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Calibration of Stochastic Radio Propagation Models Using Machine Learning

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***Abstract*—This letter proposes a machine learning based method for the calibration of stochastic radio propagation models. Model calibration is cast as a regression problem involving mapping of the channel transfer function or impulse response to the model parameters. A multilayer perceptron is trained with summary statistics computed from synthetically generated channel realizations using the model. To calibrate the model, the trained network is used to estimate the model parameters from channel statistics obtained from measurements. The performance of the proposed method is evaluated with propagation graph and Saleh-Valenzuela models using both simulated data and in-room channel measurements. Results show accurate estimation of the parameters of both models.**

***Index Terms*—Calibration, machine learning, propagation graph, measurements, neural networks, Saleh Valenzuela model**

I. Introduction

There are several applications in wireless communications where radio channel modelling is important. Channel models are used for tasks such as design and optimization of communication systems and evaluation of system performance via simulations. To this end, several models for characterizing wireless propagation channels have been developed over the last several years (see e.g., [1]–[6] and the references therein). These models can be broadly classified into deterministic, geometry based stochastic model (GSCM) and non-geometrical stochastic models (e.g., Turin [7], Saleh-Valenzuela [8] and Zwick model [9]).

For these models to be useful, they must be calibrated using real world measurements. Calibration involves estimation of the underlying parameters of the model. Historically, calibration of channel models is performed using a two stage methodology involving extraction of multipath parameters followed by model parameter estimation. While these methods have been used extensively in channel modelling literatures (see e.g., [8]–[12]), the performance is affected by the resolution and/or accuracy of the often complex multipath parameter estimation procedure such as SAGE, ESPRIT, and MUSIC.

Recently, a framework for estimating parameters of stochastic radio channel models without the intermediate multipath extraction step was introduced in [13], [14]. The authors proposed a method of moment (MoM) estimator for the Turin model and validate their proposal using both simulated and measured channel transfer functions. A MoM estimator was also developed in [15] for calibrating stochastic polarized propagation graph models [4]. Compared to the classical two stage approach, these methods appear to be more attractive since the intermediate multipath parameter extraction is completely eliminated. However, applicability of the MoM is severely limited by the need to derive new expressions for each model. This derivation could become intractable if not impossible for more complex models. These limitations can potentially be eliminated by using methods which do not involve explicit derivations of analytic expressions. An Approximate Bayesian Computation (ABC) based method is proposed in [16] for calibrating the Saleh Valenzuela model.

Machine learning paradigm provide useful tools for function approximation. For example, multilayer perceptrons (i.e., neural networks) have been proven to be capable of universal function approximation, i.e., they can be used to obtain reasonable approximations of any continuous real valued function on a bounded set in $\mathbb{R}^N$ [17].

In this letter, we propose a method for calibration of stochastic radio channel models based on a neural network (NN). To the best knowledge of the author, this is the first contribution on using machine learning tools for calibration of propagation models. The method involves synthetically generating channel realizations from the model and compressing the data into summary statistics which are used for training a NN. Parameters of the model are estimated by applying the trained network on summary statistics computed from measurements. Unlike existing methods, this method does not involve multipath parameter estimation and is applicable to any type of model. We perform experiments to evaluate the performance of the method using both synthetic data generated from the stochastic propagation graph (PG) [5] and the Saleh-Valenzuela (SV) [8] models as well as indoor channel measurements.

II. Neural Network Based Model Calibration

In this section, we present a fully data-driven calibration method for radio propagation models based on feed-forward NN with single hidden layer.

A. Neural Network Architecture

We consider a feed-forward NN with single hidden layer. The network describes the input-output mapping $f : \mathbf{s} \in \mathbb{R}^{M_i \times 1} \mapsto \boldsymbol{\theta} \in \mathbb{R}^{M_o \times 1}$ and can be represented as [18]

$$\begin{aligned} \boldsymbol{\theta} &= \rho_o(\mathbf{b}_o + \mathbf{W}_o \mathbf{v}) \\ \mathbf{v} &= \rho_h(\mathbf{b}_h + \mathbf{W}_h \mathbf{s}), \end{aligned} \tag{1}$$

where $\mathbf{s} \in \mathbb{R}^{M_i \times 1}$ and $\boldsymbol{\theta} \in \mathbb{R}^{M_o \times 1}$ are the input and output vectors, respectively. The matrix, $\mathbf{W}_h \in \mathbb{R}^{M_h \times M_i}$ contains the weight of connections between the M_i input nodes and the M_h

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hidden nodes. Similarly, the weight of connections between the hidden nodes and the M_o output nodes are contained in $\mathbf{W}_o \in \mathbb{R}^{M_o \times M_h}$. The bias of all hidden and output nodes are contained in $\mathbf{b}_h \in \mathbb{R}^{M_h \times 1}$ and $\mathbf{b}_o \in \mathbb{R}^{M_o \times 1}$, respectively. ρ_h and ρ_o are element-wise activation functions applied to the weighted sum of the inputs of each hidden and output node, respectively. In (1), $\mathbf{v} \in$ corresponds to the output of the hidden layer which is fed to the output layer.

We consider a linear activation function, $\rho_o(x) = x$ for the output layer and the non-linear sigmoid function, $\rho_h(x) = 1/(1 + e^{-x})$ for the hidden layer.

B. Model Calibration Procedure

Consider a radio propagation model which defines the channel transfer function, $\mathbf{H}(f)$;, where f denotes the frequency points, in terms of a set of M_o real valued parameters, $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_{M_o}]^T \in \mathbb{R}^{M_o \times 1}$. Given measurements of $\mathbf{H}(f)$, calibration involves finding estimates of the model parameters, $\hat{\boldsymbol{\theta}}$. The proposed ML based method for estimating the parameters is illustrated in Fig. 1. The method involves:

1) Data Generation: We generate K sets of the model parameters between a lower bound and an upper bound. It is straightforward to set these limits since model parameters are bounded in practice. For instance, an angular parameter can only take values between 0 and 2π. For each set of parameters, we generate Z realizations of CTFs. Denoting the number of frequency points in the CTF as N_s , the generated CTFs are collected into a data matrix, $\mathbf{D} \in \mathbb{C}^{K \times Z \times N_\mathrm{s}}$ with corresponding $N \times K$ parameter matrix $\boldsymbol{\theta} = [\boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_K]$.
2) Data processing: While it is possible to train a network with the generated CTF matrix, the number of input nodes and hence, network complexity can be significantly reduced by summarizing the CTFs into suitable summary statistics[1]. Motivated by the works in [13], we propose summarizing the data into mean and covariances of the first three temporal moments. For the kth parameter set, the mean, $\bar{m}_k^i$ and covariance, $\bar{c}_k^{ij}$ of temporal moments is estimated using

$$\bar{m}_k^i = \frac{1}{Z}\sum_{z=1}^{Z} m_{k,z}^i; \quad i = 0,1,2$$
$$\bar{c}_k^{ij} = \frac{1}{Z}\sum_{z=1}^{Z} (m_{k,z}^i - \bar{m}_k^i)(m_{k,z}^j - \bar{m}_k^j); \quad i = 0,1,2 \tag{2}$$

where $m_{k,z}^i$ is the ith temporal moment of the zth channel realization for the kth parameter set which is defined as

$$m_{k,z}^i = \int_0^{t_{\max}} t^i |h_{k,z}(t)|^2. \tag{3}$$

Here, $h_{k,z}(t)$ is the impulse response of the zth channel realization with the kth parameter set and $|\cdot|$ denotes absolute value of the associated complex number.
3) Network selection and training: The computed statistics and parameters are collected into K input-target pairs; $\{\mathbf{s}_k, \boldsymbol{\theta}_k\}_{k=1}^K$, where $\mathbf{s}_k = [\bar{m}_k^1, \bar{m}_k^2, \bar{m}_k^3, \bar{c}_k^{11}, \bar{c}_k^{12}, \bar{c}_k^{13}, \bar{c}_k^{22}, \bar{c}_k^{23}, \bar{c}_k^{33}]^T$. The K examples are then used to train a NN with $M_i = 9$ input, M_h hidden and M_o output nodes. While, M_o is determined by the number of model parameters, M_h is a design parameter that needs to be set to ensure reasonable performance of the model. A procedure involving comparison of network performance with different number of hidden nodes is used to set M_h in this work.
4) Model calibration: The trained network is used to estimate parameters of the model from the calibration set, $\mathbf{s}_{\text{cal}}$, which consists of statistics computed from measurements.

[1]For example, using the CTFs in $\mathbf{D}$ requires a network with ZN_s input nodes which can be considerably large depending on the bandwidth and sampling frequency. With the measurements in Section III-C for which $N_\mathrm{s} = 801$, the number of input nodes, $M_i = 801Z; Z \geq 1$.

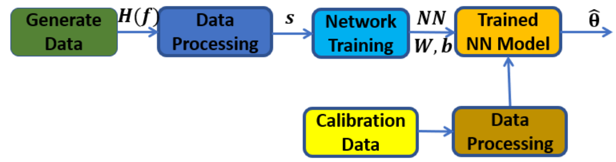


Fig. 1: Machine learning based model calibration procedure.

TABLE I: Bounds on model parameters for simulations.

Model	Parameters	Minimum	Maximum
PG	g	0.50	0.75
	N_s	5	30
	P_{vis}	0.4	1
SV	Λ[/ns]	0.01	0.40
	λ[/ns]	0.50	3.00
	Γ	1	14
	γ	1	14

C. Training Process

Based on an input vector, $\mathbf{s}_k$, the feed-forward NN yields a prediction, $\hat{\boldsymbol{\theta}}_k$ of the model parameters. The training process involves estimating the network parameters, $\{\mathbf{W}_h, \mathbf{W}_o, \mathbf{b}_h, \mathbf{b}_o\}$ such that the cost function,

$$\mathcal{L}(\hat{\boldsymbol{\theta}}, \boldsymbol{\theta}) = \sum_{k=1}^{K} (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_k)(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_k)^T, \tag{4}$$

is minimized. Several training procedures including Levenberg-Marquardt (LM), Bayesian Regularization (BR), and Gradient Descent (GD) [18] have been developed for NNs. Each of these methods has its benefits and limitations, and their usefulness is often application dependent. For example, LM provides fast computation speed at the expense of increased memory requirement. On the other hand, BR is typically slower but provides better generalization for problems with small or noisy datasets [19]. Since training data is synthetically generated in the proposed method, the amount of data is limited. Hence, BR is applied for network training in this paper.

TABLE II: Estimated parameters of PG model and RMSE.

Exp.	Parameters					RMSE		
	Parameter	True	Estimates					
			Best	Mean	Var. ($\times 10^{-4}$)	Min. ($\times 10^{-5}$)	Mean	Var. ($\times 10^{-5}$)
I	g	0.44	0.45	0.47	1.67	8.75	0.02	8.64
	$N_{\rm s}$	20	20	20	0	0	0	0
	$P_{\rm vis}$	0.75	0.75	0.72	1.03	5.64	0.02	8.91
II	g	0.46	0.46	0.47	1.63	4.73	0.01	5.22
	$N_{\rm s}$	12	12	12	0	0	0	0
	$P_{\rm vis}$	0.51	0.56	0.55	2.05	31.8	0.03	16.4
III	g	0.59	0.57	0.58	1.31	33.6	0.01	3.41
	$N_{\rm s}$	12	12	12	0	0	0	0
	$P_{\rm vis}$	0.78	0.77	0.77	1.78	31.6	0.01	4.69
IV	g	0.60	0.60	0.59	7.02	3.26	0.008	2.14
	$N_{\rm s}$	21	22	22	196	19161	0.78	1055
	$P_{\rm vis}$	0.80	0.79	0.77	1.10	19.80	0.02	9.47

TABLE III: Estimated parameters of the SV model and RMSE.

Exp.	Parameters					RMSE		
	Parameter	True	Estimates					
			Best	Mean	Var. ($\times 10^{-3}$)	Min. ($\times 10^{-4}$)	Mean	Var. ($\times 10^{-3}$)
I	Λ[1/ns]	0.101	0.118	0.111	0.281	6.712	0.164	5.296
	λ[1/ns]	2.459	2.343	2.243	7.092	0.108	0.031	0.834
	Γ	7.395	7.270	7.198	23.512	6.485	0.037	1.570
	γ	7.886	7.824	7.892	36.689	3.052	0.029	1.784
II	Λ[1/ns]	0.355	0.347	0.330	0.166	0.169	0.073	0.644
	λ[1/ns]	2.285	2.339	2.385	2.998	20.809	0.045	0.695
	Γ	8.916	8.955	8.929	14.641	2.962	0.011	0.181
	γ	8.253	8.299	8.443	15.809	2.019	0.023	0.557

III. Performance Evaluation

We present results of application of the proposed calibration method to two stochastic channel models: PG [5] and SV models [8]. These models have been used extensively for various applications. While the SV model is commonly calibrated using the classical approach involving multipath extraction, studies on calibration of the PG model have been limited. The moments based calibration method for the PG in [15] suffers from an identifiability problem.

A. Considered Channel Models

1) Propagation Graph Model: The PG [5] models the environments as a simple directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where vertex set $\mathcal{V} = \mathcal{V}_{\rm t} \cup \mathcal{V}_{\rm s} \cup \mathcal{V}_{\rm r}$ is a union of three disjoint sets: a set of $N_{\rm t}$ transmitters, $\mathcal{V}_{\rm t}$, a set of $N_{\rm s}$ scatterers, $\mathcal{V}_{\rm s}$ and a set of $N_{\rm r}$ receivers, $\mathcal{V}_{\rm r}$. Wave propagation between the vertices is modelled by edges in $\mathcal{E}$. Wave propagation in the PG is defined by the actions of the scatterers and edges and the channel transfer function is given as [5]

$$\mathbf{H}(f) = \mathbf{D}(f) + \mathbf{R}(f)[\mathbf{I} - \mathbf{B}(f)]^{-1}\mathbf{T}(f), \tag{5}$$

where $\mathbf{D}(f)$, $\mathbf{T}(f)$, $\mathbf{R}(f)$ and $\mathbf{B}(f)$ contain transfer functions of direct, transmitter to scatterers, scatterers to receivers and scatterers to scatterers edges, respectively. The edge transfer functions are defined for the stochastic PG as

$$A_e(f) = \begin{cases} g_e(f)\exp(j2\pi f\tau_e + \phi_e); & e \in \mathcal{E} \\ 0; & e \notin \mathcal{E}, \end{cases} \tag{6}$$

where $g_e(f)$ and τ_e denote the edge gain and delay, respectively. ϕ_e is the random initial phase of the edge. A procedure for stochastically generating transfer functions and impulse response from the PG is presented in [5]. This stochastic implementation requires three parameters to describe an environment: the reflection gain, g, the number of scatterers, $N_{\rm s}$, and the probability of visibility, $P_{\rm vis}$. Thus, calibration of the PG requires estimation of the parameter set $\boldsymbol{\theta} = [g, N_{\rm s}, P_{\rm vis}]$ from measurements.

2) Saleh Valenzuela (SV) Model: The SV model was originally proposed [8] to model indoor propagation with account for the inherent clustering phenomenon. The underlying principle of the model emanates from observations from indoor measurements that multipath components arrive in clusters and that arrival of the clusters as well as the rays within each cluster follow Poisson processes with different rates. The impulse response of the channel is expressed as [8]

$$h(t) = \sum_{k=0}^{\infty}\sum_{z=0}^{\infty} \beta_{k,z} e^{j\vartheta_{k,z}} \delta(t - T_k - \tau_{k,z}) \tag{7}$$

where T_k and $\tau_{k,z}$ are the arrival time of the first ray in the kth cluster and delay of the zth ray relative to T_k, respectively. $\beta_{k,z}$ and $\vartheta_{k,z}$ are the gain and uniformly distributed phase of the zth ray in the kth cluster. The mean square value of $\beta_{k,z}$ is given by

$$\overline{\beta_{k,z}^2} = \overline{\beta_{00}^2} e^{-T_k/\Gamma} e^{-\tau_{k,z}/\gamma}, \tag{8}$$

where $\overline{\beta_{00}^2}$ is the average power gain of the ray in the first cluster, and Γ and γ are exponential power-decay constants for the clusters and rays, respectively. The inter-arrival times of clusters and ray are exponential with probability distribution functions defined as

$$p(T_k|T_{k-1}) = \Lambda \exp[-\Lambda(T_k - T_{k-1})], \quad k > 0 \tag{9}$$

and

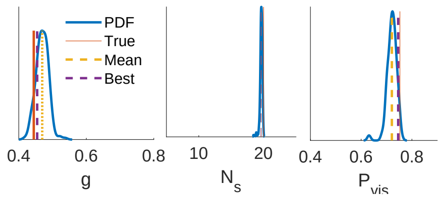
$$p(\tau_{k,z}|\tau_{k-1,z}) = \lambda \exp[-\lambda(\tau_{k,z} - \tau_{k-1,z})], \quad z > 0, \tag{10}$$

respectively. Here, $\Lambda(\lambda)$ is the cluster(ray) arrival rate. In order to describe a propagation environment, the S-V model requires four parameters viz: the cluster arrival rate, Λ; the ray arrival rate, λ, the cluster decay constant, Γ and the ray decay constant, γ. Thus, calibration of the S-V model involves estimation of the parameter set $\boldsymbol{\theta} = [\Gamma, \gamma, \Lambda, \lambda]$.

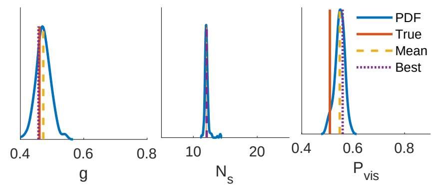
B. Results on Simulated Data

For each channel model, we generated 2000 parameter sets from a uniform distribution on the range shown in Tab. I. Model parameter bounds for the simulation were set for the SV and PG model based on values extracted from measurements in [20] and [15], respectively. A total of 500 channel realizations were generated for each parameter set. The generated CTFs are then summarized following the procedure in Section II-B.

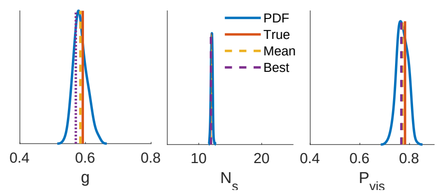
A calibration set comprising of 4 statistics - model parameter pairs was selected prior to training. The remaining data-set was randomly divided into training and test sets in the ratio 4:1. The random partitioning and stochastic nature of the training procedure often result in variations in trained models and hence, parameter estimates. To quantify this variation, the model calibration procedure was repeated 200 times with the same data-sets in our simulations. Following heuristic evaluation of NN with 5 to 30 hidden nodes, a value of $M_h = 20$ was chosen for our simulations.



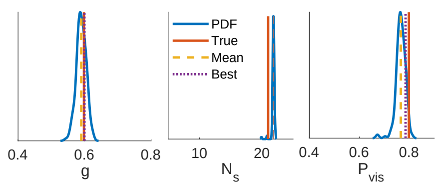
(a) I.



(b) II.



(c) III.



(d) IV.

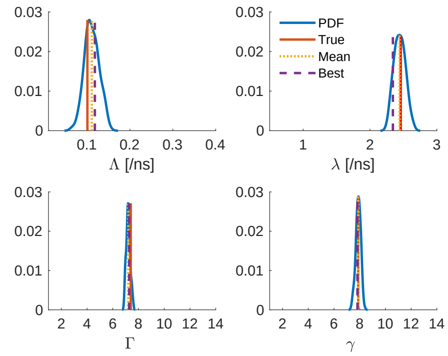
Fig. 2: Estimates of PG model parameters using a 3 - inputs network with 20 hidden nodes.

As shown in Fig. 2 and Fig. 3, estimated parameters are concentrated around the true values for the PG and SV models, respectively. For all model parameters, the best (estimates from the network with least error performance) and mean (average of 200 estimates) are close to the true parameter values resulting in very small root mean square error (RMSE) as shown in Tab. II and Tab. III for the PG and SV models, respectively.

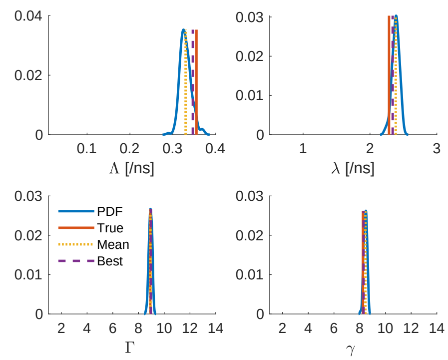
C. Application to Measured Data

We now evaluate the applicability of the proposed method by testing it on an indoor channel measurement [1]. The measurement was obtained using a VNA at 60 GHz in a $3{\times}4{\times}3\,\mathrm{m}^3$ room. The data set comprises of 625 measurements from a 25×25 virtual planar array.

The trained NN in Section III-B is applied to statistics computed from the measurements to obtain estimates of the model parameters. As shown in Fig. 4, the power level and tail decay of the measured averaged power delay profile (PDP) agree closely with those of the power delay spectrum predicted by the PG and SV model with the estimated parameters. This shows that the NN based method is able to accurately extract model parameters from real measurements. Although, exact prediction of all features of the measured instantaneous PDP is not expected from stochastic models, we also show plots of single realizations of the PG and measurements in Fig. 4.



(a) I.



(b) II.

Fig. 3: Estimates of Saleh Valenzuela model parameters using a 9 - inputs network with 20 hidden nodes.

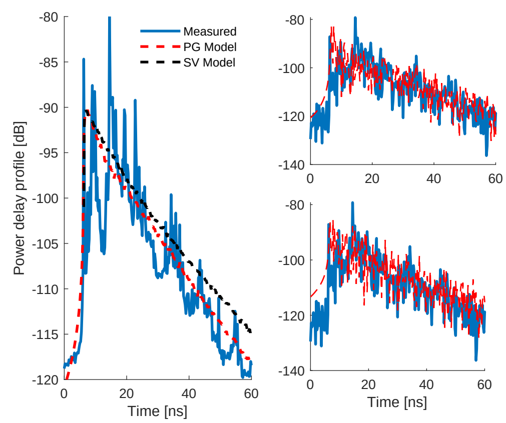


Fig. 4: Averaged (left) and instantaneous (right) PDP computed from measurements and estimated PDP from the models. Estimated model parameters are: PG ($g = 0.65$, $N_\mathrm{s} = 31$, $P_\mathrm{vis} = 0.93$) and SV ($\Gamma = 15.6$, $\gamma = 14$, $\Lambda = 0.55$, $\lambda = 2.9$).

The figure shows that the measured instantaneous PDP is well predicted by the model. Similar results were obtained from the SV model. These are not shown here due to space constraints.

IV. Conclusion

We proposed a machine learning based method for calibrating stochastic radio channel models. A neural network is trained using channel impulse response simulated from the model to be calibrated. Once training is completed, the trained network is applied to extract parameters of the model from calibration data (i.e., measurements). Results show accurate estimation of model parameters for both synthetic and measured data.

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