Using Smart Meter measurements to manage accuracy of current calculations in LV feeders

Schwefel, Hans-Peter Christian; Rasmussen, Jakob Gulddahl; Olsen, Rasmus Løvenstein; Ringgaard, Henrik; Silva, Nuno

Published in:
2019 IEEE International Conference on Communications, Control, and Computing Technologies for Smart Grids (SmartGridComm)

DOI (link to publication from Publisher):
10.1109/SmartGridComm.2019.8909723

Publication date:
2019

Document Version
Version created as part of publication process; publisher's layout; not normally made publicly available

Citation for published version (APA):
Schwefel, H.-P. C., Rasmussen, J. G., Olsen, R. L., Ringgaard, H., & Silva, N. (2019). Using Smart Meter measurements to manage accuracy of current calculations in LV feeders. In *2019 IEEE International Conference on Communications, Control, and Computing Technologies for Smart Grids (SmartGridComm)* Article 8909723 IEEE (Institute of Electrical and Electronics Engineers). https://doi.org/10.1109/SmartGridComm.2019.8909723

Using Smart Meter measurements to manage accuracy of current calculations in LV feeders

Hans-Peter Schwefel	Jakob G. Rasmussen	Rasmus L. Olsen	Henrik Ringgaard	Nuno Silva
GridData GmbH and Aalborg Univ.	*Dept. of Mathematics Aalborg University*	*Dept. of Electronic Systems Aalborg University*	*Electricity grid network Thy-Mors Energi*	*GridData GmbH*
Anger, Germany	Aalborg, Denmark	Aalborg, Denmark	Thisted, Denmark	Anger, Germany
schwefel@griddata.eu	jgr@math.aau.dk	rlo@es.aau.dk	hri@thymors.dk	nuno.silva@griddata.eu

***Abstract*—Knowledge of currents in individual Low Voltage feeders of a secondary substation is interesting for distribution system operators for a variety of purposes. Deploying measurement devices at each feeder in each substation, however, can be costly. Due to the increasing deployment of Smart Meters, the knowledge about currents at each connected customer is in principle available. This paper proposes and evaluates an approach to determine the feeder currents taking into account the impact of measurement errors of Smart Meter measurements. The developed approach makes a rigorous derivation of confidence intervals for the calculated voltage and current values utilizing a subset of measured voltages and currents as input. The approach is applied to two realistic low voltage grids and the impact of measurement errors and missing smart meter measurements is quantitatively analyzed.**

***Index Terms*—Distribution grid, measurement errors, smart meter data quality, confidence intervals in grid calculations**

I. Introduction

Measurements of electrical variables such as voltages and currents are becoming increasingly available also in the distribution grids, providing an improved opportunity for grid observability that previously was not existing. Key examples of measurement devices include smart meters and smart inverters at customer connection points or measurement devices in secondary substations. However, such measurement devices may have different measurement accuracy and time granularity. Furthermore, not all customer connections may be measured and not necessarily all deployed measurement devices can provide data for a specific time interval of interest, e.g. due to bandwidth constraints of low-bandwidth communication networks, [1], [2] or for non-technical reasons [3]. Inaccuracies or missing data from customer connections limits or reduces the benefit of grid observability applications.

One example application, for which customer-side measurements can benefit, is the feeder current calculation: The knowledge of currents of individual feeders of a secondary substation is interesting for the distribution system operator for, e.g., grid congestion detection. While installing multiple devices for feeder current measurements at each substation can be costly, the increase in deployment of Smart Meters and other smart customer side equipment provides knowledge of currents at connected customers, which can be used to estimate feeder currents. However, such estimation must take into account measurement errors and unavailability of measurements from customer connection points. This paper develops an approach to quantitatively determine the impact of measurement errors on the feeder current estimation based on a rigorous derivation of confidence intervals. The approach is applied to a realistic low voltage grid and the impact of measurement errors and unavailable measurements at customer connection points is quantitatively analyzed.

The feeder current calculation problem can be generalized to calculating voltages for all grid nodes and currents for all branches in the grid. In such calculations, it is important to quantify the measurement errors of the input data and to quantify the impact of the measurement data quality on the results of the calculation. The approach presented in this paper uses a grid model together with a subset of current (and optional voltage) measurements to calculate the feeder currents. Further, input to the approach is the standard deviation of the normally distributed error of the input measurements. Subsequently, the approach uses this quantification to derive quantitatively the error for all calculated parameters, expressed by their standard deviation. The approach assumes normally distributed measurement errors as typically resulting from measurement noise and in some cases also from time alignment errors [4].

Grid estimation has a long history, starting from transmission grids in the 70ies [5] and in the last 20 years gaining increasing attention also in the distribution grids. A large share of the work has focused on obtaining numerically tractable models from power flow equations describing AC grids [6], [7], in most cases resulting in linearized models. In contrast to this part of the existing work, this paper takes a linear model and focuses on the method to rigorously specify stochastic measurement errors on the input measurands and use those together with the linear model in order to derive confidence intervals for the calculated voltages and currents. With increasing attention on estimation of distribution grids and in particular low voltage grids, the aspect of erroneous measurements has recently started to gain momentum: approaches using weighted minimum squared error [8], [9], [10] allow in principle via the selection of the weights to mimic different input error magnitudes. However, they require another model to derive the relation of the quantitative measurement error description to the chosen weight. Furthermore, the approach does not lead to quantitative specifications of the error of the estimated values. In [11] a comprehensive overview over variants of weighted least squares estimation approaches is provided; their analysis points out that solutions by the Gauss-Newton method either become computational costly or sensitive to bad data or missing data.

Approaches based on Kalman filters [12] allow the specification of input errors via covariance matrices; however, these approaches apply to a sequence of grid estimation steps, in which the evolution of the grid behavior is described by the linear evolution model, which in low-voltage (LV) grids with potentially highly time varying loads can be problematic. In contrast to such work, this paper focuses on the calculation of currents (and voltages) of a LV grid based on measurements for a single measurement time interval.

The specific problem of feeder current calculation based on Smart Meter data can be approximated by a simple additive procedure. Section II-A introduces this approximation as comparison case and discusses qualitatively its drawbacks. III-A then describes the linear equation system resulting from a single phase representation of a Low Voltage (LV) grid. Section III-B introduces the stochastic assumptions and shows how to perform the calculation of the impact of measurement errors; this section also shows how the resulting covariance matrix can be used to obtain confidence intervals for the calculated values. Section IV applies the calculation method to two realistic low-voltage grids: a simulated reference grid model and a real LV grid. Finally, Section V summarizes the paper and presents next steps.

II. Feeder Current Calculation Scenario

Target of this paper is to obtain values of the magnitude of the currents, F_i, $i = 1, ..., \#feeders$, of the LV side feeders of a secondary substation. In order to make the presentation of the approach and analysis easier to read, we focus on a single-phase representation of LV grids, so we target to calculate one magnitude value for a current on each feeder. Generalisations to unbalanced multi-phase LV grids are however possible, while introducing a more complex structure of the grid model [13].

Input to the feeder current calculation are measurements of currents and optionally voltage at the customers that are connected to this secondary substation, e.g. obtained by Smart Meters or Smart Inverters at customer connection points. In addition, measurements of the voltage and total current of the secondary substation LV bus bar or at intermediate busbars in junction boxes may be optionally available.

All input measurements are assumed to be subject to normally distributed measurement errors, where the parameters of the normal distribution are known. The standard deviation may be obtained from the upper bounds stated in legal requirements for measurement devices, see e.g. [14]. Without loss of generality, the mean of the normally distributed measurement errors is assumed to be zero.

While more general assumptions can be included, the covariance between measurement errors of different measurement devices is in practice hard to obtain. Therefore, this paper will in the analysis later focus on independent measurement errors.

A. Comparison case: Approximation of feeder currents for radial distribution grids

The scenario in this subsection is a simplified base case that is used as initial approximation and for comparison to the enhanced method.

In a radial grid, for each feeder i, the current in the feeder from the secondary substation results as the sum of all currents of the connected prosumers, using appropriate signs:

$$F_i \approx \sum_{j \in C_i} I_j .$$

I_j is the measured complex current at customer j, where the phase angle is relative to the measured complex voltage U_j at the same measurement device and C_i is the set of customers on Feeder i. Note however that Smart Meters and other measurement devices in the LV grid are not time synchronized on small time-scales, so that there is in practice no common reference point for phase angles and only the relative phase angle at one measurement locations are known. By convention, consumption is present when the resulting active power $P = real(U \cdot I^*)$ has a negative sign, and generation for a positive sign.

This way of calculating feeder currents however only applies to a radial grid and it does not allow to include additionally available measurements, e.g. measurements of the voltage or total current at the secondary substation LV busbar into the feeder calculation.

On the other hand, such simplified calculation of feeder currents easily allows to include jointly normally distributed measurement errors via the standard formulas, here given for the assumption of independent errors:

$$Var(F_i) \approx \sum_{j \in C_i} Var(I_j).$$

Additional advantage of this simple approximation method is that it only requires very rudimentary grid topology information: only the knowledge about which customer is connected to which feeder is required.

B. Approach for general grids

The method that is presented subsequently in the paper will allow to calculate feeder currents and the corresponding standard deviations of these currents based on any number of input measurements of currents and voltages at any node in the grid. Measurements are assumed to be subject to a normally distributed error with zero mean and known standard deviation. The method will use a grid model, which requires the knowledge of the LV grid topology and of the cable impedances.

The details of this approach are introduced in the next section.

III. Grid calculation model with standard deviations

A. Linear grid model

We represent a low-voltage grid as a graph with three types of nodes: 1) the busbar on the LV side in the secondary substation is the root node as Node 1. 2) Any type of intermediate bus bar or any sleeves are represented by intermediate nodes. 3) Terminal nodes represent the connection points to customers; in case of the presence of a Smart Meter, this terminal node is the point of measurement of the Smart Meter.

The cables are represented by edges, $e_{i,j}$ which connect Nodes i and j. The total number of nodes is denoted by N, the total number of edges by E; in case of a radial grid, the resulting graph is a tree with $E = N - 1$. The resistance and reactance of the cable is obtained from cable type and cable length and represented by the complex impedance $Z_{i,j}$. We leave it for future work to also take in non-linearities in cable impedance and ageging effects.

The variables used to describe the grid status are then complex values of voltages U_i in relation to ground at each node (busbar, connection point), and complex values for the currents $I_{i,j}$ between Node i and Node j.

For simplification of the following description, we assume a tree topology (while the implementation is more general to also cover meshed LV grids). The used grid model results in three types of linear equations:

1) The Kirchhof equation for each intermediate node i:

$$I_{parent(i),i} - \sum_{j \in children(i)} I_{i,j} = 0.$$

2) Linear equations that link the voltages to the currents for each cable, i.e. for each edge $e_{i,j}$:

$$(U_i - U_j) - Z_{i,j} \cdot I_{i,j} = 0.$$

3) Measurement equations for all measured voltages $U_j = u_j$ and currents $I_{k,l} = i_{k,l}$.

We then define the complex variable vector x for the true voltages and currents by an arbitrary ordering scheme on the N voltage variables U_i and the current variables $I_{i,j}$. The first two types of equations are then summarized in the matrix notation by:

$$Cx = 0,$$

and the third type of measurement equations by:

$$Dx = d,$$

where all entries in the matrices and vectors are complex values.

Note that the above two matrix equations can be jointly written as $Bx = m$, where B is the vertical concatenation of the matrices C and D, and m contains 0 in the first components, and the last components correspond to measurement values. In the general case, B is not a square matrix. In particular, depending on the number of intermediate nodes in the LV grid topology and depending on the number of input measurements, the equation system may be over-determined, i.e. B has more rows than columns. A standard linear regression solution (or equivalently, minimization of the 2-norm of the residuals) would result from solving

$$B^H Bx = B^H m,$$

where B^H is the conjugate transposed of the matrix B.

However, using this standard linear regression has three disadvantages: (1) Some of the equations are formulated in terms of voltages, others in terms of currents; the linear regression solution would minimize the residual $||Bx - m||$, but that residual vector would contain values with different units; (2) the quantification of measurement errors cannot be included in the standard linear regression solution. (3) The approach does not provide estimation errors on the estimated voltage and current values. While the first two disadvantages could be addressed also by approaches to weight the different equations [11], [13], all three will be removed by the approach introduced in the following.

B. Confidence intervals by constrained regression

We now introduce measurement errors as follows: for each measurement d_j (which could be a voltage or a current), we assume a complex normally distributed measurement error with the real and imaginary part independent and both with standard deviation σ_j.

In the following, we use the notation:

- N: numbers of nodes in the LV grid.
- k: number of grid state variables used, i.e. N voltage values and, in a tree topology, $N - 1$ cable currents, i.e. in total in that case $k = 2N - 1$.
- n: number of available measurements, $n \leq k$
- q: number of linear equations, called constraints, that result from the grid model; here $q = N - 1 +$# intermediate nodes, $q = 2N - 2 -$# leaf nodes.

Model: Under the assumptions stated before, the model is defined by

$$d = Dx + \varepsilon, \qquad \varepsilon \sim CN_n(0, \Sigma),$$

where $d \in \mathbb{C}^n$, $D \in \mathbb{C}^{n \times k}$, $x \in \mathbb{C}^k$, $\Sigma \in \mathbb{C}^{n \times n}$, and CN_n denotes the n-dimensional circularly symmetric complex normal distribution. In our case, Σ is a diagonal matrix with the jth entry given by $2\sigma_j^2$, so it is in fact real-valued, but everything below carries through for complex valued matrices as well. Equivalently,

$$\tilde{d} = \tilde{D}x + \tilde{\varepsilon}, \qquad \tilde{\varepsilon} \sim N_n(0, I), \tag{1}$$

where $\tilde{d} = \Sigma^{-1/2} d$, $\tilde{D} = \Sigma^{-1/2} D$, and $\tilde{\varepsilon} = \Sigma^{-1/2} \varepsilon$. Furthermore, x is subjects to the constraints

$$c = Cx \tag{2}$$

where $c \in \mathbb{C}^q$ and $D \in \mathbb{C}^{q \times k}$ (in our case c is a zero vector). Thus this is a variant of a general linear model with (1) complex entries, (2) linear constraints, (3) a known, arbitrary covariance matrix, (4) a (typically) under-determined system in the sense that the number of available measurements n, is less than the number of variables to estimate, $n < k$.

Statistical Estimation: We estimate the parameters x using least squares estimation under the linear constraints (2). This is considered in the case that $n < k$ in [15], where we need to generalize the results to the complex valued case and do a few minor modifications, such as using an arbitrary, known covariance matrix.

To find the least squares estimate of $\hat{x}$ under the constraints, we minimize the Lagrange function

$$\mathcal{L}(\hat{x}, \lambda) = (\tilde{d} - \tilde{D}\hat{x})^H(\tilde{d} - \tilde{D}\hat{x}) + \mathrm{Re}(2\lambda^H(Cx - c)), \quad (3)$$

where λ denotes the complex Lagrange multiplier and H denotes the conjugate transpose. This yields the following result for the least squares estimate for x, which is similar to the real-valued case (see [15]).

Proposition 1. *Assuming the setup given by formulas 1 and 2, we get the least squares estimate for x as he solution to*

$$\begin{bmatrix} \tilde{D}^H\tilde{D} & C^H \\ C & O \end{bmatrix} \begin{bmatrix} \hat{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} \tilde{D}^H\tilde{d} \\ c \end{bmatrix}, \quad (4)$$

where O denotes a zero matrix of appropriate size (here $q \times q$)

Due to space limitation, the proofs of the propositions will be given in an extended version of this paper.

For short notation we write (4) as

$$A\tilde{x} = b. \quad (5)$$

Provided that A is invertible, the equation has a unique solution given by $\tilde{x} = A^{-1}b$.

Given that $\tilde{D}^H\tilde{D}$ is invertible we have the case where the system is fully measured, and a closed form expression for $\hat{x}$ can be obtained. If this is not the case, however with A being invertible, we have a case which allows us to use the solution above. This is the typical case that we discuss in our paper, where some data is not present for what reasons that might be. In the case A is not invertible, that is measurement and constraints equations are too few, then we do not get a unique solution and we will require additional information to find a solution that is meaningful.

Next we consider the distribution of $\hat{x}$. Firstly, following an approach in [15], notice that although the inverse of A typically does not have a nice expression, we can write it as a block matrix, i.e.

$$A^{-1} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}, \quad (6)$$

where the submatrices of A^{-1} have the following dimensions: $F_{11} \in \mathbb{C}^{k\times k}, F_{12} \in \mathbb{C}^{k\times q}, F_{21} \in \mathbb{C}^{q\times k}, F_{22} \in \mathbb{C}^{q\times q}$. This yields the following result on the distribution of $\hat{x}$.

Proposition 2. *If A is invertible, the least squares solution $\hat{x}$ given by the solution to* (4) *has the distribution*

$$\hat{x} \sim CN_k(x, F_{11}).$$

In addition to providing a nice expression for the distribution of $\hat{x}$, Proposition 2 also verifies that $\hat{x}$ is an unbiased estimate of x. Furthermore, we use the result in proving the following proposition, which is vital for creating confidence intervals for the absolute value of any (complex) entry in x, where it should be noted that the diagonal of F_{11} is real, since F_{11} is non-negative definite.

Proposition 3. *If A is invertible, then*

$$\frac{\mathrm{abs}(\hat{x}_i)^2}{(F_{11})_{ii}/2} \sim \chi^2_{nc}\left(2, \frac{\mathrm{abs}(x_i)^2}{(F_{11})_{ii}/2}\right),$$

where χ^2_{nc} denotes the non-central χ^2-distribution.

This result does not provide a closed form expression for the confidence interval of the absolute value of x_i, but using formula (11) from [16] provides an approximate confidence interval for the non-centrality parameter of a non-central χ^2-distribution, i.e. $\frac{\mathrm{abs}(x_i)^2}{(F_{11})_{ii}/2}$ in our setting. To obtain this interval, one should let the parameters x, σ and ξ in (11) in [16] be $x = \frac{\mathrm{abs}(\hat{x}_i)^2}{(F_{11})_{ii}/2}$, $\sigma = 2\sqrt{x}$, and $\xi = \pm z_\alpha/2$ to obtain the two end points, say δ_1 and δ_2, of the approximate confidence interval for $\frac{\mathrm{abs}(x_i)^2}{(F_{11})_{ii}/2}$. Transforming both end points δ_j for $j = 1, 2$ by $\sqrt{\delta_j(F_{11})_{ii}/2}$ then gives an approximate confidence interval for $\mathrm{abs}(x_i)$.

IV. Quantitative Evaluation and Comparison of Feeder Current Estimation Methods

We now apply the approach introduced in the previous section to two realistic LV grids. Main target of the numerical results is to understand, which benefits the feeder current calculation method based on the linear grid model shows, and to understand the impact of erroneous measurements and missing measurements.

Relative errors of the measurements are defined as standard deviation of the additive measurement error divided by the measurement value; unless stated explicitly to be different, the relative errors of the Smart Meter measurements at customer connections are by default assumed to be 1%, and for the measurements at the LV side busbar of the substation, if included, a measurement error of 0.1% is assumed; background for the latter is the assumption of a higher precision measurement device deployed at the substation. See the beginning of Section III-B for details on how this normally distributed error is added to the complex representation of measurements of voltage and currents.

A. Scenario and evaluation approach

We use the realistic LV grid example from [17]: The area constitutes a mixture of both industrial, commercial, agricultural and residential buildings. The specific topology and configuration of the grid we focus on in this paper is a subset of that area which contains 116 households, and is illustrated in Figure 1. This subset contains 37 residential houses, partially equipped with PV and rechargeable batteries. The topology shown has a ring between feeder on V18 and V33, which has been included to demonstrate that our methodology works even with rings, however, we do not go specifially into details of how this impacts estimates.

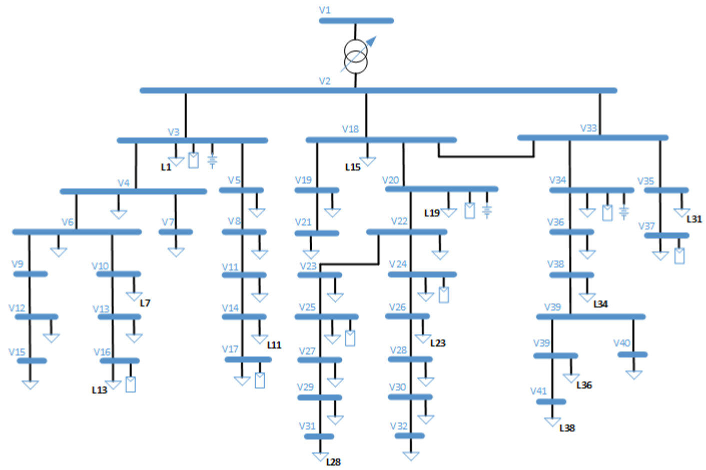

Fig. 1. Example grid from a smaller Danish town, [17].

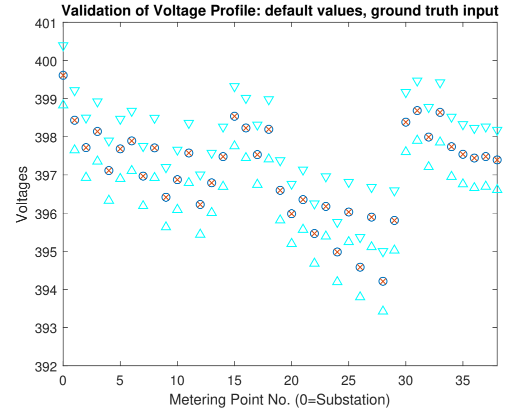

Fig. 2. Calculated voltage profile for the reference grid based on Smart Meter current values at all customers and on a voltage value at the substation; ground truth voltages from simulations are shown by the circles. The metering point IDs ≥ 1 corresponds to the load number in Figure 1.

We use a single-phase representation of this 3-phase LV grid, therefore the nominal voltage is 400V. The grid model uses the average cable length for all cables connecting junction boxes, so each of these cables has resistance $R = 0.104\Omega$ and reactance $X = 0.026\Omega$. The cable from the Smart Meter to the household is assumed to be short and have $1/100$ of that resistance and reactance; reason for this choice is hat the used simulation model does not include this cable, so that the choice of small values leads to minimal impact.

The grid has been implemented and simulated in the Matlab tool DISC [18] and the complex values of currents at each line and voltages at each node are available as output of this simulation.

We then simulate Smart Meter measurements by adding a normally distributed error to the selected set of 'available' Smart Meters and use the linear grid model together with the confidence interval approach from the previous section to calculate all voltages and currents, among them the magnitude of the 3 feeder currents, here called F_1, F_2, F_3. Before showing results on feeder currents, we first show the validation of the model via voltage profiles.

B. Validation via voltage profile calculation

The calculations from the linear grid model are compared to the output from the simulator; in reference to terminology from the Machine Learning domain, we call the latter *ground truth*. The standard deviation of the measurement error is assumed as given earlier by the default values, while the actual input values are the ground truth values (so no random error added). The calculation method works upon the following input: complex value of currents at EACH customer Smart Meter, and the magnitude of the substation voltage. Note that the substation current and also the voltages at customers are not used as input for this validation step.

Figure 2 shows the resulting voltages and the 95% confidence intervals as computed from the grid model in comparison to the ground truth voltages. The figure shows an exact match of the calculated estimate to the ground truth voltage therefore confirms the validity of the grid model and of the implementation of the calculation method.

C. Feeder estimation - Comparison with baseline approach from radial grids

In the following, erroneous measurement values are investigates as input to the grid calculation; these are obtained by adding a normally distributed error to the ground truth values from the simulation, before using these values as input to the feeder estimation approaches, see details in the beginning of Sect. III-B. The following figures show ONE random realization of this stochastic error for a varying relative error of all the customer-side Smart Meter measurements, while the relative errors of the substation measurements is fixed to the previously given default value. The dotted lines visualize the 95% confidence interval. The results in Figure 3 show that the calculated confidence intervals contain the true value in almost all cases; the size of the confidence intervals for the calculated values and also the variability of the calculated result increases strongly for smart meter measurement errors of 10% and higher.

D. Incomplete customer measurement deployments

We now investigate scenarios of incomplete measurements, i.e. when not all customers have smart meters, or when these smart meters have not provided a measurement. When there is no measurement from a Smart Meter, the following 'pseudo'-measurements are used as input to the grid calculation:

- for customers that are purely consumers: U =nominal voltage $-5V$, with a standard deviation of $50V$; $I = 7A$ with a standard deviation of $7A$.
- for customers that contain also generators: U =nominal voltage $+5V$ with a standard deviation of $50V$; $I = 0A$ with a standard deviation of $12A$.

This approach therefore uses constant values with a large standard deviation. Pseudo-measurements for current values are chosen depending on max currents of the customer type.

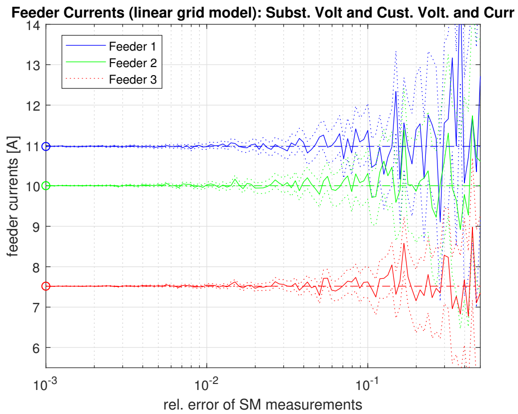

Fig. 3. Calculated feeder currents for the reference grid using the voltage at the substation together with voltages and currents measurement at all customer nodes for increasing relative error of the customer measurements.

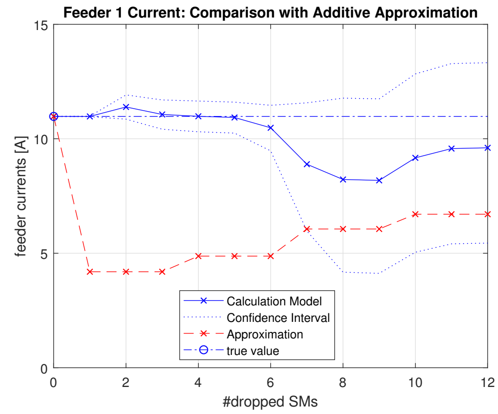

Fig. 5. Comparison of the calculated feeder estimation from the linear model with the approximation from only adding SM measurements for Feeder 1.

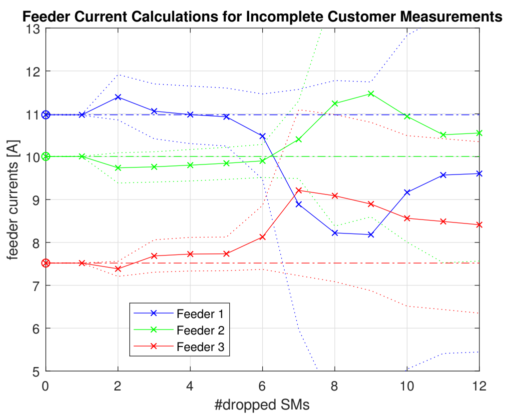

Fig. 4. Calculated feeder currents when using the substation voltage and current as well as the available voltages and current measurements from a decreasing number of customer connections.

The experiment starts from a full measurement scenario and then removes measurements by over-writing them with pseudo-measurements as explained above. The SMs that are removed for the first 12 removals in a round-robin fashion from the three feeders as follows: at loads $11, 23, 38, 7, 28, 31, 1, 15, 34, 13, 19, 36$. For this investigation, we decrease the relative measurement errors at the substation and customer Smart Meters to 10^{-4} and 10^{-3}, respectively. A normally distributed error is added to all these measurement values accordingly.

Figure 4 shows the behavior of the calculated feeder currents and their standard deviation for an increasing number of loads that are not providing Smart Meter measurements. The figure shows that for up to 6 removed Smart Meters (so two on each feeder), the calculated value is not too far from the ground truth feeder current for all 3 feeders and the calculated confidence intervals (indicated by dotted lines) remain narrow. In comparison to the simple additive approximation of the feeder current, plotted for Feeder 1 by the red dashed line in Figure 5, the calculation from the linear model has the advantage that also voltages and substation measurements can be utilized, allowing useful estimates for incomplete measurement scenarios. The simple additive approach, as demonstrated in Figure 5, however fails to provide any useful result even for a single nun-measured customer connection.

E. Application to real-life grid scenario with actual Smart Meter measurements

Finally, we check the validity of the grid model by applying it to a set of Smart meter measurements from an operational LV grid in North Denmark. This LV grid area is served by a secondary substation, which serves 10 customers; the substation is Node 1 and the Customers are the leaf nodes of the tree topology shown in Figure 7. The resistances and reactances have been obtained from the cable types and from the cable lengths that were available in the GIS system. Customers at Nodes 11 and 17 operate PVs (of 9.5 and 6kW rated power); Node 18 is a wind-turbine.

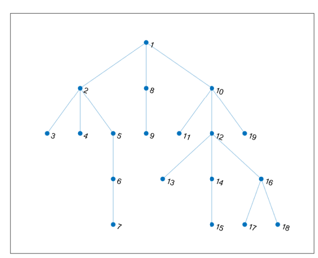

Fig. 6. Abstracted topology of a real grid from North Denmark.

The substation and all customers, except Node 3, contain Smart Meters, which provide average values over 15min intervals for the following measurands: voltage per phase, current per phase, consumed active power, generated active power, consumed reactive power, generated reactive power. The voltages are integer values (so change in steps of 1V). The currents are real positive values; in order to determine the phase angle and the sign, the net active and reactive power values are used.

Relative measurement errors of 0.5% and 1.5% are assumed for voltage and current measurements, respectively. The non-measured consumer is a low-power device which is represented by a current $I_3 = 3W/(230V)$ with a relative error of 100%.

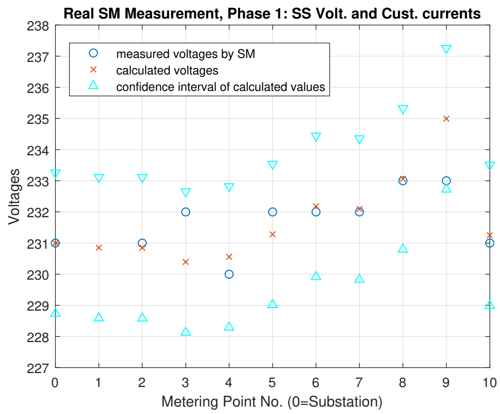

Fig. 7. Calculated voltage profile for the life LV grid based on the voltage measurement at the substation and current measurements from the nine measured customers. The measured voltages from the Smart Meters are marked by circles, but not used in the grid calculation.

Figure 7 shows the resulting voltage profile when applying the model to an individual phase (here L1) in isolation. The measured voltages are shown as circles; they are in this scenario always contained within the 95% confidence interval of the calculated voltage values. Note that the smart meter voltages are not used as input for the calculation model. The example calculation shows that the calculation model can be applied to data as provided by actually deployed Smart Meter systems.

V. Conclusion

This paper investigates how missing low-voltage grid characteristics can be obtained from substation and customer-side measurements of currents and voltages; the latter can for example be obtained from Smart Meters. The motivating example application is the calculation of LV feeder currents without deploying measurement devices at EACH feeder. The proposed approach uses a linear grid model, allows to quantify measurement errors of the input measurements, and rigorously derives measurement errors of the calculated parameters. The benefits of this approach in comparison to a simple current measurement aggregation are demonstrated quantitatively in several evaluation experiments.

Future work will analyse the obtained matrices in Section III-B to derive criteria when these are invertible. Furthermore, the approach will be extended to also allow for stochastic distributions for the grid parameters (here the elements of the impedance matrix) and to other error distributions, also covering cases of malicious modifications (e.g., tampering of Smart Meters). An extension to unbalanced 3-phase grid models should also be investigated. Finally, the obtained standard deviations and confidence intervals from the grid calculation approach can be investigated in other application contexts. Examples include the calculation of grid losses in the LV grid and the detection of voltage levels violations.

Acknowledgment

This work was supported by the European Unions Horizon 2020 research and innovation programme under grant agreement No 774145 within the project Net2DG. The authors would like to thank all project partners for their feedback and input. The authors would also like to thank the reference group members of Net2DG, specifically Josef Stadler from Stadtwerke Wels, for proposing the feeder estimation problem.

References

[1] M. Findrik, T. le Fevre Kristensen, T. Hinterhofer, R. L. Olsen, and H.-P. Schwefel, "Information-quality based lv-grid-monitoring framework and its application to power-quality control," in *Ad-hoc, Mobile, and Wireless Networks*, ser. Lecture Notes in Computer Science, S. Papavassiliou and S. Ruehrup, Eds. Springer International Publishing, 2015, vol. 9143, pp. 317–329.

[2] R. Olsen, J. T. Madsen, J. G. Rasmussen, and H.-P.Schwefel, "On the use of information quality in stochastic networked control systems," *Computer Networks*, vol. 124, pp. 157–169, Sept. 2017.

[3] D. J. Hess and J. S. Coley, "Wireless smart meters and public acceptance: The environment, limited choices, and precautionary politics," *Sage Journal*, 2012.

[4] H.-P. Schwefel, I. Antonius, and L. Lipsky, "Impact of time interval alignment on data qualitiy in electricity grids," in *IEEE SmartGridComm (2018)*.

[5] F. Schweppe and D. Rom, "Power system static-state estimation: Parts i, ii, and iii," *IEEE Transactions on Power Apparatus and Systems*, vol. 89, no. 1, pp. 125–130, 1970.

[6] S. M. S. Alam, B. Natarajan, and A. Pahwa, "Distribution grid state estimation from compressed measurements," *IEEE Transactions on Smart Grid*, vol. 5, pp. 1631–1642, 2014.

[7] N. Silva, P. Silva, L. Seca, and F. Melo, "Lv scada - how to effectively manage lv networks with limited topology and electrical characteristics data," *CIRED*, 2015.

[8] S. Bhela, V. Kekatos, and S. Veeramachaneni, "Enhancing observability in distribution grids using smart meter data," *IEEE Transactions on Smart Grid*, 2016.

[9] A. Gomez-Exposito, A. Abur, A. de la Villa Jaen, and C. Gomez-Quiles, "A multilevel state estimation paradigm for smart grids," *Proceedings of the IEEE*, 2011.

[10] A. Primadianto, "Requirements of state estimation in smart distribution grid." CIRED, 2015.

[11] K. Dehghanpour, Z. Wang, J. Wang, Y. Yuan, and F. Bu, "A survey on state estimation techniques and challenges in smart distribution systems," *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 2312–2322, 2019.

[12] M. Rana, "Smart grid state estimation and its applications to grid stabilization," Ph.D. dissertation, University of Technology Sydney, 2017.

[13] C. Lu, J. Teng, and W.-H. Liu, "Distribution system state estimation," *IEEE Transactions on Power systems*, vol. 10, no. 1, pp. 229–240, 1995.

[14] "European norm 50470: Electricity metering equipment (a.c.)," 2006.

[15] W. H. Greene and T. G. Seaks, "The restricted least squares estimator: A pedagogical note," *The Review of Economics and Statistics*, vol. 73, no. 3, pp. 563–567, 1991.

[16] W. Venables, "Calculation of confidence intervals for noncentrality parameters," *Journal of the Royal Statistical Society. Series B*, vol. 37, no. 3, pp. 406–412, 1975.

[17] H.-P. Schwefel and et. al., "Smartc2net project final report," Available online at http://smartc2net.eu/, Dec. 2015.

[18] R. Pedersen, C. Sloth, G. B. Andresen, and R. Wisniewski, "Disc: A simulation framework for distribution system voltage control," in *European Control Conference*, 2015.