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Robust Non-fragile Fuzzy Control of uncertain DC Microgrids Feeding Constant Power Loads

Navid Vafamand, Mohammad Hassan Khooban, *Senior Member, IEEE*, Tomislav Dragičević, *Senior Member, IEEE*, Frede Blaabjerg, *Fellow Member, IEEE*, and Jalil Boudjadar

Abstract—This paper investigates the problem of dynamic stabilization of DC microgrids (MGs) through a robust non-fragile fuzzy control synthesis of power buffer. The suggested robust fuzzy controller is designed to quickly stabilize the MGs by circulating the power between the DC link and an energy storage system (ESS). By employing the exponential stability analysis and Takagi-Sugeno (TS) fuzzy modeling, sufficient controller design conditions are derived in terms of linear matrix inequalities (LMIs) which bring about a simple, systematic and effective controller. The proposed approach is resilient against the uncertainties of the DC MG and energy storage system (ESS) parameters. To show the merits of the proposed approach, it is applied to a DC MG that feeds one constant power load (CPL). It is shown that the proposed approach is more robust against system and controller uncertainties compared to the existing results. Finally, experimental results are then presented that show the transient performance improvement of the closed-loop system compared to the state-of-the-art methods.

Index Terms—DC microgrid, constant power load, Takagi-Sugeno (TS) fuzzy model, fuzzy controller, exponential stability, non-fragile controller, uncertainty.

I. INTRODUCTION

With the expanding penetration of sustainable power sources into the modern power network, the idea of aggregating groups of them into microgrid (MG) architectures and thus simplify their management, has been attracting a lot of research interest over the last 15 years [1], [2]. Amid the ongoing years, MGs have been generally considered in a lot of papers. Since the traditional electric network depends on ac systems, the study on MGs is predominantly centered on alternating current (AC) architectures [3]–[6]. Be that as it may, keeping in mind the end goal to incorporate different sustainable power sources with direct current (DC) couplings, the arrangement of DC MGs has turned into a drawing in approach. As opposed to AC MGs, DC MGs are known for the benefits of higher effectiveness, nonappearance of reactive power and harmonics, and so forth [7]. These DC MGs are made out of numerous sources and dc/ac loads, which are linked together through power electronic converters and filters. Typically, the interface converters are applied in a DC MG to connect the sustainable power sources to the common bus [8], [9]. In addition, power electronic converters are

paralleled to form the distributed structure of the system. An ordinary structure of these frameworks is illustrated in Fig. 1.

Among the topics, which have been studied in the DC MG, the controlling and stabilizing of the DC MG with constant power loads (CPLs) has been attracting a lot of attention [10]–[12]. The investigation of CPLs in the DC MG is essential in marine power grids [13], automotive, and spacecraft applications [14], as well as induction motor drives.

Current sharing and voltage regulation are employed to stabilize a DC MG. The negative impedance property of the CPLs could make the overall system unstable. Due to the nonlinear nature of the DC MGs with CPLs, several nonlinear control methods have been proposed to investigate the stability [4] and alleviate the undesired effects of the CPLs in such systems [15]–[17]. In [15], a linear state-feedback controller is designed to assure the stability of the closed-loop system. Then, injecting power's reference is generated based on the obtained control law. However, this approach needs to inject a stabilizing current at all the CPLs to mitigate their effects. In [16], [17] backstepping techniques are proposed in which the derivative terms appear. However, in the presence of noise, these approaches fail to completely cancel the CPL's nonlinearity, since noise can yet amplify [18]. Recently, the simplicity and effectiveness of numerical linear matrix inequality (LMI) techniques persuaded many types of research to perform the stability analysis and controller synthesis of DC MGs with CPLs using such techniques [18], [19]. Unfortunately, in [19], only the stability analysis is investigated through the Takagi-Sugeno (TS) fuzzy model; and in [18], a robust linear controller is developed, which reduces the applicability and performance of such approaches for DC MGs. Therefore, more effort is still needed to mitigate the effect of CPLs.

In spite of the rich body of literature on DC MGs with CPLs, almost all of the existing studies assumed that the controller can be implanted without any error or inaccuracy, and the value of DC MG parameters are exactly known. However in practice, we are faced with uncertainties in the parameters of the DC MGs with CPLs, inaccuracies in the energy storage system (ESS) models, and controller errors due to computational delays, quantization effects, and computational approximations. Therefore, the designed controller must be robust against the system uncertainty and the controller implementation errors.

In this paper, a simple and systematic approach to design a robust fuzzy model-based controller for DC MGs with CPLs is proposed. The proposed approach employs the TS fuzzy model and parallel distributed compensation (PDC) scheme to design the injecting current of the storage unit to quickly stabilize the system. Through the Lyapunov stability theory, the controller design conditions are derived in terms of LMIs.

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The nonlinear controller design procedure is as simple as the linear one in [18], but the transient performance of the proposed approach is remarkably enhanced. The suggested controller is robust against the DC MG parameter uncertainties and is resilient against its implementation errors. Experimental results illustrate the settling time reduction of the proposed approach compared to the state-of-the-art methods and high efficiency when the controller and the system face the uncertainties.

II. DC MICROGRID DYNAMIC

Consider the DC MG shown in Fig. 1, which comprises several constant power loads (CPLs), a DC source, and an energy storage system (ESS). As it can be seen in Fig. 1, CPLs are introduced by tightly regulating DC or AC loads to achieve a constant power load on the input side of the converters [9]. In addition, it is assumed that the voltage of the DC sources is constant and not controllable. So, the DC link is supplied by a constant DC source [18].

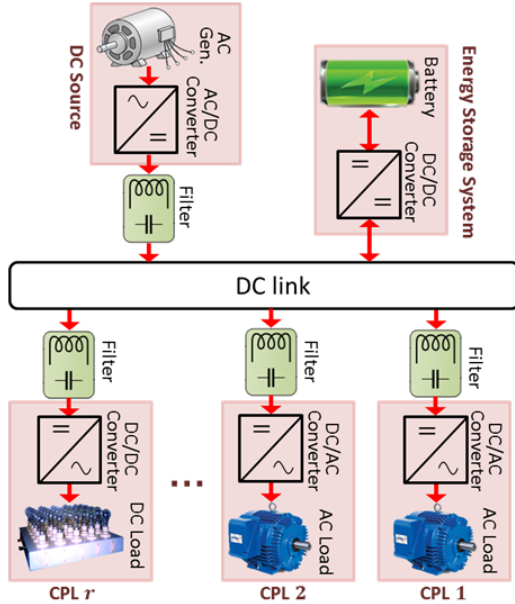


Fig. 1. Detailed DC MG of more electric aircraft, navy ships, automotive, etc.

In order to derive the overall dynamic of a DC MG with multiple CPLs, initially, the properties of one CPL with a DC source and a source subsystem are investigated. Consider the j -th CPL which is connected to the DC link, as shown in Fig. 2. A CPL is modelled by a voltage controlled current source, where its value is nonlinearly dependent to the CPL's power and voltage. Also, for simplicity, the DC link is modeled by the voltage source V_e . The dynamic equations for the j -th CPL system in series with an RLC filter and the DC link shown in Fig. 2 are given by

$$\begin{cases} \dot{i}_{L,j} = -\frac{r_{L,j}}{L_j} i_{L,j} - \frac{1}{L_j} v_{C,j} + \frac{1}{L_j} V_e \\ \dot{v}_{C,j} = \frac{1}{C_j} i_{L,j} - \frac{1}{C_j} \frac{P_j}{v_{C,j}} \end{cases} \quad (1) \quad \text{and}$$

where P_j is a constant power of the j -th CPL. For the equilibrium point $[i_{L0,j} \ v_{C0,j}]$, the constant power P_j must satisfy the constraint [18]

$$P_j < \min \left\{ \frac{V_{dc}}{4r_j}, \frac{r_j C_j v_{C0,j}^2}{L} \right\} = P_{max,j} \quad (2)$$

to guarantee that the Jacobian matrix of (1) is Hurwitz and has a negative real part and assure the existence of a real-operating point.

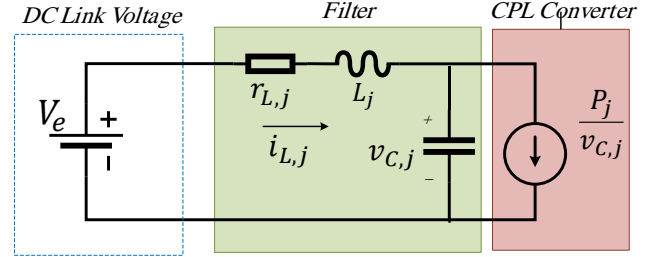


Fig. 2. Simplification of the j -th source and power converter load as a CPL.

Now, consider the voltage source connected to the ESS through an RLC filter, as shown in Fig. 3. The ESS is modeled by the current source i_{es} and the state-space representation is of the form:

$$\begin{cases} \dot{i}_{L,s} = -\frac{r_s}{L_s} i_{L,s} - \frac{1}{L_s} v_{C,s} + \frac{1}{L_s} V_{dc} \\ \dot{v}_{C,s} = \frac{1}{C_s} i_{L,s} - \frac{1}{C_s} v_{C,s} - \frac{1}{C_s} i_{es} \end{cases} \quad (3)$$

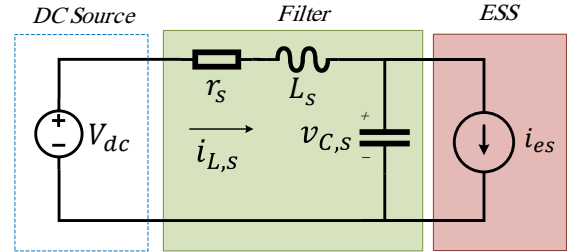


Fig. 3. Simplification of the storage energy supply.

The states of the nonlinear system (1) and (3) are shifted in such a way that a new nonlinear dynamic with origin equilibrium point is obtained. Such a transformation facilitates stability analysis and controller synthesis of nonlinear systems based on the Lyapunov stability theory. In other words, to perform the stability analysis by the Lyapunov stability method, it is necessary to have a dynamical system in which its equilibrium point is at the origin. To do this, by considering a change of coordinates, the dynamics (1) and (3) will be respectively turned to [18]

$$\begin{cases} \dot{\tilde{i}}_{L,j} = -\frac{r_{L,j}}{L_j} \tilde{i}_{L,j} - \frac{1}{L_j} \tilde{v}_{C,j} \\ \dot{\tilde{v}}_{C,j} = \frac{1}{C_j} \tilde{i}_{L,j} + \frac{P_j}{C_j v_{C0,j}} \frac{\tilde{v}_{C,j}}{\tilde{v}_{C,j} + v_{C0,j}} \end{cases} \quad (4)$$

$$\begin{cases} \dot{\tilde{i}}_{L,s} = -\frac{r}{L}\tilde{i}_{L,s} - \frac{1}{L}\tilde{v}_{C,s} \\ \dot{\tilde{v}}_{C,s} = \frac{1}{C_s}\tilde{i}_{L,s} + -\frac{1}{C_s}\tilde{v}_{C,j} - \frac{1}{C_s}\tilde{i}_{es} \end{cases} \quad (5)$$

Based on the single CPL connected to the DC link and an ESS connected to the DC source, the overall MG with multiple CPLs, an ESS, and a DC source which are connected through RLC filters as shown in Fig. 4 can be derived. As it is evident from Fig. 4, we can decouple overall DC MG into $Q + 1$ subsystems (i.e. Q CPLs and one DC source).

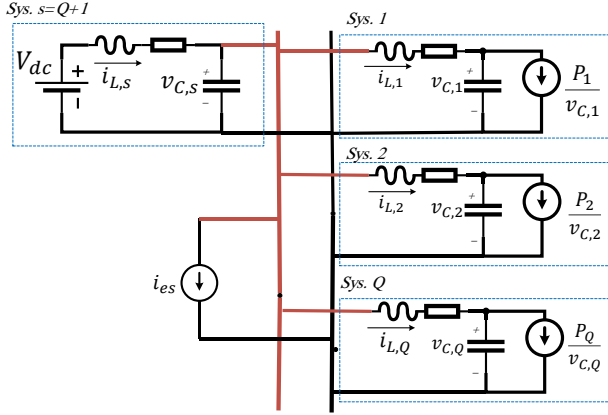


Fig. 4. A simplified version of the DC MG shown in Fig. 1 with Q CPLs.

The state-space representations of the CPLs' filters are of the form

$$\dot{x}_j = \bar{A}_j x_j + \bar{d}_j n_j + \bar{A}_{js} x_s \quad (6)$$

for $j = \{1, 2, \dots, Q\}$ and $s = Q + 1$ stands for the DC source filter's states. Also, $x_j = [i_{L,j} \ v_{C,j}]^T$ and $x_s = [i_{L,s} \ v_{C,s}]^T$ are the states of the j -th CPL's filter and the DC source's filter, respectively, and

$$\bar{A}_j = \begin{bmatrix} -\frac{r_{L,j}}{L_j} & -\frac{1}{L_j} \\ \frac{1}{C_j} & 0 \end{bmatrix}, \bar{d}_j = \begin{bmatrix} 0 \\ \frac{P_j}{C_j} \end{bmatrix}, n_j = \frac{1}{v_{C,j}}, \bar{A}_{js} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad (7)$$

The source subsystem can be described by

$$\dot{x}_s = \bar{A}_s x_s + \bar{b}_s V_{dc} + \bar{b}_e i_{es} + \sum_{j=1}^Q \bar{A}_{cnj} x_j \quad (8)$$

with matrices

$$\bar{A}_s = \begin{bmatrix} -\frac{r_s}{L_s} & -\frac{1}{L_s} \\ \frac{1}{C_s} & 0 \end{bmatrix}, \bar{b}_s = \begin{bmatrix} 1 \\ \frac{1}{C_s} \end{bmatrix}, \bar{A}_{cn} = \begin{bmatrix} 0 & 0 \\ -\frac{1}{C_s} & 0 \end{bmatrix}, \bar{b}_{es} = \begin{bmatrix} 0 \\ -\frac{1}{C_s} \end{bmatrix} \quad (9)$$

Again, assuming a coordinate change about an operating point and letting the ESS current i_{es} be the control input, we can rewrite the overall DC MG, the equilibrium point of which is at origin, in the following form [9]:

$$\dot{\tilde{X}} = \tilde{A}\tilde{X} + D h(\tilde{X}) + B_{es}\tilde{i}_{es} + B_s\tilde{V}_{dc} \quad (10)$$

where $\tilde{X} = [x_1^T \ x_2^T \ \dots \ x_Q^T \ x_s^T]^T$ and

$$A = \begin{bmatrix} \bar{A}_1 & 0 & \dots & 0 & \bar{A}_{1s} \\ 0 & \bar{A}_2 & \dots & 0 & \bar{A}_{2s} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \bar{A}_Q & \bar{A}_{Qs} \\ \bar{A}_{cn} & \bar{A}_{cn} & \dots & \bar{A}_{cn} & \bar{A}_s \end{bmatrix}, \quad (11)$$

$$D = \begin{bmatrix} \bar{d}_1 & 0 & \dots & 0 \\ 0 & \bar{d}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \bar{d}_Q \\ 0 & 0 & \dots & 0 \end{bmatrix}, B_{es} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \bar{b}_{es} \end{bmatrix}, B_s = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \bar{b}_s \end{bmatrix}$$

and $h = [h_1, \dots, h_Q]^T$ with

$$h_j(\tilde{X}) = \frac{\tilde{v}_{C,j}}{v_{C0,j}(\tilde{v}_{C,j} + v_{C0,j})} \quad (12)$$

Same as the relations (3) and (4), the equilibrium point of the system (10) is the origin. This means that the nonlinear terms h_j appearing in (10) satisfy $h_j(0) = 0$. As it can be seen in (10), the overall MG system comprises Q nonlinear terms (i.e. h_j). In order to derive the T-S fuzzy model, the so-called sector nonlinearity approach is employed to exactly represent each of the nonlinearities by an equivalent T-S model in a pre-defined local region. For the j -th CPL, define the region $R_{j,\tilde{x}} = \{\tilde{x} | -\tilde{w}_{1,j} \leq \tilde{i}_{L,j} \leq \tilde{w}_{1,j} \ \& \ -\tilde{w}_{2,j} \leq \tilde{v}_{C,j} \leq \tilde{w}_{2,j}\}$, where $\tilde{w}_{1,j}$ and $\tilde{w}_{2,j}$ are positive scalars. Consequently, the equivalent T-S fuzzy model of (10) for the region $R = \bigcap_{j=1,\dots,Q} R_{j,\tilde{x}}$ for can be systematically obtained. The selection of the parameters $\tilde{w}_{1,j}$ and $\tilde{w}_{2,j}$ is completely arbitrary and is based on the variation range of the overall DC MG states. However, in this paper, these parameters are selected based on the local stability analysis given in [18] where a systematic LMI approach to compute the maximum values for the CPLs to be locally stable is presented. Note that, in the following, the method of TS fuzzy modelling which is a function of the parameters $\tilde{w}_{1,j}$ and $\tilde{w}_{2,j}$ will be discussed. So, the obtained TS fuzzy model is valid for any selection of the parameters $\tilde{w}_{1,j}$ and $\tilde{w}_{2,j}$.

III. TS FUZZY MODELING

A TS fuzzy model is a nonlinear fuzzy blending of a finite number of local linear state-space representations of a nonlinear dynamical system [20]. Thorough the TS fuzzy modeling, a nonlinear system is represented by IF-THEN rules in which the premise variable is still nonlinear and the consequent part comprises linear dynamics [21], [22]. Therefore, the TS fuzzy model is able to exactly capture the behavior of smooth nonlinear system. Because of their linear consequent part, the TS fuzzy modeling provides a straightforward approach to applying the well-known linear control theory to the nonlinear systems without the need of linearization. The combination of the linear control theory and fuzzy concepts allows the usage of simple linear controllers to assure the semi-global stability [23]. This is the main advantage of the utilizing TS-based controllers compared to the other conventional linear and nonlinear control methods.

Designing a linear controller has a simple procedure but can only guarantee the local stability near the operating point; and, the nonlinear controllers can assure the semi-global stability at the expense of highly complex controller procedure. However,

the design procedure of the TS-based controller is simple and based on the linear control theory and the closed-loop stability is obtained by the means of fuzzy mathematics. Moreover, the smooth fuzzy membership functions of the fuzzy controllers not only lead to a smooth transition among the local linear systems, but also, the closed-loop stability of between the local models is also guaranteed. This feature distinguishes the TS-based fuzzy controllers from the piecewise controllers in which the stability among the local regions is not assured [20]. Thereby, in this paper, a novel TS-based fuzzy controller is proposed for the dynamic stability of DC MGs with CPLs.

Since the fuzzy controller is on the basis of the TS fuzzy model, it is necessary to obtain the local linear systems and the fuzzy membership functions. Among all of the presented methods, an equivalent TS fuzzy model of nonlinear systems is systematically computed by deploying the so-called sector nonlinearity approach [24], [25]. In this approach, each nonlinear term of the original system gets inside two linear sectors. Then, all groups of the two sectors are aggregated to derive the membership functions and local matrices of the TS fuzzy.

Without loss of generality, consider a MG with one CPL and one source [26]. As it is discussed in [26], a MG with several CPLs can be turned to a MG with one equivalent CPL. So, for the sake of simplicity, the procedure of a MG with one CPLs will be discussed in the following. To calculate the TS fuzzy model, one needs to model the nonlinearities of the model (10) [24]. Only one nonlinear term (i.e. h_1 defined in (12)) exists in the dynamic. For a given region $-\tilde{w}_{2,1} \leq \tilde{v}_{C,1} \leq \tilde{w}_{2,1}$, one can compute the sectors $U_{min}\tilde{v}_{C,1}$ and $U_{max}\tilde{v}_{C,1}$ so that $U_{min}\tilde{v}_{C,1} \leq h_1 \leq U_{max}\tilde{v}_{C,1}$ where the lower and upper slopes U_{min} and U_{max} . In Fig. 5, the nonlinear term h_1 and its corresponding sectors are plotted.

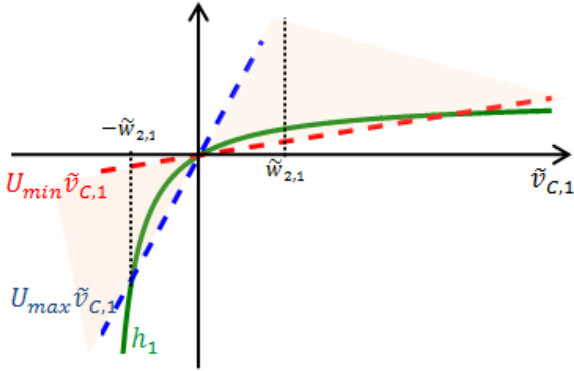


Fig. 5. Sector nonlinearity approach for the DC MG with one CPL.

The slopes are simply computed by turning the sector inequality $U_{min}\tilde{v}_{C,1} \leq h_1 \leq U_{max}\tilde{v}_{C,1}$ into a min-max inequality $U_{min} \leq \frac{h_1}{\tilde{v}_{C,1}} \leq U_{max}$, as

$$U_{min} = \frac{1}{v_{C0,1}(\tilde{w}_{2,1} + v_{C0,1})}, U_{max} = \frac{1}{v_{C0,1}(-\tilde{w}_{2,1} + v_{C0,1})} \quad (13)$$

Based on the sector nonlinearity approach, consider

$$\begin{cases} h_1 = M_1 U_{min} \tilde{v}_{C,1} + M_2 U_{max} \tilde{v}_{C,1} \\ M_1 + M_2 = 1 \end{cases} \quad (14)$$

Solving (14) obtains the membership functions M_1 and M_2 as

$$\begin{cases} M_1 = \frac{U_{max}\tilde{v}_{C,1} - h_1}{(U_{max} - U_{min})\tilde{v}_{C,1}} \\ M_2 = \frac{h_1 - U_{min}\tilde{v}_{C,1}}{(U_{max} - U_{min})\tilde{v}_{C,1}} \end{cases} \quad (15)$$

Substituting (14) into (10), the equivalent T-S fuzzy model can be obtained as

$$\dot{\tilde{X}} = \sum_{i=1}^2 M_i \{A_i \tilde{X} + B_{es} \tilde{i}_{es} + B_s \tilde{V}_{dc}\} \quad (16)$$

where

$$A_1 = \begin{bmatrix} -\frac{r_1}{L_1} & -\frac{1}{L_1} & 0 & \frac{1}{L_1} \\ \frac{1}{C_1} & \frac{P_1}{C_1} U_{min} & 0 & 0 \\ 0 & 0 & -\frac{r_s}{L_s} & -\frac{1}{L_s} \\ -\frac{1}{C_s} & 0 & \frac{1}{C_s} & 0 \end{bmatrix}, B_{es} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{C_s} \end{bmatrix}, \quad (17)$$

$$A_2 = \begin{bmatrix} -\frac{r_1}{L_1} & -\frac{1}{L_1} & 0 & \frac{1}{L_1} \\ \frac{1}{C_1} & \frac{P_1}{C_1} U_{max} & 0 & 0 \\ 0 & 0 & -\frac{r_s}{L_s} & -\frac{1}{L_s} \\ -\frac{1}{C_s} & 0 & \frac{1}{C_s} & 0 \end{bmatrix}, B_s = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_s} \\ 0 \end{bmatrix}$$

As it can be seen from (16), the TS fuzzy model comprises nonlinear membership functions M_i and linear state-space representations $A_i \tilde{X} + B_{es} \tilde{i}_{es} + B_s \tilde{V}_{dc}$ for $i = 1, 2$. This special structure, which satisfies $0 \leq M_i \leq 1$, facilitates using a fuzzy aggregation of several linear controllers, which will be discussed in the following section.

IV. MAIN RESULTS

In order to stabilize the TS fuzzy model (16), two cases will be considered. In the first case, it is assumed that the parameters of the DC MG are completely known and that system contains no uncertainties. In the second case, the uncertainty is considered in the system parameters. Also, it is assumed that ESS has some uncertainties.

A. Controller design for the nominal system without uncertainty

Since for this case, it is assumed that overall system parameters are exactly known and constant, the nominal dynamical system (16) is considered and the following fuzzy control law for designing \tilde{i}_{es} is proposed:

$$\tilde{i}_{es} = \sum_{i=1}^2 M_i K_i \tilde{X} \quad (18)$$

where M_i is defined in (15) and the matrices K_i must be designed properly to force the part converge of the MG's states to their nominal value. Note that the structure of the fuzzy control law (18) is constructed based on the associated TS fuzzy model, as both share the same fuzzy membership functions M_i . This kind of fuzzy controller is known as the PDC [20]. To design the PDC's gains K_i , the exponential Lyapunov stability scheme is used.

Definition 1 [20]: If Lyapunov function V satisfies the following inequality

$$\dot{V} + 2\sigma V < 0 \quad (19)$$

where $\sigma > 0$ is the decay rate, then the Lyapunov function exponentially converges to zero and the system is exponentially stable.

Note that, the exponential stability scheme enhances the transient performance in comparison with the conventional Lyapunov stability which is widely investigated in almost all of the existing results on DC MGs with CPLs [4], [9], [16]–[19]. Based on the definition 1, the following theorem is proposed in which the exponential stabilization conditions are derived in terms of LMIs.

Theorem 1: The nominal TS fuzzy system (16) is exponentially stabilized by the controller (18), if for a given decay rate $\sigma > 0$, there exist symmetric matrix X and matrices K_1 and K_2 such that the following LMI conditions hold:

$$P > 0 \quad (20)$$

$$XA_1^T + A_1X + B_{es}N_1 + N_1^TB_{es}^T + \sigma P < 0 \quad (21)$$

$$XA_2^T + A_2X + B_{es}N_2 + N_2^TB_{es}^T + \sigma P < 0 \quad (22)$$

Proof: Consider a quadratic Lyapunov candidate $V = \tilde{X}^T P \tilde{X}$. The constraint (20) is the sufficient condition that assures the positive definiteness of the Lyapunov candidate. Substituting time derivative of the Lyapunov function in (19) results in:

$$\dot{V} + 2\sigma V = \tilde{X}^T P \tilde{X} + \tilde{X}^T P \dot{\tilde{X}} + \sigma \tilde{X}^T P \tilde{X} \quad (23)$$

Considering (16) with $\tilde{V}_{dc} = 0$ and (18), (23) is continued as

$$\dot{V} + 2\sigma V = \sum_{i=1}^2 M_i \{ \tilde{X}^T (A_i^T P + K_i^T B_{es}^T P + P A_i^T + P B_{es} K_i + \sigma P) \tilde{X} \} \quad (24)$$

Since $M_i \geq 0$, the condition $\dot{V} + 2\sigma V < 0$ is guaranteed by $A_i^T P + K_i^T B_{es}^T P + P A_i^T + P B_{es} K_i + \sigma P < 0$, $i = 1, 2$ (25)

Pre- and post-multiplying (25) by P^{-1} and defining the variable changes $X = P^{-1}$ and $N_i = K_i P^{-1}$, the LMIs (21) and (22) are obtained. Thus, the proof is complete.

B. Robust non-fragile Controller design for the uncertain DC MG system

In practical DC MGs with CPLs, the parameters of the system can contain uncertainties and vary in time. Generally, three uncertainties can be considered for the nominal system (16): (I) *filters' parameters*, (II) *ESS parameters*, and (III) *CPLs' power value*. In this paper, only the two first sources of uncertainty are addressed. One way to describe a system with uncertainty terms is to consider additive bounded terms to the nominal TS fuzzy systems [27], as for the considered DC MG case study one has

$$\dot{\tilde{X}} = \sum_{i=1}^2 M_i \{ (A_i + \Delta A_i) \tilde{X} + (B_{es} + \Delta B_{es}) \tilde{i}_{es} + (B_s + \Delta B_s) \tilde{V}_{dc} \} \quad (26)$$

where ΔA_i , ΔB_{es} , and ΔB_s stand for overall system uncertainty matrix. Another practical issue is the uncertainties and inaccuracies in implementing the controller such as voltage variations of the ESS unit, uncertainties of the controller gains, or digital implementations of the controller. Consequently, a robust non-fragile controller should be designed to be resilient against the system parameter uncertainties and ESS unit. Without loss of generality, one can consider control input matrix uncertainty ΔB as the ESS unit uncertainty and deploys the following open-loop system

$$\dot{\tilde{X}} = \sum_{i=1}^2 M_i \{ (A_i + \Delta A_i) \tilde{X} + B_{es} \tilde{i}_{es} + (B_s + \Delta B_s) \tilde{V}_{dc} \} \quad (27)$$

and design a non-fragile controller of the form

$$\tilde{i}_{es,nfr} = \sum_{i=1}^2 M_i (K_{i,nfr} + \Delta K_{i,nfr}) \tilde{X} \quad (28)$$

where $\Delta K_{i,nfr}$ for $i = 1, 2$ are the controller gain uncertainties. The $\Delta K_{i,nfr}$ are considered in the controller design procedure to design a control signal which is robust against itself uncertainty and the control input matrix uncertainty. The uncertain closed-loop system is obtained by incorporating (27) and (28), as

$$\dot{\tilde{X}} = \sum_{i=1}^2 M_i \{ (A_i + B_{es} K_{i,nfr} + \Delta A_i + B_{es} \Delta K_{i,nfr}) \tilde{X} + (B_s + \Delta B_s) \tilde{V}_{dc} \} \quad (29)$$

Assumption 1: Assume that the uncertainty matrices are bounded [20], as

$$\Delta A_i \Delta A_i^T < \delta_a^2 I, \Delta K_{i,nfr} \Delta K_{i,nfr}^T < \delta_k^2 I \quad (30)$$

where δ_a and δ_k are given positive scalars.

Lemma 1 [28]: For any arbitrary matrices A and B and $Q > 0$, the following relation holds

$$AB^T + BA^T \leq AQA^T + BQ^{-1}B^T \quad (31)$$

In the following, the gains of the robust non-fragile controller (28) (i.e. $K_{i,nfr}$) will be computed through an LMI approach.

Theorem 2: The uncertain TS fuzzy system (27) is robustly and exponentially stabilized by the non-fragile controller (28), if for a given decay rate $\sigma > 0$ and upper bound uncertainties $\delta_a \geq 0$ and $\delta_k \geq 0$, there exist symmetric matrix X , Q_1 , and Q_2 and matrices $N_{1,nfr}$ and $N_{2,nfr}$ such that the following LMI conditions hold:

$$X > 0 \quad (32)$$

$$\begin{bmatrix} \left\{ \begin{array}{l} XA_1^T + N_{1,nfr}^T B_{es}^T + A_1^T X + B_{es} N_{1,nfr} \\ + \sigma X + Q_1 + B_{es} Q_2 B_{es}^T \end{array} \right\} & \delta_a X & \delta_k X \\ \delta_a X & -Q_1 & 0 \\ \delta_k X & 0 & -Q_2 \end{bmatrix} < 0 \quad (33)$$

$$\begin{bmatrix} \left\{ \begin{array}{l} XA_2^T + N_{2,nfr}^T B_{es}^T + A_2^T X + B_{es} N_{2,nfr} \\ + \sigma X + Q_1 + B_{es} Q_2 B_{es}^T \end{array} \right\} & \delta_a X & \delta_k X \\ \delta_a X & -Q_1 & 0 \\ \delta_k X & 0 & -Q_2 \end{bmatrix} < 0 \quad (34)$$

Proof: Recall the relation (23) and substituting (29) with $\tilde{V}_{dc} = 0$, one has

$$\dot{V} + 2\sigma V = \sum_{i=1}^2 M_i \{ \tilde{X}^T (A_i^T P + K_{i,nfr}^T B_{es}^T P + P A_i^T + P B_{es} K_{i,nfr} + \sigma P + \Delta A_i^T P + P \Delta A_i + P B_{es} \Delta K_{i,nfr} + \Delta K_{i,nfr}^T B_{es}^T P) \tilde{X} \} < 0 \quad (35)$$

Since $M_i \geq 0$, (35) is implied by

$$A_i^T P + K_{i,nfr}^T B_{es}^T P + P A_i^T + P B_{es} K_{i,nfr} + \sigma P + \Delta A_i^T P + P \Delta A_i + P B_{es} \Delta K_{i,nfr} + \Delta K_{i,nfr}^T B_{es}^T P < 0 \quad (36)$$

Considering Assumption 1 and Lemma 1, (36) is continued as

$$\begin{aligned} & A_i^T P + K_{i,nfr}^T B_{es}^T P + P A_i^T + P B_{es} K_{i,nfr} + \sigma P + P Q_1 P \\ & + \Delta A_i^T Q_1^{-1} \Delta A_i + P B_{es} Q_2 B_{es}^T P \\ & + \Delta K_{i,nfr}^T Q_2^{-1} \Delta K_{i,nfr} \\ & \leq A_i^T P + K_{i,nfr}^T B_{es}^T P + P A_i^T + P B_{es} K_{i,nfr} + \sigma P + P Q_1 P \\ & + P B_{es} Q_2 B_{es}^T P + \delta_a^2 Q_1^{-1} + \delta_k^2 Q_2^{-1} < 0 \end{aligned} \quad (37)$$

Pre- and post-multiplying (37) by P^{-1} , defining the variable changes $X = P^{-1}$ and $N_{i,nfr} = K_{i,nfr}P^{-1}$ and applying the Schur complement [20] result the LMIs (33) and (34).

Remark 1 (design procedure of the proposed approach):

The proposed approach offers a systematic procedure to design a fuzzy controller for the DC MGs with CPLs. Therefore, by changing the parameters of the DC MG, ESS, and/or number of the CPLs, the suggested method designs the gains of the fuzzy controller, simple. The overall design procedure is given in Fig. 6. As can be seen in Fig. 6, initially, the equivalent TS fuzzy representation of the nonlinear DC MG is computed to have the membership functions M_i local matrices A_i , B_{es} , and B_s . Then, the upper bounds of the uncertainty terms δ_a and δ_k are computed. Finally, based on these given matrices and scalars, Theorems 1 and 2 will be used to compute the gains K_i and $K_{i,nfr}$. These gains can be calculated through the numerical LMI solvers and there is no need to compute such gains by hand. Then, the overall controller is constructed based on the local controller gains and membership functions.

Remark 2 (advantages of the proposed approach):

To the best knowledge of the authors, this paper is the first attempt to design a robust non-fragile controller for DC MGs with CPLs that assure the exponential stability of the uncertain closed-loop system. Besides this, the proposed nonlinear control scheme has following advantages over the existing nonlinear control methods.

1. Comparing with backstepping and sliding mode controllers, the proposed approach does not need any time derivatives. So, the proposed approach is more robust against noise. Also, the controller design procedure is simpler than the mentioned conventional controllers. Since the proposed approach employs the linear control theory; meanwhile, the other approaches are established based on the nonlinear control theory. Also, the unknown gains of proposed controller are computed numerical LMI techniques; however, the other methods are designed analytically.

2. Comparing with feedback linearization, the proposed approach is more robust against the system uncertainties, because no nonlinearity cancelation is needed.

3. Comparing with conventional Mamdani-based fuzzy controllers, the proposed approach guarantees the closed-loop stability through the Lyapunov theory.

4. Comparing with the linear controllers, the proposed approach brings about semi-global stabilization and more effective transient performance. This is due to a fact that, the nonlinear fuzzy-model based controller is more compatible with nonlinear nature of the MG. Also, both the linear and proposed controllers are designed based on the linear control theory. Therefore, the design and implementation of these controllers are straightforward.

Based on the above advantages, for the first time, a fuzzy PDC control scheme is considered for the nonlinear DC MGs with CPLs and the stabilization conditions are derived in terms of LMIs.

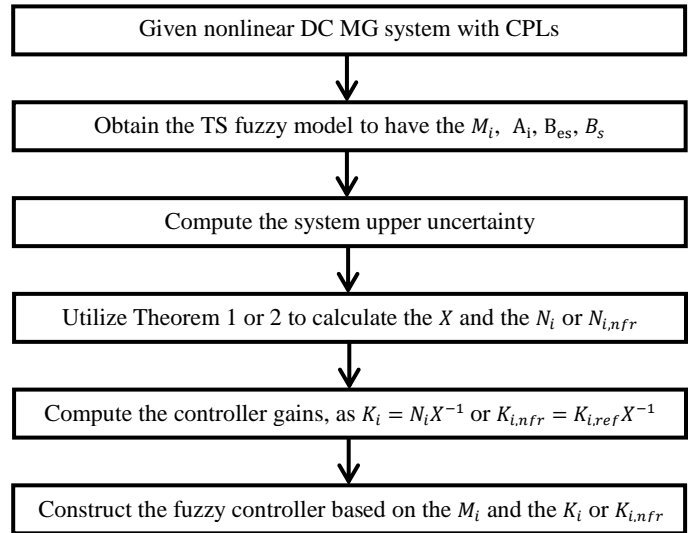


Fig. 6. The overall flowchart of controller design.

V. EXPERIMENTAL RESULTS

To investigate the robustness and fast transient performance of the proposed approach, it is tested experimentally on a test system given in Fig. 7 and the results are compared with [18]. The control algorithm is implemented in the DSpace MicroLabBox with DS1202 PowerPC Dual- Core 2 GHz processor board and DS1302 I/O board. As can be seen in Fig. 7(b), the voltages and currents of the RLC filters are measured and utilized in the fuzzy controller. The gains of the fuzzy controller are designed based on Theorems 1 and 2 and overall controller is constructed as depicted in Fig. 6. Since, only one CPL is considered, the fuzzy controller has two rules. The injecting current reference \tilde{i}_{es} is computed based on the measured states and then utilized in the energy storage unit.

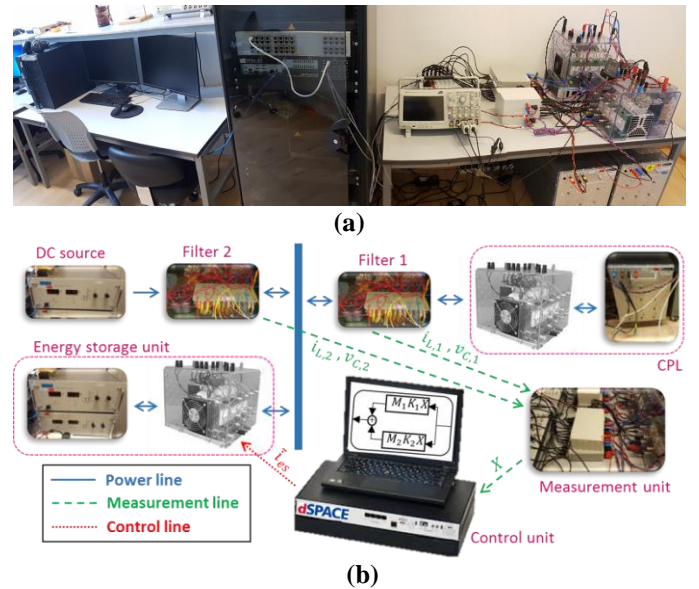


Fig. 7. Setup for testing the control approach (a). Experimental setup (b). Simplified configuration.

In the following, two scenarios are considered. In the first one, it is assumed that the all parameters of the DC MG are exactly known and determined. Therefore, the controller (18)

is designed based on the conditions of Theorem 1. Meanwhile, in the second scenario, the overall DC MG experience system uncertainty. For this case, the gains of the controller (28) are computed by the means of Theorem 2. The obtained results are compared with the robust linear controller [18]. Note that the DC MG case study of [18] comprises two CPLs and higher value for the filter inductors. However, in this paper, a DC MG with one CPL and different parameter values is utilized. Thereby, the presented controller [18] is re-designed for the case study of this paper.

Scenario 1 (nominal system): In this scenario, the values of the filters and the ESS are assumed to be exactly known without any uncertainties. These values are provided in Table I.

Table I: Parameters for DC MG with one CPL

$r_1 = 1.1 \Omega$	$v_{C0,1} = 196.64$	$C_s = 500 \mu F$
$L_1 = 3.9 mH$	$\tilde{x}_{2N} = 130.4$	$V_{dc} = 200 V$
$C_1 = 500 \mu F$	$r_s = 1.1 \Omega$	
$P_1 = 300 W$	$L_s = 3.9 mH$	

By applying the proposed Theorem 1 (T1) with the decay rate $\sigma = 50$, the following controller gains are obtained:

$$K_1 = [20.3262 \quad 1.7109 \quad -0.7600 \quad 0.3251]$$

$$K_2 = [20.3035 \quad 1.6932 \quad -0.7363 \quad 0.3241]$$

Also, employing the robust linear [18] on the MG results a linear controller $\tilde{i}_{es} = F\tilde{X}$, where

$$F = [29.8742 \quad 0.6326 \quad 1.1017 \quad 0.3556]$$

The simulation is carried out by choosing the initial conditions $\tilde{X} = [0 \quad 15 \quad 0 \quad 10]^T$. Also, the current \tilde{i}_{es} is saturated by lower and upper limits ± 10 . Fig. 8 demonstrates the closed-loop voltage and current of the CPL and storage energy and the controlled current of the storage energy supply. Experimental results show that the proposed simple nonlinear approach effectively improves the transient performance of the DC MG. Fig. 8 reveals that the settling time of the closed-loop DC MG is reduced more than 2 times compared to the robust linear [18] and more than 20 times in comparison with the DC MG without ESS.

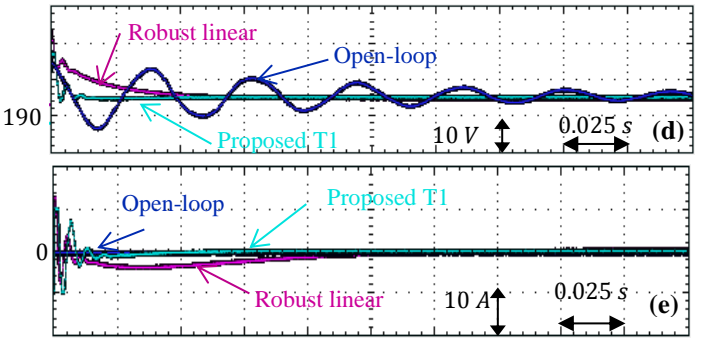
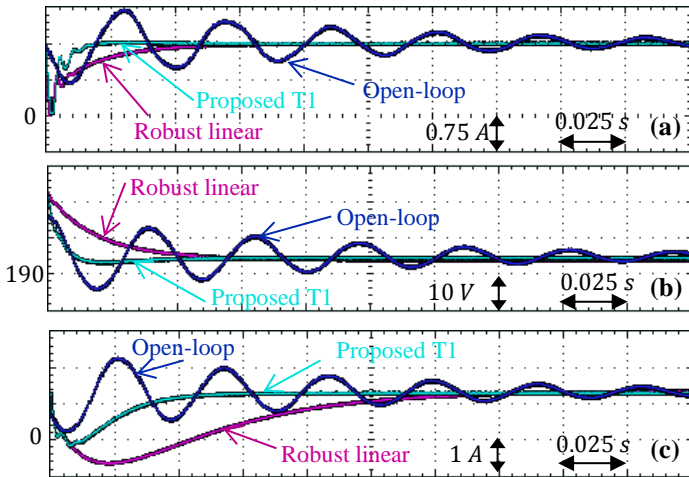


Fig 8. The DC MG states and control input_ scenario 1. (a). $i_{L,1}$ (b). $v_{C,1}$ (c). $i_{L,2}$ (d). $v_{C,2}$ (e). i_{es}

Scenario 2 (uncertain system): In this scenario, some variations are made in several DC MG's filter parameters and the ESS units to study the robustness of the designed controller based on Theorem 2 against the changes in system parameters. The percentage of the variations in values of the system parameters are provided in Table II. Note that, with this given parameters, the open-loop DC MG with CPL is unstable.

Table II: Percentage changes in uncertain parameters of the DC MG

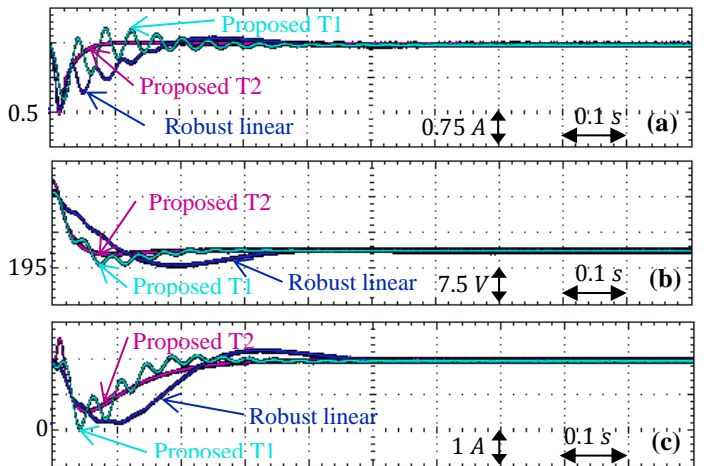
$r_1 [\Omega]$	+10%	$r_s [\Omega]$	-5%
$L_1 [mH]$	+5%	$L_s [mH]$	+10%
$C_1 [\mu F]$	-8%	$C_s [\mu F]$	+5%
ESS [A]	-3%		

By applying the proposed Theorem 2 with the decay rate $\sigma = 50$, $\delta_a = 1$, and $\delta_k = 0.1$, the following controller gains are calculated:

$$K_{1,nfr} = [31.8678 \quad 62.1220 \quad 27.2029 \quad 220.2515]$$

$$K_{2,nfr} = [31.8678 \quad 62.8396 \quad 27.2029 \quad 220.2515]$$

Fig. 8 illustrates the state evolutions and control effort of the closed-loop DC MG system obtained by Theorem 2 (T2), Theorem 1 (T2), and robust linear [18]. As can be seen in Fig. 9, the robust non-fragile controller designed based on Theorem 2 provide a fast state convergence without any chattering. Note that, there it is a shift in setting the zero on the time axis of Fig. 9.



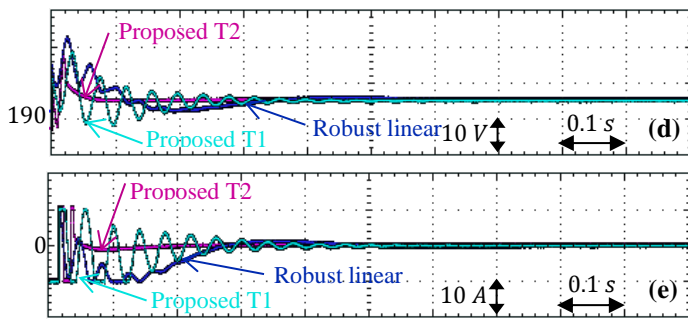


Fig 9. The DC MG states and control input_ scenario 2. (a). $i_{L,1}$ (b). $v_{C,1}$ (c). $i_{L,2}$ (d). $v_{C,2}$ (e). i_{es}

I. CONCLUSION

In this paper, a novel TS-based fuzzy control law for the regulating the current of the uncertain energy storage unit connected with multiple CPLs in an uncertain DC MG is proposed. The gain matrices of the proposed approach are computed by the convex optimization techniques through the LMI formulations. The design procedure of the presented nonlinear is as simple as the existing LMI-based linear controllers. However, the performance of the designed control method is noticeably better than the linear ones. Simulation results showed that the settling time of the closed-loop system responses is greatly improved. In addition, the proposed approach is completely resilient against the uncertainties and stabilizes the DC MG with a less oscillation and overshoot response. For the future work, considering finite-time stabilizing controller can be a good research topic. Also, the problem of charging and discharging the energy storage unit is of great importance.

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