Aalborg Universitet

Extrapolation for image interpolation

Rukundo, Olivier; Schmidt, Samuel

PROCEEDINGS OF SPIE

Extrapolation for image interpolation

Olivier Rukundo, Samuel E. Schmidt

SPIE.

Extrapolation for image interpolation

Olivier Rukundo[1a], Samuel E. Schmidt[a]

[a]Department of Health Science and Technology, Fredrik Bajers Vej 7, 9220 Aalborg, Denmark

ABSTRACT

Unlike traditional linear interpolation algorithms, which compute all kernel pixels locations, a novel image interpolation algorithm that uses the preliminary pixels kernel and extrapolated pixels adjustment has been proposed for interpolation operations. The proposed interpolation algorithm is mainly based on the weighting functions of the preliminary interpolation kernel and linearly extrapolated pixels adjustments. Experimentally, the proposed method demonstrated generally higher performance than state-of-art algorithms mentioned with objective evaluations as well as comparable performances with subjective evaluations. Potential applications may include the ultrasound scan conversion for displaying the sectored image.

Keywords: extrapolation, interpolation, extrapolated pixels adjustment, interpolation kernel, sectored image

1. INTRODUCTION

Image interpolation is a widely used operation, in digital image processing, which works by using known data or variables to estimate new pixels 'values at unknown grid points or locations, without sacrificing the original or source image quality. Image interpolation quality is, therefore, one of the aims pursued by interpolation algorithm developers and it mainly requires that source image features be reconstructed accurately, on a larger two-dimensional grid, to preserve or avoid sacrificing original image details [1], [20], [10]. Many works on image interpolation kept focusing on the minimization of visual artefacts – by either fitting the interpolation function linearly using larger interpolation kernel, as the key strategy to achieve higher performance or adaptively taking some image samples into account [2], [3]. For example, in [9], efforts have been made to minimize the relentless image interpolation artefacts by turning the interpolated value into the weighted mean between the pixel value corresponding to the smallest absolute difference and traditional bilinear interpolation value. In [5], a bilinear interpolation optimization method using ant colony algorithm has been introduced to tackle the isotropic assignment of pixels values in the effort to minimize visual artefacts related to such an assignment. In [4], a method based on the statistical selection of the pixel value closest to the traditional bilinear interpolation value has been proposed to efficiently tackle the image edge blurriness problem. In [6], the optimization scheme based on the nearest neighbor algorithm has been proposed with the main objective to improve the speed of the traditional bilinear interpolation algorithm by replacing the traditional bilinear with the nearest interpolation algorithm when the four nearest pixels have the same value. In [8], the author demonstrated the effects of rounding functions on the accuracy of the bilinear interpolation algorithm. In [7], a novel ant colony optimization-based interpolation method which, unlike in [5], uses a global weighting scheme has been proposed to smooth interpolated image edges. A novel method that rescales the bilinear interpolant pixels to achieve better image interpolation performances has been proposed in [20], and it demonstrated better performance, particularly, in terms of contrast and intensity increment and decrement as well as in decreasing the brisque-scored-distortions, in the sectored images. In [26], a cubic interpolation-based method, bicubic, was presented, with a kernel size of sixteen pixels, and generally demonstrates superior performances in terms of image surfaces smoothness than bilinear and nearest interpolation algorithms. Despite that, the determination of the optimal kernel size remains unsolved in digital image interpolation. On top of that, using a larger image pixels kernel require computing more or additional locations to locate and use pixels belonging to those distant locations. To avoid additional location computations, and to ease the determination of meaningful kernel size and information, the image interpolation algorithm, that is based on extrapolating pixels and adjusting them afterward, has been proposed in this paper. Both objective and subjective evaluations have been used to estimate the effects of extrapolated pixels adjustments for image interpolation purposes, using standard test images. Furthermore, ultrasound image data were used to estimate the decrease in distortions in the sectored image. This paper is organized as follows: Part 2 briefly defines the interpolation and extrapolation operations, focusing on their linear versions. Part 3 introduces the extrapolation-based

---

[1]orukundo@hst.aau.dk; phone 45 4294 8142; hst.aau.dk

method highlighting the determination of extrapolated pixels and corresponding weights. Part 4 presents experiments and discusses objective and subjective evaluation conducted. Part 5 gives the conclusion.

2. INTERPOLATION AND EXTRAPOLATION

Interpolation is a method for estimating the value of a function between two known variables [22]. Extrapolation is a method for estimating the value beyond a specific range of a given variable [23]. More details about interpolation and extrapolation are available (in many works found) in the literature and therefore not replicated in this paper. Here, a focus is put on linear interpolation and extrapolation. Linear interpolation involves estimating a new value by connecting two adjacent known variables with a straight line [22], as shown in Figure 1.
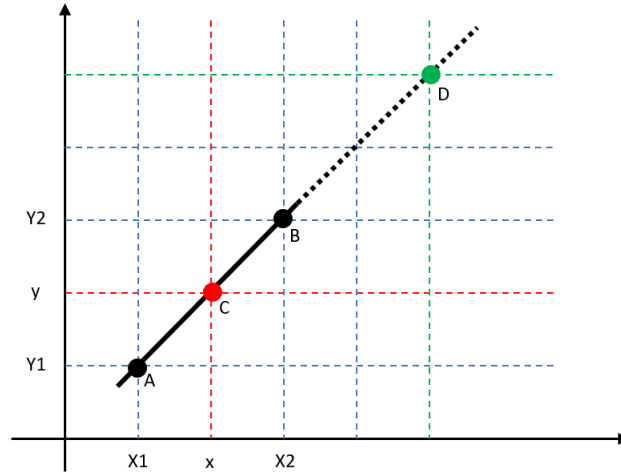

Figure 1: Interpolated C (in red) between the AB segment, Extrapolated D (in green) by extending it beyond the AB segment.

If the two known variables are (x1, y1) and (x2, y2), [22], then the *y* value for some point *x* is given by Equation 1.

$$y = y_1 + \left(\frac{x - x_1}{x_2 - x_1} \right) \times \left(y_2 - y_1 \right) \tag{1}$$

Linear extrapolation means creating a tangent line at the end of the known variable and extending it beyond that limit (see Figure 1), [23]. For linear extrapolation, if the two endpoints of a linear segment are (x1, y1) and (x2, y2) and are the nearest to the point x, then the linear extrapolation is given the function in Equation 2 (which is identical to linear interpolation's Equation 1, if $x_1 < x < x_2$), [24].

$$y(x) = y_1 + \left(\frac{x - x_1}{x_2 - x_1} \right) \times \left(y_2 - y_1 \right) \tag{2}$$

It is important to note that, linear extrapolation will only provide good results when used to extend the graph of an approximately linear function or not too far beyond the known data [24].

3. EXTRAPOLATION BASED INTERPOLATION METHOD

This is mainly based on finding values for $E1$, $E2$, $E3$ and $E4$ pixels (shown in Figure 2) without computing their corresponding locations - the same way in the $P1$, $P2$, $P3$ and $P4$ pixels kernel - to achieve a new pixels kernel, for the extrapolation-based interpolation function. Here, four nearest pixels, $P1$, $P2$, $P3$ and $P4$ form the preliminary pixels kernel and (their corresponding) four extrapolated pixels, P_e1, P_e2, P_e3 and P_e4, are linearly estimated (before adjustment); because other standard methods, used for extrapolation – such as piecewise cubic hermite interpolating

polynomial (pchip), modified akima cubic hermite interpolation (makima), cubic spline data interpolation (spline), nearest neighbor interpolation (nearest) – are subjected to greater uncertainty and/or a higher risk of producing meaningless interpolation results, as shown in Table 1 and Table 2.

Table 1: Standard methods used for extrapolation vs the adjustment-based extrapolation method

Pixels (Px)	Pchip ($P_e x$) *1e+7	Spline ($P_e x$)*1e+8	Makima ($P_e x$)*1e+7	Nearest ($P_e x$)	Linear ($P_e x$)	Ex
91	0.0967	0.1673	-0.2154	95	-5734	-63.0110
210	1.3369	2.1836	-2.7842	95	-13707	-65.2714
162	0.5981	0.9890	-1.2635	95	-10491	-64.7593
95	0.1110	0.1912	-0.2459	95	-6002	-63.1789

Next, all linearly extrapolated pixels ($P_e 1$, $P_e 2$, $P_e 3$ and $P_e 4$) are adjusted as shown in Equation 3, where x replaces the numeric number, and n is a very extremely small number, added simply to avoid a result of mathematically undefined operations (even if, it may not be necessary as the conversion to unsigned bit integer may turn such a result to zero).

$$Ex = \frac{P_e x}{Px + n} \tag{3}$$

Now, let us refer to the traditional bilinear interpolation equation, given in [20] to determine the weights WEx for the adjustment-based extrapolated pixels Ex . In [20], the distance-based-weights of the pixels, $P1$, $P2$, $P3$ and $P4$, are calculated as follows: $W(x_1, y_1) = (y2 - y) \times (x2 - x)$, $W(x_1, y_2) = (y2 - y) \times (x - x1)$, $W(x_2, y_1) = (x2 - x) \times (y - y1)$, and, $W(x_2, y_2) = (y - y1) \times (x - x1)$. Here, $W1 = W(x_1, y_1)$, $W2 = W(x_1, y_2)$, $W3 = W(x_2, y_1)$ and $W4 = W(x_2, y_2)$, (or simply, Wx for any numeric number of x varying from one to four).
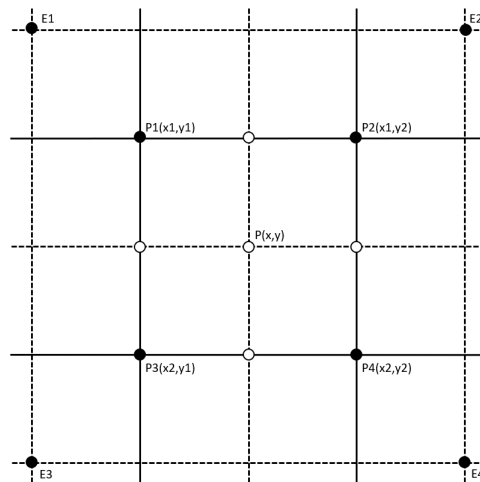
Figure 2: Extrapolated pixels (E), Image pixels (P), Interpolated pixels $P(x, y)$

The distance-based weights for extrapolated pixels adjustments, WEx , are given by Equation 4.

$$WEx = \frac{Wx}{d} \tag{4}$$

where, d is a default value set to four. It is important to note that this default value can be redefined adaptively or made variable, since, here, it was made a constant for simplicity purposes. Now, the extrapolation-based interpolation function is given by Equation 5.

$$P(x, y) = \sum_{x=1}^{4} P_x \times W_x + \sum_{x=1}^{4} E_x \times WE_x \tag{5}$$

A numerical example is shown in Table 2. As can be seen, referring to the example of the preliminary kernel of four pixels, used in Table 1, methods used for extrapolation purposes are at risk of producing meaningless interpolated pixel values. It is therefore important to adjust linearly extrapolated pixels to achieve interpolated values that are within the range of the preliminary pixels kernel.

Table 2: Numerical example

x_1	x	x_2	y_1	y	y_2	P1	P2	P3	P4	Pchip P_ex	Spline P_ex	Makima P_ex	Nearest P_ex	Linear P_ex	Eq. (5)
1	1	2	1	1	2	91	210	162	95	2.4184e+05	4.1823e+06	-5.3832e+05	114.7500	-1.3425e+03	75.2473
1	1.25	2	1	1	2	91	210	162	95	1.0170e+06	1.6785e+07	-2.1438e+06	144.5000	-1.8111e+03	104.8560
1	1.5	2	1	1	2	91	210	162	95	1.7921e+06	2.9387e+07	-3.7493e+06	174.2500	-2.2796e+03	134.4647
1	1.75	2	1	1	2	91	210	162	95	2.5672e+06	4.1989e+07	-5.3547e+06	204	-2.7482e+03	164.0734
1	2	2	1	1	2	91	210	162	95	3.3424e+06	5.4591e+07	-6.9602e+06	233.7500	-3.2168e+03	193.6821
1	1.25	2	1	1.25	2	91	210	162	95	1.0605e+06	1.7523e+07	-2.2385e+06	150.6250	-1.9075e+03	110.9317
1	1.5	2	1	1.5	2	91	210	162	95	1.3393e+06	2.2070e+07	-2.8180e+06	163.2500	-2.1064e+03	123.4862
1	1.75	2	1	1.75	2	91	210	162	95	1.0784e+06	1.7822e+07	-2.2767e+06	152.6250	-1.9390e+03	112.9107
1	2	2	1	2	2	91	210	162	95	2.7766e+05	4.7801e+06	-6.1478e+05	118.7500	-1.4055e+03	79.2053
1	2	2	1	2	2	91	91	91	91	113.7500	113.7500	113.7500	113.7500	113.7500	91
1	1.5	2	1	1.5	2	91	91	91	91	113.7500	113.7500	113.7500	113.7500	113.7500	91

The following part presents experimental results achieved using the extrapolation-based interpolation's Equation 5.

4. EXPERIMENTAL RESULTS

The proposed algorithm based on extrapolating pixels (Xtrapo) has been implemented in MATLAB-R2018a. Among many image quality metrics available in the literature, [13], [15], [17], we only chose the feature similarity index (FSIM) [16], structural similarity index (SSIM) [17], peak signal to noise ratio (PSNR) [25] and Blind/Referenceless Image Spatial Quality Evaluator (BRISQUE) [14], as well as the variance and mean based equations metrics for statistical visual representation, shown in Equation 6 and Equation 7, [18], [19]. The full reference FSIM, SSIM, and PSNR metrics have been chosen to quantify the closeness or similarity of interpolated images against a supposedly pristine reference image while BRISQUE was used for the quantification of image distortions in a referenceless way and case. Here, the aim is to measure the feature, structure, noise-level closeness between interpolated and reference images and rapidly get the idea of how such images would have consisted with subjective evaluations. Getting pristine images or simply inferring reference image for objective evaluation purposes at different interpolation ratios is another problem but not discussed here. However, note that small-sized and some reference images used, in our experiments, were inferred using the Microsoft Picture Manager application. Other two metrics, defined in Equation 6 and Equation 7, consisting of variance and mean equations, have been chosen to define the percentage of increment or decrement of contrast and intensity levels (other key image quality indicators) in each algorithm's interpolated image [19], [18].

$$C = \frac{\sigma_{out} - \sigma_{in}}{\sigma_{in}} \tag{6}$$

$$L = \frac{\mu_{out} - \mu_{in}}{\mu_{in}} \tag{7}$$

where, σ_{out} and μ_{out} are the variance and mean of interpolated images, σ_{in} and μ_{in} are the variance and mean of reference images, respectively [19]. Grayscale images, mostly, downloaded from the USC-SIPI Image database have been used as test images, while cardiac ultrasound data used belongs to our research groups. As shown by objective evaluations, for example in Figure 3 with two full-reference metrics FSIM and SSIM, the Xtrapo achieved the highest FSIM values in every case, except at the interpolation ratio that equals to five. Also, Xtrapo achieved the highest SSIM values in the first three cases and lower SSIM values than bilinear, bicubic and lanczos algorithms at interpolation ratios of five, six, seven and eight. Here, it is important to note that the higher FSIM and SSIM values, the better image quality.
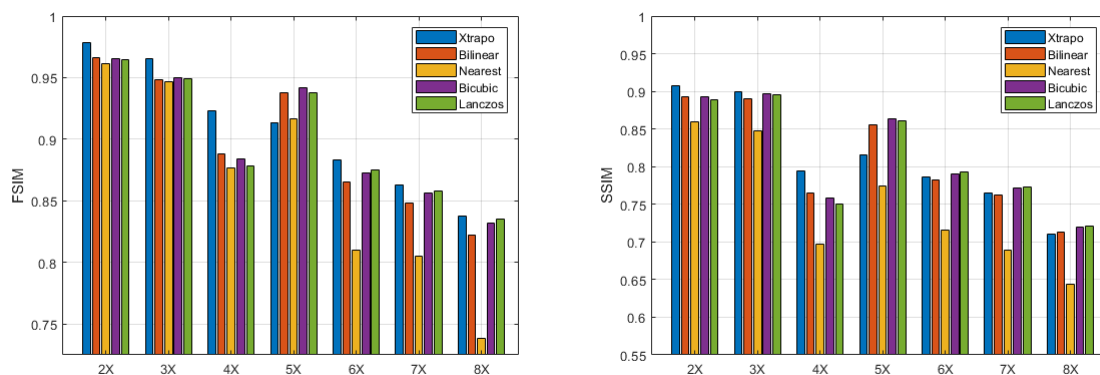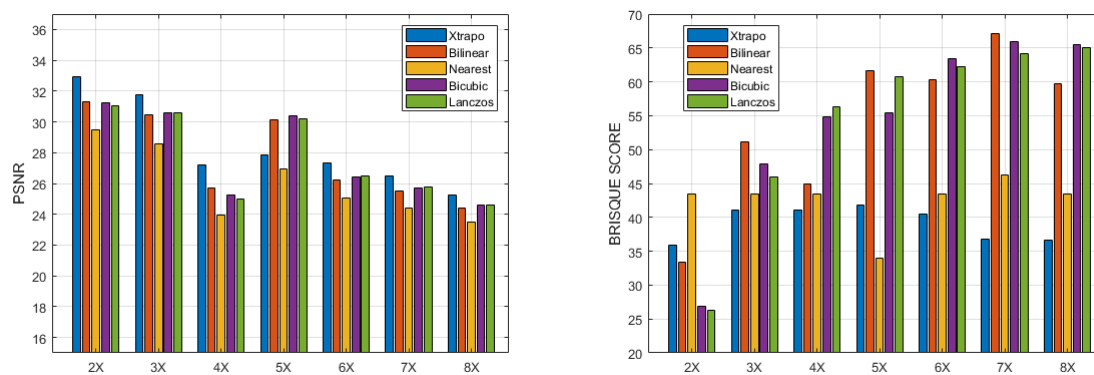
Figure 3: FSIM and SSIM

Figure 4: PSNR and BRISQUE

In Figure 4, full-reference and non-reference metrics used are PSNR and BRISQUE, respectively and, the Xtrapo achieved the highest PSNR values in most cases, except at the interpolation ratio that equals to five. Also, Xtrapo achieved the lowest BRISQUE scores in most cases except in the first and fifth cases and higher BRISQUE scores than bilinear, bicubic and lanczos at interpolation ratios of two as well as at the ratio equal to five than only the nearest interpolation algorithm. Here, it is important to note that the higher PSNR value, the better quality while the lower BRISQUE score, the better quality. In Figure 5, the results of the contrast and intensity-based metrics (of the full reference type) were presented, and the Xtrapo algorithm demonstrated a contrast decrement weaker than that of the bilinear algorithm, except at the ratio equals to five, while nearest, bicubic and lanczos achieve the weakest decrement, in most cases. However, the Xtrapo demonstrated the highest intensity increment in all cases compared to all method mentioned, except at ratio equal to five where it demonstrated lowest intensity decrement. An increment (or small decrement) in intensity and contrast generally reflects better performance or image quality.
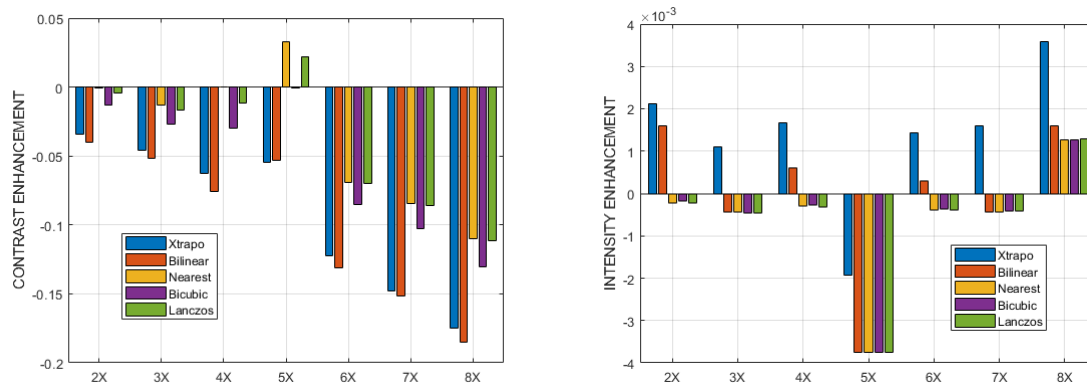
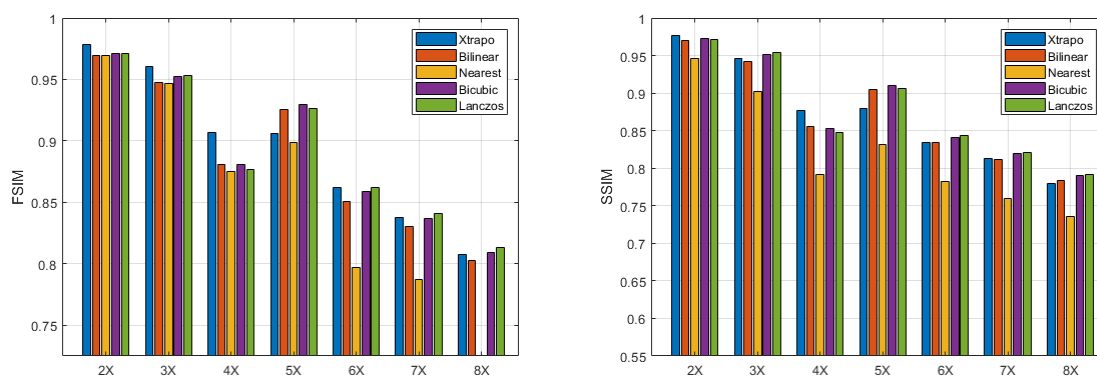Figure 5: Contrast and intensity increment/decrement
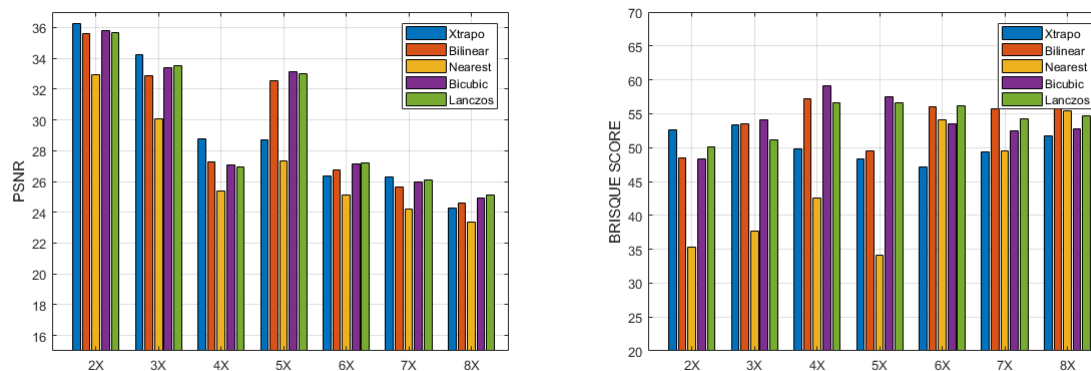

Figure 6: FSIM and SSIM

Figure 7: PSNR and BRISQUE

As can be seen in Figure 6, the Xtrapo achieved the highest FSIM values in most cases, except at the interpolation ratio that equals to five, seven and eight. Also, Xtrapo achieved the highest SSIM values only in two cases involving the ratio equal to two and four. In Figure 7, the Xtrapo achieved the highest PSNR values in the first three cases and values higher than those achieved by the nearest interpolation algorithm in the remaining cases. Furthermore, Xtrapo achieved the lowest BRISQUE scores in most cases except in the first, fourth and fifth cases and only higher BRISQUE scores than bilinear, bicubic and lanczos at the ratio equal to two as well as at the ratio equal to four and five than only the nearest interpolation algorithm. In Figure 8, the Xtrapo demonstrated a contrast decrement weaker than that of the bilinear algorithm, in all cases. Here, it is important to note that the bilinear interpolation algorithm is a commonly used method

for ultrasound scan conversion for displaying sectored image, [21]. The nearest, bicubic and lanczos algorithms achieved the weakest decrement in all cases. However, Xtrapo demonstrated the highest intensity increment in two cases involving the third and sixth ratio. Xtrapo also demonstrated a lower intensity increment than only the bilinear at the ratios equal to two, four and eight. It demonstrated the lowest intensity increment at the ratio equal to seven.
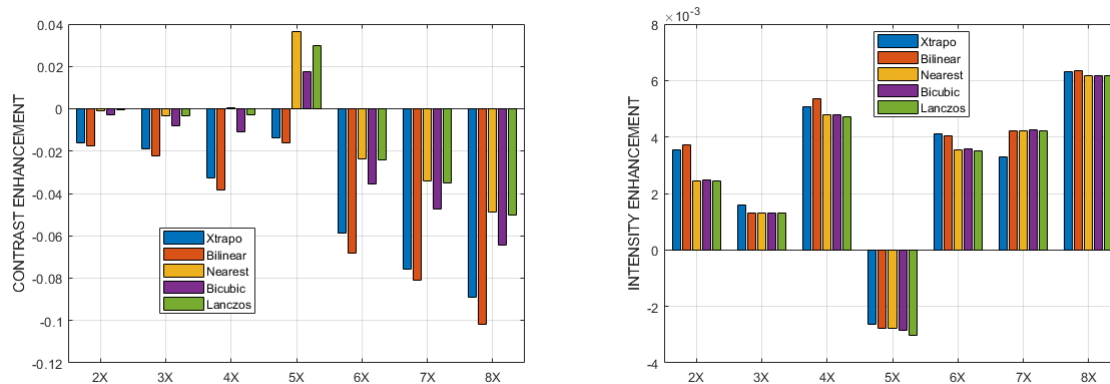

Figure 8: Contrast and Intensity increment/decrement

It is important to note that finding an objective evaluation metric that consists perfectly with subjective evaluations is not easy. As a routine, full reference metrics seek to label the better quality an interpolated image that looks like the reference image to the greatest extent, trying to consist with subjective evaluations (which is not always correct or matching). For example, in Figure 9, subjective evaluators would require that small images presented keep the same look (in terms of edges, texture, contrast, etc.) after enlarging or interpolating them. This is required because a closer view of all source details is needed to better and ease observations.

Figure 9: Source test grayscale images of the size 170 x 170

As can be seen in Figure 10, there is no big visible difference between the images interpolated using the bicubic and bilinear interpolation algorithms (which both produces blurriness artefacts, though not to the same extent thus reducing the source image quality to some extent). However, in both cases, there is a big difference between their corresponding small images shown in Figure 9. In Figure 11, there is a visible difference between the edge features of images interpolated using the lanczos algorithm (which normally creates ringing artefacts thus reducing the source image quality to some extent) and nearest algorithm (which normally creates aliasing or jaggies artefacts, even in this example, thus reducing the source image quality to some extent). Now, comparing the bicubic, lanczos, nearest and bilinear algorithm images (just mentioned) with the Xtrapo image; it can be understood that, although objective evaluation metrics, generally, demonstrated better Xtrapo algorithm performances (which can be considered as a higher accuracy in image reconstruction), it is still debatable to which extent Xtrapo images look better than other methods images, at the exception of nearest method. This repeats also in images shown in Figure 12, Figure 13, and Figure 14. It is important to note that, if the metrics produced the highest Xtrapo scores, it means that were more closely related to the corresponding reference images which may or may not match with subjective evaluations.

Figure 10: 3x-interpolated images using the bicubic algorithm (left) and bilinear algorithm (right)

Figure 11: 3x-interpolated images using the lanczos algorithm (left) and nearest algorithm (right)

Figure 12: 3x-interpolated images using the xtrapo algorithm (left) and bicubic algorithm (right)

Figure 13: 3x-interpolated images using the bilinear algorithm (left) and lanczos algorithm (right)

As previously mentioned, potential applications of the proposed method may include ultrasound scan conversion for displaying the sectored image. For example, preliminary experiments conducted using the ultrasound data to display the sectored image (on three different frames/images data), has revealed the reduction in BRISQUE-scored-distortions by 0.72%, 0.99%, 0.92% as shown in Table 3 (considering the bilinear's brisque score as the reference or original score and the xtrapo's brisque score as the target or new score). Using the same three cardiac ultrasound image data and comparing the rescaling interpolation method, in [20], to the Xtrapo method, presented here, the Xtrapo demonstrated superior performances at 0.32%, 0.57%, 0.29%, respectively. Here, note that is important to compare the proposed Xtrapo method against the bilinear interpolation algorithm because the bilinear algorithm is a commonly used interpolation method for scan conversion [21].

Figure 14: 3x-interpolated images using the nearest algorithm (left) and xtrapo algorithm (right)

Also, it is important to note that since there exists no pristine or reference ultrasound image, only a non-reference metric can be used to estimate increase or decrease in objectively quantifiable distortions with the hope that estimations would satisfy cardiologists observations.

Table 3: Bilinear and Xtrapo brisque scores

IMAGE 1		IMAGE 2		IMAGE 3	
Bilinear	Xtrapo	Bilinear	Xtrapo	Bilinear	Xtrapo
46.2410	45.9080	46.9549	46.4896	46.4631	46.0338

5. CONCLUSION

A novel image interpolation algorithm is proposed for interpolation applications. There exists in the literature, many works that demonstrated the improvement of interpolation algorithm performance by focusing on the minimization of visual artefacts. Some works on linear methods suggested the enlargement of the interpolation kernel as the key strategy to achieve higher performances which was not exactly done the same way here since we focused first on linearly extrapolating pixels and adjusting them, in the effort to ease process and improve the interpolation results. Strategies used to find the values for extrapolated pixels, the corresponding adjusted versions and weights were extensively explained in part three. Evaluations were conducted using full-reference and non-reference objective quality metrics, including variance and mean based metrics. Although objective image quality metrics demonstrated that Xtrapo algorithm achieved better performances, in some cases it remained debatable to which extent Xtrapo interpolated images looked better than the others', at the exception of the nearest algorithm images. In our future works, efforts will still be dedicated to the development of advanced algorithms improving further and particularly the quality of the sectored image. Note that, brief preliminary experiments conducted, using three cardiac ultrasound image data, demonstrated the decrease in the brisque-scored-distortions, in the sectored image.

REFERENCES

[1] Burningham, N., Pizlo, Z., Allebach, J. P., "Image Quality Metrics," In Hornak, Joseph P. Encyclopedia of imaging science and technology. New York: Wiley, (2002)

[2] Su, D., Willis, P., "Image interpolation by pixel-level data-dependent triangulation," Computer Graphics Forum, 23(2), pp. 189–201, (2004)

[3] Pan, M., Yang, X., and Tang, J., "Research on interpolation methods in medical image processing," Journal of Medical Systems, 36(2), pp. 777–807, (2010)

[4] Rukundo, O., Cao, H.Q., "Nearest neighbor value interpolation," International Journal of Advanced Computer Science and Applications, 3(4), pp. 25–30, (2012)

[5] Rukundo, O., Cao, H.Q., Huang, M.H., "Optimization of bilinear interpolation based on ant colony algorithm," Lecture Notes in Electrical Engineering 137, Springer Berlin Heidelberg, pp. 571–580, (2012)

[6] Rukundo, O., Maharaj, B.T., "Optimization of image interpolation based on nearest neighbour algorithm," International Conference on Computer Vision Theory and Applications VISAPP, pp. 641–647, (2014)

[7] Rukundo, O., Cao, H.Q., "Advances on image interpolation based on ant colony algorithm," SpringerPlus 5 (1), 403, (2016)

[8] Rukundo, O., "Effects of improved-floor function on the accuracy of bilinear interpolation algorithm," Computer and Information Science, 8(4), pp. 1–11, (2015)

[9] Rukundo, O., Wu, K.N., Cao, H.Q., "Image interpolation based on the pixel value corresponding to the smallest absolute difference," Fourth International Workshop on Advanced Computational Intelligence IWACI, pp. 432–435, (2011)

[10] Luong, H.Q., De Smet, P., Philips, W., "Image interpolation using constrained adaptive contrast enhancement techniques," IEEE International Conference on Image Processing ICIP, pp. 998–1001, (2005)

[11] Rukundo, O., "Half-unit weighted bilinear algorithm for image contrast enhancement in capsule endoscopy," Proc. SPIE 10615, Ninth International Conference on Graphic and Image Processing,106152Q, (2018)

[12] Rukundo, O., "Effects of empty bins on image upscaling in capsule endoscopy," Proc. SPIE 10420, Ninth International Conference on Digital Image Processing, 104202P, (2017)

[13] Rukundo, O., Schmidt, S., "Aliasing artefact index for image interpolation quality assessment," Proc. SPIE 10817, Optoelectronic Imaging and Multimedia Technology V, (2018)

[14] Mittal, A., Moorthy, A. K., Bovik, A. C., "No-reference image quality assessment in the spatial domain," IEEE Transaction on Image Processing 21, pp. 4695–4708, (2012)

[15] Sheikh, H.R., Sabir, M.F., Bovik, A.C., "A statistical evaluation of recent full reference image quality assessment algorithms," IEEE Transactions on Image Processing, 15(11), pp. 3440–3451, (2006)

[16] Zhang, L., Zhang, L., Mou, X., Zhang, D., "FSIM: A feature similarity index for image quality assessment," IEEE Transactions on Image Processing, 20(8), pp. 2378–2386, (2011)

[17] Wang, Z., Bovik, A.C., Sheikh, H.R., "Image Quality Assessment: From Error Visibility to Structural Similarity", IEEE Transactions on Image Processing, 13(4), pp. 600-612, (2004)

[18] Rukundo, O., Pedersen, M., Hovde, Ø., "Advanced Image Enhancement Method for Distant Vessels and Structures in Capsule Endoscopy," Computational and Mathematical Methods in Medicine, 2017, (2017)

[19] Imtiaz, M.S., Wahid, K. A., "Color enhancement in endoscopic images using adaptive sigmoid function and space variant color reproduction," Computational and Mathematical Methods in Medicine, vol. 2015, Article ID 607407, (2015)

[20] Rukundo, O., Schmidt, S., "Effects of rescaling bilinear interpolant on image interpolation quality," Proc. SPIE 10817, Optoelectronic Imaging and Multimedia Technology V, (2018)

[21] Li, X.H., "Ultrasound Scan Conversion on TI's C64x+ DSPs," Application Report SPRAB32, Texas Instruments, March, (2009)

[22] Linear Interpolation with Excel <www.datadigitization.com>, September 5, (2018)

[23] Extrapolation, <https://math.tutorvista.com>, September 5, (2018)

[24] Extrapolation, <https://en.wikipedia.org/wiki/Extrapolation>, September 5, (2018)

[25] Welstead, S.T.," Fractal and wavelet image compression techniques," SPIE Press book, pp. 155–156, (1999)

[26] Keys, R., "Cubic convolution interpolation for digital image processing," IEEE Transactions on Acoustics, Speech, and Signal Processing. 29(6), pp. 1153–1160, (1981)