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*Published in:*  
I E E Antennas and Wireless Propagation Letters

*DOI (link to publication from Publisher):*  
[10.1109/LAWP.2019.2911725](https://doi.org/10.1109/LAWP.2019.2911725)

*Publication date:*  
2019

*Document Version*  
Accepted author manuscript, peer reviewed version

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Zhang, F., Fan, W., Wang, Z., Zhang, Y., & Pedersen, G. F. (2019). Improved Over-the-air Phased Array Calibration Based on Measured Complex Array Signals. *I E E Antennas and Wireless Propagation Letters*, 18(6), 1174-1178. Article 8693562. <https://doi.org/10.1109/LAWP.2019.2911725>

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# Improved Over-the-air Phased Array Calibration Based on Measured Complex Array Signals

Fengchun Zhang, Wei Fan, Zhengpeng Wang, Yusheng Zhang, and Gert F. Pedersen

**Abstract**—This letter presents an improved phased array calibration method based on measured complex array signals with one single probe. All the phased array elements are excited simultaneously in the measurement and only phase reverse of each element is required. The error on the 1st antenna element of the reference method is analysed, which shows it is accumulated as the array size increases. One more measurement with designed element phase settings is introduced to overcome the deficiency. The improved method shows only  $N + 1$  measurements are required to calibrate  $N$  antenna elements accurately. Finally, a  $4 \times 8$  antenna array composed of 4-ridged horn antennas operating at 3.5 GHz was measured to validate the effectiveness of the improved algorithm.

**Index Terms**—Phased array calibration, complex signal measurement, over-the-air testing

## I. INTRODUCTION

Phased array, which consists of excitation-controllable antenna elements, has attracted great attention for decades in military [1]. With the development of mobile communication technologies, phased array antennas will be applied to 5G. The array radiation pattern can be controlled via setting complex weights to the antenna elements, e.g. to form a beam in a desired direction to overcome the unfavourable propagation loss, or to form a null in an interference signal direction to suppress unwanted signals. Fast and accurate calibration of radio frequency (RF) branches in phased array is essential both in massive production line and actual working condition. Integrating internal calibration circuits is one way to calibrate the RF branches [2], [3]. However, the over-the-air (OTA) test is an efficient way to calibrate the overall system [3]–[8].

The objective of phased array calibration is to obtain the relative complex electric fields (both amplitude and phase) of the individual antenna elements and to compensate for the discrepancies among antenna elements. Many measurement techniques have been proposed in the literature and can be grouped in different ways, e.g. complex signal [4] or amplitude only measurements [6], single-element or multi-element probe calibration, near-field or far field region calibration, see a summary in [5], [8]. Fast and accurate OTA calibration method is essential for 5G phased array antenna development.

In [4], a multi-element phased array calibration method by solving linear equations was proposed. The proposed technique is highly attractive, since it offers the minimum number

of measurements in the literature. Furthermore, it works with the probe antenna placed either in the near-field or far-field region and offers no restriction due to bit-number of digital phased shifters. As explained in [4], the accuracy, complexity and hardware requirements of the method depends on the phase settings, which are determined by the coefficient matrix of the linear equations. A recursive matrix-forming method was proposed in [4].

In this paper, we first review the reference calibration method presented in [4], and then mathematically demonstrate that an accumulated error exists on the 1st antenna element with the reference method. After that, a solution to tackle the problem is discussed and experimentally validated in an anechoic chamber. Our proposed solution is essentially an extension and improvement of the reference method.

## II. THEORY

### A. Signal model

The device under test (DUT) composed of  $N$  elements as the transmit (Tx) antenna and a single probe as the receive (Rx) antenna are discussed. As detailed in [4], the signal model of the phased array calibration system can be written as:

$$\begin{aligned} AX &= B \quad (1) \\ A &= \begin{bmatrix} e^{j\varphi_1(1)} & \dots & e^{j\varphi_N(1)} \\ \vdots & \ddots & \vdots \\ e^{j\varphi_1(M)} & \dots & e^{j\varphi_N(M)} \end{bmatrix} \\ X &= [a_1 S_1 \quad \dots \quad a_N S_N]^T \\ B &= [b(1) \quad \dots \quad b(M)]^T \end{aligned}$$

where  $A = \{e^{j\varphi_n(m)}\}_{M \times N}$  is the coefficient matrix with  $\varphi_n(m)$  denoting the phase shift of the  $n$ th Tx antenna during the  $m$ th measurement for  $n \in [1, N]$  and  $m \in [1, M]$ .  $M$  is the number of measurements (with  $M \geq N$ ). The specific phase shift is typically implemented with digital phase shifters. The vector  $X = \{x_n\} = \{a_n S_n\}_{N \times 1}$  can be obtained via solving linear equations (1), with  $a_n$  and  $S_n$  denoting the initial excitation of the  $n$ th Tx antenna element and scattering parameter from the  $n$ th Tx antenna element to the receive (Rx) antenna, respectively. The vector  $B = \{b(m)\}_{M \times 1}$  contains the complex array signals measured at the Rx antenna port during  $M$  measurements. As explained in [4], the objective of phased array calibration is to obtain the relative initial excitations of the antenna elements, i.e.  $a_n$  for  $n \in [1, N]$ . To achieve this goal, we need to know the relative scattering

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parameters  $S_n/S_o$ ,  $n \in [1, N]$  with  $S_o$  a reference complex value. It is typically pre-measured and stored for array calibration purpose. Alternatively, the relative scattering parameter  $S_n/S_o$  can be calculated based on Tx/Rx complex antenna radiation patterns and geometry of the measurement setup.

The key point of the array calibration method is the selection of the matrix  $A$ . According to the matrix theory, the accuracy of solving linear equations (1), in the presence of noise and quantization errors, depends on the condition number of matrix  $A$ ,  $\kappa(A)$ . The matrices  $A$  with small condition numbers are desirable, since they do not magnify errors, either those due to noise in the measured data, or those introduced by quantization errors of phase shifters. A smaller condition number, therefore, would render more accurate and robust solution. To ensure a small condition number, a recursive matrix-forming method was proposed in [4], where  $A$  can be generated based on basic matrices, i.e. three matrices with very small condition numbers and the Hadamard matrices. In [4], it was demonstrated that the proposed coefficient matrix  $A$  offers low measurement redundancy, small number of measurements (i.e. minimum  $M$ ) and it is applicable for antenna array calibration of any size.

### B. Problem statement

In this section, we intend to demonstrate the deficiency of coefficient matrix reported in [4]. For simplicity, we assume that we have a four-element phased array. The coefficient matrix  $A$  for a four-element array can be written as [4],

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (2)$$

With  $A$  given in (2),  $X = \{x_n\}$  in (1) can be directly solved:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} (b(1) + b(2) + b(3) + b(4))/4 \\ (b(1) - b(2) + b(3) - b(4))/4 \\ (b(1) + b(2) - b(3) - b(4))/4 \\ (b(1) - b(2) - b(3) + b(4))/4 \end{bmatrix} \quad (3)$$

The coefficient matrix  $A$  in (2) can be realized via setting phase states to  $0^\circ$  (for 1s in  $A$ ) and  $180^\circ$  (for -1s in  $A$ ). However, due to the non-idealities in practical setups, there exist both amplitude and phase errors for each phase shifter at various phase states. Without loss of generality, the errors introduced by the digital phase shifters can be denoted by a set of complex values, i.e.,  $\eta_n^0$  and  $\eta_n^\pi$  for the phase shift applied to the  $n$ -th DUT antenna at phase state of  $0^\circ$  and  $180^\circ$ , respectively. The implemented coefficient matrix  $\tilde{A}$  is:

$$\tilde{A} = \begin{bmatrix} 1 + \eta_1^0 & 1 + \eta_2^0 & 1 + \eta_3^0 & 1 + \eta_4^0 \\ 1 + \eta_1^0 & -(1 + \eta_2^\pi) & 1 + \eta_3^0 & -(1 + \eta_4^\pi) \\ 1 + \eta_1^0 & 1 + \eta_2^0 & -(1 + \eta_3^\pi) & -(1 + \eta_4^\pi) \\ 1 + \eta_1^0 & -(1 + \eta_2^\pi) & -(1 + \eta_3^\pi) & 1 + \eta_4^0 \end{bmatrix} \quad (4)$$

Substituting (4) in (1), we have

$$\tilde{A}X = \tilde{B} \quad (5)$$

where  $\tilde{B} = [\tilde{b}(1) \ \dots \ \tilde{b}(M)]^T$  is the observed complex signal at the Rx side during the  $M$  measurements. Calculating  $\tilde{B}$  from (5) and replacing the resultant  $\tilde{b}(m)$  for  $m \in [1, M]$  into (3), we have

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} = \begin{bmatrix} x_1 + \eta_1^0 x_1 + \sum_{i=2}^4 \frac{\eta_i^0 - \eta_i^\pi}{2} x_i \\ x_2 + \frac{\eta_2^0 + \eta_2^\pi}{2} x_2 \\ x_3 + \frac{\eta_3^0 + \eta_3^\pi}{2} x_3 \\ x_4 + \frac{\eta_4^0 + \eta_4^\pi}{2} x_4 \end{bmatrix} \quad (6)$$

As shown in (6), the errors introduced by all RF branches would accumulate on the 1st antenna element, leading to a bias in estimation on the first antenna element. The error range of the reference method is determined by the scale of the DUT array and the errors introduced by the digital phase shifters of the RF branches. For the other antenna elements, the array calibration accuracy will only be affected by the errors in the corresponding RF chain, not by the errors present in other RF chains.

### C. Proposed Algorithm

As discussed, the errors would accumulate on the 1st antenna element when coefficient matrix  $A$  in (2) is implemented. To solve this issue, an improved algorithm is introduced in this section. With the proposed algorithm, we reduce the accumulated error at the 1st element to the error level at other antenna elements by adding only one measurement in principle. For the new extra measurement, the excitation vector of the  $N$  element phased array can be designed as below. The excitation coefficient of the 1st antenna element is set to -1 (i.e. with phase state set to  $180^\circ$ ), while the phase settings for the other elements can be 1 or -1 as long as the number of 1s equals to the number of -1s. For example, for a four-element phased array, the excitation vector can be  $A_1 = [-1, -1, 1, 1]$ ,  $A_1 = [-1, 1, -1, 1]$  or  $A_1 = [-1, 1, 1, -1]$ .

Taking  $A_1 = [-1, -1, 1, 1]$  as an example, we have  $A_1 X = b_1$ .  $x_1$  can be solved as

$$x_1 = -b_1 - x_2 + x_3 + x_4 \quad (7)$$

As discussed, implementation of  $A_1$  suffers from errors in practice. The actual coefficient vector is  $\tilde{A}_1 = [-(1 + \eta_1^\pi), -(1 + \eta_2^\pi), 1 + \eta_3^0, 1 + \eta_4^0]$  and the observed complex signal is  $\tilde{b}_1 = \tilde{A}_1 X$ , i.e.,

$$-(1 + \eta_1^\pi)x_1 - (1 + \eta_2^\pi)x_2 + (1 + \eta_3^0)x_3 + (1 + \eta_4^0)x_4 = \tilde{b}_1 \quad (8)$$

The estimate of  $x_1$  can be obtained based on (7) by replacing  $b_1$  with  $\tilde{b}_1$  in (8) and substituting  $x_2$ ,  $x_3$  and  $x_4$  with their estimates in (6), respectively. That is,

$$\hat{x}_1' = x_1 + \eta_1^\pi x_1 - \sum_{i=2}^4 \frac{\eta_i^0 - \eta_i^\pi}{2} x_i \quad (9)$$

Taking the average of the two estimates of  $x_1$  obtained in (6) and in (9), we can obtain the final estimate of  $x_1$ ,  $\hat{x}_1$ , as

$$\hat{x}_1 = \frac{\hat{x}_1 + \hat{x}'_1}{2} = x_1 + \frac{\eta_1^0 + \eta_1^\pi}{2} x_1 \quad (10)$$

As shown in (10), by adding only one measurement with the designed excitation vector  $A_1$ , the accumulated error at the 1st element has been effectively reduced to the same error level as other elements. The proposed algorithm will not affect (i.e. neither improve or deteriorate) the array calibration accuracy of the other array elements. Note that other excitation vectors  $A_1$  which fulfill the requirement (e.g.  $A_1 = [-1, 1, -1, 1]$  or  $A_1 = [-1, 1, 1, -1]$ ) would render the same estimate of  $x_1$  in (9) and therefore same final estimate of  $x_1$  as in (10). To summarize, the proposed calibration algorithm can solve the inaccurate calibration problem of the first antenna element in the reference method by adding only one measurement with properly designed element phase settings.

### III. MEASUREMENT VALIDATION

#### A. Experimental Setup

To experimentally validate the proposed array calibration algorithm, a measurement campaign was performed in an anechoic chamber. The experimental system, as illustrated in Fig. 1, consists of 1) a vector network analyzer (VNA) 2) a 32 ports magnitude and phase control network which includes a power splitting circuit, 32 digital phase shifters and a programmable attenuator matrix connected to the Tx antennas 3) transmitting wideband dual polarized 4 ridged horn antennas 4) an axial grooved corrugated horn Rx antenna. The Tx/Rx antennas offer low coupling and low scattering properties, and it covers a wide frequency band ranging from 2.4 GHz to 6 GHz. The Tx antenna element has a compact antenna aperture of 72 mm and a short length of 105 mm, as illustrated in Fig. 2. A  $16 \times 16$  uniform rectangular array (URA) is constructed at the Tx side with element spacing of 108 mm. The Rx antenna was placed in the boresight direction of the Tx array with a distance of 5.85 m, as shown in Fig. 3. Digital phase shifters with 8 bits are employed, implying a phase step resolution of  $360^\circ/2^8 = 1.4^\circ$ . Programmable attenuators with 60 dB dynamic range and 1 dB tuning resolution are employed as well. According to the digital phase shifter specification, a random amplitude uncertainty within  $[-0.5, 0.5]$  dB and a random phase uncertainty within  $[-5^\circ, 5^\circ]$  might be expected for each phase state. In the measurement, the sinusoidal signal at 3.5GHz generated by the VNA is fed into the power splitter, weighted by the complex weights realized by the digital phase shifters and programmable attenuator, and then radiated by the Tx antenna elements. The array complex signal received at the Rx antenna is then recorded at the VNA. Note that mutual coupling effect is essential for phased array calibration [9], [10], especially for microstrip antenna arrays. In our measurement setup, the coupling between ridged horn array elements with a separation of 108 mm at 3.5 GHz is below  $-30$  dB, indicating a negligible mutual coupling effect. Further, array elements in the center of the  $16 \times 16$  array are utilized to reduce the edge effects, as shown in Fig. 2.

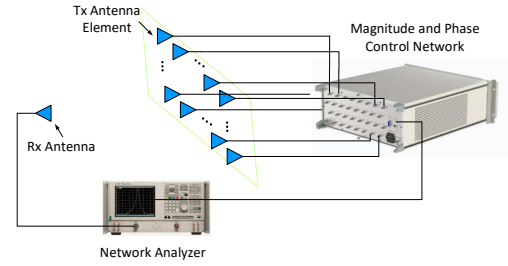


Figure 1. Schematic of the experimental system.

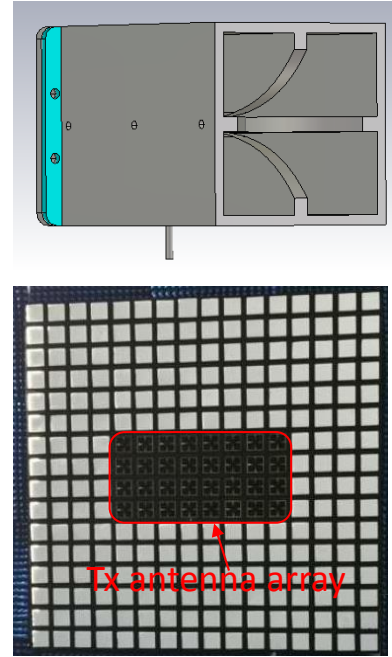


Figure 2. CST simulation model of the Tx antenna element (top) and a photo of the Tx antenna array (below).

#### B. Algorithm Validation Measurement

To evaluate the complexity and accuracy of the proposed algorithm compared to the reference algorithm, a measurement campaign was performed with a  $4 \times 8$  (i.e. 4 rows  $\times$  8 columns) URA. We first measured and stored the scattering parameters  $S_n$  between the Rx antenna and the  $n$ -th Tx antenna element for  $n \in [1, N]$ . At the Tx array side we enabled one Tx antenna (via setting 0 dB attenuation in the attenuator) with digital



Figure 3. A photo of the measurement setup in the anechoic chamber.

phase shifter set to  $0^\circ$ , and disabled all the other Tx antennas (via setting 60 dB attenuation in the associated attenuators). Then we repeated this “on/off” procedure for all the  $N$  Tx antenna elements.

To evaluate the robustness of the array calibration algorithm, two measurement campaigns were conducted. In the first campaign, the room temperature was not controlled, implying possibly larger toggling errors for the digital phase shifters. In the second campaign, the room temperature was maintained stable during measurement. In the first campaign, a deviation up to 0.6 dB in power and  $7.5^\circ$  in phase can be observed when phase states were set to  $0^\circ$  in different repeated measurements, while for the second campaign, the deviation is negligible, with power deviation up to 0.1 dB and phase deviation up to  $0.1^\circ$ .

For each measurement campaign, we measured the complex array signal at the Rx side when different excitation vectors in coefficient matrix were implemented. Two types of coefficient matrix  $A$  were implemented in the digital phase shifters in the measurements, the coefficient matrix reported in [4], and improved coefficient matrix discussed in Section II. C (i.e. coefficient matrix  $A$  and an extra measurement with designed excitation vector). In total, we carried out 33 measurements, where 32 measurements are for the reference method, and and 33 measurements for the proposed method. Note that for demonstration purpose, we intentionally set 5 dB attenuation in the 24th RF chain and 0 dB attenuation in the other RF chains. Further, we set  $45^\circ$  in the digital phase shifter in the 3rd RF branch and  $0^\circ$  in the other RF branches on purpose.

### C. Measurement results

The achieved array calibration results adopting the proposed method and reference method for the first and second measurement campaigns are shown in Fig. 4 and Fig. 5, respectively. Both algorithms are effective in detecting the initial excitation in the array, as shown in Fig. 4 and Fig. 5. The 5dB attenuation in the 24th RF chain and 0 dB attenuation in other RF chains can be clearly observed for both algorithms. Besides, the  $45^\circ$  phase shift in the 3rd RF chain and  $0^\circ$  phase shift in the other RF chains are detected. Compared with the reference method, the proposed method can effectively remove the accumulated error in the first antenna element, as shown in both measurements. For the first campaign, a power deviation is reduced from 0.8 dB to -0.3 dB, and a phase deviation is decreased from  $-4^\circ$  to  $0^\circ$  with the proposed algorithm for the first element. While for the second campaign, a power deviation from 1.1 dB to 0 dB, and a phase deviation from  $-4^\circ$  to  $0^\circ$  are achieved for the first element. Note that the array calibration accuracy for the other elements are the same for both algorithms, as expected.

With the improved calibration method, an amplitude error range of 1.5 dB and a phase error range of  $5^\circ$  are achieved for the first measurement campaign for all array elements with no temperature control, while an amplitude error range of 1 dB and phase error range of  $4^\circ$  can be achieved for the second measurement with dedicated temperature control. The measurement results indicate that the both methods are effective and robust and the proposed method can effectively improve the calibration accuracy of the first array element.

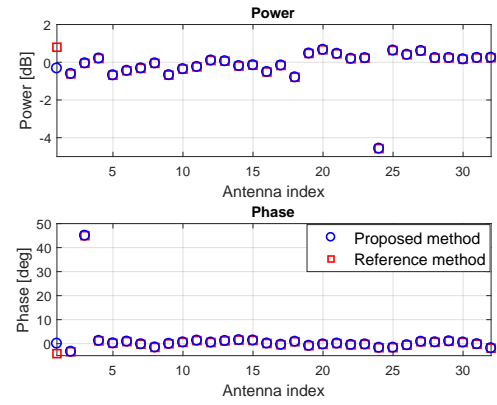


Figure 4. Array calibration results using the proposed method and reference method for the first campaign.

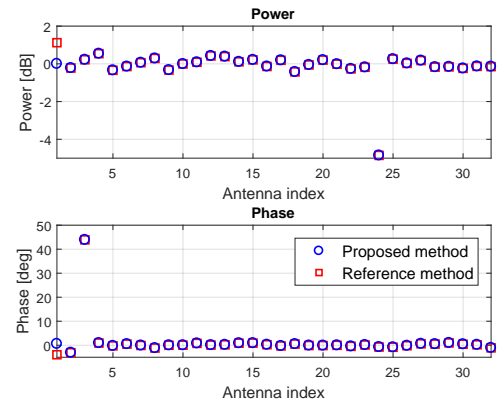


Figure 5. Array calibration results using the proposed method and reference method for the second campaign.

## IV. CONCLUSION

In the paper, an improved array calibration method was proposed. The complex transmission coefficients of the measurement system are measured when the array antenna elements are set with designed phase settings. The number of transmission coefficient measurements is limited to  $N + 1$ , with  $N$  the number of phased array antenna elements. The main contribution of the paper lies in the improvement of the reference method. Compared to the coefficient matrix in the reference method, the proposed coefficient matrix can effectively remove the accumulated error at the first antenna element via adding one more measurement with designed phase state settings in principle. The proposed method is theoretically analyzed and its effectiveness is experimentally validated.

## ACKNOWLEDGMENT

Dr. Wei Fan would like to acknowledge the financial support from Huawei Technologies and Danish Council for Independent Research under Grant DFF611100525.

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