 [\Omega]\) and \(d=0.5\) the placement of the real pole \((s_i)\) is as far from the two conjugate poles \((s_{1,2})\) that can neglect its influence on the transient response. So in this case, the system can be presented by the transfer function as shown in Fig. 8.

![Image of placement of poles and zeros of the boost transfer function](image)

Fig. 7. Placement of poles and zeros of the boost transfer function \(F_{pv, D(s)}\).

\[
\hat{d}(s) = \frac{N_1 s + N_2}{s^2 + 2\xi \omega_0 s + \omega_0^2}
\]

Fig. 8. Transfer function of a second order system.

Formulas \((11, 12)\) describe the dependence of \(\xi\) and \(\omega_0\) on the system parameters, so we will study the variations of these two parameters according to the variations of both \(r_{pv}\) and \(d\).

\[
\xi = \frac{C_{pv}(L + C_{pv}r_{pv}^2)(1 - d)^4 + L C_o(1 - d)^2 + C_o^2 r_{pv}^2}{2r_{pv}(1 - d) \sqrt{2L[C_{pv}(1 - d)^2 + C_o]^3}} \tag{11}
\]

\[
\omega_0 = (1 - d) \left( \frac{2}{L(C_{pv}(1-d)^2+C_o)} \right) \tag{12}
\]

IV. CRITERIA OF THE TRANSIENT RESPONSE

In a second order system, the factors \((\xi, \omega_0)\) characterize the step transient response (fig. 9).

![Image of transient response to a step](image)

Fig. 9. Transient response to a step.

This transient response is characterized by a first overtake \((D_1)\), reflecting the degree of damping of the system and the first peak time \((t_{pie})\), reflecting the rapid transient. It is generally desired to obtain a fast transient and well damped, therefore obtained a MPPT fast and less oscillate.

The first overtake \((D_1)\) and time to peak \((t_{pie})\) of the transient are directly related to \(\xi\) and \(\omega_0\) by the following relationships:

\[
t_{pie} = \frac{\pi}{(\omega_0 \sqrt{1 - \xi^2})} \tag{13}
\]

\[
D_1 = 100e^{-\pi \xi / \sqrt{1 - \xi^2}} \tag{14}
\]

V. RESULTS OF THE ANALYTICAL STUDY

In the following as it indicated in Fig. 8, we will show the variation influence of both \(\xi\) and \(\omega_0\) on \(v_{pv}\) as this last one should be characterized to be a well damped and fast when the transient response for the aim to implement a more efficient MPPT algorithm.
A. Discussion about the damping factor (ξ)

We know according to control theory of linear systems that a good sizing of ξ (≈ 0.703) minimizes the system’s oscillations in the transitional phase by decreasing D1 which implies a well damped system so the corresponding losses will be reduced when the MPPT action. According to (11), the obtained analytical form of ξ depend on both d and rpv. Fig. 10, shows its variations.

![Image](Fig. 10. Damping factor ξ versus some combinations of d and rpv.)

B. Discussion about the natural frequency (ω₀)

A good sizing of ω₀ reduces the system’s time response that implies fast MPPT reactions.

Based on (12), it is found that the analytical form of ω₀ depends only on d. Fig. 11, shows its variations.

![Image](Fig. 11. Natural frequency ω₀ [rad/s] versus some combinations of d and rpv.)

C. Criteria of the transient response

The criteria of the transient response (tpic, D₁) are indirectly related to the variations of both rpv and d through the variation of ξ and ω₀.

- The first peak time (tpic)

The formula (13) of tpic depends on both ξ and ω₀ so its variations are indirectly influenced by the variations of rpv and d. Fig.12, shows a specified variation interval of both rpv and d because for the rest of their intervals, tpic takes values less than 0.02 [s]. These obtained values are translated that ω₀ has more influence than ξ on tpic variations, which implies that d is the more important parameter that influences the time to the first overtake of the transient response.

![Image](Fig. 12. First peak time tpic [s] versus some combinations of d and rpv.)

VI. CONCLUSION

In this paper we have presented a PV system that is constituted with a PV module and an equivalent resistive load, which are connected together using DC/DC boost converter. The transfer function (Fv, d) is obtained and its mathematical simplification is applicable for other DC/DC topologies in the case when we want to select which topology dynamic behavior performance is the best one.

The results of analytical studies based on ξ and ω₀ of the obtained second order system, allowed us to take an overview about the transient response behaviour of the output PV voltage according to different operating conditions by studying
its first overtake $D_1$ and time to peak $t_{p_{\text{pic}}}$, for the aim to implement an efficient MPPT algorithm.

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