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Stochastic Consensus-based Control of μGs with Communication Delays and Noises

Mohammad Ali Shahab, Babak Mozafari, Soodabeh Soleymani, Nima Mahdian Dehkordi, Hosein Mohammadnezhad Shourkaei, and Josep M. Guerrero, *Fellow, IEEE*

***Abstract*—We propose a novel stochastic consensus-based secondary voltage and frequency restoration scheme with communication delays and noises for islanded microgrids (μGs). Existing distributed methods commonly design restoration layer with the assumption of ideal communication among distributed generations (DGs). Albeit, the communication channels are exposed to delay and noise, whereas any DG receives delayed and noisy measurements from its neighbors due to peripheral noises and communication delays. Delay and noise have a great impact on the control of μG, which terribly reduce the stability and quality of it. To eliminate the adverse effects of delays and noises, we propose novel consensus protocols that consider effects of the communication noises and delays simultaneously for complete plant dynamics, and study mean square consensus for frequency and voltage restoration of μGs whereas providing stringent real power sharing. To this end, we derive the mean square consensus restoration proof using rigorous Lyapunov analysis. As a result the suggested method decreases the sensitivity of the system to failures and increases its reliability. Finally, we have done several simulation scenarios in MATLAB/SimPowerSystems Toolbox to verify the proposed strategy performance.**

***Index Terms*—Cooperative control systems, mean square consensus, microgrids, networks with time-delays and noises, power sharing, secondary control, stochastic networks.**

I. Introduction

Recently, the control strategy of microgrids (μGs) is adopted based on a hierarchical droop-based control method i.e., primary, secondary, and tertiary control levels [1], [2].

The primary control level based on decentralized droop control method keeps the μG's frequency and voltages, holds plug and play capability for the distributed generations (DGs), and provides power sharing between DGs without requirements for the communication infrastructure. However, it suffers from the steady-state frequency and voltage deviations from their rated values [3].

To overcome these limitations, fully centralized control scheme which requires communication links between each DG and the central control module is proposed and it demands extensive communication system [4]. As a result, it increases the sensitivity of the system to failures and decreases its reliability leading to a single point of failure. Therefore, the distributed secondary control using sparse communication network can address this drawback [2], [5]–[8]. Unlike the complex communication network used in the centralized schemes, in the sparse communication network, each DG unit communicates with local units rather than all units. Therefore, the system reliability is improved and cost is reduced [9]–[13].

The distributed secondary control employs the sparse communication network and effectively (i) maintains the DGs output voltage at the desired values, (ii) compensates for the frequency deviation of μG, (iii) provides accurate real power sharing among DGs, and (iv) achieves accurate reactive power sharing among DGs [2], [5]–[13].

A. Related Research

The consensus-based control of continuous systems when the multiplicative noise and time delay coexist have recently been presented in the literature to control multi-agent systems [14]–[17].

The distributed control methods have been presented in the literature are almost based on assuming an ideal, noise-free communication among DGs, and few studies solve restoration problems in μGs with consideration of the communication noises or delays among DGs.

Scholars have recently proposed some types of distributed stochastic [18], [19] and/or distributed time-delay secondary control of μG to restore frequency and voltage of it. A distributed noise resilient method for the secondary control of a μG is proposed in [18], [19]. In [18], a distributed least-mean-square algorithm has been employed to consider the communication noise which has been considered only in the leader's reference signal. In [20], the technique proposed in [18] has been expanded to consider the communication noise in all the links among the DG units. Communication delays effect on an islanded μG's secondary control has been investigated by a small-signal model [21]–[24]. Note that some methods in the literature only considered the dynamics of droop control for the secondary control (for example [24]) and the others (for example [21]) linearized the complete dynamics of system, which both are small signal models. However, in this paper, the large signal model of μG has been used. Moreover, a gain scheduling method has been applied to rectify the communication delay. However, existing distributed stochastic and/or distributed time-delay secondary control approaches have limitations as:

1) they involve a system's detailed model. In practice, the parameters of the μG, such as loads, line impedances, loads,

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even the μG configuration, and etc., are uncertain. Moreover, unmodeled dynamics including unknown disturbances and uncertainties are not considered in their models, 2) these methods neglect inner control loops impact on the secondary controller thus lack global stability, which is a necessary requirement in complex networks, and 3) they are not able to undertake robust stability when confronted with both communication noises and time delays,

B. Key Contributions

Motivated by the aforementioned limitations, our work proposes a novel cooperative stochastic secondary control for the frequency and voltage restoration of μGs considering both communication time delays and noises, based on the distributed cooperative control's concept [14], [15]. The essential contributions of our work that, to the best of our knowledge, have not been presented yet are concisely provided as: 1) the communication noises and delays effects have been investigated simultaneously for complete plant dynamics, and 2) in real-world, communication delay is not constant [25], [26], the communication time delays presented in all the communication links among DG units are considered as time-varying model, while providing accurate real power sharing.

Moreover, we use a rigorous Lyapunov analysis proof for the realization of the control protocols based on a communication network graph. We show that, using the proposed method, the stability of μGs is independent of the noise and time delay i.e., the control gains in our proposed method are designed such that the stability μG is independent of the noise with time delay. Another important point is that the lower and upper bounds of the gains are determined. Therefore, our method is quantitative analysis, and is robust against noise and delay parameters variations, i.e., the gains are designed analytically to attenuate the adverse effects of delays and noises. In fact, the bounds of delays and noises are considered in the control method and parameters of the controller are designed for the wide range of environment condition deviations. Therefore, we obtain the positive definite solutions to restore voltages and frequency [14].

Comparing with existing distributed methods, the proposed method robustly attenuates the effects of both time delays and noises in communication channels, while in the conventional method the voltage and frequency waveforms become unstable.

C. Organization

The remains of the paper is organized in six parts. Section II develops preliminaries on stochastic stability to examine the multi-agent systems consentability. Section III describes modeling of μG and problem formulation, respectively. The proposed stochastic consensus-based control of μGs for voltage and frequency with communication delays and noises are given in sections IV and V, respectively. Simulation case studies along with conclusions are shown in Section VI and Section VII.

II. Preliminaries

We consider the following stochastic differential delay equation [14]

$$dy(t) = [A_0 y(t) + A_1 y(t-\tau_1)]dt + dM, \tag{1}$$

where $A_0, A_1 \in R^{n\times n}$, $M(t) = \sum_{i=1}^{d} \int_0^t f_i(y(s-\tau_2))dw_i(s)$, $\tau_1, \tau_2 \geq 0$, $f_i : R^n \to R^n$, $d > 0$, and $\{w_i(t)\}_{i=1}^d$ represent independent Brownian motions. The functions $\{f_i(x)\}_{i=1}^d$ assure the following properties.

Assumption 2.1: [14] Positive constants $\{\rho_i\}_{i=1}^d$ exist such that for any $y_1, y_2 \in R^n$, $\| f_i(y_1) - f_i(y_2) \| \leq \rho_i \| y_1 - y_2 \|$. Moreover, for any $P_0 > 0$, $D_{p0} \geq 0$ exists so that

$$\sum_{i=1}^{d} f_i^T(y) P_0 f_i(y) \leq y^T D_{p0} y. \tag{2}$$

Theorem 2.2: [14] We assume that inequality (2) along with Assumption 2.1 hold. If a positive definite matrix P exists so that

$$\bar{A}^T P + P\bar{A} + (\bar{A}^T P \bar{A} + A_1^T P A_1)\tau_1 + D_p < 0, \tag{3}$$

where $\bar{A} = A_0 + A_1$, then there exist positive constants C_0 and γ_0 so that

$$E \| y(t) \|^2 \leq C_0 e^{-\gamma_0 t},\ limsup_{t\to\infty} \frac{1}{t} log \| y(t) \| \leq -\frac{\gamma_0}{2},\ a.s. \tag{4}$$

where E represents the mathematical expectation; $\| . \|$ is the vector norm; lim is the limit; sup indicates the supremum; log is the logarithm function; and a.s. represents asymptotically stable.

III. System Models

A. μG System Large-Signal Dynamical Model

This section represents the state space formulation of μG proposed in [7]. Fig. 1 shows a typical inverter-based μG scheme in autonomous-mode of operation, where the physical, communication network, and control layers are shown.

The physical layer contains an interconnection of primary DC source interfaced with the μG via a voltage source converter (VSC), power, voltage, and current control loops, LCL filter, loads, and transmission lines. We span a spanning tree among DGs in a sparse communication network layer to simplify exchange of data among DGs. We choose it so that in case of any communication link defeat the remaining network encompasses a spanning tree.

The considered continuous-time nonlinear time-invariant system with internal control loops is modeled as

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_{i1}(x_i)u_{i1} + g_{i2}(x_i)u_{i2} + k_i(x_i)D_i \\ y_{i1} &= v_{odi} = h_{i1}(x_i) \\ y_{i2} &= \omega_i = \omega_{ni} - m_{P_i}P_i = h_{i2}(x_i) + d_i u_{i2}, \end{aligned} \tag{5}$$

in which $x_i = [\alpha_i\ P_i\ Q_i\ i_{Ldi}\ i_{Lqi}\ v_{odi}\ v_{oqi}\ i_{odi}\ i_{oqi}]^T$, α_i indicates the DG reference frame angle than the common one. i_{Ldi}, i_{Lqi}, v_{odi}, v_{oqi}, i_{odi}, and i_{oqi} indicate the direct and quadratic i_{Li}'s components, v_{oi}, and i_{oi} in Fig. 1. $y_i = [y_{i1}, y_{i2}]^T = [v_{odi}, \omega_i]^T$, $u_i = [u_{i1}, u_{i2}]^T = [V_{ni}, \omega_{ni}]^T$,

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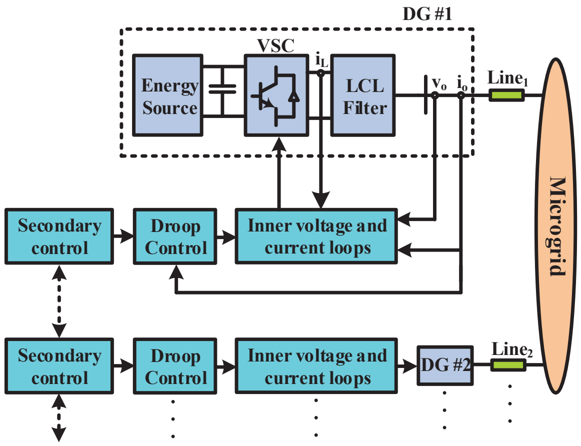

Fig. 1. An inverter-based μG scheme.

and D_i = $[\omega_{com}\ v_{bdi}\ v_{qdi}]^T$ are represented the outputs, inputs, and disturbance, respectively. The detailed model description for $f_i(x_i)$, $g_{i1}(x_i)$, $g_{i2}(x_i)$, and $k_i(x_i)$ is found in [7].

Therefore, the aim of the secondary control level is to design the protocols V_{ni} and ω_{ni} to restore the frequency and voltage to their references.

B. Communication Model

The well-known dynamic of μG including N DGs is represented as [11]

$$\begin{aligned} \dot{y}_{i1} &= y_{i1,2} \\ \dot{y}_{i1,2} &= f_i(x_i) + g_i(x_i)u_{i1}, \end{aligned} \tag{6}$$

where, $i = 1, 2, ..., N$. We define $u_{v_i} = f_i(x_i) + g_i(x_i)u_{i1}$ so that we have

$$\begin{aligned} \dot{y}_{i1} &= y_{i1,2} \\ \dot{y}_{i1,2} &= u_{v_i}, \end{aligned} \tag{7}$$

or in matrix form,

$$\dot{y}_i = Ay_i + Bu_{v_i}, \tag{8}$$

where $y_i = [y_{i1}\ \ y_{i1,2}]^T = [v_{o,magi}\ \ \dot{v}_{o,magi}]^T$, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, and $B = [0\ \ 1]^T$. Since the leader behavior is independent of the followers, therefore, the leader dynamic (reference) is shown in matrix form as follows:

$$\dot{y}_0 = Ay_0, \tag{9}$$

where $y_0 = [v_{ref}\ \ \dot{v}_{ref}]^T$.

We define $y = [y_1\ y_2\ ...\ y_N]^T$ and $u_v = [u_{v_1}\ \ u_{v_2}\ \ ...\ u_{v_N}]^T$ represents designed protocol. We model the information flow structures among DGs as a connected graph $\mathscr{G} = (\mathscr{V}, \epsilon, \mathscr{A})$ with a set of N DGs $\mathscr{V} = \{1, 2, ..., N\}$, a set of edges (communication links) ϵ, and a weighted adjacent matrix $\mathscr{A} \in R^{N\times N}$ is associated adjacency matrix. a_{ij} is the edge (i, j) weight. For $a_{ij} = 1$, i^{th} node receives information from j^{th} node and vice versa, otherwise $a_{ij} = 0$.

The graph Laplacian matrix $\mathscr{G}$ is $L = D - \mathscr{A}$ in which D represents the in-degree matrix specified as $D = Diag\{d_i\}$, and diagonal matrix with $d_i = \sum_{j\in N_i} a_{ij}$, i.e. number of incoming links at node i. Also, N_i represents the set of the node i's neighbors, $j \in N_i$, $a_{ij} = 1$. The eigenvalues of L have one zero entry $\lambda_1 = 0$ with all other have positive real parts and determine the global dynamics for the μG system, i.e., $0 < \lambda_2 \leq \lambda_3 \leq ... \leq \lambda_N$ [15].

Assumption 3.1: We suppose that the undirected graph $\mathscr{G}$ possesses a spanning tree.

IV. Distributed Secondary Voltage Control with Communication Time Delays and Noises

In this part, we design a distributed stochastic secondary voltage restoration control algorithm with time delays and multiplicative noises in communication layer to eliminate the voltage deviation. In the following, we suggest the cooperative distributed noise-resilient secondary voltage control considering communication delay

$$u_{v_i} = K \sum_{j\in N_i} e_{ji}(t), \tag{10}$$

where K denotes a positive matrix to be designed and

$$\begin{aligned} e_{ji}(t) &= y_j(t-\tau_1) - y_i(t-\tau_1) \\ &+ \sum_{l=1}^{2} f_{lji}(y_j(t-\tau_2) - y_i(t-\tau_2))\gamma_{lji}(t), \quad j \in N_i \end{aligned}$$

indicates the DG$_i$'s state measurement from its neighbor DG j; $\gamma_{ji}(t)$ = $(\gamma_{1ji}(t), \gamma_{2ji}(t))^T \in R^2$ is the measurement noise; τ_1, τ_2 are the time delays; $f_{lji}(.)$ is the noise variance. As is seen $e_{ji}(t)$ possess two sections. The protocol's first section $y_j(t-\tau_1) - y_i(t-\tau_1)$ represents the deterministic term, while $\sum_{l=1}^{2} f_{lji}(y_j(t-\tau_2) - y_i(t-\tau_2))\gamma_{lji}(t)$ represents the stochastic term. We suppose that the noises are independent Gaussian white noises, and satisfy the following assumption.

Assumption 4.1: The process noise $\gamma_{ji}(t)$ = $(\gamma_{1ji}(t), \gamma_{2ji}(t))^T \in R^2$ assures $\int_0^t \gamma_{ji}(t)(s)ds$ = $\omega_{ji}(t)$, $t \geq 0$, $i = 1, 2, ..., N$, $j \in N_i$ in which $\{\omega_{ji}(t), i = 1, 2, ..., N\}$ represent independent two-dimensional Brownian motions defined on the complete probability space (Ω, F, P) with a filtration $F_t(t \geq 0)$ satisfying the usual conditions, namely, it is right continuous and increasing while F_0 contains all P-null sets.

Moreover, we denote the set of all acceptable cooperative controls as follows

$$\begin{aligned} U(\tau_1, \tau_2, f(.)) = \{&u_v(t) | u_{v_i}(t) = K \sum_{j\in N_i} e_{ji}(t), \\ &t \geq 0, K \in R^{2\times 2}, i = 1, 2, ..., N.\} \end{aligned}$$

Remark 4.2: $\{\lambda_i^u(\mathscr{A})\}_i$ represents the unstable eigenvalues of $\mathscr{A}$, that is, $Re(\lambda_i^u(\mathscr{A})) \geq 0$. We define $\lambda_0^u = \sum_i Re(\lambda_i^u(\mathscr{A}))$.

This section's results is summarized as follows:

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Theorem 4.3: The distributed cooperative control protocol

$$\begin{aligned}
u_{i1} &= V_{ni} = \frac{1}{g_i(x)}(u_{v_i} - f_i(x)), \quad i = 1, 2, ..., N \\
u_{v_i} &= K \sum_{j \in N_i} e_{ji}(t) \\
e_{ji}(t) &= y_j(t-\tau_1) - y_i(t-\tau_1) \\
&\quad + \sum_{l=1}^{2} f_{lji}(y_j(t-\tau_2) - y_i(t-\tau_2))\gamma_{lji}(t), \quad j \in N_i
\end{aligned} \tag{11}$$

can guarantee that the DG output voltage, $v_{mag,i}$, in systems (8) and (9) synchronizes with v_{ref} irrespective of time-delay and stochastic noise in communication channels.

Note that the controller exists if the Assumptions 4.1 and 3.1 are valid, $f_{lji} = \sigma_{lji}(y) \geq 0$, $max(Re(\lambda(\mathscr{A}))) > 0$, and $4\lambda_0^u \bar{\sigma}^2 \leq \lambda_1$.

The upper bound of delays in the communication channels are calculated as

$$\tau_1^* = min(\frac{1}{2 \parallel A \parallel^2 \parallel P \parallel}, \frac{\lambda_1 - 4\bar{\sigma}(\lambda_0^u + \epsilon)}{6(\lambda_0^u + \epsilon)\lambda_N}), \tag{12}$$

where $P > 0$ is designed by solving the following Ricatti equation

$$A^T P + PA - 2\alpha PB(I + B^T PB)^{-1} B^T P + I = 0 \tag{13}$$

with $\alpha \in (\lambda_0^u, \lambda_0^u + \epsilon)$, $\epsilon \in (0, \dfrac{\lambda_1 - 4\bar{\sigma}^2\lambda_0^u}{4\bar{\sigma}^2})$, and $\bar{\sigma}^2 = \sum_{l=1}^{2} max_{i=1,j=0}^{N} \sigma_{lij}^2$.

Finally, the control protocol gain K in (11) is calculated as

$$K = k(I + B^T PB)^{-1} B^T P,$$

where $k \in (\underline{k}, \bar{k})$, $\underline{k} = \dfrac{[\lambda_1 - \sqrt{\lambda_1^2 - 2\alpha\rho}]}{\rho}$, $\bar{k} = \dfrac{[\lambda_1 + \sqrt{\lambda_1^2 - 2\alpha\rho}]}{\rho}$, and $\rho = (2\bar{\sigma}^2 + 3\lambda_N\tau_1)\lambda_1$.

Proof: Inspired from the work presented in [14], we proof the above-mentioned theorem. Let we define $\delta_i = y_i - y_0$, $i = 1, 2, ..., N$, $y = [y_1^T, ..., y_N^T]^T$, and $\delta = [\delta_1^T, ..., \delta_N^T]^T$. With the equations (11) and (6), we can obtain the closed-loop network dynamics as

$$dy(t) = (I_N \otimes A)y(t)dt - (L \otimes BK)(y(t-\tau_1)dt + dM_1), \tag{14}$$

where $\otimes$ denotes the Kronecker product, and

$$M_1 = \sum_{l=1}^{2} \sum_{i,j=1}^{N} a_{ij}\sigma_{lji} \int_0^t [S_{i,j} \otimes BK]\delta(s-\tau_2)dw_{lji}(s),$$

$S_{i,j} = [s_{kl}]_{N \times N}$ represents $N \times N$ matrix where $s_{ii} = -a_{ij}, s_{ij} = a_{ij}$ and all other elements being zero. By the definition of $\delta(t)$, then

$$d\delta(t) = (I_N \otimes A)\delta(t)dt - (L \otimes BK)(\delta(t-\tau_1)dt + dM_2), \tag{15}$$

where

$$M_2 = -\sum_{l=1}^{2} \sum_{i=1}^{N} a_{ij}\sigma_{l0i} \int_0^t [\bar{S}_i \otimes BK]\delta(s-\tau_2)dw_{l0i}(s),$$

$\bar{S}_i = [s_{kl}]_{N \times N}$ is an $N \times N$ matrix with $n_{ii} = b_i$ and all other elements being zero. In the following, we define $\bar{\delta}(t) = (\phi^{-1} \otimes I_n)\delta(t)$, we have

$$d\bar{\delta}(t) = A_0\bar{\delta}(t)dt + A_1\bar{\delta}(t-\tau_1)dt + dM_3, \tag{16}$$

where $A_0 = I_N \otimes A$, $A_1 = -\Gamma_0 \otimes BK$,

$$\begin{aligned}
M_3 &= \sum_{l=1}^{2} \sum_{i,j=1}^{N} \sigma_{lji} \int_0^t [(\phi^T S_{i,j}\phi) \otimes BK]\bar{\delta}(s-\tau_2)dw_{lji}(s) \\
&\quad - \sum_{l=1}^{2} \sum_{i=1}^{N} \sigma_{0i} \int_0^t [(\phi^T \bar{S}_i\phi) \otimes BK]\bar{\delta}(s-\tau_2)dw_{l0i}(s).
\end{aligned}$$

Now, we define $\bar{P} = I_N \otimes P$, also, we have

$$\begin{aligned}
\langle M_3, PM_3 \rangle(t) &= \sum_{l=1}^{2} \sum_{i,j=1}^{N} \sigma_{lji}^2 \int_0^t [\bar{\delta}^T(s-\tau_2)((\phi^T S_{i,j}\phi)^T \\
&(\phi^T S_{i,j}\phi)) \otimes (BK)^T PBK]\bar{\delta}(s-\tau_2)ds + \sum_{l=1}^{2} \sum_{i=1}^{N} \sigma_{l0i} \\
&\int_0^t \bar{\delta}^T(s-\tau_2)[(\phi^T \bar{S}_i\phi) \otimes (BK)^T PBK]\bar{\delta}(s-\tau_2)ds.
\end{aligned} \tag{17}$$

It should be noted that, $\sum_{i,j=1}^{N} = S_{i,j}^T S_{i,j} = 2L$ and $\sum_{i=1}^{N} \bar{S}_i^T \bar{S}_i = b$. Therefore, we obtain $\sum_{i,j=1}^{N} (\phi^T S_{i,j}\phi)^T (\phi^T S_{i,j}\phi) = 2\phi^T L\phi$ and $\sum_{i=1}^{N} (\phi^T \bar{S}_i\phi)^T (\phi^T \bar{S}_i\phi) = \phi^T D_0\phi$. So, we have

$$\langle M_3, PM_3 \rangle(t) \leq 2\bar{\sigma}^2 \bar{\delta}^T(s-\tau_2)D\bar{\delta}(s-\tau_2)dt, \tag{18}$$

where $D = \Gamma_0 \otimes (K^T B^T PBK)$. By using conditions of Theorem 4.3, under $K = k(I_m + B^T PB)^{-1}B^T P$, we obtain $\bar{A}^T\bar{P} + \bar{P}\bar{A} + (\bar{A}^T\bar{P}\bar{A} + A_1^T\bar{P}A_1)\tau_1 + \bar{\sigma}^2 D < 0$, where $\bar{A} = A_0 + A_1$. We note that it can be guaranteed by

$$\begin{aligned}
&W_i^T P + PW_i + W_i^T PW_i\tau_1 + (\lambda_i^2\tau_1 + 2\lambda_i \frac{N-1}{N}\bar{\sigma}^2) \\
&K^T B^T PBK < 0,
\end{aligned} \tag{19}$$

where $W_i = A - \lambda_i BK$. By using the following inequality

$$(z+h)^T Q(z+h) \leq 2z^T Qz + 2h^T Qh, \quad z, h \in R^n, \quad Q > 0 \tag{20}$$

therefore,

$$W_i^T PW_i \leq 2A^T PA + 2\lambda_i^2 K^T B^T PBK. \tag{21}$$

Substituting $K = k(I + B^T PB)^{-1}B^T P$ and (21) into (19), (19) is assured by

$$\begin{aligned}
\Gamma_i &= A^T P + PA + 2\tau_1 A^T PA \\
&\quad - \zeta_i PB(I_m + B^T PB)^{-1}B^T P,
\end{aligned} \tag{22}$$

where $\zeta_i = 2k\lambda_i - (2\frac{N-1}{N}\bar{\sigma}^2 + 3\lambda_N\tau_1)\lambda_i k^2$ and P is the solution to (13). Using condition (II), we have

$$\Gamma_i < (2\alpha - \zeta_i)PB(I + B^T PB)^{-1}B^T P, \tag{23}$$

where for $k \in (\underline{k}, \bar{k})$, $(2\alpha - \zeta_i) < 0$. Therefore, we conclude from (23) that $\Gamma_i < 0$. Therefore, according to the Theorem 2.2, $E \parallel \bar{\delta(t)} \parallel^2 \leq C_0 e^{-\gamma_0 t}$, $limsup_{t\to\infty} \frac{1}{t} log \parallel \bar{\delta(t)} \parallel \leq$

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$-\dfrac{\gamma_0}{2}$, a.s., where C_0 and γ_0 are positive constants. Therefore, our proposed method implies the mean square and almost sure consensus. ■

Fig. 2 illustrates the distributed secondary voltage control's schematic diagram with communication noises and delays.

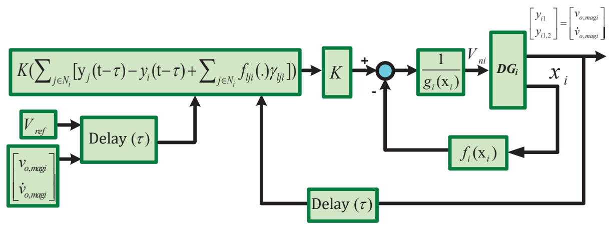

Fig. 2. An illustration of the cooperative secondary voltage control with communication noises and delays.

V. Distributed Secondary Frequency Control with Communication Time Delays and Noises

In this section, we design a distributed cooperative secondary frequency restoration control algorithm with time delays and multiplicative noises in communication layer to eliminate the frequency deviation and realize precise real power sharing. We can eliminate the effects of both time delays and noises in communication layer similar to the previous section. In the following conventional droop-controlled μGs, the primary control can attain real power sharing but cannot compensate for the frequency deviation.

$$\omega_i = \omega_{ni} - m_{P_i} P_i, \tag{24}$$

where ω_i indicates the reference frequency provided for the current and voltage control loops; m_{P_i} is the frequency droop coefficient; ω_{ni} represents the set point for the primary control in (24); and P_i is the real power of inverter.

The secondary frequency control picks out ω_{ni} in (24), to synchronize the output frequencies of each inverter to the nominal value. To do so, each DG intercommunicate data just with its neighbors on a network. Moreover, it is assumed that DGs communicate with each another through the noisy and time-delayed communication graph $\mathscr{G}$.

We can obtain control system model by applying the feedback linearization method for the system (24) as

$$\dot{\omega}_i = \dot{\omega}_{ni} - m_{P_i} \dot{P}_i = u_{\omega_i}, \tag{25}$$

where u_{ω_i} represents the virtual control input. Equation (25) represents the control system model to compute the control input ω_{ni}. It chases from (25), the secondary frequency control for a μG compromising N DGs is converted to the following tracking synchronization problem for a continuous-time linear time-invariant system multi-agent system

$$\dot{\omega}_i = u_{\omega_i}, i = 1, 2, ..., N. \tag{26}$$

We use the sparse communication network for information sharing, and a direct connection to the leader node for each DG unit is unattainable. We note that only one DG has access to the global reference values of voltage and frequency. To overcome this problem, similar to the previous section, a cooperative noise-resilient secondary frequency control considering communication delay stands on its own information and its neighbors information on the communication network can be designed as

$$u_{\omega i} = K_\omega \sum_{j \in N_i} e_{ji}(t), \tag{27}$$

where K_ω denotes a positive matrix to be designed and

$$\begin{aligned} e_{ji}(t) &= \omega_j(t-\tau_1) - \omega_i(t-\tau_1) \\ &\quad + \sum_{l=1}^{2} f_{l\omega ji}(\omega_j(t-\tau_2) - \omega_i(t-\tau_2))\gamma_{\omega lji}(t), \quad j \in N_i \end{aligned}$$

indicates the DG i's state measurement from its neighbor DG j; $\gamma_{\omega lji}(t) = (\gamma_{1\omega ji}(t), \gamma_{2\omega ji}(t))^T \in R^2$ is the measurement noise; τ_1, τ_2 are the time delays; $f_{l\omega ji}(.)$ is the noise variance. As is seen $e_{ji}(t)$ contains two parts. The first part of the controller $\omega_j(t-\tau_1)-\omega_i(t-\tau_1)$ represents the deterministic term, while $\sum_{l=1}^{2} f_{l\omega ji}(\omega_j(t-\tau_2)-\omega_i(t-\tau_2))\gamma_{\omega lji}(t)$ represents the stochastic term. We suppose that the noises are independent Gaussian white noises.

After applying the proposed protocols, real power sharing accuracy should be guaranteed:

$$\frac{P_j}{P_i} = \frac{m_{pi}}{m_{pj}}, \quad \forall i, j \in N. \tag{28}$$

Despite easy implementing, the secondary frequency control without adding the auxiliary real power control may lead to incorrect real power sharing. To realize precise real power sharing, we design the real power auxiliary control variable u_{pi} as

$$u_{pi} = K_p \sum_{j \in N_i} e_{ji}(t), \tag{29}$$

where K_p denotes a positive matrix to be designed and

$$\begin{aligned} e_{ji}(t) &= m_{p_j} P_j(t-\tau_1) - m_{p_i} P_i(t-\tau_1) \\ &\quad + \sum_{l=1}^{2} f_{lpji}(P_j(t-\tau_2) - P_i(t-\tau_2))\gamma_{plji}(t), \quad j \in N_i \end{aligned}$$

represents the DG i's state measurement from DG j (its neighbor); $\gamma_{plji}(t) = (\gamma_{1pji}(t), \gamma_{2pji}(t))^T \in R^2$ is the measurement noise; τ_1, τ_2 are the time delays; $f_{lpji}(.)$ is the noise variance. As is seen $e_{ji}(t)$ posses two sections. The protocol's first part $m_{p_j} P_j(t-\tau_1) - m_{p_i} P_i(t-\tau_1)$ represents the deterministic term, while $\sum_{l=1}^{2} f_{lpji}(P_j(t-\tau_2) - P_i(t-\tau_2))\gamma_{plji}(t)$ represents the stochastic term.

Then, the following control inputs ω_{ni} solve the frequency synchronization problem while guaranteeing the accuracy of real power sharing.

$$u_{i2} = \omega_{ni} = \int (u_{\omega_i} + u_{pi})dt, \quad i = 1, 2, ..., N. \tag{30}$$

Therefore, the proposed protocols design u_{i2} in (5) so that $y_{i2} \longrightarrow \omega_{ref}$ and $m_{pk} P_k \longrightarrow m_{pi} P_i$, $\forall i$.

This section's results is summarized as follows:

Theorem 5.1: The control protocol (30) solves the frequency restoration and real power-sharing problem in (5), irrespective of time-delay and random noise in communication channels.

Proof: Prove of theorem is similar to the previous section. ■

Since both communication time-delay and noise in the communication channels are unavoidable, comparing with the existing distributed approaches, in addition to the considering full dynamics of μG, the proposed method have better noise cancellation and robustness against time-delay in communication channels. As is seen from (30), the protocol u_{i2} contains two sections. The protocol's first part leads to the steady-state track of the nominal frequency (i.e., $\omega_i \longrightarrow \omega_{ref}$), despite communication time-delay and unavoidable noise in the communication channels, and the second part ensures real power sharing precision (i.e., $m_{pj}P_j \longrightarrow m_{pi}P_i$). Fig. 3

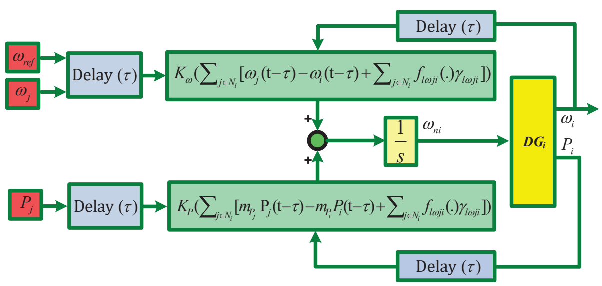

Fig. 3. An illustration of the distributed secondary frequency control with communication noises and delays.

illustrates the distributed secondary frequency control with communication noises and delays.

Remark 5.2: Since in real-world, communication delay is not constant [25], [26], in our proposed method, the delays are assumed to be time varying, and only the upper bound of time delays is important, and we have used the upper bounds of delays.

Remark 5.3: In our proposed method, it doesn't matter that which type of the noise is considered. If the noise is not white, we pass the noise through the filter to obtain the white noise.

Remark 5.4: The upper bounds of noises and delays are determined by general equation (12) and lower and upper bounds of control gain K. For under study system, the upper bound of delay and noise variance are 0.2 and 1, respectively [20], [21], [24].

VI. Study Results

In this section, to show the feasibility and effectiveness of the proposed stochastic consensus-based method with communication delays and noises, we simulate the μG presented in Fig. 4 [24], in the MATLAB/SimPowerSystem software environment for several scenarios, and we compare the results with the conventional approach in [7]. Fig. 4 shows the single line diagram of μG test system and considered communication topology. The μG is a 380 V per phase RMS and 50 Hz system, and each DG is connected to its load bus through the coupling inductance. Tables I in [24] provides the parameters of μG system.

We consider the upper bound of the time-varying communication delay 0.2 s. Moreover, the communication links are affected by an additive noise with variance 0.1.

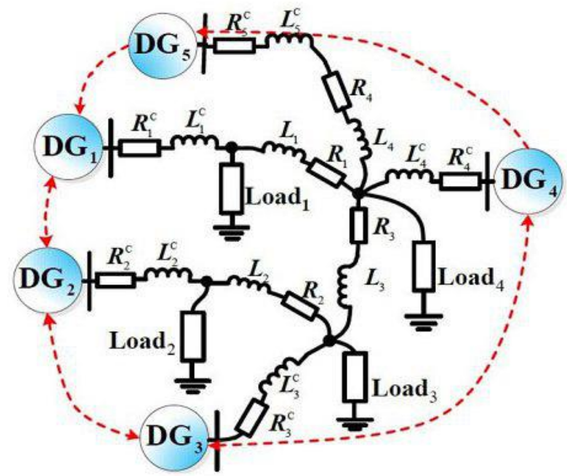

Fig. 4. The islanded μG test system and considered communication topology [24].

A. Case 1: System Performance Evaluation

Here, we present the capability of the proposed stochastic cooperative method with communication delays and noises after islanding takes place at $t = 0.0$ s. Toward this end, six test scenarios are conducted as follows:

1) $t = 1$ s. The proposed stochastic consensus-based method with communication delays and noises is activated.
2) $t = 2$ s. An additional load of 20 kW + j 14 kVar (100% load increment) is attached to DG$_1$.
3) $t = 3$ s. Additive noise with variance 0.1 is added.
4) $t = 3.5$ s. Communication delay (0.1 s) is applied.
5) $t = 4$ s. DG$_5$ is detached (plugged out).
6) $t = 5$ s. DG$_5$ is attached (plugged in).

The communication links are affected by an additive noise with variance 0.1 and 0.1 s time delay. The primary control is active whereas the secondary control is intentionally deactivated. The primary control results in voltage and frequency deviations from nominal values. In the first scenario, secondary control in (11) and (30) are activated at $t = 1$ s. As is seen from Figs. 5 and 6, the frequency and voltage both are quickly restored to their nominal references. In the second scenario, an additional RL load of 20 kW + j 14 kVar (100% load increment) is attached to DG$_1$.

In our proposed method, it doesn't matter that which type of the noise is considered. Since, we have considered the color noise and then pass the noise through the filter to obtain the white noise.

The simulation results have been shown in Figs. 5 and 6. It can be seen that the proposed stochastic consensus-based method with communication delays and noises can regulate the frequency and voltage after occurring a small-signal disturbance namely step load increase. In the third and fourth scenarios, we add the additive noise and communication delay, respectively. The proposed stochastic consensus-based method with communication delays and noises performs desirably and the voltage, real power ratio, and frequency are regulated without any fluctuation. Finally, we plug out DG_5 at $t = 4$ s and then plug in at $t = 5$ s, the recommended control scheme responds well against DG unit's disconnection and reconnection, and in addition to the robust stability, robustness performance against system topological uncertainties and moreover, plug and play (P & P) functionality of the μG is realized.

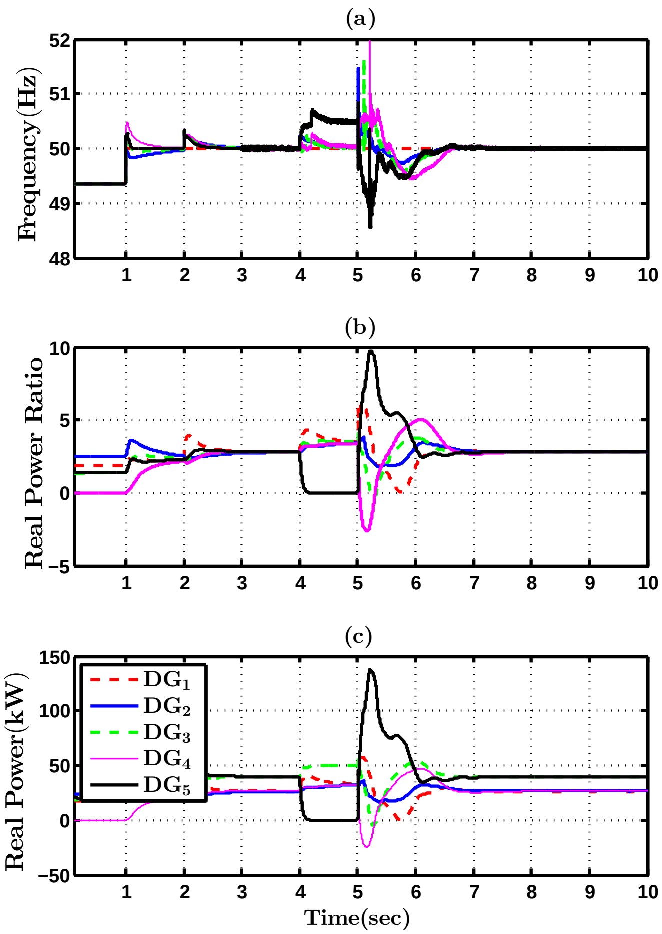

Fig. 5. Case 1: a) frequency, b) real power ratio, and c) output real power waveforms of DGs.

B. Case 2: Change of Communication Delay and Additive Noise Parameters

This test's objective is to demonstrate the robustness, resiliency, and authenticity of the proposed stochastic consensus-based method with communication delays and noises with respect to the changes of noise and communication delay parameters. Then, we repeat the Subsection V-A with the similar simulation scenario. Here, the additive noise's variance is changed from 0.1 to 0.7 (seven times) and also the

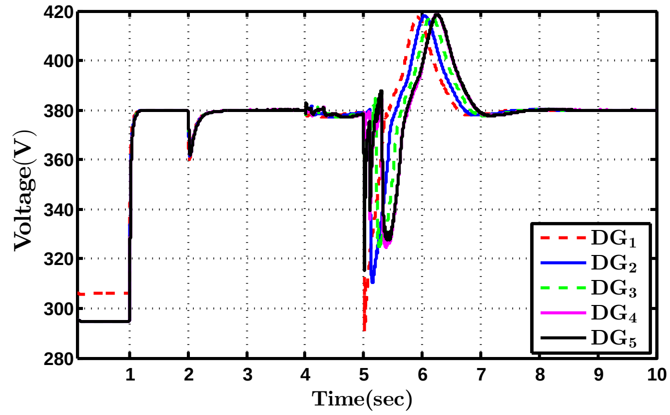

Fig. 6. Case 1: voltage magnitude waveforms of PCC buses of DGs.

communication delay from 0.1 s to 0.15 s. From Figs. 7 and 8, it is seen that the frequency and voltage and are stably restored when large and small-signal events are occurred, in the presence of the additive noise and communication delay with higher values of noise variance and time delay.

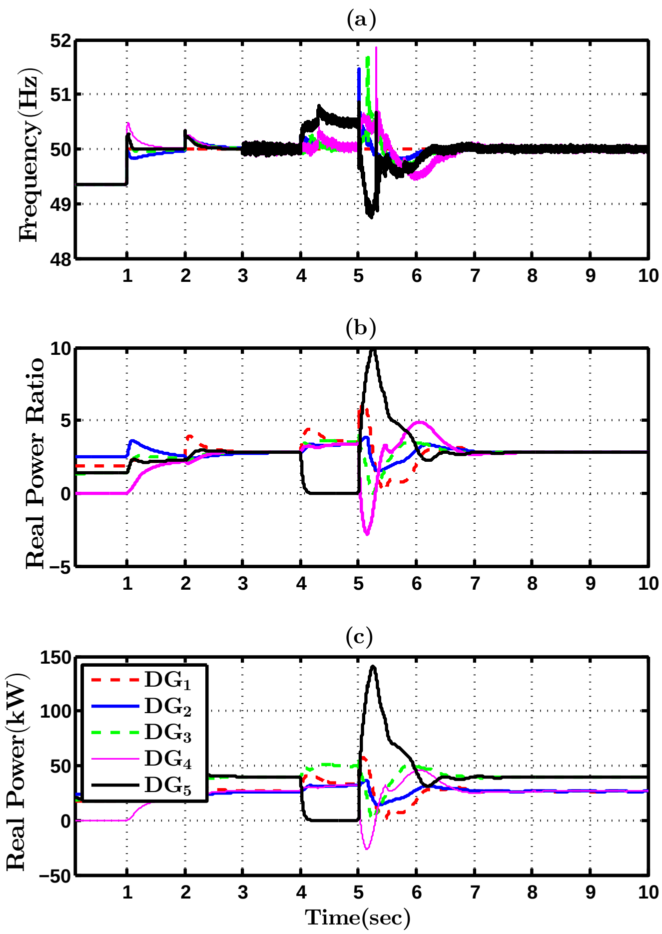

Fig. 7. Case 2: a) frequency, b) real power ratio, and c) output real power waveforms of DGs.

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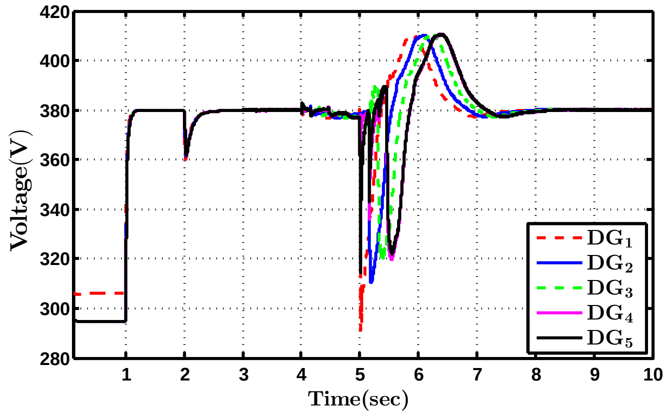

Fig. 8. Case 2: voltage magnitude waveforms of PCC buses of DGs.

C. Case 3: Comparison with a Conventional Distributed Method [7]

For this case, we compare the robust performance of the proposed stochastic consensus-based method with communication delays and noises with a conventional distributed secondary control method in [7]. This method, similar several proposed μG control schemes in the literature, includes current/voltage control loops which uses PI controllers to track reference value. The secondary method in [7] is based on the distributed cooperative control so that any DG just needs its own information and neighbours' information using a communication network graph. We carry out the simulation scenario of Subsection VI-A by employing the secondary protocol proposed in [7], and render the consequences in Figs. 9 and 10. From these Figs, it is seen that using the conventional secondary method presented in [7], when the stochastic noise is added with variance 0.1, the voltage and frequency waveforms become unstable. In contrast, as discussed in Subsection VI-B, the proposed stochastic consensus-based method with communication delays and noises has desirable performance, even in the case that the variance of the noise and time-delay are highly increased.

VII. Conclusion

In this work, we have proposed a new stochastic consensus-based secondary voltage and frequency restoration scheme with communication delays and noises for an islanded μG. To eliminate the adverse effects of delays and noises, we have proposed novel consensus protocols that considered effects of the communication noises and delays simultaneously for complete plant dynamics, and studied mean square consensus for frequency and voltage restoration of μGs while providing precise real power sharing. To this end, we have derived the mean square consensus restoration proof using rigorous Lyapunov analysis. Therefore, the proposed method increased the system reliability and decreased its sensitivity to failures. The simulation results in MATLAB/SimPowerSystems Toolbox have shown that the voltage and frequency of the μG are robustly restored to their references values in the presence of the communication delays and noisy measurements. Moreover, plug-and-play functionality of DG units has been achieved.

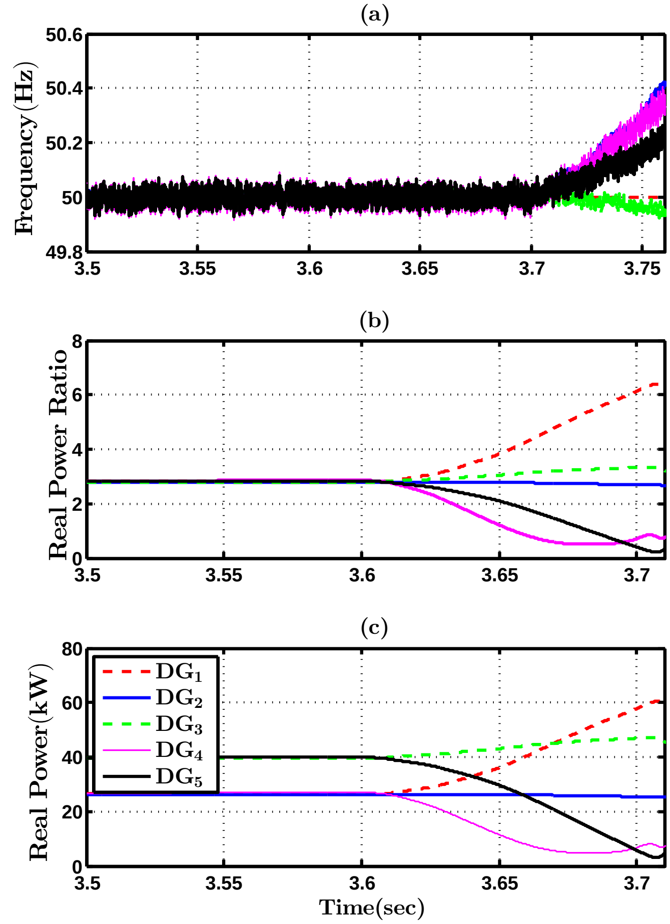

Fig. 9. Case 3: a) frequency, b) real power ratio, and c) output real power waveforms of DGs.

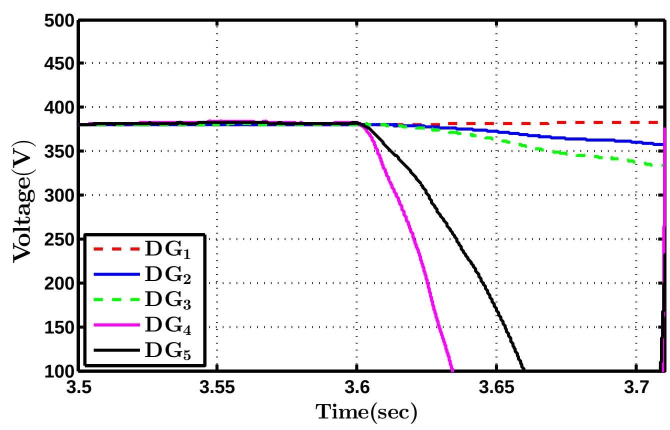

Fig. 10. Case 3: voltage magnitude waveforms of PCC buses of DGs.

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