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Published in:
Flexible Services and Manufacturing Journal

DOI (link to publication from Publisher):
10.1007/s10696-017-9292-8

Publication date:
2018

Document Version
Accepted author manuscript, peer reviewed version

Citation for published version (APA):
Reinhardt, L. B., Pisinger, D., & Lusby, R. M. (2018). Railway capacity and expansion analysis using time discretized paths. *Flexible Services and Manufacturing Journal*, *30*(4), 712-739. https://doi.org/10.1007/s10696-017-9292-8

This is a post-peer-review, pre-copyedit version of an article published in Flexible Services and Manufacturing Journal. The final authenticated version is available online at: http://dx.doi.org/10.1007/s10696-017-9292-8

Railway capacity and expansion analysis using time discretized paths *

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Abstract

When making investments in railway infrastructure it is important to be able to identify the limits for freight transportation in order to not only use the infrastructure in the best possible way, but to also guide future capacity investments. This paper presents a model to assess the capacity of railway freight transportation on a long term strategic level. The model uses an hourly time discretization and analyzes the impact of railway network expansions based on future demand forecasts. It provides an optimal macroscopic freight train schedule and can indicate the time and place of any congestion. In addition, two expansions of the primary model are developed. The first can be used to determine the minimal number of expansions needed to ensure all freight can be feasibly routed, while the second uses variable penalties to ensure additional freight trains are scheduled in such a way that they do not hinder planned passenger train movements. As part of a European Union (EU) project, all models are applied to a realistic case study that focuses on analyzing the capacity of railway network in Denmark and Southern Sweden using demand forecasts for 2030. Results suggest that informative solutions can be found quickly with the proposed approach.

1 Introduction

At the COP21 Climate Change Conference, held in Paris 2015, all member states agreed to reduce their carbon output in order to limit global warming. The transportation sector is responsible for approximately one quarter of the greenhouse gas emissions in the European Union (EU), and, while the emissions from other sectors are generally decreasing, these are much more difficult to limit. Electrified trains can be a green form of transportation when renewable energy sources are used, thus making containerized freight by electric train an environmentally friendly alternative to trucks and shipping vessels. However, before we can encourage this mode of freight transportation, it is first important to ensure that the railway can in fact accommodate the modal shift, and that the capacity of the railway is utilized in the best possible way.

In this paper we present a tool to analyze the freight capacity of a railway network using future demand forecasts under different policy assumptions. The problem we solve is a transportation problem with time dependent capacity limitations. In essence, it is a combination of train timetabling and capacity analysis. On a macroscopic level we solve a train scheduling problem. This is used to assess the capacity of the railway network and to identify any bottlenecks. We focus on a strategic planning level and consider future demand scenarios some 15-20 years into the future. As such, we do not attempt to find the exact departure and arrival times of the trains. Using forecast freight volumes in the form of trains between specified pairs of origin and destination stations, the model attempts to schedule as many trains as possible while minimizing the deviation from desired departure and arrival times. The trains are scheduled subject to known, time dependent capacities on individual railway segments and and at stations of the railway network. We demonstrate how this model can be used to assess the viability of various planned expansions of the railway network and how, through appropriate penalties, the model can avoid busy passenger train periods.

The results of this work are included in the EU project for the East West Transport Corridor II, and the model has been developed in cooperation with the Danish Traffic Authority who have provided the forecast demand and

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the data on the Danish railway infrastructure. A first version of this paper was published in the proceedings of the 6th International Conference on Computational Logistics (see [22]), and makes several contributions to the railway literature. These include the following:

1. The problem is modelled as a multi-commodity flow problem. This approach is similar to the train pathing work of Harrod [15]. In the model we propose, however, departure patterns for each origin station should be satisfied, and the objective is to schedule as many trains as possible.

2. The developed model provides an in-depth analysis of the railway capacity in Denmark and Southern Sweden, using freight volume forecasts for 2030. The analysis is used to identify bottlenecks and to study various planned expansions of the network.

3. The proposed approach provides the possibility to analyze in which parts of the network capacity needs to be increased (and perhaps how) to satisfy the forecast freight demands. Since the railway tracks are shared by freight and passenger trains, increasing the number of freight trains impacts the number of passenger trains possible. We describe a variable penalty method to increase the number of freight trains such that excess freight trains during passenger rush hour periods are punished more than those during off-peak hours.

The paper is organized as follows. Section 1.1 provides a more formal description of the problem we consider. Section 2 contains a short literature review, our primary model is presented in Section 3. In Section 4 we introduce, and model, the problem of optimizing capacity expansions. Section 5 describes our numerical results based on a case study which focuses on the railway network connecting Denmark and Southern Sweden, and uses forecast scenarios from 2010 to 2030. The paper concludes in Section 6 with a short summary of the results obtained.

1.1 Problem Description

The problem that we consider has been defined in collaboration with the Danish Traffic Authority. Given forecast freight volumes between pairs of origin and destination stations, the model must determine if all trains corresponding to the forecast volume can be routed through the network while satisfying a weekly frequency pattern for the train departures. Moreover, any bottlenecks in the network should be identified. An important requirement is that each train departs from its origin station and arrives at its destination station as close as possible to the train's desired arrival and departure times. Any deviation from this must be penalized. Trains are required to travel nonstop through the network and must return to their origin (full or empty). An overview of the considered Danish-Swedish railway network is shown in Figure 1.

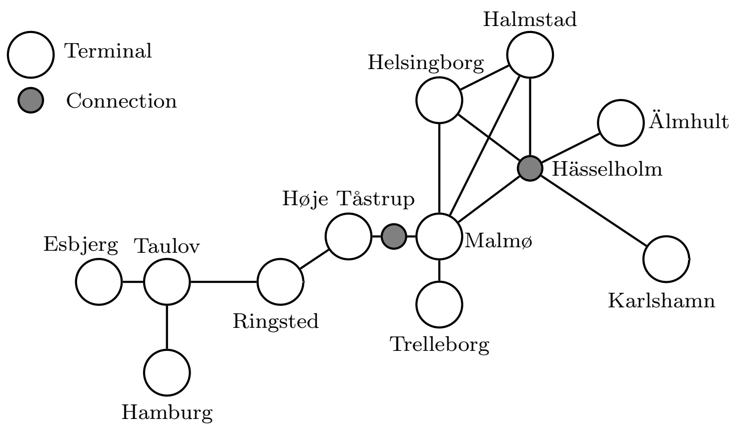


Figure 1: The Danish-Swedish railway network considered, as of 2010

As shown in Figure 1, there are two types of nodes: *terminal* nodes and *connection* nodes. A terminal node corresponds to a station where demand may originate or terminate. Demand may be routed via either terminal or connection nodes; however, it may *not* originate or terminate at a connection node.

The aim of this project is to develop a model that can analyze the capacity of rail segments in a railway network over a horizon of one week given forecasts of future freight demand under different scenarios and using business knowledge of the time dependencies of the demand.

We also address the issue of determining the time of day with the largest saturation of freight train capacity on the different macroscopic rail segments. When evaluating the network without considering time and cost, the capacity of the network may appear to be sufficient, despite the fact that a more detailed analysis would show that some time periods are overloaded. For instance if there is no need for a train between 2:00AM and 5:00AM then the capacity available at this time cannot be used to satisfy the demand in other time windows. This is a major concern when it comes to passenger trains; however, with freight trains the timing of the transportation is less important. Even though freight trains do not have as severe time restrictions as passenger trains, there are various other time constraints which do apply to the freight trains. Such constraints may include: the working hours for loading and unloading at the origin and destination stations, the cost of operating a train, and the working hours of the operators. In addition, a somewhat even distribution of the departures/arrivals over a week is generally desired for each specific origin-destination pair. This ensures a continuous flow of demand.

As mentioned earlier, a desired time window is considered for the departure and the arrival of any train. A general forecast based on market knowledge is used to set a departure time window and an arrival time window. These time windows are considered soft, with an associated penalty for breaking them. However, each soft time window is contained within a hard time window. The hard time window does not allow for any deviations in departure or arrival time exceeding a given threshold. Trains are required to travel over the network nonstop, i.e., without waiting at intermediate stations. This is due to capacity issues at stations. Furthermore, train operators would prefer not to pay for idle crew. The trains between a given origin and destination pair must have a somewhat even distribution during the week. This distribution is provided in Table 1 and depends on the number of trains required for the demand. For example, if three trains are needed per week, then one train has a preferred time window Monday, the other Wednesday, and the third Friday. No trains have preferred time windows on Saturday or Sunday; however, trains that have preferred time windows on Friday or Monday can depart on Saturday or Sunday at a penalty.

Number of Trains	Monday	Tuesday	Wednesday	Thursday	Friday
1			1		
2		1		1	
3	1		1		1
4		1	1	1	1
5	1	1	1	1	1
6	1	1	2	1	1
7	1	2	1	2	1
8	2	1	2	1	2
9	1	2	2	2	2
10	2	2	2	2	2
⋮	⋮	⋮	⋮	⋮	⋮

Table 1: Desired distribution of train departures during a week.

Using these requirements it is possible to evaluate the utilization of the network at different times. The return of trains to their origin is enforced in order to produce a schedule that can be repeated for more than a week.

Trains can travel on different routes through the network. These different routes each have a cost which depends on the time of day the train traverses the route. We therefore introduce the concept of a *path*. A path refers to a route at a specific time. Note that for each route there can be several paths.

For the problem considered, the cost of a train route from an origin to its destination is only relevant when prioritizing the different routes through the network. Therefore, only direct costs are included and socio-economic costs, see e.g. [29], are not considered. For the paths corresponding to a route a penalty for deviating from desired times is included. It is clearly always more profitable to schedule a train than to cancel it. As a consequence, only a lack of capacity in an acceptable time window can result in a canceled train. If cancellations are required, naturally trains on paths with large deviations from preferred times will be prioritized for cancellation. In contrast to most other models used for capacity analysis, where more detail is usually included, we restrict our model to much less information. This is in part due to the long time horizon considered (here, 20 years). We focus on providing useful insight into capacity analysis, while retaining a tractable approach. In Section 3 we formulate this problem as a variant of the well known integer multi-commodity flow problem.

2 Literature review

Rail transportation can be modeled as a network problem in which the nodes represent stations and the edges indicate rail (track) segments. An early survey by Assad [5] from 1980 on modelling rail transportation includes aspects concerning yard queuing and simulation of passenger movements. A more extensive review by Cordeau et al. [13] shows that since 1980 research in the area of train transportation has increased significantly, and several details and aspects of the routing and scheduling problem of rail transport have been considered. The extensive survey by Abril et al. [1] contains information on publications prior to 2008 in the area of capacity analysis. The authors divide railway capacity analysis into a theoretical analysis and a practical analysis, and perform a capacity analysis of a single track line.

Caprara et al. [12] investigate the configuration of trains and their routes on a railway corridor. This was done in connection with the EU REORIENT project (2004-2007) where a railway corridor connecting southern and northern Europe through eastern Europe is analyzed. Capacity, in the form of the number of trains, is predetermined on the route, and the assignment of demand to trains is optimized with respect to the travel time of the demand and the travel cost. In the problem we consider we do not take into account the composition of trains. This is because we do not view the problem from a train operator's perspective.

Burdett and Kozan [10] develop a model capable of a very detailed capacity analysis. The model determines the speed of the individual trains and the possible headway times. This approach is relevant when assessing the capacity of a current network and is really only applicable with a short time horizon. When considering a situation some 10 years into the future such details concerning the speed and headway of trains may be uncertain.

Different investment options for the subdivision of a network are considered by Lai and Barkan in [18]. The possible investment options are first generated, and the best one within a given budget is then selected. This work was extended by Lai and Shih in [19] to an investment plan, where the investments take place over several time periods with assumed knowledge of the development in demand at specific time periods. This method of generating investment options using unit cost is only applicable on a microscopic scale as landscape and urban structures have a great influence on the cost of investment options. In the USA, for example, where large parts of the railway maybe located on scarcely populated plains this could be applicable. However, in densely populated European countries, with many waterways the investment options cannot be generated in the same fashion using unit costs. In our case, The Danish Traffic Authority would prefer a tool that can evaluate different investment suggestions and leaves finding the exact cost of the investments until after the evaluation of their effect. Zhang [32] analyzes the capacity of the Chinese high speed passenger rail service under uncertainty. The problem considered in [32] is for the current situation and therefore includes many details on train operations. Such details, together with the uncertainty, result in the need for heuristic solution methods.

Burdett [9] considers multiple objectives, such as optimizing the flow on different network corridors, and proposes an ϵ-constraint approach. The author uses an absolute capacity value and therefore does not consider the temporal aspect. An absolute capacity flow is an abstraction level which over estimates the actual capacity. One could argue that when evaluating capacity far into the future, the temporal perspective is too uncertain. From the set of Pareto optimal solutions which the ϵ-constraint algorithm finds, the solution to implement must be selected based on some criteria or knowledge not included in the model.

Cacchiani et al. [11] optimizes train timetabling at a single railway node using desired times to ensure a good capacity usage, without significantly disrupting the train operators' schedules. Many details are known as the problem focuses on the current situation, or the very near future. The aspect of using time to ensure a desirable schedule is also used in the problem we consider, however, in a more general way, as specific knowledge of the desired times, head way times, and passenger trains is not known when looking 15- 20 years into the future. Mussone and Wolfler Calvo [21] present an analytic approach to calculate railway capacity. Timetables are not used; however, congestion at railway nodes is used to find a value for the delay.

Borndörfer et al. [7, 8] and Schlechte et al. [24] solve a macroscopic track allocation problem which considers different train types and includes train speeds and headway times. In Zhu et al. [33] blocking and service selection is combined using a heuristic method. Andersen and Christiansen [3] design a new European north south rail freight service through Poland using an exact method. The authors consider the routing of trains in more detail than we include in this paper and solve very small instances containing at most 20 services. Andersen et al. [4] study the design of rail services in an intermodal network with the aim of reducing transshipment times. Jarrah et al. [17] consider a large-scale less-than-truckload service network design. In this problem the objective is to determine origin - destination paths for expected freight volumes while minimizing overall transportation and other ancillary costs. The problem can be formulated as a capacitated multi-commodity fixed-charge network design problem.

Schöbel et al. [25] present an integrated model for designing a minimum cost infrastructure that fulfills all the requirements defined by a timetable and the operation of the railway system. A combinatorial optimization model is presented based on a graphical representation of the existing railway infrastructure and all candidate expansions of the network. Some preliminary ideas for a mixed integer programming based solution method as well as heuristic approaches are presented, although no computational results are reported. The model presented is similar to the one presented by Harrod [15] for train timetabling. In [15], the author considers trains of different speeds on a very small problem instance. In [16], the size of the problem instances was significantly increased, and two methods for train timetabling were compared. Lium et al. [20] study a service network design problem taking into account demand stochasticity. The authors show that solutions based on an uncertain demand can differ significantly from solutions based on a deterministic demand. For a thorough review of tactical planning problems in the literature related to long-distance freight distribution, we refer the reader to the review by Wieberneit [31].

As mentioned earlier, the problem we consider is a combination of train scheduling and capacity analysis. Given forecast, distant future freight volumes, the goal is to schedule the freight trains at minimum cost in such a way that all forecast demand is satisfied and all constraints concerning network capacity and any restrictions on arrival and departure time windows are respected. The cost includes a fixed track charge, an infrastructure tax per kilometer, and a penalty cost for any deviation from desired times. The problem can thus be viewed as a reduced version of that which is considered by Borndörfer et al. [7] and Schlechte et al. [24]. Due to the amount of detail considered in both [7] and [24], hours of computation time are used for solving the problems. In real-life applications, however, long running times are often undesirable. Long running times usually require a very thorough preparation of the input data and could lead to a situation in which the capacity analysis easily takes several months to perform. This is naturally too long in the political decision making process and, furthermore, any possibilities of useful extensions to the current network may be lost.

We therefore present a very simple capacity analysis which can help find and solve capacity problems for distant future demand scenarios and assist with determining network bottlenecks. The proposed model identifies the time of day the saturation occurs on the segments and tries to ensure an even distribution of the freight trains while adhering to preferred time windows on arrival and departure times. We also introduce a simple capacity expansion model which selects the least costly capacity expansions that, if used, could satisfy the forecast demand. The method presented here works with directed capacity and therefore sharing a track in both directions is indirectly represented by a low hourly capacity provided by the railway owners. This problem is relevant for distant future capacity analysis where little knowledge regarding the actual operations is available. If more knowledge about operations is available the models presented in this paper are insufficient and other methods must be used.

3 Model

In this section we formulate and describe an integer programming model that can be used to solve the capacity analysis problem specified by the Danish Traffic Authority. We first assume a set of future demands $\mathcal{K}$ is known. We further assume a set of terminals and connection nodes, $\mathcal{N}$, is available. These define the nodes of the railway network. The associated set of railway segments is denoted by $\mathcal{E} = \{(i,j) \mid i,j \in \mathcal{N}\}$. Each demand $k \in \mathcal{K}$ corresponds to a train between a specific origin station, $O_k \in \mathcal{N}$, and destination station, $D_k \in \mathcal{N}$, and has known time windows for its departure and arrival times. From a temporal perspective, we assume a set of discretized time intervals, $\mathcal{T}$, has been defined. By replicating the physical network at each time interval $t \in \mathcal{T}$, we obtain the time-space network $\hat{G} = (\hat{\mathcal{N}}, \hat{\mathcal{E}})$. That is, for each node $i \in \mathcal{N}$ and $t \in \mathcal{T}$ there exists a node $i_t \in \hat{\mathcal{N}}$ corresponding to node $i \in \mathcal{N}$ at time t. Similarly, $(i,j)_t \in \hat{\mathcal{E}}$ corresponds to edge $(i,j) \in \mathcal{E}$ at time $t \in \mathcal{T}$.

The proposed model has been developed in cooperation with the traffic authorities [28] and therefore adheres to current practices. The goal is to maximize the number of trains routed and to minimize the total deviation of the selected paths from the preferred time windows. The following is a list of constraints that must be respected.

- A train $k \in \mathcal{K}$ must be routed over the railway segments without stopping from its origin terminal $O_k \in \mathcal{N}$ to its destination terminal $D_k \in \mathcal{N}$. For each train $k \in \mathcal{K}$ we assume a set of possible routes $\mathcal{R}_k$, and corresponding set of paths $\mathcal{P}_k$, is available. Note that the set $\mathcal{P}_k$ ($\bigcup_{k\in\mathcal{K}} \mathcal{P}_k = \mathcal{P}$) can be generated so that train stopping is avoided. The cost for using path $p \in \mathcal{P}_k$ is assumed to be c_p. Clearly, for each train $k \in \mathcal{K}$ a path $p \in \mathcal{P}_k$ must be selected, otherwise the train is cancelled.
- A train's arrival time and departure time must occur within prespecified time windows. If this cannot be achieved, a train dependent penalty s^k is incurred.
- Railway segment capacity may not be exceeded. We denote the capacity of a railway segment for a given time interval to be q_{ij}^t, where $(i,j) \in \mathcal{E}$, and $t \in \mathcal{T}$. The parameter $\alpha_{ij}^{p,t}$ is used to indicate whether or not path $p \in \mathcal{P}$ uses segment $(i,j) \in \mathcal{E}$ at time $t \in \mathcal{T}$.
- For each train $k \in \mathcal{K}$ going from $O_k \in \mathcal{N}$ to $D_k \in \mathcal{N}$, there must be another train $k' \in \mathcal{K}$ going from $D_k \in \mathcal{N}$ to $O_k \in \mathcal{N}$. For notational purposes, the set $\mathcal{K}_{od}$ contains all trains having origin station $o \in \mathcal{N}$ and destination station $d \in \mathcal{N}$.

As mentioned in Section 1.1, these requirements stem from train operator desires and the physical constraints of both the railway network and the trains. To solve the problem we introduce two types of binary variables. The first, which we denote λ_p for $p \in \mathcal{P}$ governs the selection of path p; this variable takes the value of one if the path is selected and is zero otherwise. The second, which we denote h_k for $k \in \mathcal{K}$, models cancellations. The variable takes the value one if train k is cancelled and is zero otherwise.

When including time windows, one option is to use a time expanded network; however, such a network can become extremely large as each rail segment in the original data would be represented by 168 edges in the time expanded network (assuming a time discretization of one hour for one week). In our case, since trains are not allowed to take breaks and because there are also hard time windows on arrivals and departures, we are able to generate all feasible train paths through the time-space network corresponding to the train routes. The size of this network is $O(|\mathcal{T}| \cdot |\mathcal{N}| + |\mathcal{T}| \cdot |\mathcal{E}|)$, which can also be written as $O(|\hat{\mathcal{N}}| + |\hat{\mathcal{E}}|)$. Each origin-destination pair of terminals has a set of possible paths connecting them. These are generated in preprocessing phase.

When capacity analysis is performed, the rail network under consideration is often a subset of a larger network. To accommodate this, we define terminals or connection nodes as entry points to the network we wish to evaluate. The time windows are projected onto the subnetwork as part of the preprocessing phase. The entry points and exit points of the subnetwork are assumed to be provided in the forecast demand data.

We formulate the problem as an integer multi-commodity flow problem. This problem is known to be NP-hard on a directed graph, as shown by Even [14]. As such, several decomposition methods have been developed to solve it (see e.g. [6] and [2]). Barnhart et al. [6] show that Cplex 3.0 is inferior to their branch-cut-and-price algorithm; however, Alvelos and Carvalho [2], showed that in 2003 Cplex 7.5 was already competitive with several branch-and-price algorithms for small instances. One of the main problems with both the polynomially solvable splittable multi-commodity flow problem and the unsplittable multi-commodity flow problem is the number of variables. We adopt a path based formulation. However, as already stated, instead of solving it with a full branch-and-price algorithm, given the restrictions and size of the problem, we propose to enumerate all feasible paths for the trains. Our formulation is given below.

$$\texttt{Minimize:} \sum_{k\in\mathcal{K}} \left(s^k h^k + \sum_{p\in\mathcal{P}_k} c_p \lambda_p \right) \tag{1}$$

$$\texttt{subject to:} \quad \sum_{k\in\mathcal{K}} \sum_{p\in\mathcal{P}_k} \alpha_{ij}^{p,t} \lambda_p \leq q_{ij}^t \qquad \forall (i,j) \in \mathcal{E}, t \in \mathcal{T} \tag{2}$$

$$\sum_{p\in\mathcal{P}_k} \lambda_p + h^k \geq 1 \qquad \forall k \in \mathcal{K} \tag{3}$$

$$\sum_{k\in\mathcal{K}_{od}} h^k - \sum_{k\in\mathcal{K}_{do}} h^k = 0 \qquad \forall o,d \in \mathcal{N}, o \neq d \tag{4}$$

$$\lambda_p \in \{0,1\} \qquad \forall p \in \mathcal{P} \tag{5}$$

$$h^k \in \{0,1\} \qquad \forall k \in \mathcal{K} \tag{6}$$

The objective function (1) minimizes the cost of the macroscopic train schedule. By making the value of s^k, i.e., the penalty for not scheduling a train $k \in \mathcal{K}$, large enough we can ensure that trains are only cancelled if no other option is possible. Note that this ensures that the number of cancelled trains is minimized. With the defined objective function, trains which are more costly to schedule will be the ones chosen to be cancelled, if necessary. Costly trains are those that have large deviations from desired departure and arrival times. The penalty for deviating from a preferred time is assumed to be larger than the actual travel cost. Minimizing the travel cost ensures that, if all time windows can be respected, low cost paths are in a sense prioritized. In this way we try to ensure the best possible service for the operators under the restrictions of the rail network. A solution to this scheduling problem must satisfy three main types of constraints. Constraints (2) ensure that the time dependent capacity of a rail segment is never violated. Constraints (3) enforce the requirement that a train is either scheduled or cancelled. Finally, Constraints (4) ensure that the number of trains for each origin-destination pair are balanced. Variable domains are specified by Constraints (5) and (6).

As mentioned earlier, we project the preferred time windows at origins and destinations located outside the considered subnetwork onto the defined entry points of the subnetwork. This is done using the known travel time between the entry point or exit point of the subnetwork to the destination or terminal of origin. On the subnetwork a set of routes between the different start and end points of the subnetwork is generated in advance by a user with knowledge of which transportation routes are acceptable. This could be changed in future work so that the acceptability of a route is calculated from the increase in cost using some threshold value. For each of the given routes, the program generates a set of paths which all satisfy the valid time windows of the train. The cost of each path is calculated with respect to the travel cost, and the deviation from the desired time window of the train. Pseudocode for the enumeration of P_k is shown in Algorithm 1. Note that paths outside the hard time windows are eliminated. This ultimately reduces the number of paths. To avoid searching through all paths when setting up the capacity constraints, the lists for each edge of the time-space network used are updated on Line 14. As shown in Section 5.2, this results in fewer path variables due to the elimination of paths outside the acceptable time windows.

Algorithm 1 PathEnumeration

1: **procedure** PATHENUMERATION($\mathcal{K}$, $\mathcal{R}$)
2:     **for** each train k in $\mathcal{K}$ **do**
3:         **for** each route r in $\mathcal{R}_k$ **do**
4:             **for** each node O_{k_t} at origin of k **do**
5:                 **if** O_{k_t} satisfies hard time window for departure of k **then**
6:                     Add delay cost for O_{k_t} to cost of path p_t^k.
7:                     **for** each segment e on r **do**
8:                         Add to p_t^k the edge $e_{t'} \in \hat{\mathcal{E}}$ corresponding to O_{k_t}.
9:                         Add travel cost of $e_{t'}$ to cost of path p_t^k.
10:                     **end for**
11:                     **if** $D_{k_t''}$ satisfies hard time window for arrival of k **then**
12:                         Add delay cost for $D_{k_t''}$ to cost of path p_t^k.
13:                         **for** each edge $e_{t'}$ on p_t^k **do**
14:                             Add path p_t^k to the cap list of $e_{t'}$
15:                         **end for**
16:                         Add path p_t^k to the list of paths for k
17:                     **end if**
18:                 **end if**
19:             **end for**
20:         **end for**
21:     **end for**
22: **end procedure**

4 Minimum track capacity to schedule all trains

Model (1)–(6) can be used to determine the maximum number of freight trains that can be scheduled. It does not state, however, how much capacity is missing in the network. In this section we consider two modifications to the model which can be used to suggest where additional capacity (known as capacity expansions) should be placed to allow all trains to be scheduled. In particular, the two variants are:

1. Minimize the required capacity expansions to enable all trains to be scheduled.
2. Minimize the required time-dependent capacity expansions to enable all trains to be scheduled.

The two variants are discussed in turn in Sections 4.1 and 4.2 below. The second variant is motivated by the fact that passenger trains and freight trains are often required to share track capacity. As a result, it is easier to increase the number of freight trains when there are fewer passenger trains than to increase their number during passenger rush hours.

4.1 Minimizing capacity expansions

Minor modifications to Model (1)–(6) are required in order to be able to determine the minimal cost set of capacity expansions that allows all trains to be scheduled. We introduce an additional variable β_{ij} for each segment $(i,j) \in \mathcal{E}$. This variable represents the additional capacity required for segment $(i,j) \in \mathcal{E}$; in our case, the number of additional trains per hour the rail segment must be able to cater for. Since we require all trains to be scheduled, the possibility to cancel trains (and hence the variables h^k) are removed. The objective function simply minimizes the cost of the additional track capacity. To do this, we introduce the parameter f_{ij}, which denotes the cost associated with increasing the capacity of segment $(i,j) \in \mathcal{E}$ by one train per time period.

$$\texttt{Minimize:} \sum_{(i,j)\in\mathcal{E}} f_{ij}\beta_{ij} \tag{7}$$

$$\texttt{subject to:} \quad \sum_{k\in\mathcal{K}} \sum_{p\in\mathcal{P}_k} \alpha_{ij}^{p,t}\lambda_p - \beta_{ij} \le q_{ij}^t \qquad \forall (i,j) \in \mathcal{E}, t \in \mathcal{T} \tag{8}$$

$$\sum_{p\in\mathcal{P}_k} \lambda_p = 1 \qquad \forall k \in \mathcal{K} \tag{9}$$

$$\lambda_p \in \{0,1\} \qquad \forall p \in \mathcal{P} \tag{5}$$

$$\beta_{ij} \ge 0 \qquad \forall (i,j) \in \mathcal{E} \tag{10}$$

The objective function, given by (7), minimizes the cost of the additional capacity required. Constraints (8) link the new decision variables β_{ij} with existing track capacity. Constraints (9) are a modified form of Constraints (3) in which the possibility of cancelling a train has been removed so that all trains must be scheduled. Finally, the β_{ij} variables are restricted to being non-negative.

4.2 Minimizing capacity expansions with time dependent penalties

Since freight trains and passenger trains often must share track capacity, in this section we propose a model to minimize the additional capacity required subject to time dependent penalties. This is an attempt to model the situation in which some time intervals are more preferable than others when it comes to increasing the number of freight trains. For example, it is undesirable to schedule additional freight trains during passenger rush hour periods. To model this, we replace the β_{ij} variables with new variables η_{ij}^t. The variable η_{ij}^t indicates the additional capacity required in time interval $t \in \mathcal{T}$ on segment $(i,j) \in \mathcal{E}$. The parameter δ_{ij}^t is the corresponding cost of increasing the number of freight trains in the specific time interval $t \in \mathcal{T}$ by one. The resulting formulation is therefore:

$$\texttt{Minimize:} \sum_{(i,j)\in\mathcal{E}} \sum_{t\in\mathcal{T}} \delta_{ij}^t \eta_{ij}^t \tag{11}$$

$$\texttt{subject to:} \quad \sum_{k\in\mathcal{K}}\sum_{p\in\mathcal{P}_k} \alpha_{ij}^{p,t}\lambda_p - \eta_{ij}^t \le q_{ij}^t \qquad \forall (i,j)\in\mathcal{E}, t\in\mathcal{T} \tag{12}$$

$$\sum_{p\in\mathcal{P}_k} \lambda_p = 1 \qquad \forall k\in\mathcal{K} \tag{9}$$

$$\lambda_p \in \{0,1\} \qquad \forall p\in\mathcal{P} \tag{5}$$

$$\eta_{ij}^t \ge 0 \qquad \forall (i,j)\in\mathcal{E}, t\in\mathcal{T} \tag{13}$$

The objective function, (11), minimizes the total cost of the time dependent capacity expansions. The parameter δ_{ij}^t, in this case, depends on how much passenger traffic is expected at a given time $t \in \mathcal{T}$. A higher cost is imposed for rush hours, while time intervals with very few passenger trains are assigned a lower cost. Minor modifications are seen in Constraints (12), where β_{ij} has been replaced by η_{ij}^t. The domain of the η_{ij}^t variables is given by Constraints (13). In this formulation they are simply non-negative; however, in practice there may be an upper bound on how high this value can be set.

5 Numerical Analysis

The models in Section 3 and Section 4 have been used for analyzing the capacity of the Danish and Southern Swedish network. The data used for this analysis is from a flow analysis based on transportation forecasts [27] provided by Tetraplan [26]. These forecasts were generated using the program Trans-Tools [23]. The generation of the forecasts was not part of this research, and the reader is referred to [27] for further details on the generation of the forecasts.

5.1 The Network

The data contains the freight in tons per week between 47 different zones in Europe. The map of the zones is shown in Figure 2. The generated forecasts cover five different scenarios: 2010, 2030 Baseline, 2030 Green, 2030 Baseline SK, and 2030 Green SK. These are summarized below:

2010: This scenario represents current operation with an estimated 926 trains per week.

2030 Baseline: This scenario contains 2,246 trains per week with standard estimates for 2030.

2030 Green: This scenario assumes higher fees for CO_2 emission. This scenario contains 2,354 trains per week.

2030 Baseline SK: The **2030 Baseline** where all freight with origin **Sundsvall** and destination **Karlshamn** is removed, assuming a direct rail segment between **Älmhult** and **Karlshamn**.

2030 Green SK: The **2030 Green** where all freight with origin **Sundsvall** and destination **Karlshamn** is removed, assuming a direct rail segment between **Älmhult** and **Karlshamn**.

Sundsvall is the origin of some demand using the subnetwork presented; however, **Sundsvall** lies outside the subnetwork. In the last two scenarios **2030 Baseline SK** and **2030 Green SK** demand with origin **Sundsvall** and destination **Karlshamn** is removed. The last two scenarios were developed due to the large amount of freight between **Sundsvall** and **Karlshamn** which was assumed by the traffic authorities to use rail segments not included in the considered subnetwork.

The train departures are distributed somewhat evenly during the week according to the distribution pattern shown in Table 1. The pattern continues as shown for higher numbers of weekly departures. Recall that freight trains are not scheduled to depart during the weekend. Trains may travel during the weekend; however, this is outside the preferred time window, and thus will be penalized.

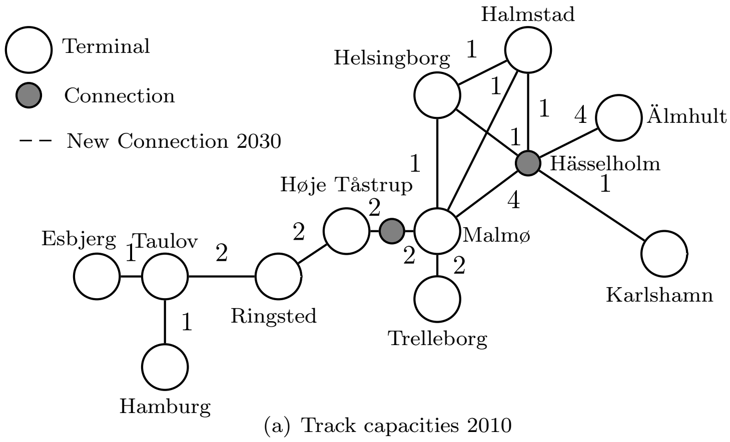
Freight is usually loaded onto the train during the afternoon, and operators therefore request departure times in the evening between 8:00PM and 2:00AM. Furthermore, since the freight is typically unloaded at the destinations during the morning hours, it is desirable for the train to arrive between 10:00PM and 6:00AM. Problems could arise for a departure if the train is scheduled to depart at 2:00PM, as there may not be enough time to load the freight before departure. This is also the case for an arrival after 6:00AM; it may not be possible to unload the freight and ensure a timely arrival at the customers. Therefore, a penalty is associated with departing early and arriving late. It is, however, not possible to arrive more than 24 hours after the end of the arrival time window, nor to depart more than 24 hours prior to the start of the departure time window; this would generate an unacceptable distribution of departures that does not adhere to the pattern stated in Table 1. This requirement may result in the cancellation of trains, even though available capacity exists on the rail segments. The penalty for deviating from a preferred time window is assumed to be 10 per minute of deviation. The penalty for cancelling a train is assumed to be 300,000 per train cancelled.

If an origin-destination pair has more than one departure on a given day, then the second departure has a preferred departure window between 10:00AM and 4:00PM and a preferred arrival window between hours 12:00 noon and 10:00PM. A third departure would then have a preferred departure window between 6:00AM and 12:00 noon and a preferred arrival window between hours 8:00AM and 2:00PM. For the fourth or more departure, the preferred departure time window is the entire day, while the preferred arrival time window is the entire day for the estimated arrival.

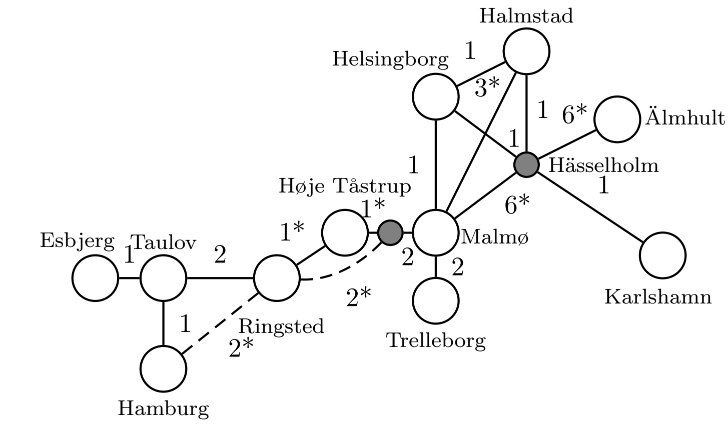
Since trains must return, the number of trains running between any origin-destination pair, must be the same as the number of trains running in the other direction (from the destination to the origin). This may imply that the freight cars on a train travel empty in one direction. Note that this does not mean that the number of trains in one direction on a specific rail segment is the same as the number of trains in the other direction on that rail segment. In fact, for a single day the number of trains on a rail segment may be very different in the two directions.

The networks used are shown in Figure 3. Figure 3 illustrates (a) the network as it is today (2010), while (b) incorporates all the politically agreed expansions of the rail network, which are estimated to be finished by 2030. The new rail segments are shown with dashed lines. The projected capacity changes on existing rail segments are marked with an "*". For each rail segment, the capacity is given in the form of the number of freight trains that can pass per hour in each direction. These capacities were provided by the Danish and the Swedish traffic authority [28, 30].

Figure 2: Map of the zones considered in the demand matrix.



(a) Track capacities 2010



(b) Track capacities 2030

Figure 3: Map of the analysed subnetwork for year 2010 and 2030. The capacity is for each rail segment expressed as freight trains per hour in each direction. Capacities which are changed from 2010 to 2030 are marked with an *.

The freight trains may have an origin and destination outside the analysed subnetwork but will pass through the subnetwork analysed by entering at a terminal of the subnetwork.

A rail segment connecting **Helsingør** and **Ringsted** was suggested by the Swedish and Danish traffic authorities. Hence, we have included a rail segment between **Helsingborg** and **Ringsted** in some of the tests of the 2030 network so that this alternative can be investigated.

Each rail segment has an associated travel time and cost. These parameters have also been made available by the Danish traffic authority [28]. The travel time is given as an integer number of hours. Non-integer travel times can also be handled; however, this requires a different temporal discretization for the underlying time space network. The cost of traversing a railway segment includes several components. In this model, only very elementary costs are included as the cost function is used for prioritizing one path over another for a given origin-destination pair. In particular, infrastructure, locomotive, and carriage charges are included in the cost of a rail segment. In addition, a capacity cost is also considered. Finally, a fixed bridge cost may be imposed on some rail segments. All other costs, such as fuel costs and external costs (e.g., air and noise pollution) are ignored; these are less important for the model since we do not compare the solutions obtained with alternative transportation modes such as trucks and shipping vessels. Note that the delay cost and the cancellation cost are the most significant costs. The cost estimates are given as parameters to the solver, and these can easily be changed to include other costs as well.

5.2 Test cases

In this section we describe the properties of the different scenarios, the tests performed, and the results obtained by the proposed models. Model (1)–(6) was first used on the five scenarios using the capacities stated in Figure 3 (b). We denote this selection of capacities as **cap1**. After analyzing the results obtained using these capacities, eight different variations were generated and investigated. These variations are generated based on political suggestions and public discussions of possible expansions. Each deals with a part of the network where a bottleneck could be relieved. The variations are constructed so that they may help suggest to traffic authorities and politicians where an investment could be beneficial. The capacity variations are summarized below:

1. **cap1 -** The capacities stated in Figure 3 (b).
2. **cap2 -** Similar to **cap1** with the exception that the capacity on the rail segment between **Vigerslev** and **Malmø** is three trains per hour (both directions).
3. **cap3 -** Similar to **cap1** with the exception that the capacity on the rail segment between **Karlshamn** and **Hässleholm** is two trains per hour (both directions).
4. **cap4 -** Similar to **cap1** with the exception that the capacity on the rail segment between **Vigerslev** and **Malmø** is four trains per hour (both directions).
5. **cap5 -** Similar to **cap1** with the exception that the capacity on the rail segment between **Karlshamn** and **Hässleholm** is three trains per hour (both directions).
6. **cap6 -** Similar to **cap1** with the exception that the capacity on the rail segment between **Karlshamn** and **Hässleholm** is two trains per hour (both directions) and the capacity on the rail segment between **Vigerslev** and **Malmø** is three trains per hour (both directions).
7. **HH -** Similar to **cap1** with the exception that a rail segment between **Helsingborg** and **Ringsted** over **Helsingør** is included with a capacity of two trains per hour (both directions).
8. **HH2 -** Similar to **cap1** with the exception that a rail segment between **Helsingborg** and **Ringsted** over **Helsingør** is included with capacity of two trains per hour (both directions) and the capacity on the rail segment between **Hässleholm** and **Helsingborg** is increased to two trains per hour (both directions).

Note that extending the capacity on an existing railway segment will not change the number of possible paths found in the enumeration; however, adding a new railway segment will increase the number of possible paths. Adding a new railway segment introduces new routing alternatives in the railway network. For example, since it includes a new railway segment, the number of paths using the **HH** capacities will be larger than when using the **cap1** capacities. Table 2 summarizes such statistics for the original five scenarios with **cap1** capacities and four scenarios with the **HH** capacities. More specifically, it lists the total number of trains, the total number of paths, and the average number of paths per train.

Scenario	$\|\mathcal{T}\|$	Total paths	Average
2010	926	71,093	76.8
2030 Baseline	2,226	247,854	111.4
2030 Baseline SK	1,794	222,158	123.8
2030 Green	2,334	248,699	106.6
2030 Green SK	1,902	223,003	117.3
2030 Baseline HH	2,226	531,096	238.6
2030 Baseline SK HH	1,794	505,400	281.7
2030 Green HH	2,334	520,257	222.9
2030 Green SK HH	1,902	494,561	260.0

Table 2: Test case statistics

From Table 2 it can be seen that the number of possible paths for the 2030 cases ranges between 220,000 and 530,000. With the exception of the 2010 scenario, we test the performance of the models on all combinations of scenarios and capacities. In other words, all capacity sets are applied to the Baseline and Green scenario of 2030. Previous discussions with the traffic authorities have highlighted the capacity of the **Øresund bridge** (the bridge connecting Denmark and Sweden and represented by the rail segment between **Vigerslev** and **Malmø**) and the suggested rail segment between **Helsingborg** and **Ringsted** over **Helsingør** as areas of interest, thus motivating us to investigate these cases. In addition, the capacity on the rail segment to **Karlshamn** is interesting due to the projected increase in transport from the East possibly through the **Kleipeda Karlshamn** sea link.

5.3 Results

Model (1)–(6) was implemented in the C++ programming language and executed on a Linux computer with an Intel Xeon 2.67GHz CPU using the commercial solver CPLEX 12.6 to solve the resulting integer programs. Table 3 shows the running times of different scenarios with two of the capacity variations (**cap1** and **cap6**). We have reported both the running time (i.e., the time required by CPLEX to solve the integer program) and the necessary setup time (i.e., the time required to enumerate all paths). Table 3 indicates that the setup time is always less than around 15 seconds, while the time used by CPLEX is always less than three minutes, except for the **2030 Green SK HH** case with capacities **cap1**. The maximum time is still, however, less than 15 minutes. This is clearly acceptable from a macroscopic planning level. It is important to remember that the considered network is relatively simple and that the set of possible routes can hence be easily calculated. Other railways may have more complex network structures, with more routes per train. In such cases, the enumeration of paths may lead to a larger model that may require more time to solve. In the worst case, enumeration may even yield a computationally intractable model. Despite the considered network's simple structure, we have shown that including the **Helsingborg-Ringsted (HH)** rail segment can result in half a million potential paths (see Table 2) and that the proposed model can easily handle instances of this size, with the described time discretization. For planning problems, hours of running time is usually not considered unacceptable and, in our opinion, Model (1)–(6) could be applied to more complex railway networks.

	cap1		**cap6**	
Scenario	Setup	Cplex	Setup	Cplex
2030 Baseline	7	12	7	35
2030 Baseline SK	7	8	7	16
2030 Green	7	10	7	38
2030 Green SK	7	8	7	16
2030 Baseline HH	15	123	-	-
2030 Baseline SK HH	14	127	-	-
2030 Green HH	15	127	-	-
2030 Green SK HH	14	859	-	-

Table 3: The time needed to setup and solve Model (1)–(6) for a selection of scenarios and capacities

Tables 4 and 5 summarize the results of Model (1)–(6) for the five scenarios and a selection of capacity variations. The results of using the capacities **HH2**, **cap4** and **cap5** are the same as the results obtained for capacities **HH**, **cap2** and **cap3** and have hence been omitted. Table 4 states the number of trains that can be routed in total, while Table 5 indicates the number of trains delayed (i.e., routed outside their preferred time windows) and the corresponding total deviation (in minutes). The results in Table 4 show that a capacity increase on the **Karlshamn Hässelholm** rail segment increases the number of scheduled trains on the network (see **cap3** and **cap6**) for the scenarios **2030 Baseline** and **2030 Green**. Furthermore, increasing the capacity by one extra train per hour on both the **Karlshamn Hässelholm** rail segment and the **Malmø Vigerslev** rail segment ensures that all trains are scheduled. For the two scenarios **2030 Baseline SK** and **2030 Green SK**, where the disputed demand between **Sundsvall** and **Karlshamn** is removed, all trains can be scheduled by simply increasing the capacity on the **Malmø Vigerslev** rail segment by one extra train per hour.

Table 4 shows that capacity variation **HH** will give the same benefits as introducing the extra capacity on

Scenario	$\lvert\mathcal{T}\rvert$	Capacity Variation **cap1**	**cap2**	**cap3**	**cap6**	**HH**
2010	926	926	-	-	-	-
2030 Baseline	2,226	1,790	2,060	1,944	2,226	2,060
2030 Baseline SK	1,794	1,512	1,794	1,512	1,794	1,794
2030 Green	2,334	1,788	2,024	2,046	2,304	2,024
2030 Green SK	1,902	1,614	1,902	1,614	1,902	1,902

Table 4: The number of routed trains under different capacity variations.

the **Malmø Vigerslev** rail segment (i.e., capacity variation **cap2**). Table 5, however, indicates that the latter incurs less total deviation from the preferred time windows. Looking at Table 5, it is also evident that a decrease in the number of trains incurring a delay does not necessarily result in a lower total deviation. The results for the **2030 Green** scenario using capacity variations **cap2** and **HH** confirm this. Note that cancelled trains are disregarded in these numbers. When compared to the number of cancelled trains (visible in Table 4) we observe that fewer cancellations does not always result in fewer trains being scheduled outside their preferred time window. In fact, the opposite is often the case. This seems reasonable; scheduling more trains may result in more total delay minutes, since more trains can be delayed. An example of this can be seen for capacity variations **cap1** and **cap2** in Table 4. The former has more cancellations than the latter, but the latter has more delayed trains than the former for both the Green and the Baseline scenario (see Table 5).

Scenario	Value	Capacity Variation **cap1**	**cap2**	**cap3**	**cap6**	**HH**
2010	Trains	242	-	-	-	-
	Minutes	71,394	-	-	-	-
2030 Baseline	Trains	1,328	1,354	1,195	1,181	1,341
	Minutes	496,558	517,717	466,345	372,346	554,011
2030 Green	Trains	1,443	1,447	1,400	1,378	1,435
	Minutes	497,791	536,583	406,387	391,269	612,756
2030 Baseline SK	Trains	1,133	1,091	1,085	1,078	1,062
	Minutes	463,209	471,017	450,256	455,694	498,150
2030 Green SK	Trains	1,208	1,222	1,220	1,213	1,213
	Minutes	520,147	546,349	498,556	524,352	574,318

Table 5: The number of trains scheduled outside preferred time windows and the total time deviation in minutes.

Table 6 provides an overview of the total number of trains traversing a given rail segment weekly in each direction in the solutions to the instances discussed in Tables 3, 4 and 5 for the **2010**, **Baseline 2030** and **Baseline 2030 SK** scenarios. Table 6 shows that in 2030 the freight to and from **Hamburg** will use the **Femern** rail segment, which is a proposed new tunnel that connects the terminals **Ringsted** and **Hamburg**. From Table 4 we can observe that increasing the capacity on the **Malmø Vigerslev** rail segment from two to three trains per hour may provide capacity for many of the cancelled trains (see **cap2**). However, when increasing this capacity to four trains per hour (**cap4**), no difference in the number of trains on the **Malmø Vigerslev** rail segment is observed. Thus, the results of **cap4**, as mentioned earlier, are not shown in the table. Increasing the capacity of both the **Malmø Vigerslev** rail segment from two to three trains per hour and the capacity of the **Karlshamn Hässelholm** rail segment from one to two trains per hour schedules all trains.

Tables 7, 8 and 9 report the results for the models presented in Section 4. These models finds the minimal number of capacity expansions under different assumptions. Table 7 shows the results from the model presented in Section 4.1. Since the costs of the different extensions are not known, all extensions have been given the same cost. The model finds the minimum number of capacity expansions, under the assumption that the increase will apply to an existing rail segment for all hours of the week (in both directions). The column "Total" reports the number

	2010	2010 SK	2030 Baseline					2030 Baseline SK				
	cap1	**cap1**	**cap1**	**cap2**	**cap3**	**cap6**	**HH**	**cap1**	**cap2**	**cap3**	**cap6**	**HH**
Malmø->Halmstad	70	70	98	170	117	170	59	97	170	97	170	59
Malmø<-Halmstad	70	70	98	170	117	171	58	97	170	97	170	58
Malmø->Helsingborg	0	0	2	2	2	2	1	2	2	2	2	1
Malmø<-Helsingborg	0	0	2	2	2	2	0	2	2	2	2	0
Malmø->Trelleborg	88	88	165	165	165	165	165	165	165	165	165	165
Malmø<-Trelleborg	88	88	165	165	165	165	165	165	165	165	165	165
Malmø->Vigerslev	150	150	336	471	336	477	336	336	477	336	477	336
Malmø<-Vigerslev	150	150	336	471	336	477	334	336	477	336	477	334
Malmø->Hasselholm	295	295	590	653	571	659	628	591	659	591	659	628
Malmø<-Hasselholm	295	295	590	653	571	658	632	591	659	591	659	632
Karlshamn->Hasselholm	71	10	147	147	230	230	147	14	14	14	14	14
Karlshamn<-Hasselholm	71	10	147	147	230	230	147	14	14	14	14	14
Halmstad->Helsingborg	2	2	5	5	5	5	118	5	5	5	5	118
Halmstad<-Helsingborg	2	2	5	5	5	5	116	5	5	5	5	116
Halmstad->Hasselholm	2	2	2	3	3	3	2	3	4	3	4	3
Halmstad<-Hasselholm	2	2	2	3	3	4	3	3	4	3	4	4
Høje Taastrup->Vigerslev	150	150	129	144	9	145	94	9	147	9	147	91
Høje Taastrup<-Vigerslev	150	150	129	143	11	144	124	11	145	11	145	125
Høje Taastrup->Ringsted	150	150	133	147	14	147	128	14	148	14	148	128
Høje Taastrup<-Ringsted	150	150	133	148	12	148	98	12	150	12	150	94
Taulov->Esbjerg	0	0	0	0	0	0	0	0	0	0	0	0
Taulov<-Esbjerg	0	0	0	0	0	0	0	0	0	0	0	0
Taulov->Ringsted	150	150	6	148	12	148	140	12	150	12	150	145
Taulov<-Ringsted	150	150	8	144	10	147	139	10	147	10	147	144
Taulov->Hamburg	147	147	8	144	5	142	139	5	142	5	142	139
Taulov<-Hamburg	147	147	6	148	7	143	140	7	145	7	145	140
Helsingborg->Hasselholm	19	19	31	31	31	31	56	31	31	31	31	62
Helsingborg<-Hasselholm	19	19	31	31	31	31	51	31	31	31	31	57
Almhult->Hasselholm	367	306	758	820	809	896	820	613	680	613	680	680
Almhult<-Hasselholm	367	306	758	820	809	896	820	613	680	613	680	680
Hamburg->Ringsted			334	327	327	332	335	327	330	327	330	335
Hamburg<-Ringsted			332	331	329	333	336	329	333	329	333	336
Vigerslev->Ringsted			207	328	325	333	212	325	332	325	332	211
Vigerslev<-Ringsted			207	327	327	332	240	327	330	327	330	243
Helsingborg->Ringsted							135					141
Helsingborg<-Ringsted							137					143

Table 6: Trains per rail segment each week for capacity variations on scenario 2010, Baseline 2030, and Baseline 2030 SK

of capacity expansions, while the column "CPLEX" reports the time taken by CPLEX to solve the problem (in seconds). Obviously, the actual cost of a capacity expansion can easily be included if known, without increasing the problem complexity. The results in Table 7 show that the rail segments **Malmø Vigerslev** and **Karlshamn Hässelholm** are the bottleneck rail segments. It is also clear from the results that all trains can be scheduled within their acceptable time window by increasing the capacity of the **Malmø Vigerslev** rail segment by one extra train per hour in each direction for the scenarios where the demand between **Sundsvall** and **Karlshamn** has been removed. CPLEX uses between 18 seconds and 65 seconds to solve the model. This is acceptable as such problems typically only need to be solved once a year when network expansions are decided. Again, we emphasize that the network we consider is quite simple, and this allows enumeration of the paths. This may not necessarily be true for more complicated networks.

Scenario	Total	CPLEX (s)	multicolumn1cRail segmets
2030 Baseline	2	65	(Malmø Vigerslev), (Karlshamn,Hasselholm)
2030 Baseline SK	1	18	(Malmø Vigerslev)
2030 Green	3	91	(Malmø Vigerslev), (Karlshamn,Hasselholm)
2030 Green SK	1	26	(Malmø Vigerslev)

Table 7: Results of the model from Section 4 with the expansion applying to every hour in both directions.

Table 8 summarizes the results for the model presented in Section 4.2. Again, we have assigned the same cost to all expansions. The main difference with this model is that the capacity expansion may only apply for a specific hour in a specific direction on a rail segment. Since there are 168 hours in one week, one extra train per hour in Table 7 corresponds to 336 extensions in Table 8. However, when considering hourly extensions, the number of expansions could be lower. In addition to the information provided in Table 7, Table 8 states the largest number of capacity expansions on any rail segment (denoted $\bar{\eta_{ij}^t}$). Note that the odd number of rail segment expansions for the case **2030 Green** shows that scheduling trains in one direction does not necessarily ensure that trains in the other direction can be scheduled in their acceptable time window. This confirms the need for the Constraints (4). The solution is found by CPLEX in less than half a minute for all cases. Even though the total number of expansions is lower than that provided by the Model in Section 4.1, the solution increases the capacity by many as six extra trains per hour at specific times. This may be achievable for hours in which no passenger trains run, but certainly not during rush hour. To prevent this, one option is to impose an upper bound on the variable indicating the maximal number of extra trains per hour per rail segment to ensure that the solution will always be feasible.

Scenario	Total	CPLEX (s)	$\bar{\eta_{ij}^t}$	Rail Segments
2030 Baseline	436	29	6	(Malmø Vigerslev), (Karlshamn,Hasselholm)
2030 Baseline SK	282	17	1	(Malmø Vigerslev)
2030 Green	593	25	5	(Malmø Vigerslev), (Karlshamn,Hasselholm)
2030 Green SK	288	18	1	(Malmø Vigerslev)

Table 8: The results of the capacity expansion model using the time-space edges.

To encourage extensions outside rush hour we have also tested the model from Section 4.2 with penalties that depend on the time of day. When doing this, we split the day into three different time periods: rush hour (6:00AM-9:00AM and 3:00PM-6:00PM), night hours (10:00PM - 6:00AM), and day hours (09:00A-3:00PM and hours 6:00PM-8:00PM); each of which has a specific penalty for a capacity expansion. The penalty weights (in the same order) are: 10, 1, and 3 per expansion. We limit the capacity per rail segment per hour to five trains. In addition to the total number of expansions broken down by period, Table 9 indicates the total cost of the expansions (column "Total").

Scenario	Extension Cost	CPLEX (s)	Night hours	Rush hour	Day hours	Total
2030 Baseline	746	27	452	14	84	540
2030 Baseline SK	590	18	296	14	84	394
2030 Green	843	24	573	14	76	663
2030 Green SK	538	15	247	14	83	344

Table 9: Results with time dependent penalties for extension

Table 9 verifies that capacity expansions are primarily assigned during night hours where the cost of the expansion is less. Furthermore, very few expansions are placed during rush hour. This means that we have been able to place the extra trains in such a way to avoid the typically busy rush hour periods, as is often desired. The total number of expansions is, not surprisingly, larger than the number of expansions in Table 8.

5.3.1 Discussion

Based on future freight transportation forecasts for the Danish and Southern Swedish region, and assuming that the departure and arrival time windows are representative of the trains, the computational tests identified network bottlenecks and showed that increasing the capacity between **Malmø** and **Vigerslev** from two to three trains per hour will make it possible to transport all of the forecast freight by train. This solution was estimated by the Danish Traffic Authority to be achievable and at a much smaller cost than many of the other possible options. However, even increasing the capacity between **Malmø** and **Vigerslev** to three trains per hour still leaves many delays in the network, and the utilization of some railway segments is close to 100% over the entire week. This could indicate that the estimate for the amount of freight to be transported by rail is much higher than expected.

An alternative to increasing the capacity of the rail segment between **Malmø** and **Vigerslev** is to build a new rail segment between **Helsingborg** and **Ringsted** which has a capacity of two trains per hour. This new connection would be much more costly and much less attractive to the Danish Traffic Authority. The results showed that the effects of the **Helsingborg Ringsted** rail segment were not significantly better than increasing capacity on the **Malmø Vigerslev** rail segment.

Comparing the results for the two scenarios **Baseline 2030** and **Green 2030**, one surprisingly sees that even though the **Green 2030** scenario has more freight transported by train, the challenges can be overcome by the same infrastructure investments required for the **Baseline 2030** scenario.

Finally, we conclude this section with a brief discussion on the advantages and disadvantages of the modelling approach. We address the long term strategic decision of capacity assessment and are primarily interested in finding schedules that are feasible on a macroscopic level. The model therefore utilizes a coarse (hourly) time discretization. This results in a tractable model which can be solved quickly, yet still provides meaningful results. It is important to emphasize that we are not seeking an operational timetable, but a reliable indication of where network bottlenecks are, and a reliable evaluation of different capacity expansion alternatives. If an operational planning tool is required, the proposed approach must be used in collaboration with a microscopic planning tool to ensure that the schedules are operationally feasible. Alternatively, one can make the time discretization of the model finer; this results in a much larger network, with potentially many more paths. This would in turn negatively impact the performance of the model as it relies on an enumerated set of train paths. We have therefore tried to produce a model which provides sound results, while remaining computationally tractable. It is worth mentioning that the proposed approach can still be used with a finer time discretization; however, in such cases we would recommend dynamically generating the train paths to avoid excessively large models.

6 Conclusion

In this paper three different models have been developed to assist with an analysis of the capacity of the Danish and Southern Swedish railway for 2030 using forecast data. The models are macroscopic and can be used for strategic, long term decisions. If operational plans are to be developed, the proposed approaches must be combined with a microscopic timetabling tool to produce a schedule which is feasible from an operational perspective. We model the capacity assessment variant as an integer multi-commodity flow problem. The first model simply determines the maximum number of freight trains that can be scheduled given an hourly discretization of the planning horizon and target time windows on when arrivals and departures should occur. The second model is an extension of the first in which it is also possible to determine the minimum number of capacity expansions which enables all trains to be scheduled. Finally, the third model includes time dependent penalties on when capacity expansion can be used and attempts to schedule the additional freight trains during some preferred time periods. Results indicate that the models can be used to find the bottlenecks of a future network under different demand scenarios.

Computational experiments show that the proposed models can be solved extremely quickly for the real-life cases considered. As such, more details could be added to the models if needed. Furthermore, it is our opinion that larger and more complex networks than the one presented should be solvable with the proposed approach. The network in the case study is fairly simple; however, the fast running times indicate that the there is the potential to study more complex networks. For the data instances considered, the solution time can increase from around three to 15 minutes when adding a rail segment to the railway network. Adding such a rail segment, increases the number of train routes (not to mention paths) in the problem. This may be a problem as these are precisely the variables of the model. Having many of these will have a detrimental impact on the performance of the algorithm.

One strength of the proposed approach is its ability to choose between a set of possible capacity expansions (including new connections) when determining how best to use the given infrastructure. Therefore, this model can be used to effectively complement the knowledge of the railway planners. This will hopefully result in investments for costly railway expansions being placed in the areas where they will be most needed. Having a tool that can reliably evaluate different possibilities reasonably quickly allows planners to evaluate many different possibilities.

The requirements given by the Traffic Authorities limit the solution, and better solutions may be achieved without these requirements. One of the limitations is the set of train routes. These route does not necessarily need to be provided by the authorities. The main issue when ignoring such input is to be able to handle all complex operational requirements to ensure that only desirable paths are generated. On the other hand, the requirements provided by the Traffic Authorities ensure that the solution found is realistic and easily implementable.

One possible future recent direction is to investigate the scalability of the model using larger networks and/or by considering a finer time discretization. Given that enumerating large sets of paths can be impractical, the application of decomposition techniques, i.e., column generation, seems like a natural next step. Enumeration is likely to generate many paths that will never appear in an optimal solution.

Acknowledgments

The authors wish to thank Mikkel Krogsgaard Niss, Eva Lindborg and Jens Brix for valuable discussions and input to the project. We would also like to thank Steven Harrod for his comments on how to improve the paper. A word of thanks also goes to the EU project *East West Transport Corridor (EWTC-II)* for their support of this project.

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