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Parameter Estimation for Stochastic Channel Models using Temporal Moments

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***Abstract*—This paper proposes a method to infer on the parameters of a stochastic channel model from observations of temporal moments without multipath extraction. The distribution of the temporal moments is approximated to be Gaussian, and sampling is carried out from the approximate posterior. The temporal moments are found to be informative about the model parameters, as the parameters can be recovered from the samples.**

I. Introduction

Parameters of stochastic multipath models, since the early works in [1] and [2], have predominantly been estimated by first extracting multipath parameters (delays, gains, etc.) and then estimating model parameters in a second step. Multipath extraction requires sophisticated algorithms which can be cumbersome to use and prone to errors [3]. In statistical terms, the multipath parameters are used as summary statistics for estimating model parameters. Other summaries, e.g. the well-known temporal moments of the received signal, can potentially be used, thereby avoiding multipath extraction altogether. Here, we propose a sampling method to estimate parameters of a stochastic multipath model based on temporal moments.

II. Signal Model

Ignoring additive noise, the received signal in a multipath channel can be written in complex baseband notation as

$$y(t) = \sum_{l} \alpha_l s(t - \tau_l), \tag{1}$$

where $s(t)$ is the transmitted signal, α_l and τ_l are the complex gain and time-delay of the l^{th} multipath component, respectively. The k^{th} temporal moment of $y(t)$ is defined as

$$m_k = \int t^k |y(t)|^2 \mathrm{dt}, \quad k = 0, 1, 2, \dots \tag{2}$$

Under the large bandwidth approximation, $|s(t)|^2 \to \delta(t)$, and the temporal moment reads

$$m_k = \sum_{l} |\alpha_l|^2 \tau_l^k, \quad k = 0, 1, 2, \dots \tag{3}$$

Here we consider a variant of Turin's model [1] where delays and gains form a homogeneous Poisson point process with arrival rate λ_0. The mark density $p(\alpha|\tau)$ is circular complex Gaussian with variance $\sigma_\alpha^2(\tau)$. For this model the power delay spectrum reads $P(\tau) = \lambda_0 \sigma_\alpha^2(\tau)$, see [4]. For

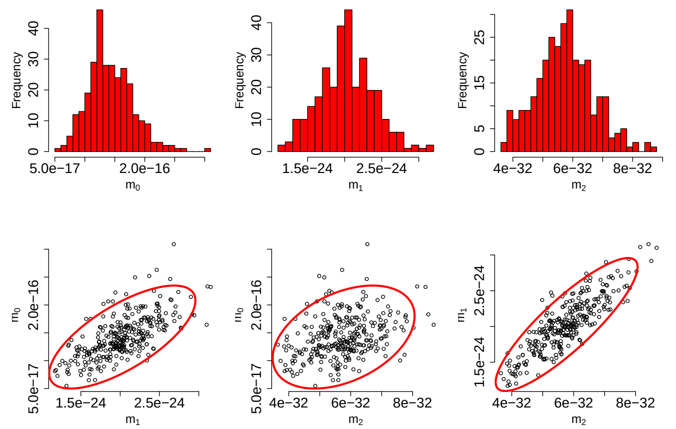

Fig. 1. Scatter plots and histograms of synthetic data of $\mathbf{m}_0$, $\mathbf{m}_1$, and $\mathbf{m}_2$. Parameter settings: $g = 0.6$, $G_0 = 10^{-8}$, $\lambda_0 = 10^9$ s^{-1}, $N = 300$, $V = 36$ m^3, $S = 66$ m^2, $c = 3 \times 10^8$ ms^{-1}, and $\tau_{\text{max}} = 200$ ns. Red ellipses are 95% probability contours of a Gaussian with parameters given by (5).

in-room scenarios, the power delay spectrum is well modelled by the reverberation model as [5]

$$P(\tau) = \begin{cases} G_0 \exp(-\frac{\tau}{T}), & \tau > 0 \\ 0 & \text{otherwise}, \end{cases} \tag{4}$$

where G_0 is the reverberant power at delay zero, and $T = -4V/cS\ln(g)$ is the reverberation time, V is the volume, S is the surface area, c is the speed of light, and g is the reflection coefficient of the room. Fig. 1 shows an example realisation drawn from the model.

The mean vector, μ, and the covariance matrix, Σ, of the first three temporal moments i.e. m_0, m_1, and m_2, can be found by invoking Campbell's theorem,

$$\mu = G_0 \begin{bmatrix} T \\ T^2 \\ 2T^3 \end{bmatrix}, \text{ and } \quad \Sigma = \frac{G_0^2}{\lambda_0} \begin{bmatrix} T & \frac{T^2}{2} & \frac{T^3}{2} \\ \frac{T^2}{2} & \frac{T^3}{2} & \frac{3T^4}{4} \\ \frac{T^3}{2} & \frac{3T^4}{4} & \frac{3T^5}{2} \end{bmatrix}. \tag{5}$$

III. Estimation Method

Let $\mathbf{m}_k$ be the N-dimensional vector of the k^{th} temporal moment, where $k = 0, 1, 2$. Samples from the posterior, $p(\Theta|\mathbf{m}_k)$, can be used to infer on the model parameters, $\Theta = [g, G_0, \lambda_0]^T$. Since the posterior and the likelihood, $p(\mathbf{m}_k|\Theta)$, are numerically unavailable, sampling is not

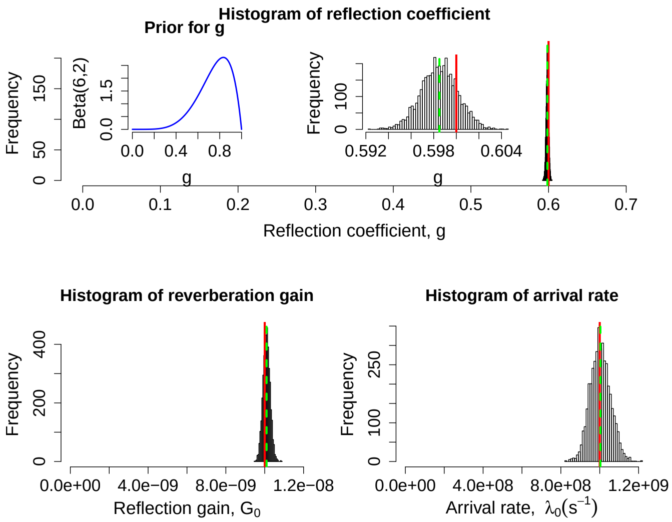

Fig. 2. Histogram of the approximate posterior samples obtained from the data in Fig. 1. A zoomed version of the histogram of g is inserted in the same plot. Red line: true value, dashed green line: MMSE estimate. Inset plot in blue represents the prior distribution, of g.

possible. However, inspired by Fig. 1, we approximate the likelihood as a Gaussian $\tilde{p}(\mathbf{m}_k|\Theta)$ with mean and covariance as in (5). Then we sample from the approximate posterior, $\tilde{p}(\Theta|\mathbf{m}_k) = \tilde{p}(\mathbf{m}_k|\Theta)p(\Theta)/p(\mathbf{m}_k)$, by using standard sampling techniques. Point estimates can then be obtained, e.g. averaging the posterior samples yield the minimum mean squared error (MMSE) estimate.

IV. Simulation and Results

We run the default sampler in [6] on synthetic data from Fig. 1 using a Beta prior for g and flat priors for G_0 and λ_0 to obtain 2000 samples from the approximate posterior. As shown in Fig. 2, samples from all three posteriors are concentrated around their respective "true" values, resulting in very small estimation errors for MMSE estimator.

Root mean square errors (RMSE) of the estimator is computed using Monte Carlo experiment as follows. In each Monte Carlo run, N realisations of temporal moments are generated with the settings in Fig. 1, and the MMSE estimate is computed. The RMSE, reported in Table I, decreases with increase in the size of the data. As expected from the relative widths of the posteriors in Fig. 2, the RMSE of $\hat{g}$ is the smallest, followed by $\hat{G}_0$ and $\hat{\lambda}_0$. Note that even with a data size of $N = 10$, the RMSE is reasonably small.

To test the applicability of the estimator, we apply it to measured data from [3]. The estimated parameters are then used to predict the power delay spectrum. The result is compared with the averaged power delay profile from the measured data in Fig. 3. Despite the employed assumptions of high bandwidth and no noise, the fit seems reasonable, indicating that the Gaussian approximation is sufficient.

V. Conclusions

We find that the parameters of the considered stochastic multipath model can be estimated by using temporal moments as summary statistics without the need for multipath extraction. Thus, temporal moments of the received signal are informative for estimating the parameters of the considered multipath model, i.e. arrival rate, reverberation gain and absorption coefficient. The proposed method is reasonably accurate despite the approximations involved (it ignores measurement noise, bandwidth limitations, and relies on a Gaussian approximation of the likelihood). Further work is needed to account for finite measurement bandwidth and noisy data.

TABLE I
RMSE of the parameter estimates for different N with 500 Monte-Carlo runs each.

N	RMSE (RMSE/True Value)		
	$\hat{g}[10^{-3}]$	$\hat{G}_0[10^{-10}]$	$\hat{\lambda}_0$[MHz]
10	9 (1.5%)	10 (10%)	373 (37.3%)
50	4.6 (0.77%)	5.5 (5.5%)	140 (14%)
100	3.6 (0.6%)	4.3 (4.3%)	97 (9.7%)
300	2.4 (0.4%)	3.1 (3.1%)	56 (5.6%)

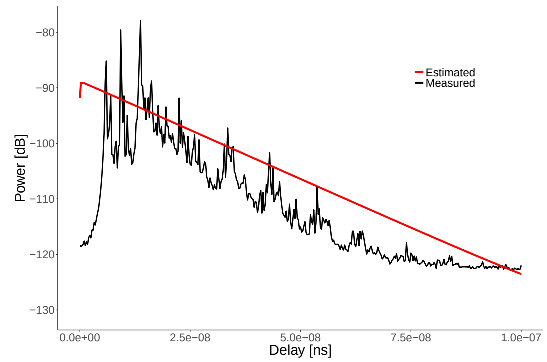

Fig. 3. Measured averaged power delay profile (black) [3] for $N = 625$ and the power delay spectrum predicted from the parameter estimates (red).

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