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Estimator for Stochastic Channel Model without Multipath Extraction using Temporal Moments

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***Abstract*—Stochastic channel models are usually calibrated after extracting the parameters of the multipath components from measurements. This paper proposes a method to infer on the underlying parameters of a stochastic multipath model, in particular the Turin model, without resolving the multipath components. Channel measurements are summarised into temporal moments instead of the multipath parameters. The parameters of the stochastic model are then estimated from the observations of temporal moments using a method of moments approach. The estimator is tested on real data obtained from in-room channel measurements. It is concluded that calibration of stochastic models can be done without multipath extraction, and that temporal moments are informative summary statistics about the model parameters.**

***Index Terms*—stochastic channel model, multipath, summary statistics, parameter estimation, method of moments**

I. Introduction

Realistic modelling of the radio channel is imperative to the design and analysis of any wireless communication system. Stochastic multipath models characterizing many different radio environments have been reported in the literature [1]–[4]. These models can be used to generate realizations of the channel in simulations, and to analyse the behaviour of communication systems. For a model to be useful for simulation, it should be calibrated, i.e. its parameters should be estimated such that the model fits to the measurement data.

Since the early works [1] and [2], parameters of a stochastic multipath channel model have usually been estimated in a two-step process as shown in Fig. 1(a). First, the multipath parameters, for example the delays and their respective gains, are estimated from the channel measurements, followed by estimation of the model parameters, e.g. [4]–[7]. Resolving multipath components using high-resolution algorithms such as CLEAN [8], SAGE [9] or RIMAX [10] is not usually trivial, and the overall estimation accuracy of the model parameters relies on how accurately the multipath parameters are obtained. One particular problem which has been considered only recently [11] is the effect that some multipath components are undetected due to noise or bandwidth constraints. This censoring effect causes significant calibration errors.

The error introduced by this intermediate multipath extraction step can potentially be eliminated by methods that bypass this step and estimate the model parameters from other summaries, as depicted in Fig. 1(b). Other summary statistics, apart from the multipath parameters, that can be found in the literature include power delay profile, root mean square delay spread, angular dispersion, temporal moments, and delay-Doppler function, among others [12]. Potentially these summaries are informative about the underlying model parameters and could thus be valuable for their estimation.

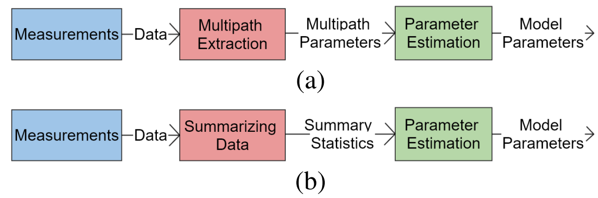

Fig. 1. Calibration procedures involving (a) multipath extraction, and (b) general summaries.

The choice of summary statistics is guided by a number of concerns. First, summary statistics should be informative about the model parameters. Ideally, a sufficient statistic should be chosen, but in most practical situations, this is not feasible or possible. In fact, the multipath parameters form a sufficient statistic for the underlying point process, provided that all multipath components are actually resolved without error, which is unlikely the case. Moreover, the summary should be easily computable. One such choice of summary statistics, that does not introduce significant computational overhead, are the temporal moments. These have been widely used in wireless communication literature since the 1970s, see e.g [13], but have, to the best of our knowledge hitherto not been considered for calibration of multipath models apart from the sampling approach proposed by the authors in [14].

In this contribution, we propose a method of moments parameter estimator for the Turin model [1] without multipath extraction. This model has recently attracted attention due to its simplicity, see e.g. [4], [20]. We compute the temporal moments from the transfer function measurements, without transforming it to time domain. Expressions for the means and covariances of the temporal moments are derived. Inserting sample means and covariances from the measurements to these expressions yield the parameter estimates. The performance of the proposed estimator is evaluated by a simulation experiment and validated using measurement data.

II. Signal Model

Consider a single-input, single-output (SISO) multipath propagation scenario where the received signal is measured using a transmit and a receive antenna at a certain frequency bandwidth, B. The received signal, Y_k, is modelled in the frequency domain as

$$Y_k = H_k + N_k, \quad k = 0, 1, \ldots, (N_s - 1), \tag{1}$$

where k is the frequency index, H_k is the transfer function, N_k is the noise contribution, and N_s is the total number of sample points. The major source of noise in such measurements is noise from the measurement equipment itself, which can be assumed independent and identically distributed (iid) at each measurement point. Here the noise is modelled as iid complex Gaussian variables, $N_k \sim \mathcal{CN}(0, \sigma_N^2)$, $k = 0, 1, \ldots, N_s - 1$.

The temporal moments can be computed in the frequency domain as follows. Discrete-frequency, continuous-time inverse Fourier transform of Y_k gives the signal in the time domain, $y(t)$

$$y(t) = \frac{1}{N_s} \sum_{k=0}^{N_s - 1} Y_k \exp(j2\pi k \Delta f t), \tag{2}$$

where Δf is the frequency separation between two samples. The time domain signal is periodic with period

$$t_{\max} = \frac{1}{\Delta f} = \frac{(N_s - 1)}{B}. \tag{3}$$

Define the i^{th} temporal moment of $y(t)$ as

$$m_i = \int_0^{t_{\max}} t^i |y(t)|^2 \mathrm{d}t, \quad i = 0, 1, 2, \ldots \tag{4}$$

Consequently, in the frequency-domain, we have

$$m_i = \frac{1}{N_s^2} \sum_k \sum_{k'} Y_k Y_{k'}^* a_i(k - k'), \tag{5}$$

with the definition

$$a_i(k - k') = \int_0^{t_{\max}} t^i \exp(j2\pi \Delta f (k - k') t) \, \mathrm{d}t. \tag{6}$$

A number of properties can be shown for this function. Since $m_i \geq 0$ for all Y_k, $a_i(k-k')$ is a positive semidefinite function. Also, $a_i(k) = a_i^*(-k)$. Note that $a_i(0) = t_{\max}^{i+1} / (i + 1)$. The magnitude of $a_i(k - k')$ decreases rapidly as $|k - k'|$ increases. In particular, $a_0(k - k') = 0$ for $k \neq k'$. Therefore, the diagonal terms in (5) are the most relevant for computing the temporal moments.

The frequency domain method in (5) is preferred over (4) here since we work with frequency domain measurements and it leads to low complexity of the estimator derived in Section III. It should be noted that no attempt has been made to remove noise and effects of finite measurement bandwidth. Thus, we avoid the employment of heuristics to set the noise floor and truncate the time domain signal at an arbitrary point. This is advantageous here, since the derived estimator and its performance does not depend on such arbitrary choices, and the results are more easily reproduced.

The temporal moments $m_i, i = 0, 1, 2, \ldots,$ are random variables with means

$$\mathbb{E}[m_i] = \frac{1}{N_s^2} \sum_k \sum_{k'} \mathbb{E}\left[Y_k Y_{k'}^*\right] a_i(k - k'), \tag{7}$$

and covariances

$$\mathrm{Cov}(m_i, m_j) = \frac{1}{N_s^4} \sum_{k,k',n,n'} \mathrm{Cov}(Y_k Y_{k'}^*, Y_n Y_{n'}^*) a_i(k - k') a_j(n - n'). \tag{8}$$

It should be noted that the first and the second moment of the temporal moments depend upon the second and fourth moment of the received signal, respectively.

Employing the uncorrelated scattering assumption [15], the second moment of Y_k can be written as

$$\mathbb{E}\left[Y_k Y_{k'}^*\right] = \int P_y(t) \exp(-j2\pi \Delta f t (k - k')) \, \mathrm{d}t + \sigma_N^2 \delta(k - k'), \tag{9}$$

where $\delta(\cdot)$ is the Kronecker delta function, and $P_y(t)$ is the power delay spectrum of the received signal[1]. The power delay spectrum reads from (2) as

$$P_y(t) = \frac{1}{N_s^2} \sum_k \sum_{k'} \mathbb{E}\left[Y_k Y_{k'}^*\right] \exp(j2\pi \Delta f (k - k') t). \tag{10}$$

Considering high bandwidths, $P_y(t)$ can be approximated as

$$P_y(t) \approx E_s P_h(t) + \sigma_N^2 / N_s, \tag{11}$$

where E_s is the energy of the transmitted signal, and $P_h(t)$ is the power delay spectrum of the channel. With the transmitted signal in frequency domain being the rectangular window of unit magnitude over the bandwidth, that gives $E_s = B$.

III. Method of Moments Estimator for Turin's Model

A. Channel model description

As an example of how the temporal moments can be utilized to calibrate stochastic channel models, we derive a method of moments estimator for the seminal model by Turin [1], applied to an in-room setting. For a multipath channel, the transfer function, H_k, reads

$$H_k = \sum_l \alpha_l \exp(-j2\pi \Delta f k \tau_l), \tag{12}$$

where α_l and τ_l are the complex gain and time-delay of the l^{th} multipath component, respectively. The delays form a homogeneous Poisson point process with arrival rate $\lambda(t) = \lambda_0$. The gains, conditioned on the delays, are modelled as independent circular complex Gaussian variables with variance $\sigma_\alpha^2(t)$. Therefore, $\{(\tau_l, \alpha_l)\}$ forms a marked Poisson point process of intensity λ_0, with points $\{\tau_l\}$ and marks $\{\alpha_l\}$. For this type of model, $P_h(t) = \lambda(t)\sigma_\alpha^2(t)$, see [16].

Typically for in-room channel measurements, the power delay spectrum has an exponentially decaying behaviour, and can be approximated using the reverberation model in [17] as

$$P_h(t) = \begin{cases} G_0 \exp(-\frac{t}{T}), & t > t_0 \\ 0 & \text{otherwise}, \end{cases} \tag{13}$$

[1]With $s(t)$ as the transmitted signal, $P_y(t)$ is defined as in the noise free case:

$$P_y(t) = \mathbb{E}[|y(t)|^2] = \int P_h(\tau) |s(t - \tau)|^2 \mathrm{d}\tau,$$

where $P_h(t)$ may be informally interpreted as $P_h(t) = \mathbb{E}[|h(t)|^2]$, where $|h(t)|^2$ is the (instantaneous) power delay profile of the channel. [16]

where G_0 is the power at delay zero called reverberation gain, T is the reverberation time, and t_0 is the delay of the first multipath component. The arrival rate, however, does not enter in (13). Substituting this model in (9) and carrying out the integration gives the autocorrelation function of Y as

$$R_Y(k,k') = \mathbb{E}\left[Y_k Y_{k'}^*\right] = \frac{G_0 T B e^{-t_0\left(\frac{1}{T}+j2\pi\Delta f(k-k')\right)}}{1+j2\pi\Delta f T(k-k')} + \sigma_N^2\delta(k-k'). \quad (14)$$

As to be expected, Y is a wide-sense stationary process in the frequency domain.

B. Estimator derivation

We follow a method of moments approach to estimate the four parameters, G_0, T, σ_N^2, and λ_0, from N observations of the summary statistic consisting of three temporal moments (m_0, m_1, m_2). Out of the four minimum equations required to solve for the parameters, three of them are taken to be the equations for the mean. Using (14) in (7) gives the three mean equations as:

$$\mu_0 = \frac{G_0 T B}{N_s^2}\beta_0(T) + \frac{\sigma_N^2}{B}, \quad (15)$$

$$\mu_1 = \frac{G_0 T B}{N_s^2}\beta_1(T) + \frac{\sigma_N^2 t_{\max}^2}{2N_s}, \quad (16)$$

$$\mu_2 = \frac{G_0 T B}{N_s^2}\beta_2(T) + \frac{\sigma_N^2 t_{\max}^3}{3N_s}, \quad (17)$$

where $\mu_i = \mathbb{E}[m_i]$, and $\beta_i(T)$ is defined as

$$\beta_i(T) = \sum_{\tilde{k}=-N_s+1}^{N_s-1} \frac{(N_s - |\tilde{k}|)e^{-t_0\left(\frac{1}{T}+j2\pi\Delta f\tilde{k}\right)}a_i(\tilde{k})}{1+j2\pi\Delta f T\tilde{k}}, \quad (18)$$

with $\tilde{k} = k - k'$ and $i = 0,1,2$. Substituting σ_N^2 from (15) into (16) and (17), and then dividing the two equations gives

$$\left(\hat{\mu}_1 - \frac{\hat{\mu}_0}{2\Delta f}\right)\left(\frac{B}{N_s^2}\beta_2(T) - \frac{e^{-\frac{t_0}{T}}}{3\Delta f^2}\right) - \left(\hat{\mu}_2 - \frac{\hat{\mu}_0}{3\Delta f^2}\right)\left(\frac{B}{N_s^2}\beta_1(T) - \frac{e^{-\frac{t_0}{T}}}{2\Delta f}\right) = 0. \quad (19)$$

Here, $\hat{\mu}_i$ are the estimates of μ_i for $i = \{0,1,2\}$, found by taking the sample mean of the temporal moments. Solving (19) numerically for T (which is easily done using standard numerical solvers) gives the estimate for the reverberation time, $\hat{T}$. The estimate for the reverberation gain, $\hat{G}_0$, is then

$$\hat{G}_0 = \frac{N_s^2\left(2\Delta f\hat{\mu}_1 - \hat{\mu}_0\right)}{2\Delta f B\beta_1(\hat{T}) - N_s^2 e^{-\frac{t_0}{\hat{T}}}}. \quad (20)$$

The noise variance is then estimated by inserting $\hat{G}_0$, $\hat{T}$ and $\hat{\mu}_0$ in (15).

The arrival rate, λ_0, does not appear in the mean equations (15)-(17), but in the covariances derived in Appendix. Any of the covariance equations can be used. The simplest is the equation for the variance of m_0:

$$\mathrm{var}(m_0) = \frac{G_0^2 T e^{-\frac{2t_0}{T}}}{\lambda_0} + \underbrace{\frac{t_{\max}^2}{N_s^4}\sum_{p=-N_s+1}^{N_s-1}(N_s - |p|)\,|R_Y(p)|^2}_{\gamma}, \quad (21)$$

where $R_Y(p)$ is the autocorrelation function of Y at lag p. Estimating γ requires the estimation of $R_Y(p)$, which is done by using $\hat{G}_0$, $\hat{T}$, and $\hat{\sigma}_N^2$ in (14). The arrival rate is then estimated as

$$\hat{\lambda}_0 = \frac{\hat{G}_0^2\hat{T}e^{-\frac{2t_0}{\hat{T}}}}{\widehat{\mathrm{var}}(m_0) - \hat{\gamma}}, \quad (22)$$

where $\widehat{\mathrm{var}}(m_0)$ is the sample variance of the zeroth temporal moment and $\hat{\gamma}$ is the estimate of γ.

Algorithm 1 Method of moments estimator

Input: $\mathbf{Y}$, t_0
1: Compute $\mathbf{D}$ from $\mathbf{Y}$ using (5)
2: Compute the sample means of $\{\mathbf{m}_0, \mathbf{m}_1, \mathbf{m}_2\}$, and the sample variance of $\mathbf{m}_0$
3: Solve (19) numerically to find $\hat{T}$
4: Use $\hat{T}$ in (20) to find $\hat{G}_0$
5: Obtain $\hat{\sigma}_N^2$ using $\hat{G}_0$ and $\hat{T}$ in (15)
6: Estimate $R_Y(p)$ from (14) using $\hat{G}_0$, $\hat{T}$, $\hat{\sigma}_N^2$, and then compute $\hat{\gamma}$
7: Obtain $\hat{\lambda}_0$ from (22) using $\hat{G}_0$, $\hat{T}$ and $\hat{\gamma}$
Output: $\hat{T}$, $\hat{G}_0$, $\hat{\sigma}_N^2$, $\hat{\lambda}_0$

C. Estimation procedure

Let the measurements of a SISO channel be stored in a matrix $\mathbf{Y} \in \mathbb{C}^{N\times N_s}$, where N is the number of realizations of the received signal. The first three temporal moments can then be computed for each realization, resulting in a data matrix $\mathbf{D} = [\mathbf{m}_0, \mathbf{m}_1, \mathbf{m}_2]$, where $\mathbf{m}_i \in \mathbb{R}^N$ for $i = 0,1,2$. The algorithm for estimating the parameters using method of moments is given in Alg. 1. Note that t_0 is considered as an input to the estimator. This could either be obtained as side information from the measurement set-up or estimated by finding the first peak in the data. No further assumptions, such as the number of multipath components to extract, are required here.

IV. Performance Evaluation

A. Numerical experiment

To evaluate the accuracy of the estimator, we perform Monte Carlo simulations for different values of SNR, defined as SNR $= G_0TB/\sigma_N^2$, and number of realizations, N, using synthetic data generated from the model. The root-mean-squared error (RMSE) of the different parameter estimates are normalized by their "true" values and shown in Fig. 2.

It can be seen from Fig. 2 that the RMSE of the estimates decrease as the number of realizations (and hence the number of data points of each temporal moment) increase. This illustrates that the method of moments estimate is consistent.

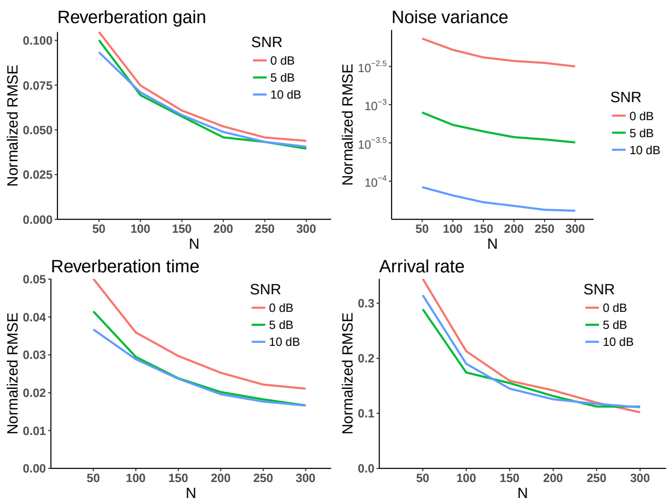

Fig. 2. Normalized RMSE of the different parameter estimates as a function of number of channel realizations, plotted for different SNR values. Parameter settings: $G_0 = 10^{-8}$, $T = 10$ ns, $\lambda_0 = 1$ ns^{-1}, $t_0 = 5$ ns, $B = 4$ GHz, $N_s = 801$, Number of Monte Carlo runs $= 500$. Note that the RMSE of noise variance is in log-scale.

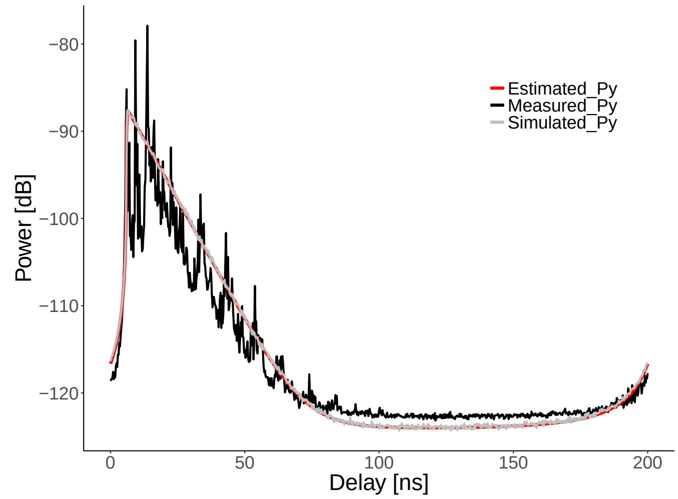

Fig. 3. Figure showing the averaged PDP of $N = 625$ realizations of the measurement data (black) and the simulated data (grey) versus the delay. The estimated $P_y(t)$ computed from (10) is shown in red. The estimates obtained are: $\hat{G}_0 = -83.9$ dB, $\hat{T} = 7.8$ ns, $\hat{\sigma}_N^2 = 2.8 \times 10^{-10}$, and $\hat{\lambda}_0 = 10.0$ ns^{-1}. The estimated noise floor is $\hat{\sigma}_N^2/N_s = -124.5$ dB.

The normalized RMSE is fairly small in all four cases with the arrival rate estimator showing the largest error. This is expected since $\hat{\lambda}$ is obtained as a combination of a variance estimate and the three other estimates. The estimation accuracy also improves with SNR. However, the improvement is less pronounced for reverberation gain due to the high bandwidth. Further analysis of the simulation data (not shown here) shows that the parameter estimates are unbiased.

B. Application to measured data

To test the estimator's applicability, we now apply it to a set of measurement data described in [11]. The dataset consists of channel transfer functions obtained using a vector network analyser (VNA) in a room of dimensions $3 \times 4 \times 3$ m^3. The set-up is SISO, with a virtual planar array of 25×25, resulting in $N = 625$ realizations of the channel. The bandwidth of the measured signal is 4 GHz, with $N_s = 801$ samples in each channel measurement, resulting in $\Delta f = 5$ MHz and $t_{\max} = 200$ ns. The delay of the first peak is $t_0 = 6$ ns, found through visual inspection of the data. The estimator was not observed to be sensitive to variations in t_0 of the order of $1/B$. For this dataset, our implementation of Alg. 1, programmed in R version 3.4.3, took around 8 s when run on a notebook with a dual-core Intel i7 processor and 24 GB RAM.

To further demonstrate the soundness of the proposed method, we now use the obtained estimates to estimate the power delay spectrum using (10). This is then compared with the averaged power delay profile (PDP) of the measurement data as shown in Fig. 3. The averaged PDP of simulated signals, i.e. the simulated power delay spectrum from the model using the estimates, is also shown. Both the theoretical and simulated power delay spectrum obtained from the estimates seem to fit the averaged PDP of the measurements, although there is a slight discrepancy between the estimated noise floor and the noise floor observed in the measurements. This demonstrates that the estimator is able to extract the model parameters accurately from real measurements without the need for multipath extraction.

V. Conclusions

The proposed method of moments estimator can be used to calibrate the Turin model with a constant arrival rate without multipath extraction. The performance evaluation shows that the parameters of the considered stochastic multipath channel model can be estimated with good accuracy. The temporal moments, used in the estimation procedure, are easy-to-compute summary statistics of the measurement data. Moreover, they are informative about the parameters of the considered model, i.e. reverberation gain, reverberation time, arrival rate and noise variance. This estimation procedure bypasses the need to truncate the measured impulse response via some heuristics to estimate the decay of the power delay spectrum.

Comparison with performance bounds, such as the Cramer-Rao bound, is hindered by the lack of a likelihood function. Furthermore, comparison with calibration procedures involving multipath extraction with a number of heuristic choices is non-trivial. These considerations are left for future work.

Appendix

The characteristic function for the received signal evaluated at an arbitrary point $\mathbf{v}$ is defined as

$$\mathcal{C}(\mathbf{v}) = \mathbb{E}\left[\exp\left(j\Re\mathbf{v}^H\mathbf{Y}\right)\right], \tag{23}$$

where $\Re$ denotes the real part. The joint moment of four frequencies are obtained by letting $\mathbf{v} = [\nu_1, \nu_2, \nu_3, \nu_4]^T$ and $\mathbf{Y} = [Y_{k_1}, Y_{k_2}, Y_{k_3}, Y_{k_4}]^T$. Since marked point process $\{(\tau_l, \alpha_l)\}$ forms a two-dimensional Poisson point process with rate $p(\alpha|\tau)\lambda(\tau)$ [18], we obtain by Campbell's theorem the cumulant generating function (log characteristic function) as

$$K(\mathbf{v}) = \int \left[\mathcal{C}_{\alpha|\tau} \left(\sum_{j=1}^{4} \nu_j \exp\left(-j2\pi\Delta f k_j \tau\right) \right) - 1 \right] \lambda(\tau) \mathrm{d}\tau, \tag{24}$$

where $\mathcal{C}_{\alpha|\tau}(\cdot)$ is the characteristic function for the circular symmetric complex Gaussian $p(\alpha|\tau)$ [20].

The covariance of the temporal moments is related to the fourth moment of the received signal, Y_k, which in turn can be written in terms of the fourth cumulant of the signal [19, Eq. (2.118)] as

$$\mathrm{Cov}(Y_k Y_{k'}^*, Y_n Y_{n'}^*) = \mathrm{Cum}(Y_k Y_{k'}^* Y_n Y_{n'}^*) + \mathbb{E}[Y_k Y_{n'}^*]\mathbb{E}[Y_n Y_{k'}^*]. \tag{25}$$

The fourth cumulant of Y is found by complex differentiation of the cumulant generating function [19]

$$\begin{aligned} \mathrm{Cum}(Y_k Y_{k'}^* Y_n Y_{n'}^*) &= \frac{2^4}{j^4} \cdot \frac{\partial^4 K(\mathbf{v})}{\partial \nu_1 \partial \nu_2^* \partial \nu_3 \partial \nu_4^*} \bigg|_{\nu_1,\nu_2,\nu_3,\nu_4=0} \\ &= 2 \int \sigma_\alpha^4(t) \lambda(t) e^{-j2\pi\Delta f (k_1 - k_2 + k_3 - k_4) t} \mathrm{dt}. \end{aligned} \tag{26}$$

With $\lambda(t) = \lambda_0 \mathbb{1}(t > t_0)$, we get

$$\begin{aligned} \mathrm{Cum}(Y_k Y_{k'}^* Y_n Y_{n'}^*) &= \frac{2G_0^2 B^2}{\lambda_0} \int_{t_0}^{\infty} e^{-\frac{2t}{T}} e^{-j2\pi\Delta f (k-k'+n-n')t} \mathrm{dt} \\ &= \frac{2G_0^2 T B^2 \psi_{(k-k'+n-n')}}{\lambda_0 \left[2 + j2\pi\Delta f (k - k' + n - n')T\right]}, \end{aligned} \tag{27}$$

where $\psi_{(k-k'+n-n')} = e^{-t_0\left(2/T + j2\pi\Delta f (k-k'+n-n')\right)}$. Now, let $\mathrm{Cov}(m_i, m_j) = \varrho_1^{ij} + \varrho_2^{ij}$. Then ϱ_1^{ij}, being the quadruple sum over the cumulant expression, can be expressed in the form of a double sum as

$$\varrho_1^{ij} = \frac{G_0^2 T B^2}{\lambda_0 N_s^4} \sum_{\tilde{k},\tilde{n}=-N_s+1}^{N_s-1} \frac{\left(N_s - |\tilde{k}|\right)\left(N_s - |\tilde{n}|\right) \psi_{(\tilde{k}+\tilde{n})} a_i(\tilde{k}) a_j(\tilde{n})}{1 + j\pi\Delta f (\tilde{k} + \tilde{n})T}. \tag{28}$$

Since $\mathbb{E}[Y_k Y_{n'}^*] = R_Y(k - n')$, ϱ_2^{ij} can be written as

$$\begin{aligned} \varrho_2^{ij} &= \frac{1}{N_s^4} \sum_{k,k',n,n'} \mathbb{E}\left[Y_k Y_{n'}^*\right] \mathbb{E}\left[Y_n Y_{k'}^*\right] a_i(k-k') a_j(n-n') \\ &= \frac{1}{N_s^4} \sum_{k,k',n,n'} R_Y(k-n') R_Y(n-k') a_i(k-k') a_j(n-n') \\ &= \frac{1}{N_s^4} \sum_{p=-N_s+1}^{N_s-1} \left(N_s - |p|\right) \left(a_j * R_Y\right)[p] \left(a_i * R_Y\right)[-p], \end{aligned} \tag{29}$$

where the convolution defined as

$$(f * g)[n] := \sum_l f[l] g^*[n-l].$$

Noticing that $a_0(k-k') = t_{\max}\delta(k-k')$, $\mathrm{var}(m_0)$ is given as in (21).

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