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On the impact of measurement errors on loss calculations in distribution grids

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***Abstract*—Power or energy losses are an important metric to describe efficiency of distribution grids. They can also be relevant input metrics for fault and theft detection approaches. When considering low-voltage grids, power losses have to be obtained from distributed low-cost measurement devices, which leads to measurement inaccuracies. When calculating power losses, these measurement inaccuracies need to be taken into account. This paper presents first steps for obtaining unbiased estimators and confidence intervals of loss calculation taking into account statistical errors on input measurands. Smart meter measurements from a real low-voltage grid are used to show the validity of approximations that are useful for efficient and effective confidence interval calculation.**

***Keywords*—Low Voltage Grid, measurement errors, grid losses**

I. Introduction

Measurements of electrical grid state variables such as voltages and currents are becoming increasingly available also in the distribution grids. Key examples of measurement devices include smart meters and smart inverters at customer connection points or measurement devices in secondary substations. These measurement devices show a stochastic measurement error, and the relevance of such measurement error for observability of distribution grids still requires to be understood.

Part of the energy flowing through the distribution grid, from the transmission grid, or generated by distributed generation (connected at distribution level), is lost in various ways in distribution systems. Electrical losses are an inevitable consequence of the transfer of energy across electricity distribution grids. On average, more than 7% of electricity transported across local distribution systems in Europe is reported as electrical losses (IEA Statistics, OECD/IEA 2014). This imposes a substantial financial cost on society both in terms of the costs of producing the electricity that is lost and the costs of transporting these units over the transmission and distribution networks.

Generally speaking, losses affect a utility's economics in two ways [1]. Firstly, they increase the power and energy demands, and thus increase the overall cost of purchasing and/or producing the total power requirements of the utility. Secondly, system losses increase the load flows through individual systems components, which then lead to additional costs being incurred due to the extra losses associated with the increased load flows. Last but not least, extra costs can also be incurred in having to increase some component ratings to cater for the additional current caused by these losses.

The recorded losses can be broken down into three main categories: variable losses, fixed losses and non-technical losses. Variable losses, often referred to as copper losses, occur mainly in lines and cables, but also in the copper parts of transformers and vary in the amount of electricity that is transmitted through the equipment. Fixed losses, or iron losses, occur mainly in the transformer cores and do not vary according to current. Both variable and fixed losses are technical losses, in the sense that they refer to units that are transformed to heat and noise during the transmission and therefore are physically lost. Non-technical losses, on the other hand, comprise of units that are delivered and consumed, but for some reason are not recorded as sales. They are lost in the sense that they are not charged for by either the suppliers or the distribution businesses.

The distribution grid measurements can be utilized for different applications, in particular to derive losses of energy or power by correlation of multiple measurement values. The obtained resulting time series of power losses can be used by the distribution system operator to detect inefficient subgrids or to detect anomalies caused by grid faults or energy theft. When calculating power losses based on Low Voltage (LV) grid measurements from different measurement devices, it is important to quantify the measurement errors of the input data and to understand the impact of the measurement data quality on the results of the calculation.

Loss calculation through the use of simulation models has been part of many research projects: The Austrian project *DG DemoNet - Smart LV Grid* (2011-2014) obtained losses by simulation/calculation in PowerFactory [2] based on models of real Low-Voltage (LV) field test grids. The project investigated the impact of inverter-based reactive power controls (e.g. Volt/VAr or Q(U) control), targeting increased hosting capacities for distributed generation, on losses in the grid. Findings include that properly designed and efficient reactive power controls can multiply reactive currents when active, while increasing annual total losses in the range of a few percent [3]. A large number of research papers uses loss calculations obtained from simulation models. Ref. [4] points out that in practice missing data makes it difficult to break down the losses into technical and non-technical losses. Simulation experiments of real grid topologies from Northern Germany and France in [5] show strong differences of the contribution of the share of losses from the LV grid (42% of the losses in Northern Germany are on LV grid level, while in France only 14%), arguing that detailed LV grid loss analysis is highly relevant. The need for simple engineering rules (not requiring detailed grid simulations) for loss calculation has been pointed out and addressed in [6]. Loss reduction has also been considered as target metric in the Energy Balancing use-case in the European research project SmartC2Net [7]. The approach in SmartC2Net distributed the adjustments of active power between multiple generation units in an MV grid and showed that losses can be reduced by approximately 10% with such method. Other work, e.g. [8] investigated the impact of placement of distributed generation in distribution systems on losses. Finally, grid losses are also used in the context of fault-detectors: for instance, [9] uses rule-based decision making with fuzzy rules on load profile measurements from the Advanced Metering Infrastructure (AMI) systems in order to detect fraud (non-technical loss) and faults. It applies the approach exemplary to a single-phase-to-ground fault of the Medium Voltage (MV) grid with subsequent restoration.

This paper discusses different scenarios of available measurements, and it derives methods to calculate properties of the resulting power loss estimator taking into account the measurement errors of the input measurements. Subsequently, the approach can use this quantification to derive confidence intervals. The approaches assume normally distributed measurement errors as typically resulting from measurement noise and in some cases also from time alignment errors [10]. Results from a case study using a real LV grid measurement are used to demonstrate the suitability of different statistical approximations.

II. Loss Calculation Scenarios

We focus on active power losses subsequently, assuming that measurements of relevant electrical variables are available over a certain known time interval, e.g. $[0, T]$. Depending on what measurands are available from the distribution grid, the calculation of active power losses can take different forms. Focus of this paper is to subsequently derive how such calculations are impacted by normally distributed measurement errors. The magnitudes of these measurement errors are influenced by the type and class of measurement device, see e.g. [11], [12]. In addition, additional measurement errors will be introduced by the use of current transformers for the measurements, see e.g. [13].

We first introduce the context in which we execute the power loss calculations: we consider a low-voltage grid with K customers (consumer, or generator, or both) that are served by a single LV-side busbar in a secondary substation. Target is to obtain the active power losses between LV-side busbar of this substation and the connected customers. Extensions to more general formulations, e.g. meshed LV grids served by multiple substations, are possible but not discussed in this paper.

We distinguish different measurement scenarios that are introduced in the following. Even though many European LV grids are unbalanced 3-phase grids using 4 wires (including neutral), for ease of notation, we introduce the equations for a single-phase representation of grids. Losses in 3-phase grids can in principle be obtained by calculating and adding up the losses from each phase, while some further attention regarding the neutral is needed in some scenarios. The latter is future work beyond the scope of this paper.

A. Losses derived from Substation and Customer Connection Measurements

In typical deployment cases, a DSO measures energy at the substation and at all customer connections for a certain time interval, here $[0, T]$, where typically $T = 15\,\text{min}$ when Smart Meters are used for customer connections. Using 2-way meters at the substation and at all customer connections, the following measurands are obtained as input - here described in terms of average active power, which translates into energy by simply scaling with the constant duration of the measurement period T:

- At LV-busbar of secondary substation: Measurement of imported (from the MV grid) and exported (to the MV grid) average active power during the time interval $[0, T]$: $P^{(0)}_{exp}$,$P^{(0)}_{imp}$.
- At each customer $i = 1, ..., K$: Measurement of generated and consumed average active power during the same time interval: $P^{(i)}_{gen}$,$P^{(i)}_{cons}$.

The calculation of active power losses in this case is just done by simply adding up the imported and generated power and subtracting the exported and consumed power. By adequate convention on the sign of the power values, positive means generated or imported, negative means consumed or exported, and by linear enumeration of all the above Power values, the loss results as a simple sum:

$$L_1 = \sum_{j=1}^{2*K+2} P_j. \tag{1}$$

The calculation of losses in this case then only requires the knowledge of which customers are connected to the specific substation transformer; so only rudimentary grid topology information is needed for its calculation.

Note that Equation 1 also captures non-technical losses (i.e. non-measured loads).

The underlying physical electrical variables are voltages and currents, while active power is only a derived variable. An alternative expression of active power losses that uses these basic physical measurements (and hence will assume that these physical variables are subject to a normally distributed measurement error) can therefore be obtained when the magnitude of the currents and voltages at the customer connections, $U_k, I_{parent(k),k}$, $k = 1, ..K$, and at the substation, U_0, I_0, have been measured:

$$L = U_0 \cdot I_0 \cdot \cos(\phi_0) - \sum_{k=customers} U_k \cdot I_{parent(k),k} \cdot \cos(\phi_k). \tag{2}$$

Currents are mapped to cables, represented by the double index, in which $parent(k)$ is the index of the house connection box that connects to customer k. $\cos(\phi_i)$, $i = 0, ..., K$, is the power factor obtained from the phase angle, ϕ_i, between current and voltage at measurement point i; for the purpose of this paper we assume this power factor to be a known constant or to have been measured without measurement error. For simplicity, we drop the $\cos\phi$ in the following, so assume $\cos\phi = 1$.

B. Loss Calculation based on Currents in All Branches

We on addition consider the scenario that values of the magnitude of the currents are available for ALL $C \geq K$ branches (i.e. cables or overhead lines) in the LV grid: I_j, where $j = 1, ..., C$ is a linear enumeration of all branches. This scenario can result from fully measured grids, in which measurement devices are also deployed at intermediate nodes such as junction boxes. Alternatively, this scenario can result when a grid model is used to calculate all electrical variables in arbitrary heterogeneous measurements scenarios. In order to be eligible for the approaches in this paper, such grid model calculations must be able to not only provide the electrical variables in each grid point, but also the covariance matrix for these calculated values, see e.g. [14], [15].

In addition, we assume in this case that the corresponding cable resistances R_j, $j = 1, ..., C$ are known. The latter are typically obtained from cable type and cable length, both of which are frequently stored in the Geographical Information System (GIS) database of the distribution system operator.

In this scenario, the technical losses caused by the branches can be calculated by

$$L_2 = \sum_{j=1}^{C} I_j^2 R_j. \tag{3}$$

III. Derivation of Loss Estimators and their Properties based on Gaussian Quadratic Forms

When the losses are derived from measurements of active power according to Eq. (1), independent normally distributed measurement errors on the power measurements can in this case be treated in a straightforward manner, since the resulting estimator in Eq. (1) is also normally distributed.

The situation is however more complicated, when the basic measurands are currents, see Eq. (3), or based on voltages and currents, see Eq. (2). In these cases, the equations are Gaussian quadratic forms, for which the rigorous analysis of the estimator and the path towards obtaining confidence intervals is more complicated and therefore discussed in this section.

A. Notation/assumptions

We introduce the following general notation in order to represent the erroneous measurements:

- k is the number of electrical variables that contribute to the loss calculation, i.e. $k = C$ and $k = 2(K+1)$ in case of Equations (3) and (2), respectively;
- $x \in R^k$, true unknown values of currents (and voltage) values;
- $\hat{x} \in R^k$ measured currents (and voltage) values,
- $\hat{x} \sim N(x, \Sigma_{\hat{x}})$ with known covariance matrix $\Sigma_{\hat{x}}$

B. Loss estimator based on quadratic forms

Using the notation from the previous subsection, both Eq. (3) and (2) can be rewritten as a Gaussian quadratic form

$$L = x^T G x \tag{4}$$

for a $k \times k$ matrix G. G results in a diagonal matrix with the line resistances as entries, when performing the loss calculation according to Equation 3. When using current and voltage measurements at the substation and customer connections according to Eq. (2), G results in a matrix with entries 1 in the appropriate places, depending on the order of the current and voltage variables in the vector x.

Since x is unknown, L is also unknown, but it can be estimated by inserting the estimates $\hat{x}$, i.e.

$$\hat{L} = \hat{x}^T G \hat{x}. \tag{5}$$

Thus the estimate $\hat{L}$ has a generalized χ^2 distribution, which depends on parameters x, $\Sigma_{\hat{x}}$ and G [16]. This we denote by $\hat{L} \sim \chi^2_{\text{gen}}(x, \Sigma_{\hat{x}}, G)$. Furthermore, $\hat{L}$ has expected value given by

$$E\hat{L} = x^T G x + \text{tr}(G\Sigma_{\hat{x}}) = L + \text{tr}(G\Sigma_{\hat{x}}) \tag{6}$$

and, provided that G is symmetric, the variance is given by

$$\sigma^2_{\hat{L}} = \text{Var}\hat{L} = 2\text{tr}(G\Sigma_{\hat{x}}G\Sigma_{\hat{x}}) + 4x^T G\Sigma_{\hat{x}}Gx, \tag{7}$$

where tr denotes the trace operator [17], [18]. The formula for the expected value shows that the estimator is biased, and suggest a bias corrected estimator given by

$$\hat{L}_{bc} = \hat{x}^T G \hat{x} - \text{tr}(G\Sigma_{\hat{x}}). \tag{8}$$

Furthermore, note that the formula for the variance contains the unknown x and therefore needs to be approximated, e.g. by

$$\hat{\sigma}^2_{\hat{L}} = 2\text{tr}(G\Sigma_{\hat{x}}G\Sigma_{\hat{x}}) + 4\hat{x}^T G\Sigma_{\hat{x}}G\hat{x}. \tag{9}$$

C. Approximation of distributions and confidence intervals

The generalized χ^2 distribution does not yield closed form expressions for the confidence intervals of L, so we consider approximations based on the normal and the (scaled) χ^2 distribution.

1) Normal approximation: Here we approximate the distribution of $\hat{L}$ by a normal distribution with expected value and variance given by (6) and (9). Assuming the approximation to be good, we get that

$$\frac{\hat{L} - (L - \text{tr}(G\Sigma_{\hat{x}}))}{\hat{\sigma}_{\hat{L}}^2} \approx N(0,1),$$

from which we immediately obtain approximate $(1-\alpha)$-confidence intervals for L given by

$$\hat{L} - \text{tr}(G\Sigma_{\hat{x}}) \pm z_{1-\alpha/2}\hat{\sigma}_{\hat{L}},$$

where z_α denotes the α-quantile for the standard normal distribution.

2) Scaled χ^2 (Gamma) approximation: As the general χ^2 distribution does not yield closed form expressions for the confidence intervals, we later investigate empirically the fit of a scaled χ^2 distribution (which is equivalent to a Gamma distribution), see Section IV-B. In this paper, we limit ourselves to the visual comparison of the distributions for the example case. A detailed investigation, whether a confidence interval for L can be completely extracted from an approximate Gamma distribution and also to justify the approximation theoretically is left for future work.

IV. Preliminary validation based on real LV grid

We show the behavior of the loss calculation for an operational LV grid in North Denmark using actual Smart Meter measurements of a real-life LV grid scenario.

A. LV Grid and Smart Meter deployment

This real LV grid area is served by a secondary substation, which serves 10 customers; the substation is Node 1 and the Customers are the leaf nodes of the tree topology shown in Figure 1. The resistances have been obtained from the cable types and from the cable lengths that were available in the GIS system. Customers at Nodes 11 and 17 operate Photo Voltaic cells (PVs) (of 9.5 and 6kW rated power); Node 18 is a wind-turbine.

The substation and all customers contain Smart Meters, which provide values of consumed or provided energy over 15min intervals. There is also a smart meter at the secondary substation, which measures energy imported from the MV grid and energy exported from the LV grid to the MV grid. Note that the latter two measurands at the substation can jointly be non-zero, since the LV grid can be importing for parts of the 15min interval and exporting for the remaining time.

Figure 2 shows the behavior of losses over 15 minute intervals, resulting from the use of the measured energy values (which are scaled into average power). The calculation is done according to Eq. (1). The figure also shows confidence intervals, which are based on the assumption of normally distributed errors on these measured power values, where the measurement error is assumed to have a standard deviation of 2% of the mean value, referring to the Measuring Instruments Directive (MID) 2004/22/CE to Class A of smart meters as defined in the EN50470-1/-3 standard, [19]. Node 3 in the LV grid actually contains an un-measured consumer, for which a value of $3W$ with 100% measurement error is used.

In practice however, the physical measurands at any measurement device are the currents and voltages. Therefore, the assumption of a normally distributed error is more likely to apply in practice on these physical measurands, which are investigated in the following.

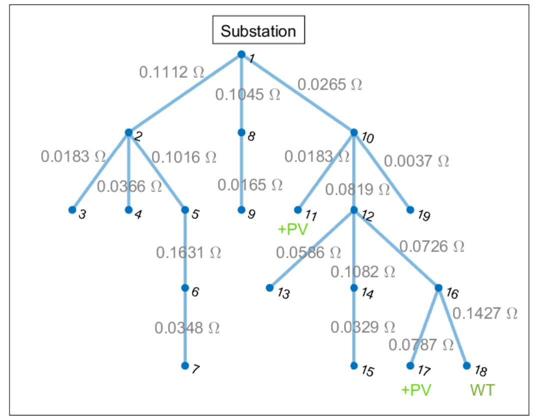

Fig. 1: Abstracted topology of a real grid from North Denmark: Node 1 is the secondary substation and all leaf nodes are consumers or consumers with generators.

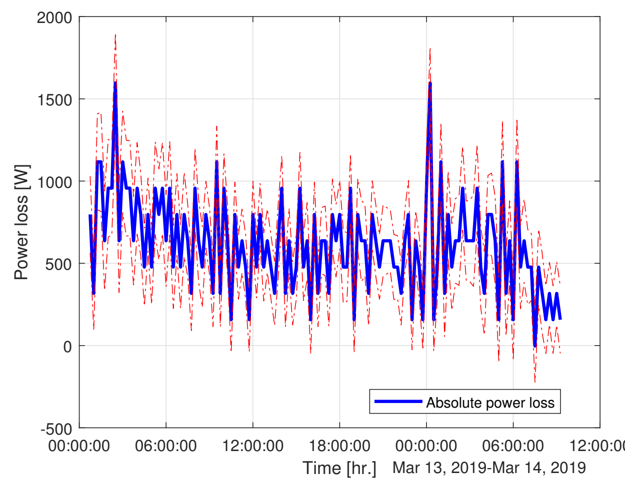

Fig. 2: Average power losses for a selected time period resulting from the processing of the energy measurements according to Eq. (1). The 95% confidence interval for the loss calculation is shown as dotted line - caused by normally distributed measurement errors with 2% relative error.

B. Behavior of loss calculation based on currents and voltages with simulated measurement errors

We use the active power values derived from the energy measurements in one selected time interval in order to calculate voltages and currents everywhere in the grid by the grid model of [14]. The resulting voltages and currents are the assumed true values; under these assumed true voltage and currents, the resulting power loss obtained from Eq. (2) is $134.77W$ in this LV grid, while the substation power is $1975W$, so the relative loss is in this measured scenario 6.82%.

We now assume the calculated voltages and currents to be the true values and add a simulated normally distributed error to the voltage and current values of the customer smart meters and to the voltage value at the substation. The ratio of the standard deviation of this simulated measurement error to the true value is called the relative measurement error and varied in these experiments. These erroneous measurements are then input into the grid model framework of [14] to calculate all voltages and currents in the grid and to obtain the technical loss from these calculated values using Eq. (3).

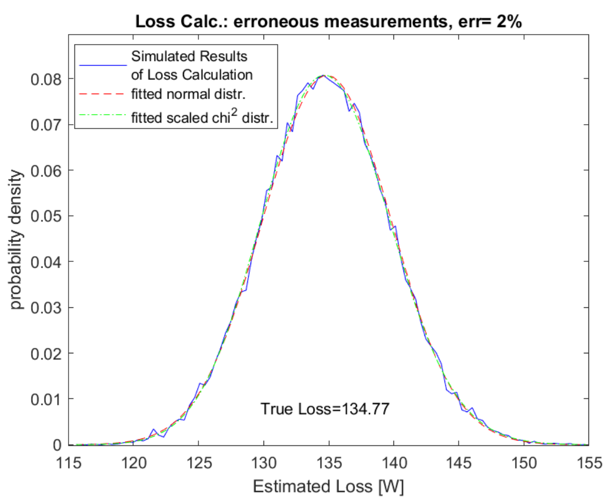

Fig. 3: Distribution of the result of the loss calculation with measurement error of 2% and fitted normal and χ^2 distribution.

The empiric probability density functions of these power losses obtained from 30000 repetitions each are shown in Figures 3 and 4 for simulated measurement errors of 2% and 20%, respectively. The result for 2% simulated measurement error show a good fit of the normal approximation and the need for correction of the mean according to Eq 8 is small. The fitting is performed by matching expected value and variance of the simulated distribution of the loss estimator.

For larger measurement error of 20%, the normal approximation in this case does not provide a good fit any more, while a scaled χ^2 distribution can be fitted to the data. These simulation results show the usefulness of the different approximations. Even though the scaled χ^2 distribution in this

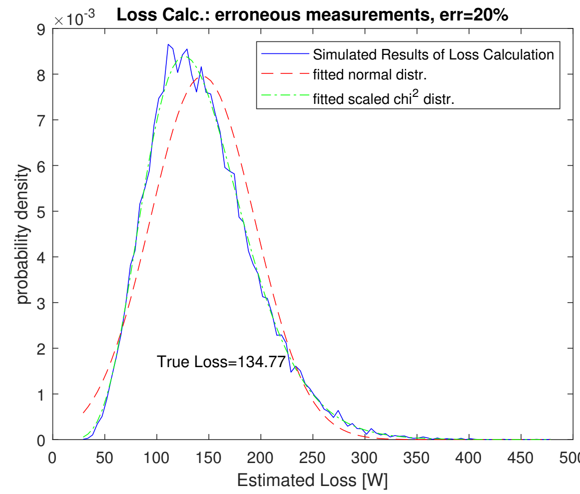

Fig. 4: Distribution of the result of the loss calculation with measurement error of 20% and fitted normal and χ^2 distribution.

scenario is only required for a very high measurement error, a detailed parametric study is needed as future work in order to investigate how grid parameters and characterization of loads and generation influence the need for the more detailed approximation with the scaled χ^2 distribution. Furthermore, a study and comparison of the obtained confidence intervals from the different approximations is needed as next step.

V. Conclusion and Outlook

This paper provides an initial direction to obtain confidence intervals for calculated values of power or energy losses, when the input measurands are voltages and currents which are subject to a normally distributed measurement error. The results show that a simple addition of line losses can lead to a bias in the calculated loss value, and a correction formula is provided. Furthermore, two approximations for the derivation of confidence intervals are outlined and the validity of the approximations is analyzed in the realistic example of a true low voltage grid with smart meter measurements.

The quantification of the measurement errors is inspired by comparable measurement device and current transformer types [11], [13], [20]. This per-device consideration in fact leads to a diagonal covariance matrix for the measurement error. Future work should investigate, in which cases a non-zero covariance between measurement errors in different points of measurement results, whether such covariance is significantly influencing the loss calculation, and for scenarios where the latter is true, how to obtain a quantification of those for the computation.

The presented directions in this paper are the starting point to make practical use of measurement-based loss calculations in low-voltage grids. Further relevant studies planned for the future are: (1) to perform a detailed parametric study with different grid scenarios and load/generation behavior in order to find out when the more detailed χ^2 approximation is required; such parametric study may also analyze cases of varying measurement errors due to heterogeneous measurement devices (the formulation in this paper already allows to include different measurement errors, but the example results do not yet use this); (2) to study the impact of averaging over 15min intervals as commonly done by many Smart Meters (and also smart inverters); this averaging may also introduce a time alignment error due to non-ideal clocks, see [10] for initial analysis of the latter; (3) to investigate the behavior of the calculated loss values for scenarios of missing measurements, e.g. customers without Smart Meters; (4) to investigate the impact of 3-phase unbalanced grids on the confidence interval calculation for losses; (5) to investigate the case when the phase-angle in Eq. (2) is a measurement that is subject to measurement errors; (6) To use the obtained confidence intervals in approaches for theft or fault detection and to investigate what improvement can be obtained from the quantification of the accuracy of the loss time series. (7) To investigate the benefit of the confidence intervals of losses when designing and analyzing loss minimization approaches. (8) In addition to the analysis of the impact of measurement errors, the consequences of inaccuracies in the grid parameters (here cable impedance, which depends on cable length and cable type, see [3], [21]) will be interesting. Future work will analyze the impact of such inaccuracies on the loss calculations.

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