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Calibration of Stochastic Channel Models using Approximate Bayesian Computation

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***Abstract*—Calibration of stochastic radio channel models is the process of fitting the parameters of a model such that it generates synthetic data similar to the measurements. The traditional calibration approach involves, first, extracting the multipath components, then, grouping them into clusters, and finally, estimating the model parameters. In this paper, we propose to use approximate Bayesian computation (ABC) to calibrate stochastic channel models so as to bypass the need for multipath extraction and clustering. We apply the ABC method to calibrate the well-known Saleh-Valenzuela model and show its performance in simulations and using measured data. We find that the Saleh-Valenzuela model can be calibrated directly without the need for multipath extraction or clustering.**

I. Introduction

Stochastic multipath models are indispensable tools for characterizing the radio channel and analysing the performance of communication systems. Typically, the models are generative, so synthetic data can be generated and used in simulation studies. Such models are particularly useful if calibrated with measurement data. Calibrating a model means estimating its parameters such that the model fits to the measurements in some metric. Traditionally, calibration problems are solved by maximizing a likelihood function, or by finding the posterior distribution of the parameters (in a Bayesian approach). In either case, access to the likelihood function is required. Unfortunately, the likelihood is intractable here as marginalization with respect to the hidden multipath components is not possible.

Instead of relying on inference frameworks, typically we resort to splitting the inference problem into subproblems and solve these independently. For example, calibration is typically done in three steps. First, multipath parameters such as delays, gains, etc., of each multipath component is extracted from the channel impulse response measurements. High-resolution algorithms such as SAGE and CLEAN, among others, are used for this step. Then, the extracted multipath components are grouped together into clusters, either manually or using automated clustering algorithms, e.g. [1]. The third step then involves fitting the extracted and clustered multipath components to the model and estimating the parameters. Such multi-step procedures involve complex algorithms that can be cumbersome to use and prone to errors such as estimation artifacts and censoring effects. Moreover, a number of arbitrary choices need to be made in order to implement these algorithms. The estimation accuracy of the parameters then relies on each of the intermediate steps, and therefore the overall performance of these methods is difficult to investigate.

Recently, calibration methods have been proposed in [2], [3] for the Turin model [4] (with constant arrival rate) that bypasses the multipath extraction step. They summarise the channel measurements into temporal moments and estimate the model parameters directly. While [2] assumes a multivariate Gaussian model for the temporal moments and samples from the approximate posterior of the parameters, [3] fits the expressions of means and covariances of the temporal moments to the parameters using a method of moments approach. However, in cases where the Gaussian assumption fails or when the model is too complicated to derive the theoretical expressions, these methods cannot be utilised. Hence, there is a need for methods to calibrate complex models that involve clustering of multipath components.

The problem of calibrating generative models with unavailable likelihoods appears in many other sciences. A potential solution is approximate Bayesian computation (ABC) [5], [6], which was first introduced in population genetics, and further developed and used in other fields. ABC relies on comparing summary statistics of simulated and measured data in some distance metric. A potential solution to calibrating stochastic multipath models could be to use ABC methods combined with informative summary statistics such as those studied in [2], [3]. To the best of our knowledge, ABC methods have not been used in radio channel characterization.

In this paper, we propose an ABC algorithm based on population Monte Carlo [7] and regression adjustment [8] for calibration of stochastic channel models. As an example, we develop summary statistics for calibrating the well-known Saleh-Valenzuela [9] model using ABC. Performance evaluation of this calibration method is carried out via a simulation experiment as well as using measurement data.

II. Estimation Problem Formulation

Consider single-input, single-output (SISO) frequency domain measurement where the received signal sampled at K equidistant frequency points within the bandwidth B reads

$$Y_k = H_k + W_k, \quad k = 0, 1, \ldots, (K-1), \tag{1}$$

where H_k is the transfer function, and W_k is independent and identically distributed circular Gaussian measurement noise with variance σ_W^2. Discrete-frequency, continuous-time inverse Fourier transform gives the time domain signal,

$$y(t) = \frac{1}{K} \sum_{k=0}^{K-1} Y_k \exp\left(j2\pi k \Delta f t\right). \tag{2}$$

The frequency separation, Δf, is related to the period of the time domain signal as

$$t_{\text{max}} = \frac{1}{\Delta f} = \frac{K-1}{B}. \tag{3}$$

A measurement campaign where N_{obs} observations of the channel transfer function are recorded, results in the data matrix $\mathbf{Y}_{\text{obs}} \in \mathbb{C}^{N_{\text{obs}} \times K}$.

A general multipath model of the transfer function may read

$$H_k = \sum_l \alpha_l \exp\left(-j2\pi \Delta f k \tau_l\right), \tag{4}$$

where τ_l and α_l are the time delay and complex gain of the l^{th} multipath component. The delays and their corresponding gains form a marked point process, $X = \{(\tau_l, \alpha_l)\}$. A stochastic channel model driven by parameters θ is a mechanism that outputs such a marked point process, i.e. $X \sim \mathcal{M}(\theta)$, where $\mathcal{M}(\cdot)$ is the generative model. The model is calibrated by estimating θ from the measurements, $\mathbf{Y}_{\text{obs}}$.

Typically, stochastic multipath models are proposed with the purpose of simulation, i.e. they are generative in nature. However, in the general case of unknown, potentially infinite, number of multipath components, the likelihood $p(X|\theta)$, and consequently, $p(\mathbf{Y}|\theta)$, are analytically and numerically intractable. Therefore, standard inference and sampling techniques methods which rely on the likelihood (or posterior) cannot be applied.

III. Approximate Bayesian Computation

ABC can be used to sample from an approximate posterior when the likelihood is numerically unavailable. Based on these samples, we can approximate the standard point estimates, e.g. the minimum mean squared error (MMSE) estimate.

ABC relies on sampling parameter values from the prior distribution, $p(\theta)$, and simulating datasets, $\mathbf{Y}$, from the model, $\mathcal{M}(\cdot)$. Summary statistics of $\mathbf{Y}$ and $\mathbf{Y}_{\text{obs}}$ are computed using a function, $S(\cdot)$. Let the vector of simulated and observed summary statistics be $\mathbf{s} = S(\mathbf{Y})$ and $\mathbf{s}_{\text{obs}} = S(\mathbf{Y}_{\text{obs}})$, respectively. The summaries are compared in some "distance" measure, $\rho(\cdot,\cdot)$. The basic accept-reject ABC method involves accepting values of θ for which $\rho(\mathbf{s}, \mathbf{s}_{\text{obs}}) < \epsilon$, where ϵ is some pre-defined tolerance threshold. Thus, samples from $\tilde{p}(\theta|\mathbf{s}_{\text{obs}})$ can be obtained, which is an approximation of the desired posterior, $p(\theta|\mathbf{Y}_{\text{obs}})$. The approximation arising here due to the substitution of $\mathbf{Y}_{\text{obs}}$ with $\mathbf{s}_{\text{obs}}$ and the use of ϵ.

Implementation of an ABC algorithm requires the specification of three quantities: the distance measure $\rho(\cdot,\cdot)$, the summary statistics $\mathbf{s}$, and the threshold ϵ. In this paper, we will use the Euclidean distance as our distance function, therefore $\rho(\cdot,\cdot) = ||\cdot||$. Moreover, we consider ϵ in terms of percentile and specify the number of accepted samples, M_ϵ instead. That is, for M samples of θ from the prior, $\epsilon = M_\epsilon/M$. Note that, an ϵ in terms of the distance measure can be used, but it leads to unpredictable run times for a certain number of accepted samples. The selection of appropriate summary statistics, however, relies heavily on the application (and model) at hand, and will be addressed later with regards to an example channel model.

There exists more advanced ABC methods in the literature than the accept-reject algorithm [5], [6], and in principle, any of them could be used for calibrating stochastic channel models. We propose to use the population Monte Carlo (PMC) ABC [7] with regression adjustment [8].

A. Proposed ABC method

The accept-reject ABC algorithm weights each of the accepted samples equally without taking into account the distance from the observed summary statistics. In this paper, we use the method proposed in [8] which improves the approximation of the posterior by weighting the accepted parameter samples, θ_j, according to $||\mathbf{s}_{\text{obs}} - \mathbf{s}_j||$ and adjusting θ_j by using linear regression model applied locally in the vicinity of $\mathbf{s}_{\text{obs}}$. Specifically, the optimisation problem being solved is [8]

$$\underset{a,\mathbf{b}}{\operatorname{argmin}} \sum_{j=1}^{M_\epsilon} \left[\theta_j - a - \left(\mathbf{s}_{\text{obs}} - \mathbf{s}_j\right)^T \mathbf{b}\right]^2 K_\epsilon\left(||\mathbf{s}_{\text{obs}} - \mathbf{s}_j||\right) \tag{5}$$

Here, $K_\epsilon(\cdot)$ is the Epanechnikov kernel. The samples, $\tilde{\theta}_j$, are then adjusted as

$$\tilde{\theta}_j = \theta_j - \left(\mathbf{s}_{\text{obs}} - \mathbf{s}_j\right)^T \hat{\mathbf{b}}. \tag{6}$$

This regression-ABC algorithm is described in Alg. 1. For details regarding the solution of (5), see [8]. The estimate of the intercept, $\hat{a}$, gives the estimate of the posterior mean, $\mathbb{E}[\theta|\mathbf{s}_{\text{obs}}]$.

After the regression adjustment, some samples of $\tilde{\theta}$ may go beyond the prior range. For example, getting a negative value after adjustment of a strictly positive parameter does not make sense. To avoid this problem, samples of θ outside its prior range are replaced by the extreme points (entrywise).

We combine the regression-ABC with the population Monte Carlo (PMC)-ABC method of [7]. PMC-ABC

Algorithm 1 Regression ABC algorithm

Input: Parameter values $(\theta_1, \dots, \theta_M)$ and corresponding simulated statistics $(\mathbf{s}_1, \dots, \mathbf{s}_M)$, observed statistics $\mathbf{s}_{\text{obs}}$, number of accepted samples M_ϵ,

Accept $(\theta_1^*, \dots, \theta_{M_\epsilon}^*) \sim \{\theta_i\}_{i=1}^M$ with the smallest $\|\mathbf{s}_{\text{obs}} - \mathbf{s}_i\|$, $i = 1, \dots, M$

Solve optimisation problem (5) with $\left\{\theta_j^*\right\}_{j=1}^{M_\epsilon}$ and corresponding $\left\{\mathbf{s}_j^*\right\}_{j=1}^{M_\epsilon}$ to get $\hat{\mathbf{b}}$

Adjust accepted samples $\left\{\theta_j^*\right\}_{j=1}^{M_\epsilon}$ using (6) to get $\left\{\tilde{\theta}_j\right\}_{j=1}^{M_\epsilon}$ in the prior range

Output: Samples $(\tilde{\theta}_1, \dots, \tilde{\theta}_{M_\epsilon})$ from approximate posterior

Algorithm 2 PMC-ABC with regression adjustment

Input: Prior $p(\theta)$, model $\mathcal{M}(\theta)$, observed statistics $\mathbf{s}_{\text{obs}}$, M_ϵ, M, T

At iteration $t = 1$,
  Sample M samples of θ from the prior, i.e. $(\theta_1, \dots, \theta_M) \sim p(\theta)$
  Generate $\mathbf{s}_i \sim \mathcal{M}(\theta_i)$, $i = 1, \dots, M$
  Perform **Algorithm** 1 on $(\theta_1, \dots, \theta_M)$ and $(\mathbf{s}_1, \dots, \mathbf{s}_M)$ to generate $(\theta_1^{(1)}, \dots, \theta_{M_\epsilon}^{(1)})$
  Set $w_i^{(1)} = 1/M_\epsilon$ for $i = 1, \dots, M_\epsilon$ and take $\sigma^{(1)}$ to be twice the sample variance of $(\theta_1^{(1)}, \dots, \theta_{M_\epsilon}^{(1)})$
for $t = 2, \dots, T$
  for $j = 1, \dots, M$
    Sample $\theta_j^* \sim (\theta_1^{(t-1)}, \dots, \theta_{M_\epsilon}^{(t-1)})$ with probability $(w_1^{(t-1)}, \dots, w_{M_\epsilon}^{(t-1)})$
    Perturb θ_j^* by sampling $\theta_j^{**}|\theta_j^* \sim \tilde{\mathcal{N}}\left(\theta_j^*, \sigma^{(t-1)}\right)$
    Generate $\mathbf{s}_j \sim \mathcal{M}(\theta_j^{**})$
  end
  Perform **Algorithm** 1 on $(\theta_1^{**}, \dots, \theta_M^{**})$ and $(\mathbf{s}_1, \dots, \mathbf{s}_M)$ to generate $(\theta_1^{(t)}, \dots, \theta_{M_\epsilon}^{(t)})$
  Set $w_i^{(t)} \propto p(\theta_i^{(t)}) / \sum_{j=1}^{M_\epsilon} w_j^{(t-1)} q\left(\theta_j^{(t-1)} | \theta_i^{(t)}, \sigma^{(t-1)}\right)$, and take $\sigma^{(t)}$ to be twice the sample variance of $(\theta_1^{(t)}, \dots, \theta_{M_\epsilon}^{(t)})$
end

Output: Samples $(\theta_1^{(T)}, \dots, \theta_{M_\epsilon}^{(T)})$ from the approximate posterior $\tilde{p}(\theta|\mathbf{s}_{\text{obs}})$

is a sequential Monte Carlo method that works with a population of θ values instead of one sample at a time. At each iteration, M samples of θ are generated from the M_ϵ accepted samples after performing regression-ABC. The overall algorithm is given in Alg. 2, where θ is assumed univariate. In the general case of θ being a vector of parameters, the same algorithm applies on each entry of θ independently. Note that $\tilde{\mathcal{N}}(\cdot, \cdot)$ denotes a Gaussian truncated to be in the prior range of θ, and $q(\cdot, \cdot)$ is a Gaussian kernel. The overall computational time of Alg. 2 depends on how many observations of the channel, say N_{sim}, are simulated to compute $\mathbf{s}$ from each value of θ.

IV. Application to Saleh-Valenzuela model

The seminal model proposed by Saleh-Valenzuela [9] has been the basis of standardized models such as IEEE 802.15.3a and IEEE 802.15.4a. It has been extended further to include the spatial properties of the channel and also been applied in millimetre wave scenarios using a MIMO system. Calibration methods for this model commonly rely on multipath extraction and clustering. For this reason, we demonstrate the utility of the proposed ABC method by applying it to the Saleh-Valenzuela model.

A. Estimation problem

The Saleh-Valenzuela model can be formulated in the frequency domain as

$$H_k = \sum_l \sum_p \beta_{pl} \exp\left(-j2\pi\Delta f k (T_l + \tau_{pl})\right), \quad (7)$$

where T_l is the delay of the l^{th} cluster, while τ_{pl} and β_{pl} are the delay and complex gain of the p^{th} ray within the l^{th} cluster, respectively. By definition in [9], $T_0 = 0$ and $\tau_{0l} = 0$, $l \in \{0, 1, \dots\}$. The arrival time of the clusters and that of the rays within the clusters are modelled as one-dimensional homogeneous Poisson point processes, i.e., $T_l \sim \text{PPP}(\mathbb{R}_+, \Lambda)$ and $\tau_{kl} \sim \text{PPP}(\mathbb{R}_+, \lambda)$ with parameters $\Lambda, \lambda > 0$. The gains β_{kl}, conditioned on T_l and τ_{kl}, are modelled as iid zero-mean complex Gaussian random variables with conditional variance

$$\mathbb{E}\left[|\beta_{kl}|^2 | T_l, \tau_{kl}\right] = Q \exp(-T_l/\Gamma) \exp(-\tau_{kl}/\gamma), \quad (8)$$

where Q denotes the average power of the first arriving multipath component, and $\Gamma, \gamma > 0$ are the cluster and ray decay constants, respectively. The expression for the power delay spectrum is given in [10]. To calibrate this model, the parameter vector, $\theta = [Q, \Lambda, \lambda, \Gamma, \gamma]^T$, should be estimated based on $\mathbf{Y}_{\text{obs}}$.

B. Development of summary statistics

In order to use the ABC method to infer on θ, informative summary statistics of $\mathbf{Y}$ are needed about the five model parameters. The selection of summaries determines the degree to which we approximate the posterior distribution by replacing $\mathbf{Y}$ with $\mathbf{s}$.

Temporal moments are widely used statistics that have been recently used to estimate the parameters of the Turin model [4] without multipath extraction in [2], [3]. We define the generalised i^{th} temporal moment as

$$m_i^{(n)} = \int_0^{t_{\max}} t^i |y(t)|^n \text{dt}, \quad i = 0, 1, 2, \dots. \quad (9)$$

Most commonly, only temopral moments with $n = 2$ are considered. The temporal moments are random variables that summarise each channel realisation into one instance for a particular n and i. So, N_{obs} realisations would lead to a vector of temporal moments, $\mathbf{m}_i^{(n)}$ for each choice of i and n. We use the first three temporal moments, i.e. $D^{(n)} = [\mathbf{m}_0^{(n)}, \mathbf{m}_1^{(n)}, \mathbf{m}_2^{(n)}]$. We further summarise the n^{th} order temporal moments by computing the sample mean vector, $\mu_D^{(n)}$, and covariance matrix, $\Sigma_D^{(n)}$. This

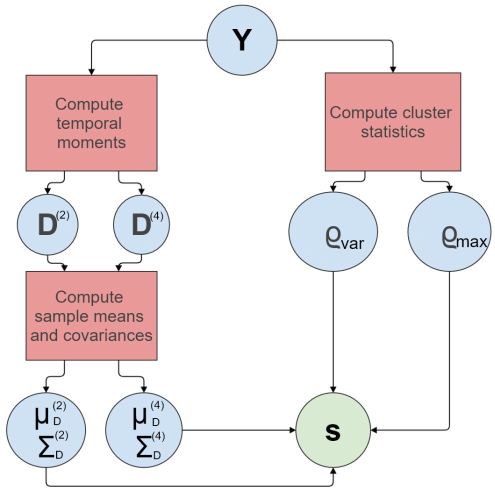

Fig. 1. Data flow from measurements to proposed summary statistics.

results in nine distinct summary statistics for each n. Since the kurtosis of channel measurements is known to be informative about the arrival rate [11], [12], we also include $\mu_D^{(4)}$ and $\Sigma_D^{(4)}$ as summaries.

We observed through simulation (not shown due to space limitations) that the temporal moments alone were not as informative about the cluster parameters (Λ, Γ). For this reason, we include two additional summaries, ϱ_{max} and ϱ_{var}, to measure the "unorderedness" of the averaged power delay profile (APDP). Sorting the discrete-time APDP with index sequence $\mathbf{v} = [1, \ldots, K]$ in descending order leads to a permutation of indices, say $\mathbf{v}'$. Then the two summaries are computed as

$$\varrho_{\text{max}} = \max\left(\mathbf{v} - \mathbf{v}'\right), \quad \text{and} \tag{10}$$

$$\varrho_{\text{var}} = \operatorname{var}\left(\mathbf{v}^{+}\right), \tag{11}$$

where $\mathbf{v}^{+}$ contains all positive entries of $(\mathbf{v} - \mathbf{v}')$.

The resulting collection of summaries yields a twenty dimensional vector, $\mathbf{s}$, as outlined in Fig. 1. The choice of summary statistics is not limited to the ones discussed here. In principle, other summaries could also be used, as long as they are informative about the model parameters.

C. Setting priors for parameters

The priors are used to sample θ values. Here we use flat priors for all the parameters as a way limit the search space in the first iteration of Alg. 2. The range should be wide enough to capture all the plausible values of the parameter, but not so wide that the computational time of the algorithm becomes excessive. The specific ranges of the priors, given in Table I, are obtained by the following reasoning.

The prior for Q is easily set from the PDP of measurements. Here, we set it to be ± 10 dB around the power of the first observed component. For Λ, on an average we expect to see a minimum of one cluster in the data. This gives the lower bound of the prior as $1/t_{\text{max}}$. We limit our search to at most 20 clusters arriving on an average, giving the upper bound as $20/t_{\text{max}}$. Similar argument is applied for λ, where the minimum and maximum number of rays arriving on an average range from one to K. The decay constants are positive parameters, so their lower bound is set to zero. The upper bound is limited to a number that will lead to a 100 dB drop in the power over half the observation window, i.e. $0.5 t_{\text{max}}/100$.

V. Performance Evaluation

A. Simulation experiment

We first test the proposed algorithm by applying it to simulated data. A synthetic data set is drawn from the Saleh-Valenzuela model with "true" parameters (denoted as θ_{true}) given in Table I, and summarised as per Fig. 1. The results obtained by applying the PMC-ABC with regression adjustment to this data are reported in Fig. 2. The approximate MMSE estimate, defined as

$$\hat{\theta} = \frac{1}{M_\epsilon} \sum_{j=1}^{M_\epsilon} \tilde{\theta}_j^{(T)}, \tag{12}$$

is also reported in Table I, along with the standard deviation of the posterior after T iterations, θ_{sd}.

It appears that the (approximate) posterior converges for all the parameters. However, the marginal posterior for Λ converges more slowly than for the other four parameters. The accepted samples are around the true value of the parameter, indicating that the method is able to estimate the parameters.

B. Application to measured data

We now apply the proposed method to a set of in-room transfer function measurements obtained using a vector network analyser and described in [13]. The set-up is SISO, with a virtual planar array of 25×25, giving $N_{\text{obs}} = 625$. The dimensions of the room was $3 \times 4 \times 3$ m^3. The signal was measured at $K = 801$ frequency points in the range 58 GHz to 62 GHz. Thus, $B = 4$ GHz, $\Delta f = 5$ MHz, and $t_{\text{max}} = 200$ ns. The results are shown in Fig. 3. The convergence behaviour of the approximate posterior for measurements is similar to that observed in simulations. The posteriors for all the parameters but Λ are concentrated in a small region of the prior range.

C. Refining the posterior for Λ

One method to speed up the convergence of $\tilde{p}(\Lambda|\mathbf{s}_{\text{obs}})$ is to limit the parameter search space by fixing a subset of the parameters to particular values, and only sampling from the remaining variables. Here, we fix all but Λ to their MMSE estimate, $\hat{\theta}$, and sample only from $p(\Lambda|\mathbf{s}_{\text{obs}})$ by applying Alg. 2 with $T = 1$. Fig. 4 shows the approximate posterior obtained with and without the regression adjustment.

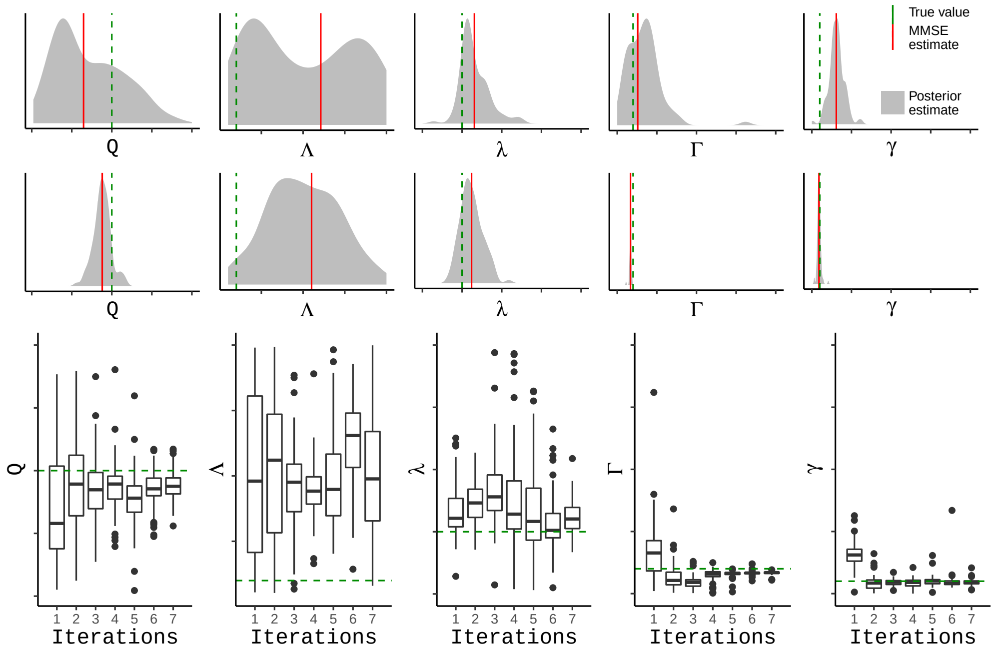

Fig. 2. Density estimate of approximate posteriors of the parameters obtained after first (top panel) and seventh (middle panel) iteration of Alg. 2 on simulated data. The bottom panel shows boxplot of accepted samples as a function of iterations. The black dots denote the outliers. The parameter axes are in their prior range. Simulation settings: $N_{\text{sim}} = 50$, $B = 4$ GHz, $K = 801$, $M = 2000$, $M_\epsilon = 100$, $T = 7$.

TABLE I
PARAMETER SETTINGS AND ESTIMATES.

θ	Prior range	Simulated			Measured	
		θ_{true}	$\hat{\theta}$	θ_{sd}	$\hat{\theta}$	θ_{sd}
Q	$(10^{-9}, 10^{-7})$	5×10^{-8}	4.4×10^{-8}	5.5×10^{-9}	9.6×10^{-9}	1.4×10^{-9}
$\Lambda[\text{ s}^{-1}]$	$(5 \times 10^6, 10^8)$	10^7	5.5×10^7	4.7×10^7	2.0×10^7	1.4×10^7
$\lambda[\text{ s}^{-1}]$	$(5 \times 10^6, 4 \times 10^9)$	10^9	1.2×10^9	2.8×10^8	7.2×10^8	3.0×10^8
Γ	$(0, 10^{-7})$	10^{-8}	8.4×10^{-9}	5.7×10^{-10}	2.8×10^{-8}	4.2×10^{-9}
γ	$(0, 10^{-7})$	10^{-9}	4.5×10^{-9}	1.2×10^{-9}	1.2×10^{-8}	4.4×10^{-9}

For the simulated data, the posterior obtained without regression adjustment is much narrower compared to Fig. 2, and the MMSE estimate agrees well with the true value. Regression adjustment narrows the posterior even further. Similar behaviour is observed in the measurements. We also observe $\hat{\Lambda}$ to be very small in the measurements. This is particularly true when the regression adjustment is applied, which shifts the posterior below the prior range. This indicates that clusters are rare in the measured data. The rarity of clusters is confirmed via visual inspection of the PDP of the data. Consequently, the lower bound of $p(\Lambda)$ could be chosen even smaller.

VI. Conclusions

We proposed the PMC-ABC with regression adjustment method for calibrating stochastic channel models with intractable likelihoods. The method is effective in calibrating the Saleh-Valenzuela model from measurement data without the need for multipath extraction and clustering. The temporal moments, combined with the proposed cluster statistics, are found to be informative about the model parameters. The summaries are observed to be less responsive to the cluster arrival rate than the other parameters. As a result, the marginal posterior of cluster arrival rate converges slower than others. However, its estimate has been shown to improve using an additional round of the ABC method. Similar convergence behaviour is observed on applying the method to measurement data. From the measurements, it seems that the cluster arrival rate is outside the prior range, suggesting the absence of clusters in this data. Non-clustered models might be a better fit for this data. In any case, a smaller lower bound for the prior of this parameter should be chosen for future studies.

Despite the fact that the method is developed without considering the measurement noise, it performs well on the noisy measurements. We conjecture that the method can be improved by including the noise variance as a parameter in the method, which may require an additional summary. This method can also be extended to

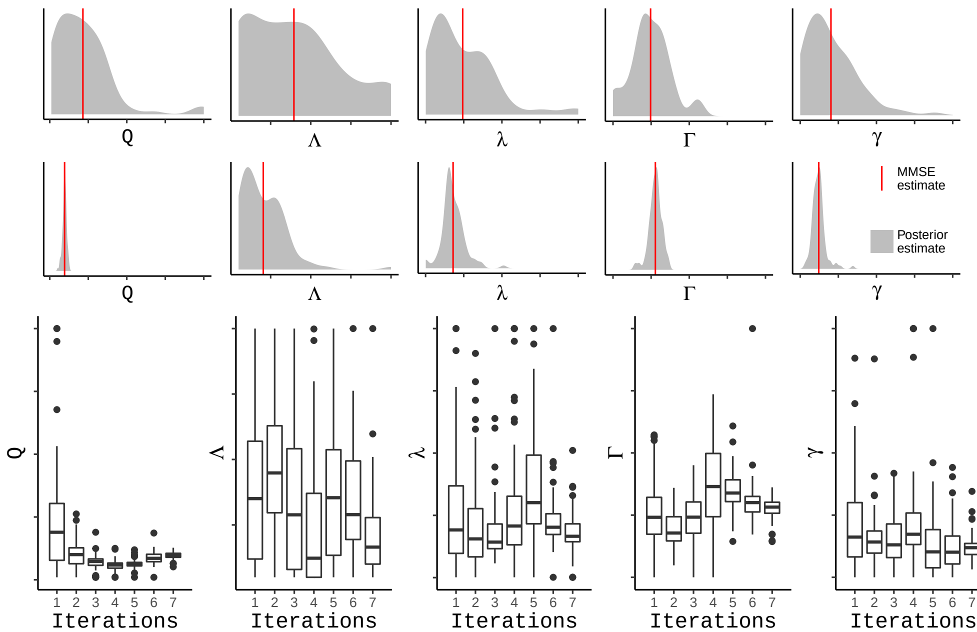

Fig. 3. Density estimate of approximate posteriors of parameters obtained after first (top panel) and seventh (middle panel) iteration of Alg. 2 on measured data. The bottom panel shows boxplot of accepted samples as a function of iterations. Simulation settings as in Fig. 2.

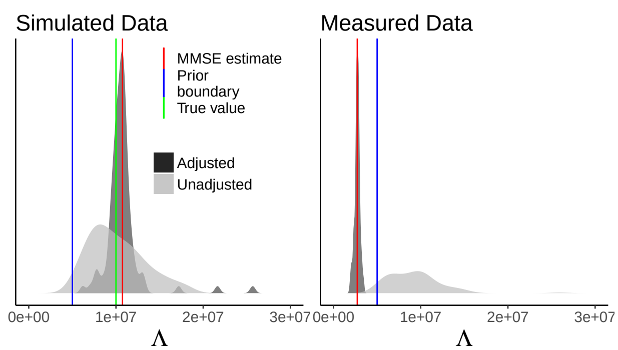

Fig. 4. Approximate posterior of Λ with and without the regression adjustment for simulated and measured data.

calibrate spatial models, provided appropriate summaries are available. As the current calibration methods involve selection and implementation of multipath extracting as well as clustering algorithms, both of which have a number of arbitrary choices, a straightforward comparison with the proposed method is not feasible.

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References

[1] N. Czink, P. Cera, J. Salo, E. Bonek, J. pekka Nuutinen, and J. Ylitalo, "A framework for automatic clustering of parametric MIMO channel data including path powers," in *IEEE Veh. Technol. Conf.*, Sep 2006.

[2] A. Bharti, R. Adeogun, and T. Pedersen, "Parameter estimation for stochastic channel models using temporal moments," in *IEEE AP-S/URSI*, 2019.

[3] A. Bharti, R. Adeogun, and T. Pedersen, "Estimator for stochastic channel model without multipath extraction using temporal moments," in *20th IEEE SPAWC*, 2019.

[4] G. L. Turin, F. D. Clapp, T. L. Johnston, S. B. Fine, and D. Lavry, "A statistical model of urban multipath propagation," *IEEE Trans. Veh. Technol.*, vol. 21, pp. 1–9, Feb 1972.

[5] B. M. Turner and T. V. Zandt, "A tutorial on approximate bayesian computation," *J. Math. Psychol.*, vol. 56, pp. 69–85, apr 2012.

[6] M. A. Beaumont, "Approximate bayesian computation in evolution and ecology," *Annu. Rev. Ecol. Evol. Syst.*, vol. 41, pp. 379–406, dec 2010.

[7] M. A. Beaumont, J.-M. Cornuet, J.-M. Marin, and C. P. Robert, "Adaptive approximate bayesian computation," *Biometrika*, vol. 96, pp. 983–990, oct 2009.

[8] M. A. Beaumont, W. Zhang, and D. J. Balding, "Approximate bayesian computation in population genetics," *Genetics*, vol. 162, no. 4, pp. 2025–2035, 2002.

[9] A. A. M. Saleh and R. Valenzuela, "A statistical model for indoor multipath propagation," *IEEE J. Sel. Areas Commun.*, vol. 5, pp. 128–137, February 1987.

[10] M. L. Jakobsen, T. Pedersen, and B. H. Fleury, "Analysis of the stochastic channel model by saleh & valenzuela via the theory of point processes," *International Zurich Seminar on Communications*, 2012.

[11] T. Pedersen, "Stochastic multipath model for the in-room radio channel based on room electromagnetics," *IEEE Trans. Antennas Propag.*, vol. 67, pp. 2591–2603, Apr 2019.

[12] C. Gentile, "Using the kurtosis measure to identify clusters in wireless channel impulse responses," *IEEE Trans. Antennas Propag.*, vol. 61, pp. 3392–3395, June 2013.

[13] C. Gustafson, D. Bolin, and F. Tufvesson, "Modeling the polarimetric mm-wave propagation channel using censored measurements," in *2016 Global Commun. Conf.*, IEEE, Dec 2016.