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TOWARDS CONTINGENT WORLD DESCRIPTIONS IN DESCRIPTION LOGICS

Abstract. The philosophical, logical, and terminological junctions between Description Logics (DLs) and Modal Logic (ML) are important because they can support the formal analysis of modal notions of 'possibility' and 'necessity' through the lens of DLs. This paper introduces functional contingents in order to (i) structurally and terminologically analyse 'functional possibility' and 'functional necessity' in DL world descriptions and (ii) logically and terminologically annotate DL world descriptions based on functional contingents. The most significant contributions of this research are the logical characterisation and terminological analysis of functional contingents in DL world descriptions. The ultimate goal is to investigate how modal operators can — logically and terminologically — be expressed within DL world descriptions.

Keywords: conditional information; contingent interpretation; contingent world description; description logics; functional contingents; knowledge representation; modality; three-valued semantics

1. Introduction

In the recent decades, knowledge representation in information and computer sciences has experienced significant improvements [see 14, 33, 63]. Underlying description logics (DLs) are now among the most widely used knowledge representation formalisms in semantics-based systems. DLs have emerged from semantic networks [47] and frame-based systems [40]. Most DLs are decidable fragments of predicate logic (PL). More specifically, DLs are PL-based terminological systems developed out of the attempt to represent knowledge, with a formal semantics, in order

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to establish a common ground for human and machine interactions [see 2, 3, 18, 48, 56].

This research deals with the notations of 'possibility' and 'necessity' within DL world descriptions, so the concepts of 'possibility' and 'necessity' have to be taken into account. The idea of thinking about possibilities as well as necessities within possible worlds complements the developments of logics with modalities, qualities, conditions, and their philosophical reflections on the world. My most central assumption based on possibilistic approaches is that the explicit statements are at least possible in the sense that they are logically and conceptually consistent. More specifically, the possibilities express what has had the potential of being valid and, subsequently, being meaningful. Accordingly, the necessities expose the certain available beliefs and, consequently, express what has been valid and meaningful in all possible worlds.

Modal logic (ML) is the most well-known logic of possibilities, necessities, and other conceptions related to them [see 10, 25]. ML provides a formal basis for expressing possibilities and necessities as well as for defining a semantics in terms of possible worlds [see 51]. Regarding the strong (i) logical dependencies between DLs and PL, and (ii) syntactical relationships between ML and DLs,[1] the foremost objective of this research is the logical and terminological analysis of knowledge representation based on conditional information. The overarching goal is investigating how modal notations can — terminologically and logically — be analysed within DL world descriptions. This research defines DL-based functional contingents in order to analyse contingent world descriptions. The most significant contribution will be the logical-terminological analysis of the interconnections between DL world descriptions and ML notations of 'possibility' and 'necessity'.[2] By defining a three-valued semantics, this article will reveal how to provide semantics for logical/terminological structures based on conditional information.

[1] The following sections will demonstrate that most DLs syntactically correspond to modal logic.

[2] DLs and ML are logical formalisms to capture, represent, and express the structure of variant forms of inferential and reasoning procedures.

2. Literature Review

Several logical approaches focus on checking validity, soundness, and completeness within knowledge analysis through possible and probable worlds. As mentioned, modal logic is the most well-known logic of possibilities, necessities, and other concepts related to them. Fuzzy logic approaches in the theory of possibility [16, 19, 36, 49, 50, 66], belief functions [9, 17, 38, 54, 55, 65], and possibility measures [1, 15, 20, 23, 34, 35, 59] are other salient approaches to the analysis of knowledge through possible worlds. Also, Doxastic Logic has been concerned with reasoning about beliefs. It has perhaps 'belief' as a modal operator [see 39, 53].

2.1. Probabilistic Approaches in Description Logics

Focusing specifically on DLs and terminological systems, there has been a strong interest in expressive probabilistic description logics [see 11, 30, 37]. The works just cited have also focused on the connections between expressive probabilistic description logics and the first-order logic of probability. Note that this research does not deal with the notation of probability, but only with possibility.

2.1.1. Possibility and probability.

The term 'probability' expresses the state or the fact of being probable of either happening or being true. The probability of event E can be seen to be equivalent to the quantificational measure of the likelihood that E will occur. Therefore, the probability of fact F is equivalent to the quantificational measure of the likelihood that F will be true and acceptable.

In contrast, the term 'possibility' is concerned with the state of being possible. Being possible has two characteristics: (i) having the potential of happening or being done and (ii) having the potential of being satisfactory and acceptable. More precisely, the possibility of event E is concerned with E's qualificational state of having the potential of happening and being done. Also, the possibility of fact F corresponds with F's qualificational state of being true (as well as satisfactory and acceptable).

2.2. Possibilistic Description Logics

There has been significant research in possibilistic approaches. [43] developed an epistemic operator for description logics. [28] focused on proof methods in possibilistic logic and on possibilistic logic applications to terminological logics, [32] modelled imprecise arguments in DLs, [21] dealt with uncertainty, possibility, and fuzzy features in DLs, [57] and [58] handled fuzziness[3] in DLs within the Semantic Web context, and [12] offered a reasoning framework based on fuzzy DL $\mathcal{SROIQ}$ [see 31].[4] [44] sees DLs through the lens of possibilistic logic and focuses on developing a possibilistic extension to DLs. It has associated a DL-based formula with a number in $[0, 1]$. Furthermore, [44] offered an adequate syntax and semantics for a possibilistic extension of DLs. In addition, [45, 46] have extended DLs with uncertainty reasoning in possibilistic logic. Furthermore, [26] and [27] offered reasoning frameworks for ontologies based on inconsistent possibilistic description logics. [41] offered a possibilistic DL extension for an uncertain geographic ontology, and [7] created a possibilistic extension of the Web Ontology Language based on [41]. Moreover, [8] designed a possibilistic description logic for uncertain geographic information.

Note that there are numerous works on DLs with rules and other normative notions (e.g., RuleML & Fuzzy RuleML [62] and SWRL [64]), as well on defeasible and non-monotonic DLs. The most common feature of possibilistic approaches in knowledge representation systems is representing the degree(s) of compatibility of an interpretation with available beliefs, where the beliefs are produced based on incomplete knowledge.

3. Description Logics

Description Logics (DLs) are the most well-known (terminological) knowledge representation formalisms in semantics-based systems [see 2, 3, 4, 13, 52, 56]. DLs represent knowledge in terms of (i) *individuals*

[3] Mentioning fuzzy DLs does not mean mixing up possibilistic and fuzzy formalisms, but taking into account that possibilistic logics fall under uncertainty theory. More specifically, the statements in a possibilistic logic are either true or false (to some possibility), whereas in fuzzy logics statements are true (to a certain degree). In this context, fuzzy DLs can show how we can formally represent the floating degrees of truth (between 0 (stands for absolute falsity) and 1 (stands for absolute truth)) within terminological systems. Undoubtedly, fuzzy-based approaches have, both formally and mathematically, supported the analysis of possibility and necessity measures [e.g. 22, 29].

[4] The $\mathcal{SROIQ}$ is the underlying description logic of the Web Ontology Language (OWL).

(that are equivalent to constant symbols in predicate logic), (ii) *concepts* (that are equivalent to unary predicates in predicate logic), and (iii) *roles* (that are equivalent to n-ary predicates in predicate logic and can be either relations or properties). More specifically, a role expresses a relationship between individuals or it assigns a property to an individual. A role is a relation defined with some valence greater than or equal to 0.

In description logics, a concept corresponds to a distinct (conceptual) entity. Also, it can be regarded as a class of other entities (e.g., objects, subjects). It shall be taken into consideration that concepts and their interrelationships are, in the form of hierarchical structures, used to create a terminology. Subsequently, the individuals are regarded as instances of concepts. For example, the individual `john` can be an instance of the concept *Student*. The predicates (either unary or n-ary) are the most important building blocks in predicate logic. The most significant fact about predicate logic (which is, terminologically, the parent of DLs) is that the unary predicate P in a formula is capable of covering something (e.g., the variable x[5]) and, in fact, P can describe x. Thus, we can have the logical term '$P(x)$'. Subsequently, the world description '$P(x)$' expresses that the variable x (that can be any possible individual) is an instance of predicate P; thus x comes under the label of P. Therefore, P can describe x. After the transformation of a predicate-based formula (in predicate logic) into a concept-based formula (in description logic), the predicate P manifests itself in the form of a (possibly specified) concept (like C).

In DLs, there are three kinds of atomic symbols: (i) individuals, e.g., `bob`, `pizza`, (ii) atomic concepts, e.g., *Person*, *Bird*, and *Food*, and (iii) atomic roles, e.g., *hasMother*, *isEating*, and *isMoving*. Atomic symbols are elementary descriptions from which we inductively build complex (more-specified) descriptions based on concept (and role) constructors. More specifically, the individual `bob` is related to itself by means of the relation of valence 0. The term 'Fred is a student' (formally: *Student*(`fred`)) is structured based on the relation of valence 1. Also, the terms 'Sebastian is married to Juliana' (formally: *marriedTo*(`sebastian`, `juliana`)), '10 is greater than 3' (formally: *greaterThan*(10, 3)), and 'Bob is the father of Alice' (formally: *hasFather*(`alice`, `bob`) are structured based on the relations of valence 2. Obviously, there are relations of greater valences as well.

[5] The variable x is a relation of valence 0. It can express any possible individual.

3.1. The Syntactic Relationship between DLs and ML

The set of the main connectors in $\mathcal{ALC}$ (Prototypical DL Attributive Language with Complements) is: {conjunction ($\sqcap$), disjunction ($\sqcup$), negation[6]($\neg$), existential restriction ($\exists$), universal restriction ($\forall$)}. In addition, we have tautology ($\top$), contradiction ($\bot$), and, as mentioned, atomic concepts and atomic roles.

As mentioned earlier, most DLs are syntactically modal logics. More specifically, $\mathcal{ALC}$ is developed using modal logic **K**[7] as its foundation. In other words, $\mathcal{ALC}$ is a syntactic variant of modal logic **K**. More precisely, I assume that the DL symbols '$\neg$' (for 'not') and '$\Rightarrow$' (for 'if ..., then ...') are the translations of the ML symbols '$\sim$' (for 'not'), '$\rightarrow$' (for 'if ..., then ...'). The DL symbols '$\sqcap$' (for 'and'), '$\sqcup$' (for 'or'), and '$\Leftrightarrow$' (for 'if and only if ..., then ...') are definable from DL '$\neg$' and '$\Rightarrow$' the same way as in propositional logic and, in fact, as in the modal logic **K**. More specifically, modal logic **K** defines the logical symbols '&' (for 'and'), '$\vee$' (for 'or'), and '$\leftrightarrow$' (for 'if ..., then ...') from '$\sim$' (for 'not') and '$\rightarrow$' (for 'if ..., then ...').

Such a syntactic similarity can be found between ML and other members of the family of DLs as well. For example, the description logic $\mathcal{SR}$ (that denotes $\mathcal{ALC}$ extended with all kinds of rule-based axioms and self-constructs) is a syntactic variant of Propositional Dynamic Logic [61]. Dynamic logics are modal logics for representing the states and the events of dynamic systems. The language of dynamic logics is both an assertion language able to express properties of computation states, and a programming language able to express properties of system transitions between these states. They are logics of programs, and permit us to describe and reason about states of affairs, processes, changes, and results [see 60]. Also, there are some DLs that are syntactic variants of the Deterministic Propositional Dynamic Logic [see 5, 6].

3.2. Knowledge Bases in DLs

A DL knowledge base usually consists of the terminological axioms (that describe the underlying terminologies and vocabularies), and assertional

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[6] In some DLs, such as in $\mathcal{AL}$, negation is permitted on atomic concepts only, while in more expressive DLs it is permitted on complex concepts as well.

[7] **K** was named after Saul Aaron Kripke, who is an American logician and philosopher. Kripke is well-known for his valuable works on the semantics of modal logic.

axioms (that describe the world). The *concept inclusion* and *role inclusion* axioms (in the form of $C \sqsubseteq D$ and $R \sqsubseteq S$, where C and D stand for two concepts, and R and S stand for two roles) are the most fundamental terminologies. In addition, (ii) the *concept equality* and *role equality* axioms (in the form of $C \equiv D$ and $R \equiv S$) are other terminological axioms and are generally defined from concept inclusions and role inclusions. Furthermore, (iii) the *concept assertions* and *role assertions* (in the form of $C(a)$ and $R(a_1, a_2, \ldots, a_n)$, where C stands for an atomic concept, R stands for an atomic role, and a_i (for $i \in [1, n]$) stands for an individual symbol) are the most fundamental descriptions of the world. Note that any specified description of the world is expressible based on fundamental descriptions of that world [see 5, 6, 42].

3.3. Terminological Interpretations in DLs

Note that the formal semantics of a term in DLs is interpretable based on individuals, concepts, and roles. In fact, they are the non-logical symbols in logical descriptions; hence, they do not independently have any logical consequence in a world description. Therefore, we need to utilise terminological interpretation in order to become involved with providing a semantics. A terminological interpretation (like $\mathcal{I}$) consists of (i) a non-empty set Δ (that is the interpretation domain and consists of any variable that occurs in any of the concept descriptions), and (ii) an interpretation function (like '${}^{\mathcal{I}}$'). The function '${}^{\mathcal{I}}$' assigns every individual symbol to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. Also, it assigns to every atomic concept A (or every atomic unary predicate) a set $A^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, and to every atomic role P (or every atomic binary predicate) a binary relation $P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. Table 1 represents the syntax and semantics of concept constructors in $\mathcal{ALC}$ [see 5, 6, 42].

Table 2 reports the terminological and assertional axioms in DLs [see 5, 6, 42]. Note that an interpretation is called a *model* of an axiom if it can satisfy the terminological axioms and fundamental world descriptions.

4. Functional Roles in Description Logics

In Description Logics, the *functional roles* (or *features*) are special kinds of roles (relations). Thus, $N_F \subseteq N_R$, where N_F and N_R stand for the 'set of functional roles' and 'set of roles', respectively. Functional roles can

Syntax	Semantics
A	$A^I \subseteq \Delta^I$
r	$r^I \subseteq \Delta^I \times \Delta^I$
$\top$	Δ^I
$\bot$	$\emptyset$
$C \sqcap D$	$(C \sqcap D)^I = C^I \cap D^I$
$C \sqcup D$	$(C \sqcup D)^I = C^I \cup D^I$
$\neg C$	$(\neg C)^I = \Delta^I \setminus C^I$
$\exists r.C$	$\{a \mid \exists b.(a,b) \in r^I \wedge b \in C^I\}$
$\forall r.C$	$\{a \mid \forall b.(a,b) \in r^I \supset b \in C^I\}$

Table 1. $\mathcal{ALC}$ syntax and semantics

Name	Syntax	Semantics
concept inclusion axiom	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
role inclusion axiom	$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
concept equality axiom	$C \equiv D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$
role equality axiom	$R \equiv S$	$R^{\mathcal{I}} = S^{\mathcal{I}}$
concept assertion	$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
role assertion	$R(a,b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$

Table 2. Terminological axioms and world descriptions in DLs

be seen as roles that are structurally (and existentially) functions and, hence, they can express functional actions, movements, and procedures. According to the functional behaviour of functional roles, a functional role associates a single value (that can be regarded as the member of a singleton[8]) to its input parameter (that is also the only member of a singleton). According to $F_R(a,b)$, we can interpret that $F_R\colon \{a\} \to \{b\}$ and, in fact, $F_R(a) \equiv b$.

The roles *motherOf*, *fatherOf*, *scoreOf*, and *lastNameOf* are functional roles, whereas *parentOf* and *childOf* are not. The term 'the father of Alice' can produce the functional role *fatherOf* (for the individual `alice`). As demonstrated above, a functional role can be applied to an individual symbol in order to make a functional relation between it and another individual symbol. Consequently, the word 'if' can be regarded as the most significant functional word in natural languages.

[8] A singleton is a set which contains exactly one element.

According to the statement 'Bob is the father of Alice', 'father of' is a functional role that relates Bob to Alice. In fact, the functional role *fatherOf* maps `bob` onto `alice`. Formally, $\mathit{fatherOf}(\texttt{alice}) \approx \texttt{bob}$. The functional role *fatherOf* is produced based on the role *hasFather*. In fact, *fatherOf* is terminologically supported by *hasFather*. Also, $\mathit{fatherOf}^-$ expresses the inverse role of *fatherOf*. Formally speaking:

$$\mathit{hasFather} \approx \mathit{fatherOf}^-$$

Proposition. Relying on the relationship *hasFather* (between the individuals `alice` and `bob`) that supports the describability of them by the concepts *Daughter* and *Father*, the functional role *fatherOf* is modelled. Subsequently, *fatherOf* maps $\mathit{Father}(\texttt{bob})$ onto $\mathit{Daughter}(\texttt{alice})$.

5. Functional Contingents in Description Logics

Assume that $A(a)$, $B(b)$, and $R(a, b)$ are three world descriptions in the knowledge base $\mathcal{K}$, where the individuals a and b are known as the instances of the concepts A and B. Also, the role R represents a binary relation between a and b. Formally speaking: $\mathcal{K} \models \{A(a),\ B(b),\ R(a, b)\}$. In fact, the fundamental world descriptions $A(a)$, $B(b)$, and $R(a, b)$ are semantically satisfied by our terminological knowledge $\mathcal{K}$. As proposed above, we can interpret that the world description $R(a, b)$ terminologically supports the production of the functional role F. F maps the concept assertion $A(a)$ onto the concept assertion $B(b)$. Therefore, F is satisfiable by $\mathcal{K}$. Note that the central focus of world description analysis will be on this functional role.

5.1. Handling the Possibility and Necessity of World Descriptions

The introduction of the *Possibility Functional Contingent* $\mathcal{P}$ and the *Necessity Functional Contingent* $\mathcal{N}$ are now due, these being structurally functional roles. They represent the possibility and necessity of a world description. The central assumption is that any functional contingent can cover its inner concept/role assertions. Actually the term 'contingent' is a property and contingency is a relationship that supports a functional role, so the following rules apply:

1. $\mathcal{P}(A(a))$ expresses that 'it is possible that $A(a)$'.
2. $\neg\mathcal{P}(A(a))$ expresses that 'it is impossible that $A(a)$'.

3. $\mathcal{N}(A(a))$ expresses that 'it is necessary that $A(a)$'.

4. $\neg\mathcal{N}(A(a))$ expresses that 'it is unnecessary that $A(a)$'.

5. $\mathcal{P}(R(A(a), B(b)))$ expresses that 'it is possible that $R(A(a), B(b))$'. *Specific Analysis:* $\mathcal{P}(R(A(a), B(b)))$ expresses the existence of, at least, one functional role (like F_i) of the concept assertion $A(a)$. Formally, the *possibility functional contingent* is — based on role assertion $R(A(a), B(b))$ — interpretable as follows:

$$\exists i[(\mathcal{K} \models \{A(a), B(b), R_i(a, b)\}) \Rightarrow F_i(A(a), B(b))].$$

This means that there exists — at least — one role assertion that is satisfied by $\mathcal{K}$ and can support the construction of a functional role of $A(a)$. Consequently, that functional role maps $A(a)$ onto $B(b)$.[9]

6. $\neg\mathcal{P}(R(A(a), B(b)))$ expresses that it is impossible that $R(A(a), B(b))$. *Specific Analysis:* according to $\neg\mathcal{P}(R(A(a), B(b)))$, there is no functional role of the concept assertion $A(a)$ that can be represented in the form of $F_i(A(a), B(b))$ and can map $A(a)$ onto $B(b)$.

7. $\mathcal{N}(R(A(a), B(b)))$ expresses that 'it is necessary that $R(A(a), B(b))$'. *Specific Analysis:* $\mathcal{N}(R(A(a), B(b)))$ expresses the existence of all possible functional roles of the concept assertion $A(a)$ in the form of $F_i(A(a), B(b)))$. Formally, the *necessity functional contingent* is, based on role assertion $R(A(a), B(b))$, interpretable as follows:

$$\forall i[(\mathcal{K} \models \{A(a), B(b), R_i(a, b)\}) \Rightarrow F_i(A(a), B(b))].$$

This means that for all possible role assertions that are satisfied by $\mathcal{K}$, there are functional roles that can map $A(a)$ onto $B(b)$.

8. $\neg\mathcal{N}(R(A(a), B(b)))$ expresses that $R(A(a), B(b))$ does not necessarily hold. *Specific Analysis:* $\neg\mathcal{N}(R(A(a), B(b)))$ expresses that the existence of the functional role '$F_i(A(a), B(b))$' that can map $A(a)$ onto $B(b)$ is not necessary.

5.2. Definability Analysis

The world description *hasFather*(`alice`, `bob`) expresses the facts that Alice has a father and Bob is Alice's father. According to the contingent world description $\mathcal{P}($*hasFather*(`alice`, `bob`)$)$, we can conclude that it is possible that Alice has a father, and it is possible that Bob is Alice's father. Actually the focus of possibility has been on (i) having a father (by

[9] The functional role $F_i(A(a), B(b))$ expresses that F_i maps $A(a)$ onto $B(b)$, formally: $F_i(A(a)) \equiv B(b)$.

Alice), (ii) being a father (by Bob) for Alice, and (iii) the interrelationships between 'having a father (by Alice)' and 'being a father (by Bob)'. We need to interpret $\mathcal{P}$ as a *functional role* of the functional role '*fatherOf*' in order to, terminologically, analyse $\mathcal{P}(\textit{hasFather}(\texttt{alice}, \texttt{bob}))$. In fact, $\mathcal{P}$ is definable at the states at which the roles 'having a father (by Alice)' and 'being a father (by Bob)' have been defined and are meaningful. Note the following consequences:

1. According to $\mathcal{P}(\textit{hasFather}(..., ...))$, the functional contingent $\mathcal{P}$ can logically be valid and meaningful at the states at which 'having a father' is defined and meaningful. Therefore, regarding $\mathcal{P}(\textit{hasFather}(\texttt{alice}, \texttt{bob}))$, the definability of the relation 'having a father' between the individual `alice` and the individual `bob` is the logical premise of the definability of the functional contingent $\mathcal{P}$.

2. According to $\mathcal{P}(\textit{hasFather}(\ldots, \ldots)) \Rightarrow \mathcal{P}(\textit{hasFather}(\texttt{alice}, \ldots))$, it is possible that someone has a father and, therefore, it is possible that Alice has a father. Informally, the possibility of the proposition 'Alice has a father' is valid and meaningful if and only if the possibility of 'having a father' is valid and meaningful.

3. According to $\mathcal{P}(\textit{hasFather}(\ldots, \ldots)) \Rightarrow \mathcal{P}(\textit{hasFather}(\ldots, \texttt{bob}))$, it is possible that someone has a father and, therefore, it is possible that Bob is the father of that person. Informally, the possibility of the proposition 'Bob is the father of someone who has a father' is valid and meaningful if and only if the possibility of 'having a father' is valid and meaningful.

4. The possibility of the proposition 'Alice has a father, and Bob is Alice's father' is valid and meaningful if and only if (i) the possibility of having a father (by Alice) and (ii) the possibility of being Alice's father (by Bob) are valid and meaningful. Then, the possibility of the proposition 'Bob is the father of Alice' is valid and meaningful if and only if (i) the possibility of having a father (by Alice), (ii) the possibility of being a father (by Bob), and (iii) the possibility of having Bob as father (by Alice) are valid and meaningful.

6. Semantic Analysis

Let $\mathcal{T} = \{\textit{Daughter} \sqsubseteq \textit{Person}, \textit{Father} \sqsubseteq \textit{Person}\}$ be the set of terminological axioms in knowledge base $\mathcal{K}$. Also, $\mathcal{W} = \{\textit{Daughter}(\texttt{alice}), \textit{Father}(\texttt{bob}), \textit{hasFather}(\texttt{alice}, \texttt{bob})\}$ is the set of fundamental world descriptions (i.e., assertional axioms) in knowledge base $\mathcal{K}$. Therefore, $\mathcal{K} = (\mathcal{T}, \mathcal{W})$ and, thus, $\mathcal{T}$ and $\mathcal{W}$ can semantically be satisfied by $\mathcal{K}$. Formally, $\mathcal{K} \models \{\mathcal{T}, \mathcal{W}\}$.

A *Contingent Assertional Axiom* (fundamental contingent world description) is representable in the form of (*world description*, *world description's value*). The values can be either 'F' (stands for Falsity) or 'T' (stands for Truth) or 'V' (stands for Vagueness). Note that the interval $[0, 1]$ has been suggested in the possibilistic logic semantics, e.g., [24, 44]. It is worth mentioning that every possibilistic logic is a weighted logic, where each classical logic formula is associated with a number in the interval $[0, 1]$. Accordingly, the semantics of possibilistic DL has usually been defined by a possibility distribution (like π) over the set I of all classical description logic interpretations, formally: $\pi\colon I \to [0, 1]$. Thus, $\pi(I)$ represents the degree of compatibility of I with available information. However, my offered semantics is (a) three-valued (based on Truth, Falsity, and Vagueness) and (b) only offered for contingent world descriptions in DLs.

In order to be more specific on this three-valued logic, we need to define *Contingent Interpretations* $\mathcal{I}_c$. As you will see, any contingent interpretation can provide a semantic basis for satisfying the functional contingents $\mathcal{P}$, $\mathcal{N}$ and, subsequently, for satisfying the fundamental contingent world descriptions. The contingent interpretation '$\mathcal{I}_c$' is provided in order to satisfy the concepts of:

- functional necessity (in the form of $\mathcal{I}_n$),
- functional possibility (in the form of $\mathcal{I}_p$), and
- functional impossibility (in the form of $\mathcal{I}_{imp}$).

In other words, the contingent interpretation $\mathcal{I}_c$ is utilised to provide semantic models of functional contingents within DL world descriptions. More specifically, if $\mathcal{I}_c$ can satisfy any of the members of $\mathcal{W}$ and, respectively, can satisfy the relevant contingent functions of that world description, then it will provide a *Contingent Model*.

6.1. Assertional Axioms for Functional Contingents

1. $\mathcal{P}(A(a))$ is a possible concept assertion. Semantically,
   (a) based on the possibility interpretation '$\mathcal{I}_p$', $|\mathcal{P}(A(a))|^{\mathcal{I}_p} = T$.
   (b) based on the impossibility interpretation '$\mathcal{I}_{imp}$', $|\mathcal{P}(A(a))|^{\mathcal{I}_{imp}} = F$.

   (c) based on the necessity interpretation '$\mathcal{I}_n$', $|\mathcal{P}(A(a))|^{\mathcal{I}_n} = V$. Actually, we only know that $A(a)$ is possible. So, there is no evidence that $A(a)$ is necessary.
2. $\mathcal{N}(A(a))$ is a necessary concept assertion. Then, semantically,
   (a) $|\mathcal{N}(A(a))|^{\mathcal{I}_n} = T$. Informally, $A(a)$ is necessary. Therefore, $A(a)$ is necessarily necessary.
   (b) $|\mathcal{N}(A(a))|^{\mathcal{I}_p} = T$. Informally, $A(a)$ is necessary. Therefore, $A(a)$ is necessarily possible.
   (c) $|\mathcal{N}(A(a))|^{\mathcal{I}_{imp}} = F$. Informally, $A(a)$ is necessary. Therefore, $A(a)$ is necessarily not impossible.
3. $\mathcal{P}(R(a,b))$ is a possible role assertion. Semantically,
   (a) $|\mathcal{P}(R(a,b)|^{\mathcal{I}_p} = T$.
   (b) $|\mathcal{P}(R(a,b)|^{\mathcal{I}_{imp}} = F$.
   (c) $|\mathcal{P}(R(a,b)|^{\mathcal{I}_n} = V$. Here, we only know that a and b are possibly related to each other (by means of R). In fact, there is no evidence that $R(a,b)$ is necessary.
4. $\mathcal{N}(R(c,d))$ is a necessary role assertion. Semantically,
   (a) $|\mathcal{N}(R(a,b))|^{\mathcal{I}_n} = T$.
   (b) $|\mathcal{N}(R(a,b))|^{\mathcal{I}_p} = T$.
   (c) $|\mathcal{N}(R(a,b))|^{\mathcal{I}_{imp}} = F$. Here, we know that a and b are necessarily related to each other (by means of R). Therefore, $R(a,b)$ is necessarily not impossible.

PROPOSITION. According to the aforementioned items, the functional contingents $\mathcal{P}$ and $\mathcal{N}$ are mappings from their central world descriptions into the values $\{T, F, V\}$.

Considering $\mathcal{K} = (\mathcal{T}, \mathcal{W})$, where $\mathcal{T} = \{\mathit{Daughter} \sqsubseteq \mathit{Person}, \mathit{Father} \sqsubseteq \mathit{Person}\}$ and $\mathcal{W} = \{\mathit{Daughter}(\texttt{alice}), \mathit{Father}(\texttt{bob}), \mathit{hasFather}(\texttt{alice}, \texttt{bob})\}$, I shall draw your attention to the following examples:

Example 1. According to the contingent role assertion '$\mathcal{P}(\mathit{hasFather}(\texttt{alice}, \texttt{bob}))$', 'it is possible that Alice has a father, and it is possible that Bob is Alice's father'. In this example, $(\mathcal{P}(\mathit{hasFather}(\texttt{alice}, \texttt{bob}))^{\mathcal{I}_p} = T$, $(\mathcal{P}(\mathit{hasFather}(\texttt{alice}, \texttt{bob}))^{\mathcal{I}_{imp}} = F$, and $(\mathcal{P}(\mathit{hasFather}(\texttt{alice}, \texttt{bob}))^{\mathcal{I}_n} = T$. Therefore, $\mathcal{I}_p$ is the possibility model of $\mathit{hasFather}(\texttt{alice}, \texttt{bob})$ and $\mathcal{I}_n$ is the necessity model of $\mathcal{P}(\mathit{hasFather}(\texttt{alice}, \texttt{bob}))$.

Example 2. According to the contingent concept assertion '$\mathcal{N}(\mathit{Father}(\texttt{bob}))$', it is necessary that Bob is a father. In this example, $(\mathit{Father}(\texttt{bob}))^{\mathcal{I}_n} = T$, $(\mathit{Father}(\texttt{bob}))^{\mathcal{I}_p} = T$, and $(\mathit{Father}(\texttt{bob}))^{\mathcal{I}_{imp}} = F$. Therefore, $\mathcal{I}_n$ is the necessity model of $\mathcal{N}(\mathit{Father}(\texttt{bob}))$. Obviously,

$\mathcal{N}(Father(\texttt{bob})) \Rightarrow \mathcal{P}(Father(\texttt{bob}))$, then $\mathcal{I}_n$ is the necessity model of the possibility world description '$\mathcal{P}(Father(\texttt{bob}))$' as well.

Example 3. According to the contingent concept assertion '$\mathcal{P}(Daughter(\texttt{maria}))$', it is possible that Maria is a daughter. In this example, $(Daughter(\texttt{maria}))^{\mathcal{I}_p} = T$, $(Daughter(\texttt{maria}))^{\mathcal{I}_{imp}} = V$, and $(Daughter(\texttt{maria}))^{\mathcal{I}_{imp}} = V$. Therefore, $\mathcal{I}_p$ is the possibility model of $\mathcal{P}(Daughter(\texttt{maria}))$.

Example 4. According to the contingent role assertion '$\mathcal{P}(hasFather(\texttt{alice}, \texttt{john}))$', it is possible that Alice has a father and it is possible that John is Alice's father. In this example, $(\mathcal{P}(hasFather(\texttt{alice}, \texttt{john})))^{\mathcal{I}_p} = T$, $(\mathcal{P}(hasFather(\texttt{alice}, \texttt{john})))^{\mathcal{I}_n} = V$, and $(\mathcal{P}(hasFather(\texttt{alice}, \texttt{john})))^{\mathcal{I}_{imp}} = V$.

6.2. Axiomatisation

Axiom 1. $\mathcal{N}(A(a)) \Rightarrow A(a)$.
This axiom expresses that necessary concept assertions are definitely valid. It is axiomatised based on the concept of 'reflexivity'.

Specific Analysis: We are certain that the individual a is necessarily an instance of the concept A. Therefore, $A(a)$ is necessarily valid. In fact, $A(a)$ is valid.

Axiom 2. $\mathcal{N}(R(a, b)) \Rightarrow R(a, b)$.
This axiom expresses that necessary role assertions are definitely valid. It is axiomatised based on the concept of 'reflexivity'.

Specific Analysis: We are certain that the individuals a and b are necessarily connectable to each other (by means of R). Therefore, $R(a, b)$ is necessarily valid. In fact, $R(a, b)$ is valid.

Axiom 3. $\mathcal{N}(A(a)) \Rightarrow \mathcal{P}(A(a))$.
This axiom expresses that necessary concept assertions are possible. It is axiomatised based on the concept of 'seriality'.

Specific Analysis: We are certain that 'it is necessary that $A(a)$' is valid. Then, the individual a is necessarily describable under the label of the concept A. Therefore, it is definitely possible to describe a as an instance of the concept A. Then, it is possible to describe a by A. So, $A(a)$ is possible.

Axiom 4. $\mathcal{N}(R(a, b)) \Rightarrow \mathcal{P}(R(a, b))$.
This axiom expresses that necessary role assertions are possible. It is axiomatised based on the concept of 'seriality'.

Specific Analysis: We are certain that 'it is necessary that $R(a,b)$' is valid. Then, the individuals a and b are necessarily relatable to each other (by means of R). Therefore, it is definitely possible to relate a and b by means of R. Then, $R(a,b)$ is possible.

Axiom 5. $A(a) \Rightarrow \mathcal{N}(\mathcal{P}(A(a)))$.
This axiom expresses that any valid concept assertion is necessarily possible. It is axiomatised based on the concept of 'symmetry'.

Specific Analysis: We know (and are certain) that $A(a)$ is valid. Therefore, the concept A can describe the individual a. Thus, it is definitely possible to describe a by A. Then, a is necessarily possibly described by A. Hence, $A(a)$ is necessarily possible.

Axiom 6. $R(a,b) \Rightarrow \mathcal{N}(\mathcal{P}(R(a,b)))$.
This axiom expresses that any valid role assertion is necessarily possible. It is axiomatised based on the concept of 'symmetry'.

Specific Analysis: We know (and are certain) that $R(a,b)$ is valid. Therefore, it is definitely possible to relate a and b by means of R. Then, a and b are necessarily possibly related by R. Hence, $R(a,b)$ is necessarily possible.

Axiom 7. $\mathcal{N}(A(a)) \Rightarrow \mathcal{N}(\mathcal{N}(A(a)))$.
This axiom represents the iteration (and transitivity) of necessary concept assertions. It expresses that necessary concept assertions are necessary.

Specific Analysis: We are certain that the individual a is necessarily an instance of the concept A. So, a can necessarily be described by A. Thus, 'a can necessarily be described by A' is definitely valid. Therefore, it is necessary that a can necessarily be described by A.

Axiom 8. $\mathcal{N}(R(a,b)) \Rightarrow \mathcal{N}(\mathcal{N}(R(a,b)))$.
This axiom represents the iteration (and transitivity) of necessary role assertions. It expresses that necessary role assertions are necessary.

Specific Analysis: We are certain that the individuals a and b are necessarily related to each other by means of R. So, a and b can necessarily be related to each other (based on R). In fact, 'a and b can necessarily be related by means of R' is definitely valid. Therefore, it is necessary that a and b can necessarily be related to each other (by means of R).

Axiom 9. $\mathcal{N}(\mathcal{N}(A(a))) \Rightarrow \mathcal{N}(A(a))$.
This axiom expresses that the necessity of a necessary concept assertion implies the necessity of that concept assertion. It is axiomatised based on the concept of 'density'.

Specific Analysis: This axiom is analysable based on axiom 1. We are certain that it is necessary that the individual a can necessarily be described by the concept A. Therefore, a can necessarily be described by A. Hence, it is necessary that $A(a)$.

Axiom 10. $\mathcal{N}(\mathcal{N}(R(a,b))) \Rightarrow \mathcal{N}(R(a,b))$.
This axiom expresses that the necessity of a necessary role assertion implies the necessity of that role assertion. It is axiomatised based on the concept of 'density'.

Specific Analysis: This axiom is analysable based on axiom 2. We are certain that it is necessary that the individuals a and b can necessarily be related to each other (by means of R). Therefore, a and b can necessarily be related to each other (by means of R). Hence, it is necessary that $R(a,b)$.

Axiom 11. $\mathcal{P}(A(a)) \Rightarrow \mathcal{N}(\mathcal{P}(A(a)))$.
This axiom expresses that possible concept assertions are necessarily possible.

Specific Analysis: We know that the individual a might be an instance of the concept A. Thus, it is possible to represent a under the label of A. Hence, it is definitely possible to represent a under the label of A. So, it is necessarily possible to describe a by A. In fact, $A(a)$ is necessarily possible.

Axiom 12. $\mathcal{P}(R(a,b)) \Rightarrow \mathcal{N}(\mathcal{P}(R(a,b)))$.
This axiom expresses that possible role assertions are necessarily possible.

Specific Analysis: We know that the individuals a and b might be connected to each other by means of R. Thus, it is possible to represent $R(a,b)$. Hence, it is definitely possible to relate a and b by means of R. So, it is necessarily possible to relate a and b by means of R. In fact, $R(a,b)$ is necessarily possible.

Axiom 13. $\mathcal{N}[\mathcal{N}(A(a)) \Rightarrow A(a)]$.
This axiom expresses that the necessity of a concept assertion necessarily implies the validity of that concept assertion. It is axiomatised based on the concept of 'shift-reflexivity'.

Specific Analysis: This axiom is analysable based on axiom 1. We know that the individual a is necessarily describable by the concept A. Therefore, 'a is necessarily describable by A' necessarily implies that 'a can be described by A'. This means that it is necessary that $A(a)$ necessarily implies $A(a)$. Hence, $A(a)$ is valid.

Axiom 14. $\mathcal{N}[\mathcal{N}(R(a,b)) \Rightarrow R(a,b)]$.
This axiom expresses that the necessity of a role assertion necessarily implies the validity of that role assertion. It is axiomatised based on the concept of 'shift-reflexivity'.

Specific Analysis: This axiom is analysable based on axiom 2. We know that the individuals a and b are necessarily connectable (by means of R). Therefore, 'a and b are necessarily connectable (by means of R)' necessarily implies that 'a and b are connectable (by means of R)'. This means that it is necessary that $R(a,b)$ necessarily implies $R(a,b)$. Hence, $R(a,b)$ is valid.

Axiom 15. $\mathcal{P}(\mathcal{N}(A(a))) \Rightarrow \mathcal{N}(\mathcal{P}(A(a)))$.
This axiom expresses that the possibility of the necessity of a concept assertion implies the necessity of its possibility. It is axiomatised based on the concept of 'convergency'.

Specific Analysis: Suppose that it is possible that $A(a)$ is necessary. Then, it is not necessary that $A(a)$ is necessary. Therefore, $A(a)$ is necessarily possible (and not necessary). In fact, $\mathcal{N}(\mathcal{P}(A(a)))$. Note that this axiom has a strong correlation with axiom 3.

Axiom 16. $\mathcal{P}(\mathcal{N}(R(a,b))) \Rightarrow \mathcal{N}(\mathcal{P}(R(a,b)))$.
This axiom expresses that the possibility of the necessity of a role assertion implies the necessity of its possibility. It is axiomatised based on the concept of 'convergency'.

Specific Analysis: Suppose that it is possible that $R(a,b)$ is necessary. Then, $R(a,b)$ is not necessarily necessary. Therefore, $R(a,b)$ is necessarily possible (and not necessary). In fact, $\mathcal{N}(\mathcal{P}(R(a,b)))$. Note that this axiom has a strong correlation with axiom 4.

Axiom 17. $[\mathcal{N}(A(a) \Rightarrow B(b))] \Rightarrow [\mathcal{N}(A(a)) \Rightarrow \mathcal{N}(B(b))]$.
This axiom represents the distribution of necessity over concept assertions.

Specific Analysis: We know that $A(a)$ and $B(b)$ are valid. We also know that $A(a)$ necessarily implies $B(b)$. Therefore, the necessity of $B(b)$ is deduced from the necessity of $A(a)$. For example, we know that *Father*(`bob`) and *Daughter*(`alice`). We also know that *Father*(`bob`) necessarily implies *Daughter*(`alice`). Therefore, the necessity of *Daughter* (`alice`) is deduced from the necessity of *Father*(`bob`). Informally, we know that Bob is a father and Alice is a daughter. We also know that 'being a father by Bob' necessarily implies 'being a daughter by Alice'. In fact, we know that Bob has become a father and, subsequently, Alice has become a daughter. Therefore, the necessity of 'being a daughter by Alice' is deduced from the necessity of 'being a father by Bob'.

Axiom 18. $[\mathcal{N}(R(a,b) \Rightarrow S(c,d))] \Rightarrow [\mathcal{N}(R(a,b)) \Rightarrow \mathcal{N}(S(c,d))]$.
This axiom represents the distribution of necessity over role assertions.

Specific Analysis: We know that $R(a,b)$ and $S(c,d)$ are valid. We also know that $R(a,b)$ necessarily implies $S(c,d)$. Therefore, the necessity of $S(c,d)$ is deduced from the necessity of $R(a,b)$. For example, we know that *hasMother*($\texttt{bob}, \texttt{mary}$) and we know that *hasFather*($\texttt{alice}, \texttt{bob}$). We also know that *hasMother*($\texttt{bob}, \texttt{mary}$) necessarily implies *hasFather* ($\texttt{alice}, \texttt{bob}$). Therefore, the necessity of *hasFather*($\texttt{alice}, \texttt{bob}$) is deduced from the necessity of *hasMother*($\texttt{bob}, \texttt{mary}$). Informally, we know that Bob has a mother and Mary is his mother and we know that Alice has a father, and Bob is Alice's father. We also know that 'being the mother of Bob (by Mary)' necessarily implies 'being the father of Alice (by Bob)'. Therefore, the necessity of 'being the father of Alice (by Bob)' is deduced from the necessity of 'being the mother of Bob (by Mary)'.

6.3. Semantics of Negative Contingent World Descriptions

Regarding the following items, the notions of 'satisfiability', 'logical consequence', and 'tight logical consequence for possibilistic knowledge bases' are definable as similar to [24, 28].

1. $\mathcal{P}(A(a)) \equiv \neg\mathcal{N}(\neg A(a))$.

Specific Analysis. If we know that A can possibly describe a, then it will not be necessary that A cannot describe a. This means that it will not be necessary that A cannot describe a. Similarly, if we know that it is not necessary that A cannot describe a, then it will not be necessary that A cannot describe a. This means that A will possibly describe a. In fact, $[\mathcal{P}(A(a)) \Rightarrow \neg\mathcal{N}(\neg A(a))] \sqcap [\neg\mathcal{N}(\neg A(a)) \Rightarrow \mathcal{P}(A(a))]$. This means that: $\mathcal{P}(A(a)) \equiv \neg\mathcal{N}(\neg A(a))$.

2. $\mathcal{P}(R(a,b)) \equiv \neg\mathcal{N}(\neg R(a,b))$.

Specific Analysis. If we know that R can possibly relate a and b, then it will not be necessary that R does not relate a and b with each other. This means that it will not be necessary that R does not relate a and b with each other. Similarly, if we know that it is not necessary that R does not relate a and b with each other, then it will not be necessary that R does not relate a and b with each other. Thus, R will possibly relate a and b with each other. In fact, $[\mathcal{P}(R(a,b)) \Rightarrow \neg\mathcal{N}(\neg R(a,b))] \sqcap$

$[\neg\mathcal{N}(\neg R(a,b)) \Rightarrow \mathcal{P}(R(a,b))]$. This means that: $\mathcal{P}(R(a,b)) \equiv \neg\mathcal{N}(\neg R(a,b))$.

3. $\mathcal{N}(A(a)) \equiv \neg\mathcal{P}(\neg A(a))$.

Specific Analysis. If we know that A necessarily describes a, then there will be no possibility that A cannot describe a. Hence, it will be impossible that A cannot describe a. Similarly, if we know that it is impossible that A cannot describe a, then there will be no possibility that A cannot describe a. Therefore, A will necessarily be able to describe a. In fact, $[\mathcal{N}(A(a)) \Rightarrow \neg\mathcal{P}(\neg A(a))] \sqcap [\neg\mathcal{P}(\neg A(a)) \Rightarrow \mathcal{N}(A(a))]$. This means that: $\mathcal{N}(A(a)) \equiv \neg\mathcal{P}(\neg A(a))$.

4. $\mathcal{N}(R(a,b)) \equiv \neg\mathcal{P}(\neg R(a,b))$.

Specific Analysis. If we know that it is necessary that $R(a,b)$, then there will be no possibility that $R(a,b)$ will not be valid. In other words, it will be impossible that $R(a,b)$ will not be valid. Similarly, if we know that it is impossible that $R(a,b)$ is not valid, then there will be no possibility that $R(a,b)$ will not be valid. Therefore, $R(a,b)$ will necessarily be valid. In fact, $[\mathcal{N}(R(a,b)) \Rightarrow \neg\mathcal{P}(\neg R(a,b))] \sqcap [\neg\mathcal{P}(\neg R(a,b)) \Rightarrow \mathcal{N}(R(a,b))]$. This means that: $\mathcal{N}(R(a,b)) \equiv \neg\mathcal{P}(\neg R(a,b))$.

6.3.1. Semantic representation of negative contingent world descriptions

1. $\neg(\mathcal{P}(A(a)), T) \equiv (\neg\mathcal{P}(A(a)), F)$.
2. $\neg(\mathcal{P}(A(a)), F) \equiv (\neg\mathcal{P}(A(a)), T)$.
3. $\neg(\mathcal{P}(A(a)), V) \equiv (\neg\mathcal{P}(A(a)), V)$.
4. $\neg(\mathcal{P}(R(a,b)), T) \equiv (\neg\mathcal{P}(R(a,b)), F)$.
5. $\neg(\mathcal{P}(R(a,b)), F) \equiv (\neg\mathcal{P}(R(a,b)), T)$.
6. $\neg(\mathcal{P}(R(a,b)), V) \equiv (\neg\mathcal{P}(R(a,b)), V)$.
7. $\neg(\mathcal{N}(A(a)), T) \equiv (\neg\mathcal{N}(A(a)), F)$.
8. $\neg(\mathcal{N}(A(a)), F) \equiv (\neg\mathcal{N}(A(a)), T)$.
9. $\neg(\mathcal{N}(A(a)), V) \equiv (\neg\mathcal{N}(A(a)), V)$.
10. $\neg(\mathcal{N}(R(a,b)), T) \equiv (\neg\mathcal{N}(R(a,b)), F)$.
11. $\neg(\mathcal{N}(R(a,b)), F) \equiv (\neg\mathcal{N}(R(a,b)), T)$.
12. $\neg(\mathcal{N}(R(a,b)), V) \equiv (\neg\mathcal{N}(R(a,b)), V)$.

Consider the following examples:

Example 5. We know that Martin is not a father. Therefore, $|\mathcal{P}(\mathit{Father}(\texttt{martin}))|^{\mathcal{I}_p} = (\mathcal{P}(\mathit{Father}(\texttt{martin})), F)$. Hence, it is impossible to describe Martin by the concept *Father*. In fact, the term 'Martin is a father' is impossible. Consequently, it is deducible that $\neg\mathit{Father}(\texttt{martin})$.

Example 6. Bob is the father of Alice. So $|\mathcal{P}(hasFather(\texttt{alice}, \texttt{john}))|^{\mathcal{I}_p} = (\mathcal{P}(hasFather(\texttt{alice}, \texttt{john})), F)$. Thus, it is impossible to relate Alice and John by the relation *hasFather*. In fact, the term 'Alice has a father and John is Alice's father' is impossible. Consequently, it is deducible that $\neg hasFather(\texttt{alice}, \texttt{john})$.

Example 7. We are not certain that James is a father (or not). So, the term 'James is a father' is semantically vague. In fact, based on our conditional information, it is unnecessary that 'James is a father'. However, it is possible that 'James is a father'. Consequently, we can represent our knowledge by either:

1. $|\neg\mathcal{N}(Father(\texttt{james}))|^{\mathcal{I}_n} = (\mathcal{N}(Father(\texttt{james})), V)$, or
2. $|\neg\mathcal{N}(Father(\texttt{james}))|^{\mathcal{I}_p} = (\mathcal{P}(Father(\texttt{james})), T)$.

Example 8. Suppose that we are not certain that $hasMother(\texttt{alice}, \texttt{ann})$ (i.e., we are not certain that 'Alice has a mother and Ann is Alice's mother'). Thus, we can conclude that the term 'Alice has a mother and Ann is Alice's mother' is possible. Formally, $|\mathcal{P}(hasMother(\texttt{alice}, \texttt{ann}))|^{\mathcal{I}_p} = (\mathcal{P}(hasMother(\texttt{alice}, \texttt{ann})), T)$. In addition, based on our conditional information, it is unnecessary that 'Alice has a mother and Ann is Alice's mother'. Formally: $|\neg\mathcal{N}(hasMother(\texttt{alice}, \texttt{ann}))|^{\mathcal{I}_n} = (\mathcal{N}(hasMother(\texttt{alice}, \texttt{ann})), V)$. Note that the term 'Alice has a mother and Ann is Alice's mother' is based on the conjunction of the terms 'Alice has a mother' and 'Ann is Alice's mother'. Alice (as a human being) certainly has (or has had) a mother. So, 'Alice has a mother' is necessarily valid. In fact, we can conclude that the vagueness of the term 'Alice has a mother and Ann is Alice's mother' is because of the vagueness of the term 'Ann is Alice's mother'. In fact, $(\mathcal{N}(hasMother(\texttt{alice}, \texttt{ann})), V) \Rightarrow [(\mathcal{N}(Daughter(\texttt{alice}), T) \wedge \mathcal{N}((Mother(\texttt{ann})), V)]$. Obviously, a contingent role assertion has been expressed which is based on the conjunction of two contingent concept assertions in order to be semantically interpreted.

7. Conclusions

The main focus of this research has been on possibility and necessity of DL world descriptions. This paper has initially focused on DL fundamental world descriptions (that are in the form of either concept assertions or role assertions) and has introduced functional contingents $\mathcal{P}$ (for possibility) and $\mathcal{N}$ (for necessity) over fundamental world descriptions.

$\mathcal{P}$ and $\mathcal{N}$ are interpreted as the functional roles of the most central concept assertions and role assertions in every world description. Functional contingents support structural analyses of the concepts of 'being-functionally-possible' and 'being-functionally-necessary'. Based on the role assertion $R(a, b)$, the functional role F (that is supported by the role R) associates the singleton $\{b\}$ (that consists of the individual b) with the singleton $\{a\}$ (that consists of the individual a).

According to $W = \{A(a), B(b), R(a, b)\}$, the individuals a and b can be described by the concepts A and B. This research has proved that the validity of the possibility of $R(a, b)$ is equivalent to the existence of — at least — one functional role of $A(a)$, like $F_i(A(a))$, that can be mapped onto $B(b)$. Therefore, there is — at least — one possible R that can functionally relate a and b with each other. In addition, it has been proved that the validity of the necessity of $R(a, b)$ is equivalent to the existence of all possible functional roles of $A(a)$ (that can be mapped onto $B(b)$). The concept of functional necessity means that there is always a functional relation that can functionally relate a and b with each other.

This research has introduced contingent interpretations (or $\mathcal{I}_c$) in order to handle the semantics of functional contingency. The basic assumption is that any $\mathcal{I}_c$ can provide a semantic basis for satisfying functional contingents $\mathcal{P}$ and $\mathcal{N}$ and, subsequently, for satisfying fundamental contingent world descriptions. Contingent interpretations are utilised to provide semantic models for functional possibilities and functional necessities within DL world descriptions. Relying on contingent interpretations, a three-valued semantics (based on Truth, Falsity, and Vagueness) has been analysed. It is concluded that the functional contingents $\mathcal{P}$ and $\mathcal{N}$ are two kinds of mappings from their central world descriptions into the values $\{T, F, V\}$.

This paper has finally offered several axioms and annotated DL world descriptions based on functional contingents. Furthermore, a semantic analysis of negative contingent world descriptions has been offered.

References

[1] Mohamed Nazih Omri Amen Ajroud, Salem Benferhat and Habib Youssef, "On the use of guaranteed possibility measures in possibilistic networks", in H. C. Lane and H. W. Guesgen (eds.), *Proceedings of the Twenty-Second International FLAIRS Conference*, Florida, AAAI Press.

[2] Franz Baader et al., *The Description Logic Handbook: Theory, Implementation and Applications*, Cambridge University Press, New York, 2nd edition, 2010.

[3] Carsten Lutz Franz Baader, Ian Horrocks and Ulrike Sattler, *An Introduction to Description Logic*, Cambridge University Press, 2017. DOI: 10.1017/9781139025355

[4] Carsten Lutz Franz Baader, Ian Horrocks and Ulrike Sattler, "Introduction", pages 1–9 in [3]. DOI: 10.1017/9781139025355.001

[5] Carsten Lutz Franz Baader, Ian Horrocks and Ulrike Sattler, "A basic description logic", pages 10–49 in [3] DOI: 10.1017/9781139025355.002

[6] Carsten Lutz Franz Baader, Ian Horrocks and Ulrike Sattler, "Description logic terminology", pages 228–233 in [3]. DOI: 10.1017/9781139025355.009

[7] Safia Bal-Bourai and Aïcha Mokhtari, "Poss-owl 2: Possibilistic extension of owl 2 for an uncertain geographic ontology", pages 407–416 in P. Jedrzejowicz et al. (ed.), *18th International Conference on Knowledge-Based and Intelligent Information & Engineering Systems – KES 2014*, Procedia Computer Science, Elsevier, 2014. DOI: 10.1016/j.procs.2014.08.121

[8] Safia Bal-Bourai and Aïcha Mokhtari, "π-sroiq(d): Possibilistic description logic for uncertain geographic information", pages 818–829 in H. Fujita et al. (eds.), *IEA/AIE 2016: Trends in Applied Knowledge-Based Systems and Data Science*, vol. 9799 of *Lecture Notes in Computer Science*, Springer, 2016. DOI: 10.1007/978-3-319-42007-3_69

[9] Laurent Mouchard Pierre Vera Benoit Lelandais, Isabelle Gardin and Su Ruan, "Using belief function theory to deal with uncertainties and imprecisions in image processing", pages 197–204 in T. Denoeux and M.-H. Masson (eds.), *Belief Functions: Theory and Applications*, Advances in Intelligent and Soft Computing, Springer, Berlin, Heidelberg, 2012. DOI: 10.1007/978-3-642-29461-7_23

[10] Patrick Blackburn et al. (eds.), *Handbook of Modal Logic*, vol. 3 of *Studies in Logic and Practical Reasoning*, Elsevier Science, 2006.

[11] Miguel Delgado Fernando Bobillo and Juan Gómez Romero, "Crisp representations and reasoning for fuzzy ontologies", *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* 17, 4 (2009): 501–530. DOI: 10.1142/s0218488509006121

[12] Fernando Bobillo and Umberto Straccia, "Reasoning with the finitely many-valued Łukasiewicz fuzzy description logic sroiq", *Information Sciences* 181 4 (2011): 758–778. DOI: 10.1016/j.ins.2010.10.020

[13] Alex Borgida, “On the relationship between description logic and predicate logic queries”, pages 219–225 in *Proceedings of the Third International Conference on Information and Knowledge Management*, CIKM ’94, New York, 1994. DOI: 10.1145/191246.191283

[14] Ronald J. Brachman and Hector J. Levesque, *Readings in Knowledge Representation*, Morgan Kaufmann Pub, 1985.

[15] Romano Scozzafava Giulianella Coletti and Barbara Vantaggi, “Possibility measures in probabilistic inference”, pages 51–58 in *Soft Methods for Handling Variability and Imprecision*, Advances in Soft Computing, Springer, Berlin, Heidelberg, 2008. DOI: 10.1007/978-3-540-85027-4_7

[16] Abdelkarim Ben Ayed Fatma Dammak, Leila Baccour and Adel M. Alimi, “Electre method using interval-valued intuitionistic fuzzy sets and possibility theory for multi-criteria decision making problem resolution”, pages 1–6 in *2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, 2017. DOI: 10.1109/FUZZ-IEEE.2017.8015408

[17] Jean Dezert and Jean-Marc Tacnet, “Sigmoidal model for belief function-based electre tri method”, pages 401–408 in T. Denoeux and M.-H. Masson (eds.), *Belief Functions: Theory and Applications*, Advances in Intelligent and Soft Computing, Springer, Berlin, Heidelberg, 2012. DOI: 10.1007/978-3-642-29461-7_47

[18] Francesco M. Donini, Martin Buchheit and Andrea Schaerf, “Decidable reasoning in terminological knowledge representation systems”, in *Computing Research Repository (CoRR)*, 1993. DOI: 10.1613/jair.21

[19] Isabela Neves Drummond, “Imprecise data classification based on fuzzy logic and possibility theory”, PhD thesis, National Institute for Space Research, São José dos Campos, Brazil, 2007.

[20] Henri Prade Didier Dubois and Agnes Rico, “Representing qualitative capacities as families of possibility measures”, *International Journal of Approximate Reasoning* 58, C (2015): 3–24. DOI: 10.1016/j.ijar.2014.11.005

[21] Jérôme Mengin Didier Dubois and Henri Prade, “Possibilistic uncertainty and fuzzy features in description logic: a preliminary discussion”, 2006. DOI: 10.1016/S1574-9576(06)80008-0

[22] Didier Dubois, “Fuzzy measures on finite scales as families of possibility measures” pages 822–829 in S. Galichet et al. (eds.), *Proceedings of the 7th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT-11)*, Advances in Intelligent Systems Research, Aix-les-Bains, Atlantis Press, 2011.

[23] Didier Dubois and Henri Prade, "Updating with belief functions, ordinal conditioning functions and possibility measures", *Computing Research Repository (CoRR)*, 2013. http://arxiv.org/abs/1304.1118

[24] J. Gallagher and P. M. Hill, "Meta-programming in logic programming", pages 421–497, Oxford Science Publications, Oxford University Press, 1994.

[25] James Garson, "Modal logic", *The Stanford Encyclopedia of Philosophy*, 2014.

[26] Sergio Alejandro Gómez, "Reasoning with inconsistent possibilistic description logics ontologies with disjunctive assertions", in ISTEC RedUNCI (ed.), *XXI Argentinian Conference of Computer Science (CACIC 2015)*, Buenos Aires, 2015.

[27] Sergio Alejandro Gómez, "Reasoning with inconsistent possibilistic ontologies by applying argument accrual", *Journal of Computer Science and Technology* 17, 2 (2017): 117–126. DOI: 10.24215/16666038.17.e16

[28] Bernhard Hollunder, "An alternative proof method for possibilistic logic and its application to terminological logics", *International Journal of Approximate Reasoning* 12 (1994): 85–109. DOI: 10.1016/0888-613X(94)00015-U

[29] Kosuke Kato, Takeshi Uno and Hideki Katagiri, "A stackelberg location with fuzzy random demands based upon possibility and necessity measures", *International Journal of Knowledge Engineering and Soft Data Paradigms* (2014): 249–260. DOI: 10.1504/IJKESDP.2014.064263

[30] Pavel Klinov and Bijan Parsia, "Understanding a probabilistic description logic via connections to first-order logic of probability", pages 41–58 in Fernando Bobillo et al. (eds.), *Uncertainty Reasoning for the Semantic Web*, Lecture Notes in Computer Science, Springer, Berlin, Heidelberg, 2013. DOI: 10.1007/978-3-642-35975-0_3

[31] Oliver Kutz, Ian Horrocks and Ulrike Sattler, "The even more irresistible sroiq", pages 57–67 in *Proceedings of the 10th International Conference on Principles of Knowledge Representation and Reasoning 2006*, Lake District, AAAI Press, 2006.

[32] Ioan A. Letia and Adrian Groza, "Modelling imprecise arguments in description logic", pages 94–99 in *Advances in Electrical and Computer Engineering*, 2009. DOI: 10.4316/AECE.2009.03017

[33] H. J. Levesque and R. J. Brachman, *A Fundamental Tradeoff in Knowledge Representation and Reasoning*, Readings in Knowledge Representation, Morgan Kaufmann, 1985.

[34] Jun Li, Tao Chen, Radko Mesiar and Andrea Stupnanova, "Possibility and necessity measures and integral equivalence", *International Journal of Approximate Reasoning* (2017): 62–72. DOI: 10.1016/j.ijar.2017.04.008

[35] Yongming Li, "Quantitative model checking of linear-time properties based on generalized possibility measures", *Fuzzy Sets and Systems* (2017): 17–39. DOI: 10.1016/j.fss.2017.03.012

[36] Sebastian Link and Henri Prade, "Possibilistic functional dependencies and their relationship to possibility theory", *IEEE Transactions on Fuzzy Systems* 24, 3 (2016): 757–763. DOI: 10.1109/TFUZZ.2015.2466074

[37] Thomas Lukasiewicz, "Expressive probabilistic description logics", *Artificial Intelligence* 172. 6–7 (2008): 852–883. DOI: 10.1016/j.artint.2007.10.017

[38] Andrzej Matuszewski and Mieczyslaw A. Klopotek, "What does a belief function believe in?", *Computing Research Repository (CoRR)*, 2017. http://arxiv.org/abs/1706.02686

[39] John-Jules Ch. Meyer, "Modal epistemic and doxastic logic", *Handbook of Philosophical Logic*, vol. 10, 2003. DOI: 10.1007/978-94-017-4524-6_1

[40] Marvin Minsky, "A framework for representing knowledge", Technical Report 306, MIT-AI Laboratory, 1974.

[41] Aïcha Mokhtari, Safia Bal-Bourai and Faiza Khellaf, "Poss-sroiq(d): Possibilistic description logic extension toward an uncertain geographic ontology", pages 277–286 in B. Catania et al. (eds.), *New Trends in Databases and Information Systems*, vol. 241 of *Advances in Intelligent Systems and Computing*, Springer, 2013. DOI: 10.1007/978-3-319-01863-8_30

[42] Daniele Nardi and Ronald J. Brachman, *An Introduction to Description Logics*, pages 1–42. Cambridge University Press, New York, 2010.

[43] D. Nardi, W. Nutt, F. M. Donini, M. Lenzerini and A. Schaerf, "An epistemic operator for description logics", *Artificial Intelligence* 100, 1 (1998): 225–274. DOI: 10.1016/S0004-3702(98)00009-5

[44] Jeff Z. Pan, Guilin Qi and Qiu Ji, "A possibilistic extension of description logics", in D. Calvanese et al. (eds.), *In Proceedings of DL'07*, vol. 250 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2007

[45] Jeff Z. Pan, Guilin Qi and Qiu Ji, "Extending description logics with uncertainty reasoning in possibilistic logic", pages 828–839 in K. Mellouli (ed.), *ECSQARU 2007: Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, vol. 4724 of *Lecture Notes in Computer Science*, Springer, Berlin, Heidelberg, 2007. DOI: 10.1007/978-3-540-75256-1_72

[46] Jeff Z. Pan, Guilin Qi, Qiu Ji and Jianfeng Du, *International Journal of Intelligent Systems* 26, 4 (2011): 353–381. DOI: 10.1002/int.20470

[47] Ross Quillian, "Semantic memory", pages 216–270 in *Semantic Information Processing*, MIT Press, 1968.

[48] Sebastian Rudolph, "Foundations of description logics", in *In Reasoning Web, volume 6848 of LNCS*, 2011. DOI: 10.1007/978-3-642-23032-5_2

[49] Jose L. Ruiz and Inmaculada Sirvent, *Fuzzy Optimization and Decision Making*, 2017.

[50] Enrique H. Ruspini, "Possibility as similarity: The semantics of fuzzy logic", *Computing Research Repository (CoRR)*, 2013. http://arxiv.org/abs/1304.1115

[51] Klaus Schild, "A correspondence theory for terminological logics: Preliminary report", pages 101–104 in *IJCAI'91 Proceedings of the 12th International Joint Conference on Artificial intelligence*, vol. 1, San Francisco, CA, 1991.

[52] Manfred Schmidt-Schaulss and Gert Smolka, "Attributive concept descriptions with complements", in *Artificial Intelligence* Elsevier, 1991.

[53] Krister Segerberg, "Default logic as dynamic doxastic logic", *Erkenntnis* 50, 2–3 (1999): 333–352. DOI: 10.1023/A:1005546526502

[54] Glenn Shafer, "Belief function and possibility measures", *The Analysis of Fuzzy Information* 1 (1987): 51–84.

[55] Glenn Shafer, "Belief functions and parametric models", pages 265–290 in R. R. Yager and L. Liu (eds.), *Classic Works of the Dempster-Shafer Theory of Belief Functions*, Studies in Fuzziness and Soft Computing, Springer, Berlin, Heidelberg, 2008. DOI: 10.1007/978-3-540-44792-4_10

[56] Leslie F. Sikos, *Description Logics in Multimedia Reasoning*, Springer International Publishing, 2017. DOI: 10.1007/978-3-319-54066-5

[57] Umberto Straccia, "Managing uncertainty and vagueness in description logics, logic programs and description logic programs", in *Reasoning Web*, Lecture Notes in Computer Science. Springer, Berlin, Heidelberg, 2008. DOI: 10.1007/978-3-540-85658-0_2

[58] Umberto Straccia, *Foundations of Fuzzy Logic and Semantic Web Languages*, Chapman & Hall/CRC, 2013.

[59] Matthias C. M. Troffaes, Enrique Miranda and Sebastien Destercke, "On the connection between probability boxes and possibility measures", *Information Sciences* (2013): 88–108. DOI: 10.1016/j.ins.2012.09.033

[60] Nicolas Troquard and Philippe Balbiani, “Propositional dynamic logic”, in *Stanford Encyclopedia of Philosophy*, 2015. https://plato.stanford.edu/entries/logic-dynamic

[61] Nicolas Troquard and Philippe Balbiani, “Propositional dynamic logic”, *The Stanford Encyclopedia of Philosophy*, 2015.

[62] Vassilis Tzouvaras, Giorgos Stoilos, Giorgos B. Stamou and Jeff Z. Pan, “Uncertainty and ruleml rulebases: A preliminary report”, in Asaf Adi et al. (eds.), *RuleML 2005: Rules and Rule Markup Languages for the Semantic Web*, vol. 3791 of *Lecture Notes in Computer Science*, Springer, Berlin, Heidelberg, 2005. DOI: 10.1007/11580072_17

[63] Frank van Harmelen et al. (eds.), *Handbook of Knowledge Representation*, Foundations of Artificial Intelligence, Elsevier Science, San Diego, 2007.

[64] World Wide Web Consortium (W3C). Swrl: A semantic web rule language combining owl and ruleml, 2004. http://www.w3.org/Submission/SWRL

[65] Ronald R. Yager and Naif Alajlan, “Maxitive belief structures and imprecise possibility distributions”, *IEEE Transactions on Fuzzy Systems* 25, 4 (2017): 768–774. DOI: 10.1109/TFUZZ.2016.2574930

[66] Lotfi A. Zadeh, “Fuzzy sets as a basis for a theory of possibility”, *Fuzzy Sets and Systems* 100 (1999): 9–34. DOI: 10.1016/S0165-0114(99)80004-9

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