Aalborg Universitet



Procurement planning in a multi-period supply chain

An epiphany Nielsen, Izabela Ewa; Saha, Subrata

Published in: **Operations Research Perspectives**

DOI (link to publication from Publisher): 10.1016/j.orp.2018.11.003

Creative Commons License CC BY-NC-ND 4.0

Publication date: 2018

Document Version Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA): Nielsen, I. E., & Saha, S. (2018). Procurement planning in a multi-period supply chain: An epiphany. *Operations Research Perspectives*, *5*, 383-398. https://doi.org/10.1016/j.orp.2018.11.003

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain You may freely distribute the URL identifying the publication in the public portal -

Take down policy

If you believe that this document breaches copyright please contact us at vbn@aub.aau.dk providing details, and we will remove access to the work immediately and investigate your claim.



Contents lists available at ScienceDirect

Operations Research Perspectives



journal homepage: www.elsevier.com/locate/orp

Procurement planning in a multi-period supply chain: An epiphany

Izabela Ewa Nielsen, Subrata Saha*

Department of Materials and Production, Aalborg University, Aalborg East DK 9220, Denmark

ARTICLE INFO

Keywords: Multi-period supply chain Inventory Procurement

ABSTRACT

Pricing and inventory distribution strategy in a multi-period supply chain environment has not yet been comprehensively explored. We derive closed-form solutions for both two-and three-echelon supply chains under a manufacturer-stackelberg game framework where the supply chain members execute integrated procurement planning by taking account upto four consecutive selling periods. Optimal pricing and inventory distribution policy is identified in the perspective of each member among three pragmatic procurement scenarios. Our results demonstrate that both integrated procurement strategies outperform conventional single-period decision and reduce double-marginalization effect. But, the retailer can prefer bulk procurement to earn maximum profit. The distributor acts as a catalyst, prevents the retailer from executing integrated multi-period procurement planning and creates conflict among supply chain members. Procurement decisions in presence of strategic inventory may lead to suboptimal profits compared to bulk procurement and supply chain members can face some implementation issues. Supply chain members always receive higher profits if the retailer distributes inventory strategically in a multi-period supply chain.

1. Introduction and Literature Review

Systematic procurement and inventory management create the interfaces of efficient retail supply chains. Efficient inventory management helps to reduce transportation costs, ensures continuity of selling activities, and evades variations in wholesale price and demand. In contrast, an effective procurement strategy ensures to maximize return on investment. For example, [10] found that inventories need to place downstream in the supply chain, especially when the demand uncertainty is high. Inventory can be used to minimize the expected time at the retail store and reduces the expected number of backorders [34]. It is well established in operations management literature that effective pricing, procurement and inventory decisions are never-ending research issue [4,14,15,23,24,28,33,35,38,39,41]. We refer the recent review of [8,22,29] for more detail discussions on the importance of inventory on retail operations and a supply chain environment.

However, [21] explored an alluring correlation between procurement and inventory management in a two-period procurement setting. The authors noted that the manufacturer's second-period wholesale price decreases proportionally with the amount of products the retailer carries from the first period. [2] also noted similar evidence and stated that the retailer can force the manufacturer to reduce the wholesale price of forthcoming periods by maintaining products as inventory in between two consecutive selling periods. The authors call it as *Strategic*

Inventory(SI). [6] proved that the manufacturer may introduce consumer rebates to curtail strategic advantage of the retailer in building SI, although both supply chain members can receive more profits in presence of SI. [1] also reported that strategic use of inventory is practiced in real world. [5] highlighted the benefits of SI in a single manufacturer and multiple retailers supply chain. [18] conducted empirical investigation to explore the effect of SI and found that the retailer can immensely induce differentiated wholesale pricing behavior by building up SI. [26] analyzed the impact of SI from the perspective of supply chain coordination. They found that the optimal supply chain profit cannot be achieved by implementing a quadratic quantity discount contract mechanism. [13] proved that the retailer's decision to maintain SI can improve the greening level for the product. [16] also suggested that the retailer should withhold SI to receive higher profits. [12] showed that the manufacturer's decision to produce development or marginal-cost intensive product is also correlated with the retailer's decision in maintaining SI. The literature cited above acknowledges the benefits of SI by analyzing outcomes of two-consecutive selling periods only. Therefore, the supremacy of SI is required to scrutinize by considering more number of consecutive selling periods within an integrated procurement planning. In this study, the outcomes of two integrated procurement decisions in presence of inventory are explored by considering upto four consecutive selling periods in the perspective of improving profits of each participating member in a supply chain.

* Corresponding author.

E-mail addresses: izabela@mp.aau.dk (I.E. Nielsen), subrata.scm@gmail.com (S. Saha).

https://doi.org/10.1016/j.orp.2018.11.003

Received 8 August 2018; Received in revised form 22 November 2018; Accepted 22 November 2018 Available online 24 November 2018

2214-7160/ © 2018 Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/BY-NC-ND/4.0/).

This study demonstrates that procurement decision in the presence of SI or conventional single-period procurement, where the retailer procures products to satisfy demand in each selling period may not be an optimal procurement strategy. We analyze the impact of a new procurement strategy mixed with inventory distribution, and find that an appropriate combination of pricing-inventory distribution strategy can lead to a higher profit for every member. We call it procurement Scenario BP (bulk procurement). The concept of large-volume procurement arrangements is practiced among small retailer in country such as Chile, Malaysia, Indonesia, the Philippines due to rapid supermarket expansion in Latin America and Asia [9,31]. Additionally, inventories in between two consecutive selling periods can serve as buffers for responding to demand shocks. Unlike the procurement scenario where the retailer maintains SI and determines the retail price and how much additional products to be carried forwarded, the retailer needs to decide the volume of additional products to be procured in first selling period and formulate an optimal distribution planning of those products in forthcoming selling periods. We consider both two and threeechelon supply chains to identify the influence of distributor also. The consequence of associated pricing behaviour is also discussed. Results are compared with procurement Scenario SPI, which is similar to [2,13,18], where the retailer can maintain SI in between two consecutive selling periods. Therefore, in each selling period, the retailer decides the price and the amount of product to be carried forward as SI to enforce the upstream member to reduce wholesale price in forthcoming selling periods. We prove that the former can outperform later.

Third procurement Scenario BM is similar to the conventional singleperiod decision model. It is found that the single-period procurement decision always leads to a suboptimal solution. The supply chain members and consumers can receive higher benefits if the retailer can mix procurement, pricing, and inventory management decision systematically. Supply chain members can be able to sell a higher amount of products.

2. Problem description

We explore the interactions in two-and three-echelon supply chains under linear price-sensitive [18] demand. Therefore, the demand for each selling period is $a - bp_r$, where a, b and p_r represent market potential, price sensitivity, and retail price, respectively. Linear pricesensitive demand is one of the fundamental demand functions studied in literature.

We derive optimal decisions where the supply chain member can execute integrated procurement planning by taking account upto four consecutive selling periods. Good procurement strategy is essential to reduce total purchasing cost. In a recent study by [7], the author mentioned that "even modest improvements in procurement policies can provide enormous savings" by comparing two different procurement models formulated under large-scale integer programming formulation. The procurement Scenario BM is similar to conventional single-period decision making model where impact of inventory is ignored. In procurement Scenario BP, the retailer procures in bulk at first



а





Fig. 1. a. Procurement Scenario BP under two-echelon supply chain under four-period integrated planning; b. Procurement Scenario SPI under two-echelon supply chain under four-period integrated planning.

Table 1

Optimal decisions for two-echelon supply chain in procurement Scenarios BP.

	Two-period $(t=2)$	Three-period $(t=3)$	Four-period $(t=4)$
p_{mt4}^{2bp}	-	-	$\frac{12a + 58bh}{33b}$
p_{mt3}^{2bp}	-	$\frac{54a + 176bh}{150b}$	$\frac{12a + 36bh}{33b}$
p_{mt2}^{2bp}	$\frac{6a + 10bh}{17b}$	$\frac{54a + 76bh}{150b}$	$\frac{12a + 14bh}{33b}$
p_{mt1}^{2bp}	$\frac{9a - 2bh}{17b}$	$\frac{81a - 36bh}{150b}$	$\frac{18a - 12bh}{33b}$
p_{rt4}^{2bp}	-	-	$\frac{45a + 58bh}{66b}$
p_{rt3}^{2bp}	-	$\frac{204a + 176hb}{300b}$	$\frac{45a + 36bh}{66h}$
p_{rt2}^{2bp}	$\frac{23a + 10bh}{34b}$	$\frac{204a + 76bh}{300b}$	$\frac{45a + 14bh}{66b}$
p_{rt1}^{2bp}	$\frac{26a-2bh}{34b}$	$\frac{231a - 36bh}{300b}$	$\frac{51a - 12bh}{66b}$
I_{t3}^{2bp}	-		$\frac{9a - 116bh}{66}$
I_{t2}^{2bp}	-	$\frac{21a - 176bh}{150}$	$\frac{9a - 72bh}{66}$
I_{t1}^{2bp}	$\frac{5(a-4bh)}{34}$	$\frac{21a - 76bh}{150}$	$\frac{9a - 28bh}{66}$
π^{2bp}_{rt4}	-	-	$\frac{219a^2 - 1392abh - 3364b^2h^2}{1452b}$
π^{2bp}_{mt4}	-		$\frac{2(6a+29bh)^2}{1080b}$
π_{rt3}^{2bp}	-	$\frac{573a^2 - 2376abh - 3872b^2h^2}{2750h}$	$\frac{219a^2 - 1227abh - 1054b^2h^2}{726h}$
π^{2bp}_{mt3}	-	$\frac{(27a + 88bh)^2}{11250b}$	$\frac{2(72a^2 + 564abh + 1165b^2h^2)}{1089b}$
π^{2bp}_{rt2}	$\frac{181a^2 - 360abh - 300b^2h^2}{1156b}$	$\frac{1146a^2 - 3927abh - 194b^2h^2}{3750b}$	$\frac{(3a - 2bh)(219a - 916bh)}{1452b}$
π^{2bp}_{mt2}	$\frac{2(3a+5bh)^2}{289b}$	$\frac{729a^2 + 3402abh + 4594b^2h^2}{5625b}$	$\frac{4(54a^2 + 324abh + 607b^2h^2)}{1089b}$
π_{rt1}^{2bp}	$\frac{155a^2 - 118abh + 304b^2h^2}{1156b}$	$\frac{6219a^2 - 8928abh + 37184b^2h^2}{30000b}$	$\frac{102a^2 - 213abh + 1226b^2h^2}{363b}$
π^{2bp}_{mt1}	$\frac{9a^2 - 4abh + 8b^2h^2}{34b}$	$\frac{729a^2 - 648abh + 2144b^2h^2}{1800b}$	$\frac{2(27a^2 - 36abh + 166b^2h^2)}{99bb}$
Q_t^{2bp}	$\frac{19a - 8bh}{34}$	$\frac{3(29a - 24bh)}{100}$	$\frac{13a - 16bh}{11}$

selling period and distributes those products in forthcoming selling periods associated with the integrated planning. For example, in a three-period integrated procurement planning, the supply chain members need to set their respective prices for three consecutive selling periods, and in addition, the retailer needs to make an inventory distribution planning for those three periods. Where as in procurement Scenario SPI, the retailer needs to develop an inventory distribution planning for two consecutive selling periods. We explore optimal decisions under manufacturer-Stackelberg game model.

The retailer in the supply chain has a downstream retail monopoly and relies solely on the upstream manufacturer in a two-echelon setting or distributor and manufacturer in a three-echelon setting for the retailed product. For feasibility of the optimal solution, it is assumed that the retail (p_r) and wholesale prices (p_m and p_d) at each selling period satisfy the relations $p_r > p_d > p_m > 0$. The holding cost per unit for the retailer is h.

For analytical simplicity, the marginal costs are normalized to zero and all the parameters related to market demand are common knowledge between supply chain members. Shortages are not allowed. It is assumed that a > 4hb. Otherwise, the retailer cannot buildup SI [5,18]. To distinguish the outcomes in different scenarios, the following additional subscripts and superscripts are used.

The following notations are used to develop the models:

i	number of echelon (distribution structures), $i = 2, 3$
j	procurement scenarios, $j = bm$, bp , spi
t	number of consecutive selling periods to be taken account in a
	integrated procurement planning, $t = 1, 2, 3, 4$
S	<i>sth</i> selling period under integrated procurement planning $t, s \le t, \forall t$
p_{mts}^{ij}	wholesale price per unit determined by the manufacturer
p_{dts}^{ij}	wholesale price per unit determined by the distributor
p_{rts}^{ij}	retail price per unit determined by the retailer
$I_{t(s-1)}^{ij}$	amount of SI $(I_{t(s-1)}^{ij} \ge 0)$

π_{mts}^{ij} prof	t of	the	manufacturer
-----------------------	------	-----	--------------

profit of the distributor

 π_{rls}^{ij} profit of the retailer

 Q_t^{ij} cumulative sales volume under tth procurement planning

2.1. Benchmark model

To establish the necessity of multi-period integrated procurement strategies, we consider a conventional single-period procurement decision model as a benchmark. In procurement Scenario BM, the retailer does not maintain SI or procure products in bulk. Therefore, profit functions of the retailer and manufacturer will be identical for each selling period and their values are $\pi_{r11}^{2bm} = (p_{r11}^{2bm} - p_{m11}^{2bm})(a - bp_{r11}^{2bm})$ and $\pi_{m11}^{2bm} = p_{m11}^{2bm}(a - bp_{r11}^{2bm})$, respectively. One may obtain the optimal response function for the retailer by solving first-order condition of optimization as $p_{r11}^{2bm}(p_{m11}^{2bm}) = \frac{a + bp_{m11}^{2bm}}{2b}$. Substituting optimal response, the manufacturer's profit function is obtained as follows, $\pi_{m11}^{2bm} = \frac{p_{m11}^{2bm}(a - bp_{m11}^{2bm})}{2}$ and corresponding optimal wholesale price is $p_{m11}^{2bm} = \frac{a}{2b}$. Therefore, profits for the retailer and manufacturer, and sales volume in each selling period are obtained as, $\pi_{r11}^{2bm} = \frac{a^2}{16b}$, $\pi_{m11}^{2bm} = \frac{a^2}{3b}$, and $Q_1^{2bm} = \frac{a}{4}$, respectively. Similarly, profit functions for a single-period three-echelon supply chain consisting of a Stackelberg-manufacturer, a distributor, and a retailer are $\pi_{m11}^{3bm} = p_{m11}^{3bm}(a - bp_{r11}^{3bm})$, $\pi_{d11}^{3bm} = (p_{d11}^{3bm} - p_{m11}^{3bm})(a - bp_{r11}^{3bm})$, and $\pi_{m11}^{3bm} = p_{m11}^{3bm}(a - bp_{r11}^{3bm})$, respectively. The corresponding optimal prices and sales volume are $p_{r11}^{3bm} = \frac{ra}{8}$, $p_{d11}^{3bm} = \frac{a}{4}$, $p_{m11}^{3bm} = \frac{a}{2b}$, and $Q_1^{3bm} = \frac{a}{8}$, $p_{d11}^{3bm} = \frac{a}{4}$, $p_{m11}^{3bm} = \frac{a}{2b}$, and $Q_1^{3bm} = \frac{a}{8}$, $p_{m11}^{3bm} = \frac{a}{2b}$, and $Q_1^{3bm} = \frac{a}{8}$, $p_{m11}^{3bm} = \frac{a}{2b}$, and $Q_1^{3bm} = \frac{a}{8}$, $p_{m11}^{3bm} = \frac{a}{2b}$, and $Q_1^{3bm} = \frac{a}{8}$, $p_{m11}^{3bm} = \frac{a^2}{16b}$, $\pi_{m11}^{3bm} = \frac{a^2}{32b}$, and $\pi_{m11}^{3bm} = \frac{a^2}{ab}$, respectively. Using optimal prices, profits of the manufacturer, d

2.2. Optimal decisions in Scenario BP

We derive optimal decision for a four-period integrated procurement planning of a two-echelon supply chain consisting of a Stackelberg manufacturer and a retailer. In first selling period, the manufacturer determines wholesale price (p_{m41}^{2bp}) . Based on the manufacturer's decisions, the retailer sets retail price (p_{r41}^{2bp}) and decides the amount of inventory to be distributed in forthcoming periods Therefore, $(I_{4(s-1)}^{2bp} \ge 0, \quad s = 2, 3, 4).$ the retailer procures $q_{41}^{2bp} = a - bp_{r41}^{2bp} + \sum_{s=2}^{4} I_{4(s-1)}^{2bp}$ units of product from the manufacturer. In forthcoming selling periods, the manufacturer sets the wholesale price (p_{m4s}^{2bp}) , and then retailer sets the retail price (p_{r4s}^{2bp}) , (s = 2, 3, 4). Therefore, the retailer procures $(q_{4s}^{2bp} = a - bp_{r4s}^{2bp} - I_{4(s-1)}^{2bp})(s = 2, 3, 4)$ units of product in forthcoming periods and needs to invest more in first period. We use a backward substitution method to find an optimal decision. The profit functions for the supply chain members in four consecutive selling periods are obtained as follows:

$$\begin{aligned} \pi_{r41}^{2bp} &= p_{r41}^{2bp}(a - bp_{r41}^{2bp}) - p_{m41}^{2bp}(a - bp_{r41}^{2bp} + \sum_{s=2}^{4} I_{4(s-1)}^{2bp}) - h \sum_{s=2}^{4} I_{4(s-1)}^{2bp} \\ &+ \pi_{r42}^{2bp} \\ \pi_{m41}^{2bp} &= p_{m41}^{2bp}(a - bp_{r41}^{2bp} + \sum_{t=s}^{4} I_{4(s-1)}^{2bp}) + \pi_{m42}^{2bp} \end{aligned}$$

$$\begin{split} \pi^{2bp}_{r42} &= p^{2bp}_{r42}(a - bp^{2bp}_{r42}) - p^{2bp}_{m42}(a - bp^{2bp}_{r42} - I^{2bp}_{41}) - h\sum_{s=3}^{4}I^{2bp}_{4(s-1)} + \pi^{2bp}_{r43} \\ \pi^{2bp}_{m42} &= p^{2bp}_{m42}(a - bp^{2bp}_{r42} - I^{2bp}_{41}) + \pi^{2bp}_{m43} \\ \pi^{2bp}_{r43} &= p^{2bp}_{r43}(a - bp^{2bp}_{r43}) - p^{2bp}_{m43}(a - bp^{2bp}_{r43} - I^{2bp}_{42}) - hI^{2bp}_{43} + \pi^{2bp}_{r44} \\ \pi^{2bp}_{m43} &= p^{2bp}_{m43}(a - bp^{2bp}_{r43} - I^{2bp}_{42}) + \pi^{2bp}_{m44} \\ \pi^{2bp}_{r44} &= p^{2bp}_{r44}(a - bp^{2bp}_{r44} - I^{2bp}_{43}), \quad \pi^{2bp}_{m44} \\ &= p^{2bp}_{m44}(a - bp^{2bp}_{r44} - I^{2bp}_{43}) \end{split}$$

The graphical representation of procurement Scenario BP under four-period planning is presented in Fig. 1a.

Table 2

Optimal decisions for three-echelon supply chain in procurement Scenario BP.

μ_{n}^{2n} - $1000000000000000000000000000000000000$		Two-period (t=2)	Three-period (t=3)	Four-period (t=4)
p_{ab}^{N} · $\frac{0.8982 + 0.9738 h}{h}$ $\frac{0.9982 + 0.9738 h}{h}$ $\frac{0.9982 + 0.9738 h}{h}$ p_{ab}^{N} $\frac{0.9982 + 0.9738 h}{h}$ $\frac{0.9982 + 0.9738 h}{h}$ $\frac{0.9982 + 0.9738 h}{h}$ $\frac{0.9982 + 0.9738 h}{h}$ p_{ab}^{N} $\frac{0.9982 + 0.9738 h}{h}$ $\frac{0.9882 + 0.9738 h}{h}$ $\frac{0.9892 + 0.9738 h}{h}$ $\frac{0.9982 + 0.9738 h}{h}$ p_{ab}^{N} $\frac{0.9982 + 0.9738 h}{h}$ $\frac{0.9882 + 0.9738 h}{h}$ $\frac{0.9882 + 0.9738 h}{h}$ $\frac{0.9882 + 0.9738 h}{h}$ p_{ab}^{N} $\frac{1.918 + 0.773 h}{h}$ $\frac{0.9484 - 0.9630 h}{h}$ $\frac{0.9438 + 0.9933 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ p_{ab}^{N} $\frac{1.918 + 0.773 h}{h}$ $\frac{0.9472 + 0.9873 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ p_{ab}^{N} $\frac{1.918 + 0.773 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ p_{ab}^{N} $\frac{0.9872 + 0.9873 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ p_{ab}^{N} $\frac{0.9872 + 0.9873 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ $\frac{0.9872 + 0.9873 h}{h}$ p_{ab}^{N} $\frac{0.9872 + 0.978 h}{h}$ <th>P_{mt4}^{3bp}</th> <th>-</th> <th></th> <th>$\frac{0.40967a + 1.07268bh}{b}$</th>	P _{mt4} ^{3bp}	-		$\frac{0.40967a + 1.07268bh}{b}$
p_{abc}^{bbc} $p_{abc}^$	P _{mt3} ^{3bp}	-	$\frac{0.36284a + 0.63757bh}{h}$	$\frac{0.40967a + 0.53934bh}{b}$
μ_{n}^{NR} $\frac{28579 - 6730h}{h}$ $\frac{68542 - 6437h}{h}$ $\frac{68552 - 6437h}{h}$ $\frac{68552 - 6437h}{h}$ $\frac{66551 + 16000h}{h}$ μ_{n}^{NR} $ \frac{66551 + 16000h}{h}$ $\frac{66551 + 16000h}{h}$ $\frac{66551 + 16000h}{h}$ μ_{n}^{NR} $ \frac{66551 + 16000h}{h}$ $\frac{66551 + 16000h}{h}$ $\frac{66551 + 16000h}{h}$ μ_{n}^{NR} $\frac{7713 + 6730h}{2040h}$ $\frac{66512 + 16000h}{h}$ $\frac{66512 + 16000h}{h}$ $\frac{66512 + 16000h}{h}$ μ_{n}^{NR} $\frac{7713 + 6730h}{2040h}$ $\frac{66001 - 60355h}{h}$ $\frac{60372 + 6085hh}{h}$ $\frac{60372 + 6085hh}{h}$ μ_{n}^{NR} $\frac{7713 + 6731h}{2040h}$ $\frac{60372 + 6085hh}{h}$ $\frac{60372 + 6085hh}{h}$ $\frac{60372 + 6085hh}{h}$ μ_{n}^{NR} $\frac{7713 + 6731h}{2040h}$ $\frac{60372 + 6085hh}{h}$ $\frac{60372 + 6085hh}{h}$ $\frac{60372 + 6085hh}{h}$ $\frac{60372 + 6085hh}{h}$ μ_{n}^{NR} $\frac{7713 + 6731h}{2040h}$ $\frac{60372 + 6035hh}{h}$	p ^{3bp} _{mt2}	$\frac{5452a + 4648bh}{12472b}$	0.36284a + 0.10424bh	0.40967a + 0.00601bh
NumNumNumNum μ_{33}^{NP} - $\frac{64249 + 0.9508h}{h}$ $\frac{04541 + 0.9003h}{h}$ μ_{33}^{NP} - $\frac{64249 + 0.9508h}{h}$ $04541 + 0.9003h$ μ_{34}^{NP} $\frac{11921b}{11921b}$ $\frac{04541 + 0.9003h}{h}$ $04511 + 0.9003h$ μ_{34}^{NP} $\frac{20452 + 0.1508h}{11921b}$ $04511 + 0.9003h$ $04511 + 0.9003h$ μ_{34}^{NP} $\frac{20452 - 0.1508h}{11921b}$ $06552 + 0.0005h$ $06552 + 0.0005h$ μ_{34}^{NP} $\frac{20452 - 0.0005h}{11921b}$ $06552 + 0.0005h$ $06552 + 0.0005h$ μ_{34}^{NP} $\frac{20601 + 0721h}{20000}$ $06557 + 0.0005h$ $05752 + 0.0005h$ μ_{34}^{NP} $\frac{10001 + 0721h}{20000}$ $000031 - 0.0005h$ $000032 - 0.0005h$ μ_{34}^{NP} $\frac{10001 + 0721h}{20000}$ $000032 - 0.0005h$ $000032 - 0.0005h$ μ_{34}^{NP} $\frac{10001 + 0721h}{20000}$ $000032 - 0.0005h$ $000032 - 0.0005h$ μ_{34}^{NP} $\frac{10001 + 0721h}{20000}$ $000032 - 0.0005h$ $000032 - 0.0005h$ μ_{34}^{NP} $1000000000000000000000000000000000000$	p_{mt1}^{3bp}	20927a – 95720bh 416113b	<u>0.86642a - 0.45474bh</u>	$\frac{0.50584a - 0.81034bh}{b}$
p_{ab}^{3d} - $\frac{9.442a + 0.0585bh}{b}$ $\frac{9.444a + 0.0585bh}{b}$ $\frac{9.444a + 0.0585bh}{b}$ p_{ab}^{3d} $\frac{177a + 072bh}{13422b}$ $\frac{9.447a + 0.1585bh}{b}$ $\frac{9.447a + 0.1585bh}{b}$ $\frac{9.447a + 0.0585bh}{b}$ p_{ab}^{3d} $\frac{9.447a - 0.416bh}{20.404b}$ $\frac{9.447a + 0.0585bh}{b}$ $\frac{9.0037a + 0.0585bh}{b}$ $\frac{9.0037a + 0.0615bh}{b}$ p_{ab}^{3d} $\frac{9.144a - 0.0785bh}{20.404b}$ $\frac{9.0037a + 0.0781bh}{b}$ $\frac{9.077a + 0.0615bh}{b}$ p_{ab}^{3d} $\frac{9.144a - 0.0785bh}{20.404b}$ $\frac{9.077a + 0.0615bh}{b}$ $\frac{9.077a + 0.0615bh}{b}$ p_{ab}^{3d} $\frac{9.144a - 0.0781bh}{20.404b}$ $\frac{9.077a + 0.0615bh}{b}$ $\frac{9.077a + 0.0615bh}{b}$ p_{ab}^{3d} $\frac{9.160a + 0072bh}{20.404b}$ $\frac{9.077a + 0.0715bh}{b}$ $\frac{9.077a + 0.0615bh}{b}$ p_{ab}^{3d} $\frac{9.160a + 0072bh}{20.404b}$ $\frac{9.077a + 0.0715bh}{b}$ $\frac{9.077a + 0.0615bh}{b}$ p_{ab}^{3d} $\frac{9.0003a - 1.0725bh}{20.404b}$ $\frac{9.0003a - 1.0725bh}{b}$ $\frac{9.0003a - 1.0725bh}{b}$ p_{ab}^{3d} $\frac{9.0003a - 0.0726bh}{20.404b}$ $\frac{9.0003a - 0.0726bh}{20.40005bh}$ $\frac{9.0003a - 0.0726bh}{b}$ p_{ab}^{3d} $\frac{9.0003a - 0.0726bh}{20.4000bh}$ <	p_{dt4}^{3bp}	-	-	$\frac{0.61451a + 1.60902bh}{b}$
p_{11}^{31} $11_{12,12}$ $11_{12,12}$ $0.4151 - 0.0002h$ p_{11}^{31} $21_{12,12}$ $0.0002h$ $0.0002h$ $0.0002h$ p_{11}^{31} $20.0055h$ $0.00055h$ $0.0002h$ $0.0002h$ p_{12}^{31} $0.0002h$ $0.0002h$ $0.0002h$ $0.0002h$ p_{12}^{31} $0.0002h$ $0.0002h$ $0.0002h$ $0.0002h$ p_{12}^{31} $0.0001+0072h$ $0.0001+0072h$ $0.0002h$ $0.0002h$ p_{12}^{31} $0.0001+0072h$ $0.0001+0072h$ $0.0002h$ $0.0003a - 0.0002h$ p_{12}^{31} $0.0001+0072h$ $0.0003a - 0.0002h$ $0.0003a - 0.0002h$ p_{12}^{310} $0.0003a - 0.00002h$ $0.0003a - 0.0002h$ $0.0003a - 0.0002h$ p_{12}^{310} $0.0003a - 0.00002h$ $0.0003a - 0.00002h$ $0.0003a - 0.00002h$ p_{13}^{310} $0.0003a - 0.00002h$ $0.0003a - 0.00002h$ $0.0003a - 0.00002h$ p_{13}^{310} $0.0003a - 0.00002h$ $0.0003a - 0.00002h$ $0.00003a - 0.00002h$ p_{13}^{310} $0.0003a - 0.00002h$ $0.00003a - 0.00002h$ $0.00003a - 0.00002h$ p_{13}^{310} $0.00003a - 0.00002h^2$ $0.00000a^2 - 0.0070h^2$ $0.00000a^2 - 0.0070h^2$ p_{13}^{310} $0.00000a^2 - 0.0070h^2$ $0.0000a^2 - 0.0070h^2$ $0.0000a^2 - 0.0070h^2$ p_{13}^{310} $0.00000a^2 - 0.0070h^2$ $0.0000a^2 - 0.0070h^2$ $0.0000a^2 - 0.0070h^2$ p_{13}^{310} $0.0000a^2 - 0.0070h^2$ $0.0000a^2 - 0.0070h^2$ $0.0000a^2 - 0.0070h^2$ p_{13}^{310} $0.0000a^2 - 0.0070h^2$ <th< th=""><th>Pdt3</th><th>-</th><th>$\frac{0.54426a + 0.95636bh}{b}$</th><th>$\frac{0.61451a + 0.80902bh}{b}$</th></th<>	Pdt3	-	$\frac{0.54426a + 0.95636bh}{b}$	$\frac{0.61451a + 0.80902bh}{b}$
p_{11}^{hln} $\frac{20453 - 410hc}{8440}$ $\frac{00333 - 00355hc}{b}$ $\frac{07313 - 0.0352hc}{b}$ p_{13}^{hln} - $\frac{02731 + 0.731bc}{b}$ $\frac{00372 + 0.0351bc}{b}$ p_{13}^{hln} - $\frac{02721 + 0.731bc}{b}$ $\frac{00372 + 0.0351bc}{b}$ p_{13}^{hln} - $\frac{02721 + 0.731bc}{b}$ $\frac{00372 + 0.0351bc}{b}$ p_{13}^{hln} $\frac{1021 + 0772bc}{20440}$ $\frac{00372 + 0.0351bc}{b}$ $\frac{00372 + 0.0351bc}{b}$ p_{13}^{hln} $\frac{1021 + 0772bc}{20440}$ $\frac{00372 + 0.0351bc}{b}$ $\frac{00372 + 0.0351bc}{b}$ p_{13}^{hln} $\frac{1721a - 0.0731bc}{20440}$ $\frac{00372 + 0.0351bc}{b}$ $\frac{00372 + 0.0351bc}{b}$ p_{13}^{hln} $\frac{1721a - 0.0731bc}{20440}$ $\frac{0.0072 + 0.0351bc}{b}$ $\frac{0.0072 + 0.0351bc}{b}$ p_{13}^{hln} $\frac{2721a - 0.731bc}{20440}$ $\frac{0.0072 + 0.0351bc}{b}$ $\frac{0.0072 + 0.0351bc}{b}$ p_{13}^{hln} $\frac{2721a - 0.0351bc}{2040}$ $\frac{0.0372 - 0.0372bc}{b}$ $\frac{0.00933 - 0.0061bb}{b}$ r_{13}^{hln} $\frac{2738 - 0.0393bc}{b}$ $\frac{0.0393c^2 - 0.0323bc}{b}$ $\frac{0.0393c^2 - 0.0323bc}{b}$ r_{13}^{hln} $\frac{0.0393c^2 - 0.0373bc}{b}$ $\frac{0.0393c^2 - 0.0373bc}{b}$ $\frac{0.0393c^2 - 0.0373bc}{b}$ r_{13}^{hln} $\frac{0.0393c^2 - 0.0373bc}{b}$ $\frac{0.0393c^2 - 0.0373bc}{b}$ $\frac{0.0393c^2 - 0.0373bc}{b}$ r_{13}^{hln} $\frac{0.0393c^2 - 0.0373bc}{b}$ $\frac{0.0393c^2 - 0.0373bc}{b}$ $\frac{0.0393c^2 - 0.0373bc}{b}$ r_{13}^{hln} $\frac{0.0393c^2 - 0.0373bc}{b}$ $\frac{0.0393c^2 - 0.0373bc}{b}$ $\frac{0.0393c^2 - 0.0373bc}{b}$ r_{13}^{hln} $\frac{0.0393c^2 - 0.0373bc}{b}$ <td< th=""><th>p_{dt2}^{3bp}</th><th>$\frac{8178a + 6972bh}{13473b}$</th><th><u>0.54427a + 0.15636bh</u> <u>b</u></th><th>$\frac{0.61451a + 0.00902bh}{b}$</th></td<>	p_{dt2}^{3bp}	$\frac{8178a + 6972bh}{13473b}$	<u>0.54427a + 0.15636bh</u> <u>b</u>	$\frac{0.61451a + 0.00902bh}{b}$
p_{14}^{14} p_{17}^{12} $p_$	p_{dt1}^{3bp}	20445a - 9416bh 2884b	$\frac{0.68033a - 0.80455bh}{b}$	$\frac{0.76813a - 0.98872bh}{b}$
p_{13}^{3b} · $0.0721s_1 + 0.4918bh$ $0.0721s_1 - 0.4918bh$ $0.0721s_1 - 0.4918bh$ p_{13}^{3b} $2101 + 0.972bh$ $0.0721s_1 + 0.07818bh$ $0.0721s_1 - 0.0913bh$ p_{13}^{3b} $2101 + 0.972bh$ $0.0913s_1 - 0.0913bh$ $0.0913s_1 - 0.0913bh$ p_{13}^{3b} $1.972bh$ $0.09033s_1 - 0.1928bh$ $0.09033s_1 - 0.1928bh$ p_{13}^{3b} $1.972bh$ $0.09033s_1 - 0.1928bh$ $0.09033s_1 - 0.1928bh$ p_{13}^{3b} $1.972bh$ $0.09033s_1 - 0.0903bh$ $0.09033s_1 - 0.0903bh$ p_{13}^{3b} $1.972bh$ $0.09033s_1 - 0.0903bh$ $0.09033s_1 - 0.0903bh$ p_{14}^{3b} $2.519s_1 - 929bh$ $0.0903s_1 - 0.1924bh$ $0.0903s_1 - 0.0903bh$ p_{14}^{3b} $1.972bh$ $0.0903s_1 - 0.0924bh$ $0.0903s_1 - 0.0903bh$ p_{14}^{3b} $1.972bh$ $0.0903s_1 - 0.0903bh$ $0.0903bh$ p_{14}^{3b} $1.972bh$ $0.0903s_1 - 0.0903bh$ $0.0903bh$ p_{14}^{3b} $1.972bh$ $0.0903s_1 - 0.0903bh$ $0.0903bh$ p_{14}^{3b} $1.972bh$ $0.0903bh$ $0.0903bh$ $0.0903bh$ p_{14}^{3b} <th>p_{rt4}^{3bp}</th> <th>-</th> <th>-</th> <th>$\frac{0.80725a + 0.80451bh}{b}$</th>	p_{rt4}^{3bp}	-	-	$\frac{0.80725a + 0.80451bh}{b}$
p_{n2}^{32} $\frac{2101 + 0773h}{360}$ $0.8772h + 0.0051hh}{h}$ p_n^{32} $\frac{4723h - 9416hh}{36020}$ $0.94017a - 0.40277hh}{h}$ $0.88107a - 0.40375h$ p_n^{33} $\frac{4723h - 9416hh}{36020}$ $0.94017a - 0.40277hh$ $0.88107a - 0.40376hh$ p_n^{33} $ 0.09033a - 1.07268hh$ l_1^{33} $ 0.09033a - 0.05385hh$ l_1^{33} $219a - 9296hh$ $0.09033a - 0.0001hh$ n_n^{34} $2386h$ $0.09033a - 0.0001hh$ n_n^{34} $ 0.0025ac^2 - 0.00001hh$ n_n^{34} $ 0.0005ac^2 - 0.0000hh$ n_n^{34} $ 0.0000c^2 n^2 - 0.0000hh^2$ n_n^{34} $0.0000c^2 n^2 - 0.0000hh^2$	p _{rt3} ^{3bp}	-	$\frac{0.77213a + 0.47818bh}{h}$	$\frac{0.80725a + 0.40451bh}{b}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	p_{rt2}^{3bp}	$\frac{21601a + 6972bh}{26846h}$	$\frac{0.77213a + 0.07818bh}{b}$	$\frac{0.80725a + 0.00451bh}{b}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	p_{rt1}^{3bp}	47291a - 9416bh 5360b	<u>0.84017a - 0.40227bh</u>	0.88407a - 0.49436bh
I_1^{3p} -0.13716a - 0.63757bh0.09033a - 0.53935bh I_1^{3p} $\frac{2519a - 9236bh}{26846}$ 0.13716a - 0.10424bh0.09033a - 0.00601bh π_1^{3p} $\frac{00926a^2 - 0.82396abh - 1.07873b^2h^2}{b}$ π_1^{3p} $\frac{00926a^2 - 0.82396abh - 1.07873b^2h^2}{b}$ π_{ath}^{3p} $\frac{00926a^2 - 0.82396abh - 1.07873b^2h^2}{b}$ π_{ath}^{3p} $\frac{00936a^2 - 0.05396abh + 0.14383b^2h^2}{b}$ π_{ath}^{3p} $\frac{00936a^2 - 0.05396abh + 0.14383b^2h^2}{b}$ π_{ath}^{3p} $\frac{01956a^2 - 0.4376abh - 0.38109b^2h^2}{b}$ $\frac{00938a^2 - 0.1587abh - 0.2876b^2h^2}{b}$ π_{ath}^{3p} - $\frac{01646a^2 + 0.05784abh + 0.0581b^2h^2}{b}$ $\frac{0194196a^2 + 0.07747abh + 1.25287b^2h^2}{b}$ π_{ath}^{3p} - $\frac{01956a^2 - 0.1576abh + 0.0581b^2h^2}{b}$ $\frac{0194196a^2 + 0.07747abh + 1.25287b^2h^2}{b}$ π_{ath}^{3p} - $\frac{01956a^2 - 0.1576abh + 0.0581b^2h^2}{b}$ $\frac{0194196a^2 + 0.07747abh + 1.25287b^2h^2}{b}$ π_{ath}^{3p} $\frac{7(981587a^2 - 2715096babh - 1157352b^2h^2)}{72070716b}$ $\frac{05291a^2 - 0.1587abh + 0.05217b^2h^2}{b}$ $\frac{008936a^2 - 0.0758abh + 1.35232b^2h^2}{b}$ π_{ath}^{3p} $\frac{1031347a^2 - 0.75936bb + 1.15332b^2h^2}{18017692b}$ $\frac{005291a^2 - 0.0558abh + 1.05217b^2h^2}{b}$ $\frac{008936a^2 - 0.0758abh + 1.25288b^2h^2}{b}$ π_{ath}^{3p} $\frac{1091347a^2 - 0.75936bb + 1.133336b^2h^2}{19047692b}$ $\frac{005291a^2 - 0.7592abh + 0.05217b^2h^2}{b}$ $008326a^2 - 0.1587a^2 - 0.1587a^2$	I_{t3}^{3bp}	-	-	0.09033a - 1.07268bh
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	I_{t2}^{3bp}		0.13716a - 0.63757bh	0.09033a - 0.53935bh
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	I_{t1}^{3bp}	<u>2519a – 9296bh</u> 26846	0.13716a - 0.10424bh	0.09033a - 0.00601bh
	π^{3bp}_{rt4}	-	-	$\frac{0.09266a^2 - 0.82396abh - 1.07873b^2h^2}{b}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	π^{3bp}_{dt4}		-	$\frac{0.02098a^2 + 0.10986abh + 0.14383b^2h^2}{h}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	π^{3bp}_{mt4}	-	-	$\frac{0.04196a^2 + 0.21972abh + 0.28766b^2h^2}{b}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	π^{3bp}_{rt3}		$\frac{0.12657a^2 - 0.43376abh - 0.38109b^2h^2}{h}$	$\frac{0.18532a^2 - 1.32858abh - 0.27876b^2h^2}{b}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	π^{3bp}_{dt3}	-	$\frac{0.01646a^2 + 0.05784abh + 0.05081b^2h^2}{h}$	$\frac{0.04196a^2 + 0.07477abh + 1.25287b^2h^2}{b}$
$ \pi_{n12}^{3bp} \qquad \underbrace{\frac{7(9815827a^2 - 27150960abh - 11573520b^2h^2)}{720707716b}} \\ \underline{0.25315a^2 - 0.61184abh + 0.24629b^2h^2}{b} \qquad \underbrace{\frac{0.27798a^2 - 1.5136abh + 1.33323b^2h^2}{b}}{b} \\ \pi_{d12}^{3bp} \qquad \underbrace{\frac{2(1363a + 1162bh)^2}{180176929b}}_{180176929b} \\ \underline{0.12587a^2 + 0.07538abh + 1.25288b^2h^2}{b} \\ \underline{0.06583a^2 + 0.14348abh + 0.01434b^2h^2}_{b} \\ \underline{0.12587a^2 + 0.31143abh + 0.36039b^2h^2}_{b} \\ \underline{0.12587a^2 + 0.31143abh + 0.36039b^2h^2}_{b} \\ \pi_{n11}^{3bp} \qquad \underbrace{\frac{3542327a^2 - 1776808abh + 1331536b^2h^2}{9294544b}}_{1994544b} \\ \underline{0.09207a^2 - 0.06218abh + 0.55311b^2h^2}_{b} \\ \underline{0.08326a^2 - 0.15941abh + 1.59587b^2h^2}_{b} \\ \underline{0.08326a^2 - 0.15941abh + 1.59587b^2h^2}_{b} \\ \pi_{n11}^{3bp} \qquad \underbrace{\frac{109133473a^2 - 50085968abh + 1333880b^2h^2}{1441415432b}} \\ \underline{0.17236a^2 - 0.73052abh + 0.36506b^2h^2}_{b} \\ \underline{0.12587a^2 - 0.2587abh + 1.45332b^2h^2}_{b} \\ \underline{0.16442a^2 - 0.2885abh + 1.45332b^2h^2}_{b} \\ \underline{0.16442a^2 - 0.2885abh + 1.45332b^2h^2}_{b} \\ \underline{0.32159a^2 - 0.55052abh + 1.2579b^2h^2}_{b} \\ \underline{0.32159a^2 - 0.55052abh + 1.27096b^2h^2}_{b} \\ \underline{0.61557a - 0.15409bh} \\ \underline{0.69418a - 0.71917bh} \\ \underline{0.69518a - 0.5505ab}_{b} \\ 0.6155$	π^{3bp}_{mt3}	-	$\frac{0.03291a^2 + 0.11567abh + 0.10163b^2h^2}{h}$	$\frac{0.08391a^2 + 0.33019abh + 0.36038b^2h^2}{b}$
$ \pi_{d12}^{3bp} \qquad \frac{2(1363a + 1162bh)^2}{180176929b} \qquad \frac{0.03291a^2 + 0.06729abh + 0.05217b^2h^2}{b} \qquad \frac{0.062936a^2 + 0.0753abh + 1.25288b^2h^2}{b} \\ \pi_{mt2}^{3bp} \qquad \frac{4(1363a + 1162bh)^2}{180176929b} \qquad \frac{0.06583a^2 + 0.13458abh + 0.10434b^2h^2}{b} \qquad \frac{0.12587a^2 + 0.33143abh + 0.36039b^2h^2}{b} \\ \pi_{n11}^{3bp} \qquad \frac{3542327a^2 - 1776808abh + 13313536b^2h^2}{19294544b} \qquad \frac{0.09207a^2 - 0.06218abh + 0.55311b^2h^2}{b} \qquad \frac{0.08326a^2 - 0.1594bh + 1.55987b^2h^2}{b} \\ \pi_{d11}^{3bp} \qquad \frac{109133473a^2 - 50085968abh + 51338880b^2h^2}{144145432b} \qquad \frac{0.17236a^2 - 0.73052abh + 0.35606b^2h^2}{b} \qquad \frac{0.16442a^2 - 0.28836abh + 145332b^2h^2}{b} \\ \pi_{d11}^{3bp} \qquad \frac{247009a^2 - 107600abh + 115328b^2h^2}{164452b} \qquad \frac{0.44198a^2 - 0.35703abh + 0.25875b^2h^2}{b} \qquad \frac{0.32159a^2 - 0.55052abh + 1.27096b^2h^2}{b} \\ Q_{d}^{3bp} \qquad \frac{16891a - 4528bh}{53692} \qquad 0.61557a - 0.15409bh \qquad 0.69418a - 0.71917bh \\ \end{array}$	π_{rt2}^{3bp}	$\frac{7(9815827a^2 - 27150960abh - 11573520b^2h^2)}{720707716h}$	$\frac{0.25315a^2 - 0.64184abh + 0.24629b^2h^2}{h}$	$\frac{0.27798a^2 - 1.51386abh + 1.33323b^2h^2}{b}$
$ \pi_{mt2}^{3bp} \qquad \frac{4(1363a + 1162bh)^2}{180176929b} \qquad \frac{0.06583a^2 + 0.13458abh + 0.10434b^2h^2}{b} \qquad \frac{0.12587a^2 + 0.33145abh + 0.36039b^2h^2}{b} \\ \pi_{r11}^{3bp} \qquad \frac{3542327a^2 - 177608abh + 13313536b^2h^2}{92994544b} \qquad \frac{0.09207a^2 - 0.06218abh + 0.55311b^2h^2}{b} \qquad \frac{0.08326a^2 - 0.15941abh + 1.59587b^2h^2}{b} \\ \pi_{dt1}^{3bp} \qquad \frac{109133473a^2 - 50085968abh + 51338880b^2h^2}{1441415432b} \qquad \frac{0.17236a^2 - 0.73052abh + 0.36506b^2h^2}{b} \qquad \frac{0.16442a^2 - 0.2885abh + 1.45332b^2h^2}{b} \\ \pi_{mt1}^{3bp} \qquad \frac{24709a^2 - 107600abh + 115328b^2h^2}{1664452b} \qquad \frac{0.44198a^2 - 0.35703abh + 0.25875b^2h^2}{b} \qquad \frac{0.32159a^2 - 0.55052abh + 1.27096b^2h^2}{b} \\ Q_{t}^{3bp} \qquad \frac{16891a - 4528bh}{53692} \qquad 0.61557a - 0.15409bh \qquad 0.69418a - 0.71917bh \\ \end{array} $	π^{3bp}_{dt2}	$\frac{2(1363a + 1162bh)^2}{180176929h}$	$\frac{0.03291a^2 + 0.06729abh + 0.05217b^2h^2}{h}$	$\frac{0.062936a^2 + 0.07538abh + 1.25288b^2h^2}{b}$
$ \pi_{r11}^{3bp} \qquad \frac{354237a^2 - 1776808abh + 13313536b^2h^2}{9294544b} \qquad \frac{0.09207a^2 - 0.06218abh + 0.55311b^2h^2}{b} \qquad \frac{0.08326a^2 - 0.15941abh + 1.59587b^2h^2}{b} \\ \pi_{d1}^{3bp} \qquad \frac{109133473a^2 - 50085968abh + 51338880b^2h^2}{1441415432b} \qquad \frac{0.17236a^2 - 0.73052abh + 0.36506b^2h^2}{b} \qquad \frac{0.16442a^2 - 0.28836abh + 1.45332b^2h^2}{b} \\ \pi_{m11}^{3bp} \qquad \frac{247009a^2 - 107600abh + 115328b^2h^2}{1664452b} \qquad \frac{0.44198a^2 - 0.35703abh + 0.25875b^2h^2}{b} \qquad \frac{0.32159a^2 - 0.5502abh + 1.27096b^2h^2}{b} \\ Q_t^{3bp} \qquad \frac{16891a - 4528bh}{53692} \qquad 0.61557a - 0.15409bh \qquad 0.69418a - 0.71917bh \\ \end{array} $	π_{mt2}^{3bp}	$\frac{4(1363a+1162bh)^2}{180176929b}$	$\frac{0.06583a^2 + 0.13458abh + 0.10434b^2h^2}{b}$	$\frac{0.12587a^2 + 0.33143abh + 0.36039b^2h^2}{b}$
$\pi_{dt1}^{3bp} \qquad \underbrace{ \begin{array}{c} 10913473a^2 - 50085968bh + 51338880b^2h^2 \\ 1441415432b \end{array}} \\ \pi_{mt1}^{3bp} \qquad \underbrace{ \begin{array}{c} 247009a^2 - 107600abh + 115328b^2h^2 \\ 164442b \end{array}} \\ \frac{247009a^2 - 107600abh + 115328b^2h^2 \\ 164442b \end{array}} \\ \frac{247009a^2 - 107600abh + 115328b^2h^2 \\ 164442b \end{array}} \\ \frac{247009a^2 - 107600abh + 115328b^2h^2 \\ 164442b \end{array}} \\ 0.61557a - 0.15409bh \\ 0.61557a - 0.15409bh \end{array} \\ \underbrace{ \begin{array}{c} 0.16442a^2 - 0.28836abh + 1.45332b^2h^2 \\ b \\ $	π_{rt1}^{3bp}	$\frac{3542327a^2 - 1776808abh + 13313536b^2h^2}{92994544b}$	$\frac{0.09207a^2 - 0.06218abh + 0.55311b^2h^2}{b}$	$\frac{0.08326a^2 - 0.15941abh + 1.59587b^2h^2}{b}$
$ \frac{\pi_{mt1}^{3bp}}{q_{l}^{3bp}} \frac{\frac{247009a^2 - 107600abh + 115328b^2h^2}{1664452b}}{\frac{0.44198a^2 - 0.35703abh + 0.25875b^2h^2}{b}} \frac{\frac{0.32159a^2 - 0.55052abh + 1.27096b^2h^2}{b}}{0.61557a - 0.15409bh} $	π^{3bp}_{dt1}	$\frac{109133473a^2 - 50085968abh + 51338880b^2h^2}{1441415432b}$	$\frac{0.17236a^2 - 0.73052abh + 0.36506b^2h^2}{b}$	$\frac{0.16442a^2 - 0.28836abh + 1.45332b^2h^2}{b}$
$Q_t^{3bp} \qquad \frac{16891a - 4528bh}{53692} \qquad 0.61557a - 0.15409bh \qquad 0.69418a - 0.71917bh$	π_{mt1}^{3bp}	$\frac{247009a^2 - 107600abh + 115328b^2h^2}{1664452b}$	$\frac{0.44198a^2 - 0.35703abh + 0.25875b^2h^2}{b}$	$\frac{0.32159a^2 - 0.55052abh + 1.27096b^2h^2}{b}$
	Q_t^{3bp}	<u>16891a - 4528bh</u> 53692	0.61557a - 0.15409bh	0.69418a - 0.71917bh

Therefore, the retailer needs to determine optimal retail price and how much inventory to be distributed in forthcoming periods. Consequently one needs to consider the recursive impact of inventory distribution for finding optimal profit. The detail derivations are provided in Appendix A. Table 1 represents the simplified values of equilibrium outcomes under different scenarios.

Similarly, we also derive optimal decisions for the three-echelon supply chains under different scenarios. Profit functions are presented in Appendix A, and the optimal outcomes are presented in Table 2. The detail derivation of the optimal decisions are similar to the two-echelon supply chain, and thus omitted.

We propose following propositions to explore the characteristics of optimal decisions:

Proposition 1. In any of the multi-period procurement planning upto fourperiod under Scenario BP:

- (i) the retail and wholesale prices increase from the second selling period.
- (ii) the amount of products distributed by the retailer decreases as the selling period increases.

Proof. Please see Appendix B.

Proposition 2. Irrespective of number of echelons, participating supply chain members receive higher profits by executing integrated procurement planning upto four consecutive periods in Scenario BP compared to BM if $\frac{0.138158a}{h} > h$.

Proof. Please see Appendix C.

Proposition 3. In the multi-period procurement planning under Scenario *BP*:

- (i) the average profits for the manufacturer and retailer increase under integrated procurement strategy upto four-period if holding cost satisfies $\frac{0.027431a}{h} > h$ and $\frac{0.035817a}{h} > h$, respectively.
- (ii) the average profits for three-echelon supply chain members are not increasing under integrated procurement strategy upto four-period.

Proof. Please see Appendix D.

Proposition 4. In the multi-period procurement planning upto four consecutive periods under Scenario BP:

(i) sales volumes under both two- and three-echelon supply chains are



Fig. 2. a. Wholesale prices in Scenario BP under two-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Black); b. Retail prices in Scenario BP under two-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Black); c. Retailer's profit in Scenario BP under two-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Black); d. Manufacturer's profit in Scenario BP under two-echelon supply chain Two-period (Green) Threeperiod (Blue) Four-period (Black).

higher compared to Scenarios BM if $\frac{0.125a}{b} > h$. (ii) average sales volumes in two-echelon supply chains will increase if $\frac{0.044117a}{1} > h$, however, in three-echelon supply chain sales volumes do not increase.

Proof. Please see Appendix E.

The graphical representation of wholesale and retail prices, the profits for supply chain members are shown in Figs. 2a, b, c, d and 3 a, b, c, d, e, f respectively. The following parameter values are used for illustration: a = 200, b = 0.5, h = 5.

Above figures justify analytical findings. Pricing behaviors are not effected by supply chain structures. In both two and three-echelon supply chain, every member charges a higher price in first selling period. However, prices decrease in second-period and then increase. Due to bulk procurement and inventory distribution trend for the retailer, the prices show such a pattern. The retailer needs to increase retail prices as the number of selling periods increase from the second period. In this way, the retailer can compensate increasing holding cost. However, the most important observation from the above derivations is that all the participants receive higher profits due to the businesslike bulk procurement planning compared to single-period benchmark procurement decision. However, the presence of an intermediary prevents the retailer from executing an integrated bulk procurement decision by taken account upto four consecutive selling periods. The results demonstrate that the average profits for all the supply chain members decrease from the fourth selling period under three-echelon supply chain. Although, the average profits for the retailer and manufacturer remain increasing under the two-echelon supply chain. The presence of the distributor reduces the flexibility to earn higher profits. Moreover, demand decreases with increasing retail price. Therefore, the profits also decrease. Note that, the retailer utilizes the additional

products in second period and the distributed amount of products are always higher from first period. Therefore, the profit earned in second period is always high for the retailer. However, the procurement volumes decrease from the second period. Consequently, profits for the manufacturer and distributor decrease. The trends of average sales volumes and profits in three-echelon supply chain are also analogous.

2.3. Optimal decisions in Scenario SPI

At the beginning of each selling period ($s = 1, \dots, 4$) under integrated procurement planning upto four consecutive periods, the manufacturer determines a wholesale price (p_{max}^{2spi}) and posts it to the retailer. The retailer then procures (Q_{4s}^{2spi}) amounts of product and sets retail price (p_{r4s}^{2spi}) to satisfy market demand $(q_{4s}^{2spi} = a - bp_{r4s}^{2spi})$, $\forall s$). If the purchased quantity at each period is larger than the quantity sold in that period is $Q_{4s}^{2spi} = q_{4s}^{2spi} = q_{4s$ in procurement Scenario SPI are obtained follows: $\pi_{r41}^{2spi} = p_{r41}^{2spi}(a - bp_{r41}^{2spi}) - p_{m41}^{2spi}(a - bp_{r41}^{2spi} + I_{41}^{2spi}) - hI_{41}^{2spi} + \pi_{r42}^{2spi}$ $\pi_{m41}^{2spi} = p_{m41}^{2spi}(a - bp_{r41}^{2spi} + I_{41}^{2spi}) + \pi_{m42}^{2spi}$ as

$$\begin{aligned} \pi^{2spi}_{r42} &= p^{2spi}_{r42}(a - bp^{2spi}_{r42}) - p^{2spi}_{m42}(a - bp^{2spi}_{r42} - I^{2spi}_{41} + I^{2spi}_{42}) - hI^{2spi}_{42} + \pi^{2spi}_{r43} \\ \pi^{2spi}_{m42} &= p^{2spi}_{m42}(a - bp^{2spi}_{r42} - I^{2spi}_{41} + I^{2spi}_{42}) + \pi^{2spi}_{m43} \end{aligned}$$

$$\begin{aligned} \pi^{2spi}_{r43} &= p^{2spi}_{r43}(a - bp^{2spi}_{r43}) - p^{2spi}_{r43}(a - bp^{2spi}_{r43} - I^{2spi}_{42} + I^{2spi}_{43}) - hI^{2spi}_{43} + \pi^{2spi}_{r44} \\ \pi^{2spi}_{m43} &= p^{2spi}_{m43}(a - bp^{2spi}_{r43} - I^{2spi}_{42} + I^{2spi}_{43}) + \pi^{2spi}_{m44} \end{aligned}$$

 $\pi_{r44}^{2spi} = (p_{r44}^{2spi} - p_{m44}^{2spi})(a - bp_{r44}^{2spi} - I_{43}^{2spi}), \quad \pi_{m44}^{2spi} = p_{m44}^{2spi}(a - bp_{r44}^{2spi} - I_{43}^{2spi})$ The graphical representation of procurement Scenario SPI under four-period planning is presented in Fig. 1b.



Fig. 3. a. Manufacturer's prices in Scenario BP under three-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Black); b. Distributor's prices in Scenario BP under three-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Black); c. Retailer's prices in Scenario BP under three-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Black); d. Retailer's profit in Scenario BP under three-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Black); e. Distributor's profit in Scenario BP under three-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Black); f. Manufacturer's profit in Scenario BP under three-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Black).

Therefore, the retailer needs to decide how much additional products to be carried as a SI in between two consecutive selling periods. It should be noted that optimal outcomes in Scenarios BP and SPI are identical in two-period procurement planning. Table 3 represents the simplified values of equilibrium outcomes for three and four-period integrated procurement planning under both two and three-echelon supply chains. The detail derivations for optimal decisions in twoechelon supply chain and profit functions for three-echelon supply chain are presented in Appendix F. Similar to Scenario BP, one needs to consider the recursive impact to obtain optimal decision because the retailer carries forward inventory from the present selling period to immediate forthcoming period.

We propose the following propositions to explore the nature of optimal decisions.

Proposition 5. In any of the multi-period procurement planning upto four consecutive selling period under Scenario SPI:

(i) the retailer and manufacturer respectively set maximum retail and wholesale prices in first selling period and prices decrease onwards. (ii) the amount of SI decreases as the selling period increases.

Proof. Please see Appendix G.

Proposition 6. Irrespective of number of echelon, participating supply chain members receive higher profits by executing procurement planning upto four consecutive periods in Scenario SPI compared to BM if $\frac{0.073661a}{h} > h.$

Proof. Please see Appendix H. □

Proposition 7. In the multi-period procurement planning under Scenario SPI:

- (i) in two-echelon supply chain, the average profits for the manufacturer and retailer for four consecutive selling periods are increasing if holding cost satisfies $\frac{0.056581a}{h} > h$ and $\frac{0.03148a}{h} > h$, respectively.
- (ii) in three-echelon supply chain, the average profits for supply chain members are increasing if they execute integrated procurement planning upto four consecutive periods.

Proof. Please see Appendix I.

Proposition 8. In the multi-period integrated procurement planning upto four-period under Scenario SPI:

- (i) sales volumes under both two- and three-echelon supply chains are
- (ii) average sales volumes in two- and three-echelon supply chains are $\frac{0.144873a}{b} > h$ (ii) average sales volumes in two- and three supply chains are increasing if $\frac{0.084559a}{b} > h$ and $\frac{0.24785a}{b} > h$, respectively.

Proof. Please see Appendix J.

	Two-echelon (i=2)		Three-echelon (i=3)	
	Three-period $(t=3)$	Four-period (t=4)	Three-period $(t=3)$	Four-period $(t=4)$
p_{mt4}^{ispi}	-	$\frac{0.20394a + 1.64584bh}{h}$	-	$\frac{0.28719a + 1.01973bh}{b}$
p_{mt3}^{ispi}	0.26359a + 1.14132bh	0.30592 <i>a</i> + 1.46876 <i>bh</i>	0.33845a + 0.68803bh	$\frac{0.35565a + 0.60395bh}{h}$
p ^{ispi} pmt2	0.39539a + 0.71197bh	0.42288a + 0.79505bh	0.41912a + 0.19319bh	0.42886a + 0.03777bh
p _{ispi}	b 0.54657 <i>a</i> - 0.25109 <i>bh</i>	b 0.55817a - 0.39554bh	b 0.50539a - 0.45753bh	b 0.50729a - 0.68431bh
nispi	-	-	-	b 0.43079 <i>a</i> + 1.52957 <i>bh</i>
Pat4	-	-	0.50768a + 1.03205bh	b 0.53849a + 0.91199bh
Pdt3	-		b 0.63459a + 0.29006bh	b = 0.65322a + 0.05087bh
P _{di2} ispi			b 0.76979 <i>a</i> - 0.70357 <i>bb</i>	b 0.77595a - 1.05867bb
$P_{dt1}^{(p)}$	-		<u>b</u>	b
Prt4	-	<u>b</u>	-	<u>0.71539a + 0.76479bn</u> b
P _{rt3} ^{ispi}	$\frac{0.63179a + 0.57066bh}{b}$	$\frac{0.65296a + 0.73438bh}{b}$	$\frac{0.75384a + 0.51603bh}{b}$	$\frac{0.76925a + 0.45599bh}{b}$
P _{rt2} ^{ispi}	$\frac{0.69769a + 0.35599bh}{b}$	$\frac{0.71144a + 0.39753bh}{b}$	$\frac{0.81729a + 0.14503bh}{b}$	$\frac{0.82661a + 0.02544bh}{b}$
P_{rt1}^{ispi}	$\frac{0.77329a - 0.12555bh}{h}$	$\frac{0.77908a - 0.19777bh}{b}$	$\frac{0.88489a - 0.35178bh}{b}$	$\frac{0.88798a - 0.52933bh}{h}$
I ^{ispi}	-	0.29606a - 1.64584bh	-	0.21281a – 1.01973bh
I_{t2}^{ispi}	0.23641a - 1.14132bh	0.42216a — 2.99655bh	0.16155a - 0.68803bh	0.29291a - 1.65808bh
I_{t1}^{ispi}	0.25315a - 1.56706bh	0.33945a — 2.87103bh	0.16672a - 0.84143bh	0.22715a - 1.47007bh
π_{rt4}^{ispi}	-	$\frac{0.21881a^2 - 0.50349abh - 2.03158b^2h^2}{b}$	-	$0.17267a^2 - 0.54912abh - 0.97486b^2h^2$
π^{ispi}_{dt4}	-	-	-	$\frac{0.01031a^2 + 0.07322abh + 0.12998b^2h^2}{b}$
π^{ispi}_{mt4}	-	$\frac{1.35442(0.12391a + bh)^2}{b}$	-	$\frac{0.02062a^2 + 0.14643abh + 0.25996b^2h^2}{b}$
π_{rt3}^{ispi}	$\frac{0.19789a^2 - 0.45127abh - 0.97695b^2h^2}{b}$	$\frac{0.37782a^2 - 1.53725abh - 1.83031b^2h^2}{b}$	$\frac{0.14261a^2 - 0.43662abh - 0.44381b^2h^2}{b}$	$\frac{0.26906a^2 - 1.24314abh - 0.32937b^2h^2}{b}$
π^{ispi}_{dt3}	-	-	$\frac{0.01432a^2 + 0.05822abh + 0.05918b^2h^2}{b}$	$\frac{0.03786a^2 + 0.15296abh + 0.18615b^2h^2}{b}$
π_{mt3}^{ispi}	$\frac{0.65131(0.23096a + bh)^2}{b}$	$\frac{0.08839a^2 + 0.84871abh + 2.25964b^2h^2}{b}$	$\frac{0.02864a^2 + 0.11643abh + 0.11835b^2h^2}{b}$	$\frac{0.07423a^2 + 0.30227abh + 0.37009b^2h^2}{b}$
π_{rt2}^{ispi}	$\frac{0.29589a^2 - 1.05932abh - 0.01203b^2h^2}{b}$	$\frac{0.42611a^2 - 2.20151abh + 1.42406b^2h^2}{b}$	$\frac{0.17927a^2 - 0.74701abh + 0.22077b^2h^2}{b}$	$\frac{0.25617a^2 - 1.42532abh + 1.33892b^2h^2}{b}$
π^{ispi}_{dt2}	•	-	$\frac{0.05257a^2 + 0.07722abh + 0.05998b^2h^2}{b}$	$\frac{0.09151a^2 + 0.10821abh + 0.18336b^2h^2}{b}$
π_{mt2}^{ispi}	$\frac{0.14765a^2 + 0.53174abh + 0.70097b^2h^2}{b}$	$\frac{0.24539a^2 + 0.92268abh + 1.84379b^2h^2}{b}$	$\frac{0.10304a^2 + 0.15424abh + 0.11996b^2h^2}{b}$	$\frac{0.17676a^2 + 0.21976abh + 0.36203b^2h^2}{b}$
π_{rt1}^{ispi}	$\frac{0.20893a^2 - 0.33547abh + 1.17731b^2h^2}{b}$	$\frac{0.28545a^2 - 0.71681abh + 3.19862b^2h^2}{b}$	$\frac{0.06418a^2 - 0.06772abh + 0.59395b^2h^2}{b}$	$\frac{0.09246a^2 - 0.15269abh + 1.53287b^2h^2}{b}$
π^{ispi}_{dt1}	-	-	$\frac{0.12709a^2 - 0.12158abh + 0.18045b^2h^2}{b}$	$\frac{0.18264a^2 - 0.27152abh + 0.53554b^2h^2}{b}$
π_{mt1}^{ispi}	$\frac{0.40993a^2 - 0.37664abh + 1.06292b^2h^2}{b}$	$\frac{0.55817a^2 - 0.79107abh + 2.90115b^2h^2}{h}$	$\frac{0.24548a^2 - 0.22217abh + 0.34399b^2h^2}{h}$	$\frac{0.34882a^2 - 0.48956abh + 1.00578b^2h^2}{b}$
Q_t^{ispi}	0.89722a - 0.80112bh	1.25455a — 1.75706bh	0.54397a – 0.30927bh	0.80077a - 0.71687bh

The graphical representation of the profits, wholesale and retail prices in Scenario SPI, are shown in Figs. 4a, b, c, d and 5 a, b, c, d, e, f. The parameter values remain unchanged.

Unlike the procurement Scenario BP, optimal results show different characteristics. In both two and three-echelon supply chains, the prices for the supply chain members demonstrate a sharp decreasing trend. This result is consistent with the existing literature. The retailer's strategic decision to carry SI always enforces upstream members to reduce wholesale prices [13,18]. However, average profits for all the supply chain members remain higher compared to single-period procurement planning models. The profits for the retailer in second-period show sharp increments due to additional procurement in first period. However, due to serial distribution of inventory, profits decreases but in a steady pattern. Although the market demand increases with the decrement of retail price, but the profits do not increase continuously. The presence of the distributor cannot prevent the retailer from preparing a four-period profitable procurement planning. In Scenario BP, the average profits for the three-echelon supply chain members demonstrate decreasing trend. However, if the holding cost for the retailer is not large enough then the retailer can execute a four-period integrated procurement planning to obtain higher profits.

3. Managerial Implications

A growing number of researchers formulate supply chain models to explore the influence of inventory [11]. In this study, we explore the correlation between procurement and inventory carrying decisions. Procurement strategies discussed in this study can be implemented in practice because the profits of the supply chain members are always higher compared to single-period procurement decisions. However, pricing strategy for the retailer in Scenario SPI is consistent with a price skimming strategy; the retailer charges higher initial price and lowers it over forthcoming selling periods, suitable for consumer electronics products. In contrast, the pricing strategy in Scenario BP is consistent with a mix of skimming and penetration pricing strategy, which is practiced in FMCG products extensively. We compare total profits of the supply chain members under a three-period integrated procurement planning and the profit differences are obtained as follows:

$$\begin{aligned} \pi_{r31}^{2bp} &- \pi_{r31}^{2spi} = \frac{634791a^{2} - 1695192abh + 8466176b^{2}h^{2}}{8670000b} > 0\\ \pi_{m31}^{2bp} &- \pi_{m31}^{2spi} = \frac{4293a^{2} - 7416abh + 29248b^{2}h^{2}}{36600b} > 0 \end{aligned}$$



Fig. 4. a. Wholesale prices in Scenario SPI under two-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Red); b. Retail prices in Scenario SPI under two-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Red); c. Retailer's profit in Scenario SPI under two-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Blue) Four-period (Blue) Four-period (Blue) Four-period (Blue); d. Manufacturer's profit in Scenario SPI under two-echelon supply chain Two-period (Blue) Four-period (Blue); d. Manufacturer's profit in Scenario SPI under two-echelon supply chain Two-period (Blue); four-period (Blue); d. Manufacturer's profit in Scenario SPI under two-echelon supply chain Two-period (Blue); four-period (Blue); d. Manufacturer's profit in Scenario SPI under two-echelon supply chain Two-period (Blue); four-period (Blue); d. Manufacturer's profit in Scenario SPI under two-echelon supply chain Two-period (Blue); four-period (Blue); d. Manufacturer's profit in Scenario SPI under two-echelon supply chain Two-period (Blue); four-period (Black); d. Manufacturer's profit in Scenario SPI under two-echelon supply chain Two-period (Blue); four-period (Black); d. Manufacturer's profit in Scenario SPI under two-echelon supply chain Two-period (Blue); four-period (Black).

$$\begin{aligned} \pi_{r31}^{3bp} &- \pi_{r31}^{3spi} = \frac{0.00165a^2 + 0.20231abh - 0.48961b^2h^2}{b} > 0 \\ \pi_{d31}^{3bp} &- \pi_{d31}^{3spi} = \frac{0.04527a^2 - 0.60893abh + 0.18461b^2h^2}{b} > 0 \quad \text{if} \quad \frac{0.07609a}{b} > h \\ \pi_{d31}^{3bp} &- \pi_{d311}^{3spi} = \frac{0.19651a^2 - 0.13486abh - 0.08524b^2h^2}{b} > 0 \end{aligned}$$

Note that the profits of the supply chain members are identical in two-period procurement scenarios. Additionally, average profits of the supply chain members decrease in four-period procurement scenario, this motivates us to keep the number of consecutive selling period upto four and develops the models under integrated procurement planning. One can observe that the supply chain members always receive higher profits in Scenario BP under two-echelon supply chain. However, the channel members may prefer Scenario SPI under three-echelon setting. The graphical representation of the profits for the supply chain members under two and three-echelon setting are shown in Figs. 6 and 7, respectively. The following parameters values are used: h = 10, $a \in (50, 500)$, and $b \in (0, 2)$.

Above figures demonstrate that the optimal decision in Scenario BP outperforms SPI for two-echelon supply chain. Retailer and manufacturer prefer Scenario SPI, not the distributor under three-echelon supply chain. However, there are implementation issues related to the two procurement policies. The supply chain members need to imply dynamic pricing policy to receive higher profits.

The findings of this research indicate that the procurement planning is creating a new bottleneck. The supply chain members are facing a dilemma due to *conventional versus integrative thinking*. In the history of operation research literature, an integrated production-inventory-distribution problem has been extensively studied to find outcomes in the long run. [3,17,20,27,32,37]. If we look at pricing behavior, the supply chain members need to decrease prices in Scenario SPI or increase in Scenario BP. Moreover, optimal profits decrease progressively. One cannot ignore the effect of price sensitivity for pragmatic planning, because price sensitivity is a critical factor in retailing [19,36]. Therefore, we need to analyze in micro-level before considering number of selling periods for those problems mainly formulated under mixedinteger programming settings. The results demonstrate that, in the presence of inventory, supply chain members can receive negative profits if the number of consecutive selling period is large, although the average profits remain high. The results also support rational decision making, because different marketing tools are commonly employed to maintain flow of products to earn profits in long run.

4. Summary and concluding remarks

Efficient procurement planning, strategic utilization of resource, and smart marketing can be a fun yet challenging way to make profit. Commonly, a convenience store retailer buys products from a manufacturer or distributor, and markets them to consumers and keeps a reasonable amount of inventory to eradicate possibility of shortages. This study explores the pricing and integrated procurement decisions in two and three-echelon supply chains. Under linear price-sensitive demand, two pragmatic integrated procurement strategies are analyzed to explore multi-period interaction among supply chain members. Recently, some researchers have discussed properties of two-period supply chain models [25,42] to explore some pragmatic business frameworks without considering impact of inventory. Comparative analysis among equilibrium outcomes from the perspective of profits of the supply chain members demonstrate how the procurement decisions in the presence of inventory are influencing the overall preference of supply chain members.

Contribution of the study to existing literature are as follows: First, two multi-period integrated procurement strategies discussed in this research can help supply chain members to make a profitable procurement planning, because both can outperform single period decision model. Without considering impact of inventory, researchers have





Fig. 5. a. Manufacturer's prices in Scenario SPI under three-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Red); b. Distributor's prices in Scenario SPI under three-echelon supply chain Two-period (Green) Three-period (Blue) Four-period (Red); c. Retailer's prices in Scenario SPI under three-echelon supply chain Two-period (Blue) Four-period (Red); d. Retailer's profit in Scenario SPI under three-echelon supply chain Two-period (Blue) Four-period (Blue) Four-period (Blue) Four-period (Green) Three-period (Blue) Four-period (Blue)

suggested that two-period models are more efficient to obtain practical decision; for example, electronic goods [30]; short-life-cycle products [40]. Our study also demonstrates that a multi-period integrated procurement decision always provides more robust decision. Second, [2,18] reported that procurement planning in presence of SI is an optimal procurement strategy for the retailer. However, we proved that the supply chain members can receive higher profits in procurement Scenario BP. Moreover, the supply chain members needs to reduce the wholesale and retail prices significantly in the presence of SI. Therefore, supply chain members can face a serious implementation issue. Third, the results suggest that the presence of the intermediary prevents the retailer from accomplishing a profitable procurement planning. The retailer's decision to mix inventory distribution, pricing, and procurement in a multi-period setting not only improves their own profits, but also those of other members. However, the retailer needs to imply a product distribution strategy based on product categories. For example, price skimming is suitable for innovative electronic products or products which have a "status-indicating quality, which is associated with the procurement Scenario SPI. Fourth, this study suggests that supply chain members, especially the retailer needs to think about the mix of demand-enhancing marketing tools such as rebate or sales effort, alone with procurement and inventory distribution policy, in anticipating profits at later selling periods. We need to revisit the outcomes of production-distribution planning problem in presence of inventory and determine the optimal number of consecutive selling periods to be considered in an integrated procurement planning.

The present analysis can be extended to incorporate several important features. For the analytical tractability, we consider four



Fig. 6. a. Retailer's profit in two-echelon supply chain, Scenarios BP (Blue) and SPI (Black);b. Manufacturer's profit in two-echelon supply chain Scenarios BP (Blue) and SPI (Black).



Fig. 7. a. Retailer's profit in three-echelon supply chain, Scenarios BP (Blue) and SPI (Black); b. Distributor's profit in three-echelon supply chain Scenarios BP (Blue) and SPI (Black); c. Manufacturer's profit in three-echelon supply chain, Scenarios BP (Blue) and SPI (Black).

consecutive selling periods and linear price dependent demand. We predict the characteristics of optimal decisions remained unchanged for a greater number of consecutive selling periods. The proposed study can be extended by considering the effect of reference-price in between two consecutive selling periods. One can explore the outcomes of two procurement decisions under retailer-Stackelberg or the Nash game or introduce factors such as sales-effort, rebate, green-sensitivity in the demand function.

Appendix A. Derivation of optimal decision in Scenario BP

The optimal solution for the retailer's four-period optimization problem is obtained by solving $\frac{d\pi_{ret}^{2bp}}{dp_{ret}^{2bj}} = 0$. On simplification, we have $p_{r44}^{2bp} = \frac{a + bp_{m44}^{2bp}}{2b}$. The profit function of the retailer in fourth-period is concave because $\frac{d^2 \pi_{r44}^{2bp}}{dp_{s}^{2bp^2}} = -2b < 0$. The optimal solution for the manufacturer's four-period optimization is obtained by solving $\frac{d\pi_{m44}^{2bp}}{dp_{m44}^{2bp}} = 0$. On simplification, one can obtain $p_{m44}^{2bp} = \frac{a - 2I_{43}^{2bp}}{2b}$. The profit function of the manufacturer in fourth-period is concave because $\frac{d^2 p_{m42}^{2bp}}{dp_{m42}^{2bp^2}} = -b < 0$. One can observe that the wholesale price is a function of I_{43}^{2bp} , therefore, we need to consider the impact of forth-period profit function to obtain optimal decision in third selling period.

Substituting the optimal response obtained in fourth-period, the profit function for the retailer in third-period is obtained as follows:

$$\pi_{r43}^{2bp} = \frac{a^2 + 12aI_{43}^{2bp} - 12I_{43}^{2bp}}{16b} + p_{r43}^{2bp}(a - bp_{r43}^{2bp}) - (a - I_{43}^{2bp} - bp_{r43}^{2bp})p_{m43}^{2bp} - hI_{43}^{2b}$$

Corresponding optimal retail price in third-period is $p_{r43}^{2bp} = \frac{a + bp_{m43}^{2bp}}{2b}$ and third-period optimization problem for the manufacturer is $\pi_{m43}^{2bp} = \frac{(a-2l_{43}^{2bp})^2 + 4bp_{m43}^{2bp}(a-2l_{43}^{2p}-bp_{m43}^{2bp})}{8b}$. One may verify that the wholesale price of the manufacturer is $p_{m43}^{2bp} = \frac{a-2l_{43}^{2bp}}{2b}$. The profit functions for the manufacturer and retailer in third-period are concave because $\frac{d^2\pi_{r43}^{2bp}}{dp_{r43}^{2bp}} = -2b < 0$ and $\frac{d^2\pi_{m43}^{2bp}}{dp_{m43}^{2bp}} = -b < 0$, respectively. Substituting the optimal response obtained in third-period, the profit function for the retailer in second-period is obtained as follows:

$$\pi_{r42}^{2bp} = \frac{a^2 + 6a(I_{42}^{2bp} + I_{43}^{2bp}) - 2(3I_{42}^{2bp^2} + I_{43}(4bh + 3I_{43}^{2bp}))}{8b} + p_{r42}^{2bp}(a - bp_{r42}^{2bp}) - (a - I_{41}^{2bp} - bp_{r42}^{2bp})p_{m42}^{2bp} - h(I_{42}^{2bp} + I_{43}^{2bp})$$

Corresponding optimal retail price is $p_{r42}^{2bp} = \frac{a + bp_{m42}^{2bp}}{2b}$ and the second-period optimization problem for the manufacturer is $\pi_{m42}^{2bp} = \frac{a^2 - 2a(I_{42}^{2bp} + I_{43}^{2bp} - bp_{m42}^{2bp}) + 2(I_{42}^{2bp^2} + I_{43}^{2bp^2} - bp_{m42}^{2bp^2} - bp_{m42}^{2bp^2} + I_{43}^{2bp^2} - bp_{m42}^{2bp^2} - bp_{m42}^{2bp^2} + I_{43}^{2bp^2} - bp_{m42}^{2bp^2} - bp_{m42}^{2b^2} - bp_{m42}^{2b^2} - bp_{$

functions for the manufacturer and retailer in second-period are concave because $\frac{d^2 \pi_{r42}^{2bp}}{dp_{r42}^{2bp^2}} = -2b < 0$ and $\frac{d^2 \pi_{r42}^{2bp}}{dp_{r42}^{2bp^2}} = -b < 0$, respectively. Substituting the optimal response obtained in second-period, the profit function for the retailer in first selling period is obtained as follows: $\pi_{r41}^{2bp} = \frac{3a^2 + 12a(l_{41}^{2bp} + l_{42}^{2bp} + l_{43}^{2bp}) - 4(3l_{41}^{2bp^2} + 4bh(l_{42}^{2bp^2} + l_{43}^{2bp^2}))}{16b}$

$$+ p_{r41}^{2bp}(a - bp_{r41}^{2bp}) - (a + I_{41}^{2bp} + I_{42}^{2bp} + I_{43}^{2bp} - bp_{r41}^{2bp})p_{m41}^{2bp} - h(I_{41}^{2bp} + I_{42}^{2bp} + I_{43}^{2bp})$$

In procurement Scenario BP, the profit function of the retailer is a function of I_{41}^{2bp} , I_{42}^{2bp} , I_{43}^{2bp} , and p_{41}^{2bp} . Therefore, optimal solution for the retailer in first-period optimization problem can be obtained by solving the following first order conditions simultaneously:

$$\frac{\frac{\partial \pi_{r41}^{2D}}{\partial p_{r41}^{2Dp}}}{\frac{\partial p_{r41}^{2Dp}}{\partial l_{p}}} = a - 2bp_{r41}^{2bp} + bp_{m41}^{2bp} = 0, \\ \frac{\frac{\partial \pi_{r41}^{2Dp}}{\partial l_{42}^{2bp}}}{\frac{\partial I_{42}^{2Dp}}{\partial l_{42}}} = 12a - 24I_{41}^{2bp} - 16b(p_{m41}^{2bp} + h) = 0 \\ \frac{\frac{\partial \pi_{r41}^{2Dp}}{\partial l_{42}}}{\frac{\partial I_{42}^{2Dp}}{\partial l_{42}}} = 12a - 4(4bh + 6I_{42}^{2bp}) - 16b(p_{m41}^{2bp} + h) = 0, \\ \frac{\frac{\partial \pi_{r41}^{2Dp}}{\partial l_{42}}}{\frac{\partial I_{42}^{2Dp}}{\partial l_{43}}} = 12a - 4(8bh + 6I_{43}^{2bp}) - 16b(p_{m41}^{2bp} + h) = 0 \\ \text{After solving we obtain:} \\ p_{r41}^{2bp} = \frac{a + bp_{m41}^{2bp}}{2b}, \\ I_{42}^{2bp} = \frac{3a - 4b(h + p_{m41}^{2bp})}{6}, \\ I_{42}^{2bp} = \frac{3a - 4b(h + p_{m41}^{2bp})}{6}, \\ I_{43}^{2bp} = \frac{3a - 4b(3h + p_{m41}^{2bp})}{6} \\ \text{We compute Hessian matrix} (H^{2bp}) \text{ as follows:}$$

$$H^{2bp} = \begin{pmatrix} \frac{\partial^2 \pi_{r41}^{2bp}}{\partial p_{r41}^{2bp^2}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial p_{r41}^{2bp} \partial l_{41}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial p_{r41}^{2bp} \partial l_{42}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial p_{r41}^{2bp} \partial l_{42}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial p_{r41}^{2bp} \partial l_{42}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial p_{r41}^{2bp} \partial l_{41}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{41}^{2bp} \partial l_{42}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{41}^{2bp} \partial l_{42}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{41}^{2bp} \partial l_{42}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial p_{r41}^{2bp} \partial l_{41}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{41}^{2bp} \partial l_{42}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{42}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{42}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial p_{r41}^{2bp} \partial l_{42}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{41}^{2bp} \partial l_{42}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{42}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial p_{r41}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{41}^{2bp} \partial l_{42}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{42}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{42}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} & \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} \\ \frac{\partial^2 \pi_{r41}^{2bp}}{\partial l_{42}^{2bp} \partial l_{43}^{2bp}} &$$

Values of principal minors of Hessian matrix (H^{2bp}) are $\Delta_4 = \frac{27}{4b^2} > 0$; $\Delta_3 = -\frac{9}{2b} < 0$; $\Delta_2 = 3 > 0$; and $\Delta_1 = -2b < 0$, respectively. Therefore, profit function for the retailer is concave. Substituting the optimal response, the profit function for the manufacturer is obtained as $\pi_{m41}^{2bp} = 2ap_{m41}^{2bp} + \frac{b(56h^2 - 24hp_{m41}^{2bp} - 33p_{m41}^{2bp^2})}{18}$. Solving the first order condition, one can obtain the wholesale price as $p_{m41}^{2bp} = \frac{2(3a - 2bh)}{11b}$. By using back substitution, we obtain the optimal decision shown in Table 1. Similarly, we derive optimal decisions under three- and two-period integrated procurement models.

The profit functions for the supply chain members in three-echelon structure are obtained as follows:

$$\begin{aligned} \pi_{r41}^{3bp} &= p_{r41}^{3bp}(a - bp_{r41}^{3bp}) - p_{d41}^{3bp}(a - bp_{r41}^{3bp} + \sum_{s=2}^{4} I_{4(s-1)}^{3bp}) - h \sum_{s=2}^{4} I_{4(s-1)}^{3bp} + \pi_{r42}^{3bp} \\ \pi_{d41}^{3bp} &= (p_{d41}^{3bp} - p_{m41}^{3bp})(a - bp_{r41}^{3bp} + \sum_{s=2}^{4} I_{4(s-1)}^{3bp}) + \pi_{d42}^{3bp} \\ \pi_{m41}^{3bp} &= p_{m41}^{3bp}(a - bp_{r41}^{3bp} + \sum_{s=2}^{4} I_{4(s-1)}^{3bp}) + \pi_{m42}^{3bp} \\ \pi_{r42}^{3bp} &= p_{r42}^{3bp}(a - bp_{r42}^{3bp}) - p_{d42}^{3bp}(a - p_{r42}^{3bp} - I_{42}^{3bp}) - h \sum_{s=3}^{4} I_{4(s-1)}^{3bp} + \pi_{r43}^{3bp} \\ \pi_{d42}^{3bp} &= (p_{d42}^{3bp} - p_{m42}^{3bp})(a - bp_{r42}^{3bp} - I_{42}^{3bp}) + \pi_{d43}^{3bp} \\ \pi_{m42}^{3bp} &= p_{m42}^{3bp}(a - p_{r42}^{3bp}) + \pi_{m43}^{3bp} \\ \pi_{m43}^{3bp} &= p_{m42}^{3bp}(a - p_{r42}^{3bp}) + \pi_{d43}^{3bp} - h I_{44}^{3bp} + \pi_{r44}^{3bp} \end{aligned}$$

 $\begin{aligned} \pi^{3bp}_{d43} &= (p^{3bp}_{d43} - p^{3bp}_{m43})(a - p^{3bp}_{r43} - I^{3bp}_{43}) + \pi^{3bp}_{d44} \\ \pi^{3bp}_{m43} &= p^{3bp}_{m43}(a - p^{3bp}_{r43} - I^{3bp}_{43}) + \pi^{3bp}_{m44}, \\ \pi^{3bp}_{a44} &= (p^{3bp}_{r44} - p^{3bp}_{r44})(a - p^{3bp}_{r44} - I^{3bp}_{44}); \\ \pi^{3bp}_{d44} &= (p^{3bp}_{d44} - p^{3bp}_{a44})(a - p^{3bp}_{r44} - I^{3bp}_{44}); \\ \pi^{3bp}_{d44} &= (p^{3bp}_{r44} - p^{3bp}_{a44})(a - p^{3bp}_{r44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{d44} &= (p^{3bp}_{r44} - I^{3bp}_{a44})(a - p^{3bp}_{r44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{d44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - p^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44})(a - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^{3bp}_{a44} - I^{3bp}_{a44} - I^{3bp}_{a44}); \\ \pi^{3bp}_{a44} &= (p^$

Appendix B. Proof of Proposition 1

If the supply chain members execute integrated procurement planning for four consecutive selling periods then the retail and wholesale prices, and amount of products distributed by the retailer, satisfy the following relations:

d amount of products distributed by the retailer, satisfy the following retainer, satisfy the following retainer $p_{r41}^{2bp} - p_{r42}^{2bp} = \frac{3a-13bh}{33b} > 0; p_{r42}^{2bp} - p_{r43}^{2bp} = p_{r43}^{2bp} - p_{r44}^{2bp} = -\frac{h}{3} < 0$ $p_{m41}^{2bp} - p_{m42}^{2bp} = \frac{2(3a-13bh)}{33b} > 0; p_{m42}^{2bp} - p_{m43}^{2bp} = p_{m43}^{2bp} - p_{m44}^{2bp} = -\frac{2h}{3} < 0$ $I_{41}^{2bp} - I_{42}^{2bp} = I_{42}^{2bp} - I_{43}^{2bp} = \frac{2bh}{3} > 0$ $p_{r41}^{3bp} - p_{r42}^{3bp} = \frac{4565a-29648bh}{59430b} > 0; p_{r42}^{3bp} - p_{r43}^{3bp} = p_{r43}^{3bp} - p_{r44}^{3bp} = -\frac{2h}{5} < 0$ $p_{441}^{3bp} - p_{422}^{3bp} = \frac{4565a-29648bh}{29715b} > 0; p_{422}^{3bp} - p_{433}^{3bp} = p_{433}^{3bp} - p_{444}^{3bp} = -\frac{4h}{5} < 0$ $p_{m41}^{3bp} - p_{m42}^{3bp} = \frac{2(90025a-764128bh)}{1872045b} > 0; p_{m42}^{3bp} - p_{m43}^{3bp} = p_{m43}^{3bp} - p_{m44}^{3bp} = -\frac{8h}{15} < 0$ $I_{41}^{3bp} - I_{42}^{3bp} = I_{42}^{3bp} - I_{43}^{3bp} = \frac{8bh}{15} > 0$

Similarly, if the supply chain members execute integrated procurement planning for three consecutive selling periods then the retail and wholesale prices, and amount of products distributed by the retailer, satisfy the following relations:

$$p_{731}^{2bp} - p_{r32}^{2bp} = \frac{21a - 112bn}{300b} > 0; \quad p_{r32}^{2bp} = -\frac{n}{3} < 0; \quad p_{m31}^{2bp} - p_{m32}^{2bp} = \frac{21a - 112bn}{150b} > 0; \\ p_{m32}^{3bp} - p_{m33}^{2bp} = \frac{-2h}{3} < 0; \quad I_{31}^{2bp} - I_{32}^{2bp} = \frac{2bh}{3} > 0 \\ p_{r31}^{3bp} - p_{r32}^{3bp} = \frac{116775a - 824672bh}{1716440b} > 0; \quad p_{r32}^{3bp} - p_{r33}^{3bp} = -\frac{2h}{5} < 0 \\ p_{331}^{3bp} - p_{d32}^{3bp} = \frac{30468825a - 33821344bh}{6504510b} > 0; \quad p_{332}^{3bp} - p_{m33}^{3bp} = -\frac{4h}{5} < 0 \\ p_{m31}^{3bp} - p_{m32}^{3bp} = \frac{30468825a - 33821344bh}{66504510b} > 0; \quad p_{m32}^{3bp} - p_{m33}^{3bp} = -\frac{8h}{15} < 0; \quad I_{31}^{2bp} - I_{32}^{2bp} = \frac{8bh}{15} > 0 \\ Finally, \text{ for procurement planning of two consecutive selling periods, the following relations are obtained: \\ p_{r21}^{2bp} - p_{r22}^{2bp} = \frac{3(a - 4bh)}{34b} > 0; \quad p_{m21}^{2bp} - p_{m22}^{2bp} = \frac{3(a - 4bh)}{17b} > 0; \quad p_{m32}^{3bp} - p_{m32}^{3bp} = -\frac{9}{1000} > 0; \\ p_{321}^{3bp} - p_{322}^{2bp} = \frac{4089a - 23360bh}{53692b} > 0; \quad p_{m32}^{3bp} - p_{m22}^{3bp} = \frac{40285a - 239808bh}{416113b} > 0; \\ p_{321}^{3bp} - p_{322}^{2bp} = \frac{4089a - 23360bh}{53692b} > 0; \quad p_{m21}^{3bp} - p_{m22}^{3bp} = \frac{40285a - 239808bh}{416113b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4089a - 23360bh}{53692b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{40285a - 239808bh}{416113b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4089a - 23360bh}{53692b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4089a - 23360bh}{53692b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4089a - 23360bh}{53692b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4089a - 23360bh}{53692b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4089a - 23360bh}{53692b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4089a - 23360bh}{53692b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4089a - 23360bh}{53692b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4089a - 23360bh}{53692b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4088a - 2360bh}{53692b} > 0; \\ P_{321}^{3bp} - P_{322}^{3bp} = \frac{4088a - 2360bh}{53692b} > 0; \\ P_{322}^{3bp} - P_{32}^{3bp} = \frac{4088a$$

Appendix C. Proof of Proposition 2

By comparing average profits for the manufacturer, we obtain following inequalities:

 $\begin{aligned} \pi_{ab}^{2p}(4 - \pi_{ab}^{2b}) &= \frac{9\pi^2 - 144abb + 664b^2h^2}{72b} &= \frac{9(a - 8bb)^2 + 88b^2h^2}{72b} > 0 \\ \pi_{ab}^{2p}(3 - \pi_{ab}^{2b}) &= \frac{27\pi^2 - 23babb + 102b^2h^2}{2700b} &= \frac{27(a - 6bb)^2 + 10b^2h^2}{2700b} > 0 \\ \pi_{ab}^{2p}(3 - \pi_{ab}^{2b}) &= \frac{(a - 4bb)^2}{12b} > 0 \\ \pi_{ab}^{2p}(3 - \pi_{ab}^{2b}) &= \frac{(a - 4bb)^2}{12b} > 0 \\ \pi_{ab}^{2p}(3 - \pi_{ab}^{2b}) &= \frac{(a - 4bb)^2}{12b} > 0 \\ \pi_{ab}^{2p}(3 - \pi_{ab}^{2b}) &= \frac{(a - 4bb)^2}{12b} > 0 \\ \pi_{ab}^{2p}(3 - \pi_{ab}^{2b}) &= \frac{23838075\pi^2 - 2061200abb + 47585728b^2h^2}{1200} > 0 \\ \pi_{ab}^{2p}(3 - \pi_{ab}^{2b}) &= \frac{23838075\pi^2 - 2061200abb + 47585728b^2h^2}{1200} > 0 \\ \pi_{ab}^{2p}(3 - \pi_{ab}^{2b}) &= \frac{23838075\pi^2 - 2061200abb + 493921088b^2h^2}{120046862b^2h^2} > 0 \\ \pi_{ab}^{2p}(3 - \pi_{ab}^{2b}) &= \frac{23838075\pi^2 - 23520abb + 1920278772b^2h^2}{1200468628b^2h^2} > 0 \\ Similarly, the following relations ensure that the average profits for the distributor always greater compere to the profit earns in Scenario BM: \\ \pi_{ab}^{2p}(3 - \pi_{ab}^{2b}) &= \frac{2477374305x^2 - 327283120b}{121046868712051a} + 10320714112b^2h^2} > 0 \text{ if } \\ 12104686812054 + 104217881212b - 121048887212051a \\ 40332728228b - 12014888712051a \\ 40337282282b - 12014888712051a \\ 4033728272828b - 12014888712051a \\ 4033728272828b - 12014888712051a \\ 403372872828b - 12014888712051a \\ 40337287282b - 12014888712051a \\ 40337287282b - 12014888712051a \\ 403378410 \\ 40337287282b - 12014888712051a \\ 4033787128 - 12014888712051a \\ 40337814 \\ 403787128 - 12014888712051a \\ 403787128 - 12014888712051a \\ 40387128 - 118928480 \\ 40337128 - 118928480 \\ 40337127814 \\ 4037814 \\ 4037814 \\ 40373147 \\ 403714 \\ 40373147 \\ 4037147 \\ 4037147 \\ 4037147 \\ 4037147$

Note that a quadratic expression $ax^2 + bx + c$ can be written as $a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$. Therefore, if a and $\Gamma = 4ac - b^2$ are both positive, then the expression as a whole is positive. For example, the values of $\Gamma = 4ac - b^2$ for the expressions $\pi_{r41}^{3bp}/4 - \pi_{r11}^{3bm}$, $\pi_{r21}^{3bp}/3 - \pi_{r11}^{3bm}$, $\pi_{r21}^{3bp}/2 - \pi_{r11}^{3bm}$ are 39582674790400, 33992604694789171200, 122903077263104, respectively. Consequently, the expressions are considered as positive as a whole. The above inequalities ensure proof.

Appendix D. Proof of Proposition 3

By comparing average profits for the retailer, one can obtain the following inequalities: $\pi_{rat}^{2b}/4 - \pi_{rst}^{2b}/3 = \frac{1250(a^{2} - 517212abh + 4095756^{2}h^{2}}{10800000} > 0 \text{ if } \min\left\{\frac{544310 - 2275\sqrt{3003}a}{247684b}, \frac{544310 + 275\sqrt{3003}a}{247684b}\right\} = \frac{0.0358169a}{b} > h$ $\pi_{rst}^{2b}/3 - \pi_{rtt}^{2b}/2 = \frac{(9a - 104b)(5396a - 70444b)}{20010000} > 0 \text{ if } \min\left\{\frac{5444a}{152b}, \frac{4a}{10a}\right\} = \frac{0.0484501a}{b} > h$ $\pi_{rtt}^{2b}/2 - \pi_{rt1}^{2b} = \frac{(21a - 152b)(a^{2} - 1074b)}{6240} > 0 \text{ if } \min\left\{\frac{121a}{152b}, \frac{a}{a}\right\} = \frac{0.135815a}{b} > h$ $\pi_{rtt}^{2b}/4 - \pi_{rtt}^{3b}/3 = \frac{-99222441518775a^{2} - 17286015423000abch + 199328140360472b^{2}h^{2}}{63579604124215460} < 0$ $\pi_{rtt}^{2b}/2 - \pi_{rt1}^{3bm} = \frac{1272495a^{2} - 35550164424800abch + 79564901179140096b^{2}h^{2}}{635796941242145460} > 0$ $\pi_{rtt}^{3b}/3 - \pi_{rt2}^{3b}/3 = \frac{-36444101097240275a^{2} + 38220188927387900abch + 5888015567383232b^{2}h^{2}}{22207795795060113800} > 0$ Similarly, by comparing average profits of the distributor, one can obtain the following inequalities: $\pi_{rtt}^{3b}/3 - \pi_{rt2}^{3b}/2 = \frac{9615975358097256a^{2} - 10282201889273879000abch + 5888015567383232b^{2}h^{2}}{2220795795595060113800}} < 0$ $\pi_{rtt}^{3b}/3 - \pi_{rt1}^{3b}/3 = \frac{-3644410109740275a^{2} + 8822018892738790020abch + 5888015567383232b^{2}h^{2}}{220795795595060113800}} < 0$ $\pi_{rtt}^{3b}/3 - \pi_{rt1}^{3b}/3 = \frac{-3644941097240275a^{2} + 8822018892738790020abch + 5888015567383232b^{2}h^{2}}{22077575595060113800} > 0$ Similarly, by comparing average profits for the manufacturer, we obtain the following inequalities: $\pi_{rtt}^{3b}/3 - \pi_{rt1}^{3b}/3 = \frac{-36449410972402756a^{2} + 10277560b^{2}h^{2}}{2207757559509021084} > 0$ finally, by comparing average profits for the manufacturer, we obtain the following inequalities: $\pi_{rtt}^{3b}/3 - \pi_{rt1}^{3b}/2 = \frac{364941027760b^{2}h^{2}}{91000} > 0$ $\pi_{rtt}^{3b}/3 - \pi_{rt1}^{3b}/3 = \frac{36494-5616b^{2}h^{2}}{91000} > 0$ $\pi_{rtt}^{3b}/3 - \pi_{rtt}^{3b}/3 = \frac{3649-5616abh + 25648b^{2}h^{2}}{90} 0$ finally, 9(

Appendix E. Proof of Proposition 4

By comparing sales volumes in procurement Scenarios BP and BM, one can obtain the differences as follows: $\begin{aligned} Q_4^{2bp}/4 - Q_1^{2bm} &= \frac{a - 8bh}{22} > 0 \\ Q_3^{2bp}/3 - Q_1^{2bm} &= \frac{a - 4bh}{25} > 0 \\ Q_2^{2bp}/2 - Q_1^{2bm} &= \frac{a - 4bh}{34} > 0 \\ Q_3^{3bp}/4 - Q_1^{3bm} &= \frac{577a - 2137bh}{11886} > 0 \\ Q_3^{3bp}/3 - Q_1^{3bm} &= \frac{3441a - 2204bh}{42911} > 0 \\ Q_2^{3bp}/2 - Q_1^{3bm} &= \frac{367a - 1132bh}{26846} > 0 \\ Similarly, the differences among average sales volumes in procurement Scenarios BP and BM are obtained as follows: <math display="block">\begin{aligned} Q_4^{2bp}/4 - Q_3^{2bp}/2 - Q_1^{3bm} &= \frac{3a - 68bh}{550} > 0 \\ Q_3^{3bp}/3 - Q_2^{2bp}/2 &= \frac{9a - 104bh}{550} > 0 \\ Q_2^{3bp}/3 - Q_2^{2bp}/2 &= \frac{9a - 104bh}{550} > 0 \\ Q_2^{3bp}/4 - Q_3^{3bp}/3 &= \frac{-16140079a - 65504063bh}{510040146} < 0 \\ Q_3^{3bp}/3 - Q_2^{3bp}/2 &= \frac{5173249a - 1059332bh}{1151988706} > 0 \\ Q_3^{3bp}/2 - Q_1^{3bm} &= \frac{867a - 1132bh}{26846} > 0 \\ D_2^{3bp}/2 - Q_1^{3bm} &= \frac{867a - 1132bh}{26846} > 0 \end{aligned}$

Appendix F. Derivation of optimal decision in Scenario SPI

The optimal solution for the retailer in fourth-period optimization problem is obtained by solving $\frac{d\pi_{t44}^{22pi}}{dp_{t44}^{22pi}} = 0$. On simplification, $p_{r44}^{2spi} = \frac{a + bp_{m44}^{2spi}}{2b}$. The profit function of the retailer in fourth-period is concave because $\frac{d^2\pi_{r44}^{2spi}}{dp_{r44}^{2spi^2}} = -2b < 0$. The optimal solution for the manufacturer in fourth-period optimization problem is obtained by solving $\frac{d\pi_{m44}^{2spi}}{dp_{m44}^{2spi}} = 0$. On simplification, $p_{m44}^{2spi} = \frac{a - 2I_{45}^{2spi}}{2b}$. The profit function of the manufacturer in fourth period is concave because $\frac{d^2\pi_{m44}^{2spi}}{dp_{m44}^{2spi}} = \frac{a - 2I_{45}^{2spi}}{2b}$. The profit function of the manufacturer in fourth period is concave because $\frac{d^2\pi_{m44}^{2spi}}{dp_{m44}^{2spi}} = -b < 0$.

Similarly, the profit function for the retailer in third-period is obtained as follows:

$$\pi_{r43}^{2spi} = \frac{a^2 + 12aI_{43}^{2spi} - 12I_{43}^{2spi}}{16b} + p_{r43}^{2spi}(a - bp_{r43}^{2spi}) - (a - I_{42}^{2spi} + I_{43}^{2spi} - bp_{r43}^{2spi})p_{m43}^{2spi} - hI_{43}^{2spi}$$

Corresponding optimal retail price and amount of SI are $p_{r43}^{2spi} = \frac{a + bp_{m43}^{2spi}}{2b}$ and $I_{43}^{2spi} = \frac{3a - 4b(h + p_{m43}^{2spi})}{6}$, respectively. Substituting optimal response, the profit function for the manufacturer in third-period is $\pi_{m43}^{2spi} = \frac{b(4h^2 - 4hp_{m43}^{2spi} - 17p_{m43}^{2spi})}{18} + (a - I_{42}^{2spi})p_{m43}^{2spi}$, and the corresponding wholesale price is $p_{m43}^{2spi} = \frac{9a - 2bh - 9I_{42}^{2spi}}{17b}$. Note that the third period optimization problem for the retailer and manufacturer are concave because $\frac{\partial^2 \pi_{r43}^{2spi}}{\partial p_{r43}^{2spi^2}} = -2b < 0$ and

$$\frac{\partial^2 \pi_{r43}^{2spi}}{\partial p_{r43}^{r_{2}spi}} \times \frac{\partial^2 \pi_{r3}^{2pi}}{\partial I_{43}^{2spi}} - \left(\frac{\partial^2 \pi_{r3}^{2spi}}{\partial I_{43}^{2spi} \partial p_{r43}^{2spi}}\right)^2 = 3 > 0; \text{ and } \frac{d^2 \pi_{m3}^{2spi}}{d p_{m43}^{2spi}} = -\frac{17b}{9} < 0.$$

The second-period profit function for the retailer is obtained as follows:
$$\pi_{r42}^{espi} = \frac{155a^2 + (76bh - 423I_{42}^{2spi})(4bh + I_{42}^{2spi}) - a(118bh - 846I_{42}^{2spi})}{a(118bh - 846I_{42}^{2spi})} + ap_{r42}^{2spi} - bp_{r42}^{2spi2} - (a - I_{41}^{2spi} + I_{42}^{2spi} - bp_{r42}^{2spi})p_{m42}^{2spi})$$

Corresponding optimal retail price and amount of SI are $p_{r42}^{2spi} = \frac{a + bp_{m42}^{2spi}}{2b}$ and $I_{42}^{2spi} = a - \frac{2b(404h + 289p_{m42}^{2spi})}{423}$. Substituting optimal response, the profit function for the manufacturer in second-period is obtained as follows:

$$p_{m42}^{2spi} = \frac{19881(3a - 2I_{41}^{2spi})p_{m42}^{2spi} + b(38824h^2 - 27400hp_{m42}^{2spi} - 54561p_{m42}^{2spi})}{39762}$$
 and the corresponding wholesale price is $p_{m42}^{2spi} = \frac{59643a - 27400h - 39762I_{41}^{2spi}}{109122b}$. Note that the

 $\frac{\frac{\partial^2 \pi_{r42}}{\partial n^{2spi^2}}}{\frac{\partial n^{2spi^2}}{\partial n^{2spi^2}}} = -2b < 0$ second-period optimization problem for the retailer and manufacturer are concave because and

$$\frac{\partial^2 \pi_{r42}^{22pi}}{\partial p_{r42}^{22pi}} \times \frac{\partial^2 \pi_{r42}^{22pi}}{\partial l_{r42}^{22pi}} - \left(\frac{\partial^2 \pi_{r42}^{22pi}}{\partial l_{r42}^{22pi} \partial p_{r42}^{22pi}}\right)^2 = \frac{423}{289} > 0; \text{ and } \frac{d^2 \pi_{m42}^{22pi}}{d u_{r42}^{22pi} d p_{r42}^{22pi}} = -\frac{18187b}{6627} < 0.$$

Finally, the first-period profit function for the retailer is obtained as follows:

 $\pi_{r41}^{2spi} = \frac{0.285445a^2 + 0.714555aI_{41}^{2spi} - 0.178639I_{41}^{2spi} - bI_{41}^{2spi}(2.1416h + p_{m41}^{2spi}) + ab(p_{r41}^{2spi} - p_{m41}^{2spi} - 0.716806h) + b^2(3.19862h^2 - p_{r41}^{2spi} - p_{m41}^{2spi}) + b^2(2.1416h + p_{m41}^{2spi}) + ab(p_{r41}^{2spi} - p_{m41}^{2spi} - 0.716806h) + b^2(3.19862h^2 - p_{r41}^{2spi} - p_{m41}^{2spi}) + b^2(2.1416h + p_{m41}^{2spi}) + ab(p_{r41}^{2spi} - p_{m41}^{2spi} - 0.716806h) + b^2(3.19862h^2 - p_{r41}^{2spi} - p_{m41}^{2spi}) + b^2(2.1416h + p_{m41}^{2spi}) + b^2(2.1416h + p_{m41}^{2spi} - p_{m41}^{2spi} - 0.716806h) + b^2(3.19862h^2 - p_{r41}^{2spi} - p_{m41}^{2spi}) + b^2(2.1416h + p_{m41}^{2spi} - p_{m41}^{2spi} - 0.716806h) + b^2(3.19862h^2 - p_{r41}^{2spi} - p_{m41}^{2spi} - p_{m41}^{2spi} - 0.716806h) + b^2(3.19862h^2 - p_{r41}^{2spi} - p_{m41}^{2spi} - p_{m41}^{2spi} - 0.716806h) + b^2(3.19862h^2 - p_{r41}^{2spi} - p_{m41}^{2spi} - p_{m41}^{2spi}$

In contrast to procurement Scenario BP, the retailer needs to determine price-inventory pair for each selling period. Although profit function of each member depends of profit function of previous period. Corresponding optimal retail price and amount of SI are $p_{r41}^{2spi} = \frac{a + b p_{m41}^{2spi}}{2b}$ and $I_{41}^{2spi} = 2a - 5.99421bh - 2.79895bp_{m41}^{2spi}$. Substituting optimal response for the retailer, the profit function for the manufacturer in first-period is obtained as follows:

 $\pi_{m41}^{2spi} = 2.5ap_{m41}^{2spi} + b(5.54404h^2 - 2.41898hp_{m41}^{2spi} - 2.20576p_{m41}^{2spi^2})$ and the corresponding wholesale price is $p_{m41}^{2spi} = \frac{0.566697a - 0.548333bh}{h}$. Note that the $\frac{\frac{\partial^2 \pi_{r41}^{2spi}}{\partial p_{r41}^{2spi^2}} = -2b < 0$ optimization and first-period problem the manufacturer because for retailer and concave are

$$\frac{\partial^2 \pi_{r41}^{2spi}}{\partial p_{r41}^{2spi}} \times \frac{\partial^2 \pi_{r41}^{2spi}}{\partial l_{41}^{2spi}} - \left(\frac{\partial^2 \pi_{r41}^{2spi}}{\partial l_{41}^{2spi} \partial p_{r41}^{2spi}}\right)^2 = 0.714555 > 0; \text{ and } \frac{d^2 \pi_{m41}^{2spi}}{d p_{m41}^{2spi}} = -4.41153b < 0.$$

By using back substitution one may obtain the optimal decision shown in Table 3.

follows: three-echelon are obtained as

By using back substitution one may obtain the optimal decision shown in Table 3. The profit functions for the supply chain members in three $\pi_{r41}^{3spi} = p_{r41}^{3spi}(a - bp_{r41}^{3spi}) - p_{l41}^{3spi}(a - bp_{r41}^{3spi}) - h_{l41}^{3spi} + \pi_{r42}^{3spi}$ $\pi_{r41}^{3spi} = p_{r41}^{3spi}(a - bp_{r41}^{3spi}) - p_{r41}^{3spi} + I_{41}^{3spi}) - h_{l41}^{3spi} + \pi_{r42}^{3spi}$ $\pi_{r42}^{3spi} = p_{r42}^{3spi}(a - bp_{r42}^{3spi}) - p_{r42}^{3spi} + I_{41}^{3spi}) + \pi_{r42}^{3spi}$ $\pi_{r42}^{3spi} = p_{r42}^{3spi}(a - bp_{r42}^{3spi}) - p_{r42}^{3spi}(a - bp_{r42}^{3spi} - I_{41}^{3spi}) + \pi_{d42}^{3spi}$ $\pi_{r42}^{3spi} = p_{r42}^{3spi}(a - bp_{r42}^{3spi}) - p_{r42}^{3spi}(a - bp_{r42}^{3spi} - I_{41}^{3spi}) + \pi_{d42}^{3spi}$ $\pi_{r42}^{3spi} = p_{r42}^{3spi}(a - bp_{r42}^{3spi} - I_{41}^{3spi} + I_{42}^{3spi}) + \pi_{d42}^{3spi}$ $\pi_{d42}^{3spi} = p_{r42}^{3spi}(a - bp_{r42}^{3spi} - I_{41}^{3spi} + I_{42}^{3spi}) + \pi_{d42}^{3spi}$ $\pi_{d42}^{3spi} = p_{r42}^{3spi}(a - bp_{r42}^{3spi} - I_{41}^{3spi} + I_{42}^{3spi}) + \pi_{d42}^{3spi}$ $\pi_{d42}^{3spi} = p_{r42}^{3spi}(a - bp_{r42}^{3spi} - I_{41}^{3spi} + I_{42}^{3spi}) + \pi_{d42}^{3spi}$ $\pi_{d43}^{3spi} = p_{r43}^{3spi}(a - bp_{r43}^{3spi} - I_{41}^{3spi} + I_{42}^{3spi}) + \pi_{d43}^{3spi}$ $\pi_{d43}^{3spi} = p_{r43}^{3spi}(a - bp_{r43}^{3spi} - I_{41}^{3spi} + I_{42}^{3spi}) + \pi_{d44}^{3spi}$ $\pi_{d43}^{3spi} = p_{r43}^{3spi}(a - bp_{r44}^{3spi} - I_{42}^{3spi} + I_{43}^{3spi}) + \pi_{d44}^{3spi}(a - bp_{r44}^{3spi} - I_{43}^{3spi} - I_{42}^{3spi} + I_{43}^{3spi}) + \pi_{d44}^{3spi}$ $\pi_{d44}^{3spi} = p_{r43}^{3spi}(a - bp_{r44}^{3spi} - I_{43}^{3spi}) + \pi_{m44}^{3spi}, \pi_{r44}^{3spi}(a - bp_{r44}^{3spi} - I_{43}^{3spi})$ $\pi_{d44}^{3spi} = (p_{d44}^{3spi} - p_{m44}^{3spi})(a - bp_{r44}^{3spi}); \pi_{33}^{3spi} = p_{m44}^{3spi}(a - bp_{r44}^{3spi} - I_{43}^{3spi}); \pi_{33}^{3spi} = p_{344}^{3spi}(a - bp_{r44}^{3spi} - I_{43}^{3spi}); \pi_{33}^{3spi} = p_{344}^{3spi}(a - bp_{r44}^{3spi} - I_{43}^{3spi})$

Appendix G. Proof of Proposition 5

If the supply chain members execute integrated procurement planning upto four consecutive selling periods, then the retail and wholesale prices, and amount of SI, satisfy the following relations:

$$\begin{array}{l} p_{r41}^{2spi} - p_{r42}^{2spi} = \frac{0.06764a - 0.59529bh}{b} > 0; \ p_{r42}^{2spi} - p_{r43}^{2spi} = \frac{0.05848a - 0.33685bh}{b} > 0; \\ p_{r43}^{2spi} - p_{r44}^{2spi} = \frac{0.05099a - 0.08854bh}{b} > 0; \ p_{m41}^{2spi} - p_{m42}^{2spi} = \frac{0.13528a - 1.19059bh}{b} > 0; \\ p_{m42}^{2spi} - p_{m43}^{2spi} = \frac{0.11697a - 0.67371bh}{b} > 0; \ p_{m43}^{2spi} - p_{m44}^{2spi} = \frac{0.10197a - 0.17708bh}{b} > 0; \\ l_{41}^{2spi} - l_{42}^{2spi} = -0.08271a + 0.12552bh < 0; \ and \ l_{42}^{2spi} - l_{43}^{2spi} = 0.12611a - 1.35071bh > 0; \\ p_{r41}^{3si} - p_{r42}^{3si} = \frac{0.061365a - 0.55477bh}{b} > 0; \ p_{r42}^{3si} - p_{r43}^{3si} = \frac{0.05736a - 0.43056bh}{b} > 0; \\ p_{r43}^{3si} - p_{r44}^{3si} = \frac{0.05385a - 0.30881bh}{b} > 0; \ p_{341}^{3si} - p_{342}^{3si} = \frac{0.1273a - 1.10954bh}{b} > 0 \\ p_{a41}^{3si} - p_{a42}^{3si} = \frac{0.11473a - 0.86113bh}{b} > 0; \ p_{d41}^{3bp} - p_{d44}^{3si} = \frac{0.10772a - 0.61759bh}{b} > 0 \\ p_{a41}^{3si} - p_{a42}^{3si} = \frac{0.07843a - 0.72208bh}{b} > 0; \ p_{a43}^{3si} - p_{m43}^{3si} = \frac{0.07321a - 0.5618bh}{b} > 0 \\ p_{a41}^{3si} - p_{a42}^{3si} = \frac{0.07843a - 0.72208bh}{b} > 0; \ p_{a42}^{3si} - p_{a43}^{3si} = \frac{0.07321a - 0.5618bh}{b} > 0 \\ p_{a41}^{3si} - p_{a42}^{3si} = \frac{0.06846a - 0.41578bh}{b} > 0 \\ p_{a41}^{3si} - p_{a42}^{3si} = \frac{0.06846a - 0.41578bh}{b} > 0 \\ p_{a41}^{3si} - p_{a42}^{3si} = 0.06576a + 0.18801bh < 0; \ l_{431}^{3si} - l_{433}^{3si} = 0.08011a - 0.63835bh > 0 \\ p_{a41}^{3si} - p_{a44}^{3si} = 0.06576a + 0.18801bh < 0; \ l_{431}^{3si} - l_{433}^{3si} = 0.08011a - 0.63835bh > 0 \\ p_{a41}^{3si} - p_{a42}^{3si} = 0.06576a + 0.18801bh < 0; \ l_{432}^{3si} - l_{433}^{3si} = 0.08011a - 0.63835bh > 0 \\ p_{a41}^{3si} - l_{431}^{3si} = -0.06576a + 0.18801bh < 0; \ l_{432}^{3si} - l_{433}^{3si} = 0.08011a - 0.63835bh > 0 \\ p_{a41}^{3si} - l_{431}^{3si} = 0.08011a - 0.63835bh > 0 \\ p_{a41}^{3si} - l_{431}^{3si} = 0.08011a - 0.63835bh > 0 \\ p_{a41}^{3si} - l_{431}^{3si} = 0.08011a - 0.63835bh > 0 \\ p_{a41}^{3si} - l_{431}^{3si} = 0.08011a - 0.63835bh > 0 \\ p_{a41$$

If the supply chain members execute integrated procurement planning upto three consecutive selling periods, then the retail and wholesale prices, and amount of SI, satisfy the following relations:

$$\begin{array}{l} p_{r31}^{2spi} & -p_{r32}^{2spi} = \frac{47(27a-172bh)}{1678bb} > 0; \ p_{r32}^{2spi} - p_{r33}^{2spi} = \frac{21573a-70276bh}{32736bb} > 0 \\ p_{m13}^{2spi} & -p_{m23}^{2spi} = \frac{47(27a-172bh)}{8394b} > 0; \ p_{m23}^{2spi} - p_{m33}^{2spi} = \frac{21573a-70276bh}{163683b} > 0 \\ I_{31}^{3spi} & I_{32}^{2spi} = \frac{7(2349a-59732bh)}{982098} > 0 \\ p_{r31}^{3spi} & -p_{r32}^{3spi} = \frac{0.06759a-0.49682bh}{b} > 0; \ p_{r32}^{3spi} - p_{r33}^{3spi} = \frac{0.06346a-0.37099bh}{b} > 0 \\ p_{a31}^{3spi} & -p_{a32}^{3spi} = \frac{0.13521a-0.99363bh}{b} > 0; \ p_{a32}^{3spi} - p_{a33}^{3spi} = \frac{0.12692a-0.74199bh}{b} > 0 \\ p_{a31}^{3spi} & -p_{a32}^{3spi} = \frac{0.08627a-0.65072bh}{b} > 0; \ p_{m32}^{3spi} - p_{m33}^{3spi} = \frac{0.08067a-0.49484bh}{b} > 0 \\ I_{31}^{3spi} & -I_{32}^{3spi} = 0.00517a-0.15339bh > 0 \end{array}$$

The above inequalities ensure proof.

Appendix H. Proof of Proposition 6

The following relations ensure that the average profits of the manufacturer always greater compere to the profit earned by the manufacturer in Scenario BM:

$$\begin{aligned} \pi_{231}^{23p1/4} - \pi_{211}^{23m1} &= \frac{0.01434a^2 - 0.01777abh + 0.72529b^2h^2}{b} > 0 \\ \pi_{231}^{23p1/3} - \pi_{211}^{23m1} &= \frac{(a - 4bh)^2}{136b} > 0 \\ \pi_{231}^{23p1/2} - \pi_{2111}^{23m1} &= \frac{(a - 4bh)^2}{136b} > 0 \\ \pi_{231}^{32p1/2} - \pi_{2111}^{33m1} &= \frac{0.02471a^2 - 0.02239abh + 0.25145b^2h^2}{b} > 0 \\ \pi_{231}^{32p1/2} - \pi_{2111}^{33m1} &= \frac{0.002471a^2 - 0.02239abh + 0.25145b^2h^2}{b} > 0 \\ \pi_{2311}^{32p1/2} - \pi_{3111}^{33m1} &= \frac{0.002471a^2 - 0.0236bb^2h^2}{b} > 0 \\ \text{Similarly, the following relations ensure that the average profits for the distributor always greater compere to the profit earned in Scenario BM: \\ \pi_{241}^{32p1/2} - \pi_{3111}^{33m1} &= \frac{0.0144a^2 - 0.06788abh + 0.06015b^2h^2}{b} > 0 \\ \pi_{241}^{32p1/2} - \pi_{3111}^{33m1} &= \frac{0.0144a^2 - 0.06788abh + 0.06015b^2h^2}{b} > 0 \\ \pi_{341}^{32p1/2} - \pi_{4111}^{33m1} &= \frac{0.0111a^2 - 0.0403abh + 0.06015b^2h^2}{b} > 0 \\ \pi_{431}^{32p1/2} - \pi_{4111}^{33m1} &= \frac{0.0111a^2 - 0.0403abh + 0.06015b^2h^2}{b} > 0 \\ \pi_{431}^{32p1/2} - \pi_{4111}^{33m1} &= \frac{0.0011a^2 - 0.01792abh + 0.06015b^2h^2}{b} > 0 \\ \pi_{431}^{32p1/2} - \pi_{4111}^{3m1} &= \frac{0.0011a^2 - 0.0403abh + 0.06015b^2h^2}{b} > 0 \\ \pi_{431}^{32p1/2} - \pi_{4111}^{3m1} &= \frac{0.0011a^2 - 0.0403abh + 0.06015b^2h^2}{b} > 0 \\ \pi_{431}^{32p1/2} - \pi_{4111}^{3m1} &= \frac{0.0011a^2 - 0.0403abh + 0.06015b^2h^2}{b} > 0 \\ \pi_{431}^{3p1/4} - \pi_{4111}^{2m1} &= \frac{0.0088a^2 - 0.01721abh + 0.020760b^2h^2}{b} > 0 \\ \pi_{431}^{3p1/4} - \pi_{4111}^{3m1} &= \frac{0.0074a^2 - 0.01721abh + 0.020760b^2h^2}{b} > 0 \\ \pi_{431}^{3p1/4} - \pi_{4111}^{3m1} &= \frac{0.0074a^2 - 0.01323abh + 0.0324b^2h^2}{b} > 0 \\ \pi_{431}^{3p1/4} - \pi_{411}^{3m1} &= \frac{0.0074a^2 - 0.01323abh + 0.0324b^2h^2}{b} > 0 \\ \pi_{431}^{3p1/4} - \pi_{411}^{3m1} &= \frac{0.0074a^2 - 0.0137abh + 0.38322b^2h^2}{b} > h \\ \pi_{431}^{3p1/4} - \pi_{411}^{3m1} &= \frac{0.0074a^2 - 0.0317abh + 0.38322b^2h^2}{b} > 0 \\ \pi_{431}^{3p1/4} - \pi_{411}^{3m1} &= \frac{0.0074a^2 - 0.0317abh + 0.38322b^2h^2}{b} > 0 \\ \pi_{431}^{3p1/4} - \pi_{411}^{3m1} &= \frac{0.0077a^2 - 0.03277abh + 0.38322b^2h^2}{b} > 0 \\ \pi_{431}^{3p1/4} - \pi_{$$

Appendix I. Proof of Proposition 7

By comparing average profits of the retailer, we obtain the following inequalities: $\pi_{r41}^{2spl}/4 - \pi_{r31}^{2spl}/3 = \frac{0.00172a^2 - 0.06738cdh + 0.04722b^2h^2}{b} > 0 \text{ if } \frac{0.03148a}{0.03148a} > h$ $\pi_{r31}^{2spl}/3 - \pi_{r21}^{2spl}/2 = \frac{0.00261a^2 - 0.06078cdh + 0.26095b^2h^2}{37160330911408b} > 0 \text{ if } \frac{0.05654a}{b} > h$ $\pi_{r21}^{2spl}/2 - \pi_{r11}^{2hm} = \frac{(21a - 152bh)(a - 40h)}{4624b} > 0 \text{ if } \frac{0.138158a}{b} > h$ $\pi_{r31}^{2spl}/3 - \pi_{r31}^{3pl}/3 = \frac{0.00172a^2 - 0.0158cdh + 0.18524b^2h^2}{4624b} > 0$ $\pi_{r31}^{3spl}/3 - \pi_{r11}^{3pl}/3 = \frac{0.00235a^2 - 0.0158cdh + 0.12641b^2h^2}{371978176b} > 0$ $\pi_{r21}^{3spl}/2 - \pi_{r11}^{3bm} = \frac{1272495a^2 - 3553616abh + 26627072b^2h^2}{371978176b} > 0$ Similarly, by comparing average profits of the distributor and manufacturer, we obtain the following inequalities: $\pi_{d41}^{3spl}/4 - \pi_{d31}^{3spl}/3 = \frac{0.00329a^2 - 0.0235abh + 0.0073b^2h^2}{371978176b} > 0$ $\pi_{331}^{3spl}/3 - \pi_{d31}^{3spl}/2 = \frac{0.00451a^2 - 0.0235abh + 0.0073b^2h^2}{b} > 0$ $\pi_{d31}^{3spl}/3 - \pi_{d31}^{3spl}/2 = \frac{0.00451a^2 - 0.02355abh + 0.04234b^2h^2}{b} > 0$ $\pi_{d31}^{3spl}/3 - \pi_{d31}^{3spl}/3 = \frac{0.00239a^2 - 0.02355abh + 0.04234b^2h^2}{b} > 0$ $\pi_{d31}^{3spl}/3 - \pi_{d31}^{3spl}/3 = \frac{0.00239a^2 - 0.02355abh + 0.04234b^2h^2}{b} > 0$ $\pi_{d31}^{3spl}/3 - \pi_{d31}^{3spl}/3 = \frac{0.00239a^2 - 0.02355abh + 0.04234b^2h^2}{b} > 0$ $\pi_{d31}^{3spl}/3 - \pi_{d31}^{3spl}/3 = \frac{0.00239a^2 - 0.02355abh + 0.04234b^2h^2}{b} > 0$ $\pi_{d31}^{3spl}/3 - \pi_{d31}^{3spl}/3 = \frac{0.00239a^2 - 0.02355abh + 0.04234b^2h^2}{b} > 0$ $\pi_{d31}^{3spl}/3 - \pi_{d31}^{3spl}/3 = \frac{0.00239a^2 - 0.02355abh + 0.0378b^2h^2}{b} > 0$ $\pi_{m31}^{3spl}/3 - \pi_{m31}^{3spl}/3 = \frac{0.00258a^2 - 0.04335abh + 0.13678b^2h^2}{b} > 0$ $\pi_{m31}^{3spl}/3 - \pi_{m31}^{2spl}/3 = \frac{0.00538a^2 - 0.0433abh + 1.03678b^2h^2}{200347992b} > 0$ $\pi_{m31}^{3spl}/3 - \pi_{m31}^{3spl}/3 = \frac{0.00538a^2 - 0.0433abh + 0.13678b^2h^2}{b} > 0$ $\pi_{m31}^{3spl}/3 - \pi_{m31}^{3spl}/3 = \frac{0.00538a^2 - 0.0433abh + 0.0367b^2h^2}{b} > 0$ $\pi_{m31}^{3spl}/3 - \pi_{$

Appendix J. Proof of Proposition 8

By comparing sales volume in Scenarios SPI and BM, the following inequalities are obtained: $Q_4^{2spi}/4 - Q_1^{2bm} = 0.06364a - 0.43927bh > 0$ if $\frac{0.144873a}{b} > h$ $Q_3^{3spi}/3 - Q_1^{2bm} = 0.049074a - 0.26703bh > 0$ if $\frac{0.133774a}{b} > h$ $Q_2^{2spi}/2 - Q_1^{2bm} = \frac{a-4bh}{34} > 0$ if $\frac{a}{4b} > h$ $Q_4^{4spi}/4 - Q_1^{3bm} = 0.07519a - 0.17923bh > 0$ if $\frac{0.419541a}{b} > h$ $Q_3^{3spi}/3 - Q_1^{3bm} = 0.05632a - 0.10309bh > 0$ if $\frac{0.546332a}{b} > h$

 $Q_2^{2spi}/2 - Q_1^{3bm} = \frac{867a - 1132bh}{26846} > 0$ if $\frac{867a}{1132b} > h$ Therefore, sales volume in Scenarios SPI will be greater compared to sales volume in BM if $\frac{0.144873a}{b} > h$. Similarly, the difference among average sales volumes under different scenarios are:

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.orp.2018.11.003.

References

- [1] Martínez-de Albéniz V, Simchi-Levi D. Supplier-buyer negotiation games: Equilibrium conditions and supply chain efficiency. Production and Operations Management 2013;22(2):397-409.
- Anand K, Anupindi R, Bassok Y. Strategic inventories in vertical contracts. [2] Management Science 2008;54(10):1792-804
- [3] Ancarani A, Mauro CD, DUrso D. Measuring overconfidence in inventory management decisions. Journal of Purchasing and Supply Management 2016;22(3):171-80.
- [4] Arcelus FJ, Kumar S, Srinivasan G. Risk tolerance and a retailer pricing and ordering policies within a newsvendor framework. Omega 2012;40(2):188-98.
- [5] Arya A, Frimor H, Mittendorf B. Decentralized procurement in light of strategic inventories. Management Science 2014;61(3):578-85.
- [6] Arya A, Mittendorf B. Managing strategic inventories via manufacturer-to-consumer rebates. Management Science 2013;59(4):813-8.
- [7] Balakrishnan A, Natarajan HP. Integrated procurement planning in multi-division firms. Production and Operations Management 2014;23(10):1795-810.
- Balcik B, Bozkir CD, Kundakcioglu OE. A literature review on inventory manage-[8] ment in humanitarian supply chains. Surveys in Operations Research and Management Science 2016;21(2):101–16.
- Benaning, M. N., (2014). Indon agri supply-chain problems similar to those of PHL. [9] https://businessmirror.com.ph/indon-agri-supply-chain-problems-similar-to-thoseof-phl/.
- [10] Boulaksil Y. Safety stock placement in supply chains with demand forecast updates. Operations Research Perspectives 2016:3:27-31.
- Darvish M, Coelho L. Sequential versus integrated optimization: Production, loca-[11] tion, inventory control, and distribution. European Journal of Operational Research 2018;268:203-14.
- Dey K, Roy S, Saha S. The impact of strategic inventory and procurement strategies [12] on green product design in a two-period supply chain. International Journal of Production Research 2018. https://doi.org/10.1080/00207543.2018.1511071.
- Dev K. Saha S. Influence of procurement decisions in two-period green supply [13] chain. Journal of Cleaner Production 2018:190:388-402.
- [14] Federgruen A, Heching A. Combined pricing and inventory control under uncertainty. Operations Research 1999;47:454-75.
- Fumero F, Vercellis C. Synchronized development of production, inventory, and [15] distribution schedules. Transportation Science 1999;33(3):257-340.
- [16] Guan H, Gurnani H, Geng X, Luo Y. Strategic inventory and supplier encroachment. Manufacturing and Service Operations Management 2018. https://doi.org/10. 1287/msom.2018.0705.
- Haq AN, Vrat P, Kanda A. An integrated production-inventory-distribution model [17] for manufacture of urea: a case. International Journal of Production Economics 1991:25(1-3):39-49.
- [18] Hartwig R, Inderfurth K, Sadrieh A, Voigt G. Strategic inventory and supply chain behavior. Production and Operations Management 2015;24(8):1329-45.
- [19] Huang A, Dawes A, Lockshin L, Greenacre L. Consumer response to price changes in higher-priced brands. Journal of Retailing and Consumer Services 2017;39:1-10.
- [20] Kaur H, Singh SP. Heuristic modeling for sustainable procurement and logistics in a supply chain using big data. Computers and Operations Research 2018.98(2018).301-21
- [21] Keskinocak P, Chivatxaranukul K, Griffin PM. Strategic inventory in capacitated supply chain procurement. Managerial and Decision Economics 2008;29(1):23-36.
- [22] Kienzler M, Kowalkowski C. Pricing strategy: A review of 22 years of marketing research. Journal of Business Research 2017;78:101-10.
- [23] Lee CY, Lee D. An efficient method for solving a correlated multi-item inventory

system. Operations Research Perspectives 2018;5:13-21.

- [24] Liu W, Song S, Li B, Wu C. A periodic review inventory model with loss-averse retailer, random supply capacity and demand. International Journal of Production Research 2015;53(12):3623-34.
- [25] Maiti T, Giri BC. Two-period pricing and decision strategies in a two-echelon supply chain under price-dependent demand. Applied Mathematical Modelling 2017:42:655-74.
- [26] Moon I, Dey K, Saha S. Strategic inventory: manufacturer vs. retailer investment. Transportation Research Part E: Logistics and Transportation Review 2018.109.63-82
- Moon I, Jeong YJ, Saha S. Fuzzy bi-objective production-distribution planning [27] problem under the carbon emission constraint. Sustainability 2016:8(8):798.
- [28] Moon I, Sarmah SP, Saha S. The impact of online sales on centralised and decentralised dual-channel supply chains. European Journal of Industrial Engineering 2018.12(1).67-92
- Mou S. Robb DJ. DeHoratius N. Retail store operations: Literature review and re-[29] search directions. European Journal of Operational Research 2018;265:399-422.
- [30] Pan K, Lai KK, Liang L, Leung SCH. Two-period pricing and ordering policy for the dominant retailer in a two-echelon supply chain with demand uncertainty. Omega 2009:37:919-29
- [31] Reardon T, Timmer CP, Berdegué J. Supermarket expansion in latin america and asia: implications for food marketing systems. In: Regmi A, Gehlhar M, editors. New directions in global food markets, agriculture information bulletin number 794. United States Department of Agriculture; 2005. p. 47-61.
- [32] Safaei AS, Moattar Husseini SM, Z. Farahani R, Jolai F, Ghodsypour SH. Integrated multi-site production-distribution planning in supply chain by hybrid modeling. International Journal of Production Research 2010;48(14):4043-69.
- [33] Saha S, Nielsen I, Moon I. Optimal retailer investments in green operations and preservation technology for deteriorating items. Journal of Cleaner Production 2018:140:1514-27.
- [34] Srivathsan S, Kamath M. Understanding the value of upstream inventory information sharing in supply chain networks. Applied Mathematical Modelling 2018:54:393-412.
- Transchel S, Minner S. The impact of dynamic pricing on the economic order de-[35] cision. European Journal of Operational Research 2009;198(3):773-89.
- [36] Walters RG, Bommer W. Measuring the impact of product and promotion-related factors on product category price elasticities. Journal of Business Research 1996;36:203-16.
- Wichapa N, Khokhajaikiat P. Solving multi-objective facility location problem using [37] the fuzzy analytical hierarchy process and goal programming: a case study on infectious waste disposal centers. Operations Research Perspectives 2017;4:39-48.
- [38] Wu S, Liu Q, Zhang RQ. The reference effects on a retailer dynamic pricing and inventory strategies with strategic consumers. Operations Research 2015;63(6):1320-35.
- [39] Xiao G, Yang N, Zhang R. Dynamic pricing and inventory management under fluctuating procurement costs. Manufacturing and Service Operations Management 2015:17(3):321-34.
- Yang H, Zhuo W, Zha Y, Wan H. Two-period supply chain with flexible trade credit [40] contract. Expert Systems With Applications 2016;66:95-105.
- [41] Zhang F. Procurement mechanism design in a two-echelon inventory system with price-sensitive demand. Manufacturing and Service Operations Management 2010;12(4):608-26.
- [42] Zhang S, Zhang J. Contract preference with stochastic cost learning in a two-period supply chain under asymmetric information. International Journal of Production Economics 2018;196:226-47.