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Impact of Lorentz forces on Fe$_3$O$_4$-water ferrofluid entropy and exergy treatment within a permeable semi annulus

Mohsen Sheikholeslami $^{a,b}$, Ahmad Arabkoohsar $^c$, Ilyas Khan $^{d,e,1}$, Ahmad Shafee $^{f,g}$, Zhixiong Li $^h,i$

$^a$ Department of Mechanical Engineering, Babol Noshirvani University of Technology, Babol, Iran
$^b$ Renewable energy systems and nanofluid applications in heat transfer Laboratory, Babol Noshirvani University of Technology, Babol, Iran
$^c$ Department of Energy Technology, Aalborg University, Denmark
$^d$ Department of Mathematics, College of Science Al-Zulfi, Majmaah University, Al-Majmaah, 11952, Saudi Arabia
$^e$ Faculty of Mathematics and Statistics, Ton Duc Thang University, Ho Chi Minh City, Vietnam.
$^f$ FAST, University Tun Hussein Onn Malaysia, 86400, Parit Raja, Batu Pahat, Johor State, Malaysia
$^g$ Public Authority of Applied Education and Training, College of Technological Studies, Applied Science Department, Shuwaikh, Kuwait
$^h$ School of Engineering, Ocean University of China, Qingdao 266110, China
$^i$ School of Mechanical, Materials, Mechatronic and Biomedical Engineering, University of Wollongong, Wollongong, NSW 2522, Australia

Abstract

Challenge of energy will be increase in whole world by augmenting relevance of industry with fossil energy. According to this fact, renewable energies become popular in recent years. Employing nanofluids can help scientists to improve the performance of such systems. The impact of iron oxide–water nanofluid, as working fluid, was employed to evaluate entropy generation in an enclosure in existence of magnetic force. To analyze the performance of heating unit, both view of first and second law of thermodynamic should be involved. In current research, environment-friendly magnetic fluid namely Fe$_3$O$_4$-water ferrofluid has been studied which is useful in magnetic nanostructured materials have been found to be very efficient in wastewater decontamination. More exactly, the behavior of magnetic nanofluid through a porous space with innovative computational method is

1 *Corresponding Author:

Emails: ilyaskhan@tdt.edu.vn (Ilyas Khan); First author: mohsen.sheikholeslami@nit.ac.ir (Mohsen Sheikholeslami)
displayed. To involving porous media, non-Darcy approach was considered. Outcomes are obtained via Control volume based finite element method (CVFEM) to portray the impacts of Hartmann, Rayleigh numbers and permeability. Results display that dispersing nanoparticles leads to augment in thermal performance and decrease in entropy generation. As permeability enhances, Bejan number improves. As Lorentz forces augments, impact of adding nanoparticles reduces and exergy loss detracts. Dispersing nanoparticles are more beneficial in lower values of permeability.

**Keywords:** Nanofluid; Entropy; Porous; Heat transfer; Exergy and CVFEM; Magnetic field.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$S_{gen}$</td>
<td>Entropy generation</td>
</tr>
<tr>
<td>$X_d$</td>
<td>Exergy loss</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>$Ha$</td>
<td>Hartmann number</td>
</tr>
<tr>
<td>$g$</td>
<td>gravity</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
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<tr>
<td>$Be$</td>
<td>Bejan number</td>
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<tr>
<td>$B$</td>
<td>Magnetic field</td>
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<tr>
<td>$Ra$</td>
<td>Rayleigh number</td>
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**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\sigma$</td>
<td>Electrical conductivity</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>vorticity</td>
</tr>
<tr>
<td>$\theta$</td>
<td>temperature</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinetic viscosity</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Thermal expansion coefficient</td>
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**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
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<tr>
<td>$M$</td>
<td>magnetic</td>
</tr>
<tr>
<td>$p$</td>
<td>porous</td>
</tr>
<tr>
<td>$nf$</td>
<td>Nanofluid</td>
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</table>
1. Introduction

Nanofluids are the greatest popular tool to augment the efficiency of thermal equipment. Various kinds of nanoparticles have been employed because they can improve conductivity. This kind of working fluids can be used in renewable energy systems for various applications. Bellos et al. (2018) scrutinized different applications of nanofluid in renewable energies. They focus on solar technologies and presented variation of thermal performance in each cases. Nazari et al. (2019) provided solar experimental set up to examine the thermal performance of nanofluid. They utilized cooper oxide nanoparticle for single slope solar still. The productivity of fresh water augments with adding nanoparticles. Nan et al. (2019) scrutinized clean way for producing magnetic nanoparticles. Sheikholeslami et al. (2019a) utilized nanoparticles for solar heat storage unit. They suggested new shapes for metallic fin. Hayat et al. (2017) scrutinized nanofluid concentration analysis in a three dimensional enclosure. Entropy generation of nanoparticles within a porous space was demonstrated by Sheikholeslami (2019a). He considered magnetic force influence on exergy loss. Qi et al. (2017) scrutinized the silver nanoparticle migration in a cavity by using numerical method. Working fluid can be considered as non-Newtonian fluid when nanoparticles have been added (Khan et al. (2017), Hashim et al. (2018), Abro and Khan (2017)).

Sharafeldin and Gróf (2018) presented an application of CeO2/water nanofluid. They indicated that the outlet temperature augments when nanofluids are utilized. Sheikholeslami et al. (2018a) employed two temperature approaches for porous medium to discover ferrofluid behavior due to magnetic. Utilizing magnetic and electric fields are common ways for controlling flow direction (Mishra et al. (2015), Sheikholeslami and Mahian (2019), Sheikholeslami et al. (2018b,c), Moatimid and Hassan (2018), Muhammad et al. (2018)). Said et al. (2016) carried out the exergy performances of solar collector in existence of
alumina nanoparticles. They showed the impact of nanoparticles' size on thermal performance. If domain is porous space, several models can be used for simulation (Zin et al. (2017), Sheikholeslami et al. (2019b), Soomro et al. (2019), Sheikholeslami (2019b)). Ali et al. (2017) employed the fractional model for analyzing nanofluid flow due to magnetic force. They utilized polar coordinate for circular tube. The pollution of water these days has become one of a critical issue throughout the world. However, several water treatment technologies are in continuous efforts for improvement. Amongst them nanomaterials are regarded as an efficient strategy for water decontamination, and for environment protection. However, it is of central focus that the water treatment methodologies themselves should not produce additional harmful materials but should use instead non-toxic biodegradable ones. In this work, ferrofluid have been involved which is indeed a potential candidate for water remediation and for the homogenous dispersion of magnetite nanoparticles (NPs) in aqueous solution. Recent years, to enhance the thermal performance, nanoparticles and other passive ways have been utilized (Sheikholeslami (2018), Lee et al. (2018), Sheikholeslami et al. (2019c), Qi et al. (2011), Fengrui et al. (2018)).

There is few papers in which, nanofluid exergy and entropy analysis have been done. To reach best design of renewable energy unit, minimizing entropy generation is vital factor. In current text, as an application of magnetic nanoparticles, entropy and exergy analysis of ferrofluid due to magnetic forces within a permeable medium was scrutinized. Powerful numerical method was employed to display the energy and exergy analysis for different values of permeability, Lorentz and buoyancy forces.
2. Geometry explanation

Boundary condition and geometry of current paper has been provided in Fig. 1. Permeable space is full of ferrofluid. Constant heat flux was employed on inner cylinder. Horizontal magnetic field was employed. Both energy and exergy views have been included to reach the best design. Selecting nanofluid causes to improve thermal treatment of system.

3. Formulation and CVFEM

3.1. Governing

The aim of article is to simulate ferrofluid convective flow inside a two dimensional (2D) permeable space with magnetic force. Gravity force is included as buoyancy forces. Non-Darcy model for porous space has been selected. Moreover, for estimating ferrofluid properties, homogeneous model has been assumed. Related formulations are:

\[ \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0 \]  

\[ \rho_{nf} \left( \frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} \right) = \left[ \sigma_{nf} B_y B_s v - \frac{\mu_{nf}}{K} u - \sigma_{nf} B_y^2 u - \frac{\partial P}{\partial x} + \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} \right) \mu_{nf} \right], \]  

\[ \left( B_y, B_s \right) = B_s \left( \sin \gamma, \cos \gamma \right) \]  

\[ \rho_{nf} \left( v \frac{\partial v}{\partial y} + \frac{\partial v}{\partial x} \right) = \mu_{nf} \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) + g \left( T - T_c \right) \rho_{nf} B_{nf} \]  

\[ -B_x v B_s \sigma_{nf} - \frac{\mu_{nf}}{K} v + B_x u \sigma_{nf} B_y \frac{\partial P}{\partial y}, \]  

\[ \left( B_y, B_s \right) = B_s \left( \sin \gamma, \cos \gamma \right) \]
\[ k_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \left( \rho C_p \right)_{nf} \left( \frac{\partial u}{\partial x} + \frac{\partial T}{\partial y} \right) \] (4)

\[ (\rho C_p)_{nf}, (\rho \beta)_{nf}, \rho_{nf}, k_{nf} \] and \( \sigma_{nf} \) are predicted as:

\[ (\rho C_p)_{nf} = (\rho C_p) (1-\phi) + (\rho C_p) \phi \] (5)

\[ (\rho \beta)_{nf} = \phi (\rho \beta) + (1-\phi) (\rho \beta) \] (6)

\[ \rho_{nf} = (1-\phi) \rho_f + \rho_f \phi \] (7)

\[ k_{nf} = k_f \left( \frac{2k_f + k_s + 2\phi (k_s - k_f)}{-(k_s - k_f) \phi + k_s + 2k_f} \right) \] (8)

\[ \sigma_{nf} = 1 + \frac{3(-1+\Delta) \phi}{(2+\Delta)(-1+\Delta) \phi} \] (9)

\[ \frac{\sigma_{nf}}{\sigma_f} = 1 + \Delta \]

\( \mu_{nf} \) is estimated as (Wang et al. (2016)):

\[ \mu_{nf} = \left( 3.1B - 27886.4807 \phi^2 + 0.035B^2 + 4263.02 \phi + 316.0629 \right) e^{-0.001T} \] (10)

Properties of ferrofluid have been listed in Table 1. To eliminate pressure terms, below definitions should be involved:

\[ \frac{\partial \psi}{\partial y} = u, \quad \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\alpha_k, \quad v = -\frac{\partial \psi}{\partial x}. \] (11)

Defining non-dimensional quantities:

\[ \Theta = \frac{T - T_c}{\Delta T}, U = \frac{uL}{\alpha_f}, Q = qL/k_f, \quad \frac{V}{L} = \frac{(x, y)}{L}. \] (12)

So, following equations can be derived:
\[
V \frac{\partial \Theta}{\partial Y} + U \frac{\partial \Theta}{\partial X} = \frac{\partial^2 \Theta}{\partial Y^2} + \frac{\partial^2 \Theta}{\partial X^2}
\] (13)

\[
\frac{\partial^3 \Psi}{\partial X^2} + \frac{\partial^3 \Psi}{\partial Y^2} = -\Omega
\] (14)

\[
\text{Pr} \left( A_2 A_2 \frac{\partial^2 \Theta}{\partial Y^2} + A_1 A_4 \frac{\partial^2 \Theta}{\partial X^2} \right) + Ra \text{Pr} \left( \frac{\partial \Theta}{\partial X} \right) A_1 A_2^2
\]

\[
+ Ha^2 \text{Pr} A_1 A_4 \left[ - (\sin \gamma) (\cos \gamma) \frac{\partial V}{\partial Y} + (\cos \gamma) \frac{\partial U}{\partial X} (\cos \gamma) - \frac{\partial V}{\partial X} (\cos \gamma)^2 + (\sin \gamma)^2 \frac{\partial U}{\partial Y} \right]
\]

\[
- \frac{Pr}{Da} \left( A_2 A_4 \right) \Omega = U \frac{\partial \Omega}{\partial X} + \frac{\partial \Omega}{\partial Y} V
\] (15)

Following definitions should be mentioned for dimensionless variables:

\[
Da = \frac{K}{L}, Ra = g \beta_\ell q^* L^4 / (k_f v_f \alpha_f)
\]

\[
A_2 = \frac{(\rho C_p)_{nf}^*}{(\rho C_p)_f}, Ha = LB_0 \sqrt{\sigma_f / \mu_f}
\]

\[
A_1 = \frac{\rho_f}{\rho_f^*}, A_6 = \frac{\sigma_f}{\sigma_f^*}, A_5 = \frac{\mu_f}{\mu_f^*},
\]

and current boundary conditions can be presented as:

\[
\Theta = 0.0 \quad \text{@ cold surface}
\] (17)

\[
\Psi = 0.0 \quad \text{@ all walls}
\]

\[
\frac{\partial \Theta}{\partial n} = 1.0 \quad \text{@ hot surface}
\]

\[
Nu_{loc}, Nu_{ave} \text{ and } En \text{ are determined from:}
\]

\[
Nu_{loc} = \frac{1}{\Theta} \left( \frac{k_{nf}}{k_f} \right)
\] (18)
\[ Nu_{ave} = \frac{1}{S} \int_{S_0}^s Nu_{loc} \, ds \]  \hspace{1cm} (19) \\
\[ En = \frac{Nu_{ave}\big|_{\theta=0.04} - Nu_{ave}\big|_{\theta=0}}{Nu_{ave}\big|_{\theta=0}} \times 100 \]

Definitions of entropy generation, exergy loss and Bejan number are (Sheikholeslami et al. (2019c)): 

\[ S_{gen, total} = \frac{\mu_{st}}{T} \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]^2 + 2 \left( \frac{\partial u}{\partial x} \right)^2 + \frac{\sigma_{st}}{T^2} B \dot{\theta}^2 + \frac{\mu_{st}}{KT} \left( u^2 + v^2 \right) \]  \hspace{1cm} (20) \\
\[ \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2} \]

\[ X_d = T_0 S_{gen, total} \]  \hspace{1cm} (21) \\
\[ Be = S_{gen,th} / S_{gen, total} \]  \hspace{1cm} (22)

### 3.2. CVFEM

Innovative method has been applied in current article. The first code of this method was written by Sheikholeslami (2019c). He employed the mention method for various heat transfer problems. Finite element method (FEM) has been merged with Finite volume method (FVM) to generate this new algorithm. Researchers can find more details of this approach in new reference book (Sheikholeslami (2019c)). Current approach uses triangular element for 2D problems (see Fig. 1(b)).
4. Mesh independency and verification

To obtain unique results, various mesh sizes should be tested. One instance exists in table2. Also, Fig. 2 and table3 prove the accuracy of this code. Both nanofluid flow and magnetohydrodynamic (MHD) flow have been checked (Rudraiah et al. (1995), Calcagni et al. (2003), Khanafer et al. (2005)).

5. Results and discussion

Iron oxide-water ferrofluid free convection inside a permeable space was scrutinized in current text. To estimate the viscosity, Lorentz forces effect has been involved. Not only energy analysis but also exergy and entropy treatment have been reported. CVFEM has been employed to depict the results for various Darcy number ($Da = 0.01$ to $100$), Magnetic field ($Ha = 1$ to $40$) and Rayleigh number ($Ra = 10^3$ to $10^4$).

Figs. 3, 4, 5 and 6 are presented to display the influences of $Ra$, $Da$ and $Ha$ on energy, entropy and exergy behavior of ferrofluid. According to definition of $Ra$ and $Da$, augmenting such variables lead greater heat transfer. Graphs indicate this fact and it can be seen that convection enhances with rise of buoyancy and permeability. Furthermore, Lorentz forces make the conduction to augment and dispersing nanoparticles have more benefit. Surface temperature reduces with augment of permeability but it improves with augment of magnetic force. $|\psi_{mix}|$ augments with augment of $Da$ and $Ra$ while it declines with rise of $Ha$. Temperature along the inner surface reduces with decrease of magnetic force. Magnetic entropy generation declines with reduce of $Ra$ and $Ha$. As Lorentz forces augments, Bejan number augments.
Figs. 7, 8 and 9 illustrate the changes of $Be$, $Nu_{ave}$, $X_d$ with variation of $Ra$, $Ha$, and $Da$. Eqs. (23-25) has been derived from simulation data:

\[
Nu_{ave} = 0.067Da \log(Ra) - 0.068 \log(Ra)Ha - 0.044Ha Da \\
+ 1.82 + 0.17 \log(Ra) + 0.081Da - 0.08Ha
\]  

(23)

\[
Be = 0.97 - 8.84 \times 10^{-3}Da - 0.028 \log(Ra) + 8.87 \times 10^{-3}Ha \\
- 8.5 \times 10^{-3}Da \log(Ra) + 4.17 \times 10^{-3}Da Ha + 8.5 \times 10^{-3} \log(Ra) Ha
\]  

(24)

\[
X_d = 105.29 - 2.94Da - 5.31 \log(Ra) + 2.86Ha \\
- 2.2Da \log(Ra) + 1.61Da Ha + 2.19 \log(Ra) Ha
\]  

(25)

Convective mode has been boosted with rise of $Da$ and $Ra$. Also, augmenting magnetic force causes $Nu_{ave}$ to detract. Exergy loss and Bejan number have reverse treatment in comparison with $Nu_{ave}$. Exergy loss detracts with augment of permeability. Bejan number improves with increase of $Ha$. Fig. 10 displays the variation of heat transfer augmentation ($En$) due to changing $Ra$, $Ha$ and $Da$. Dispersing nanoparticles has greater impact in cases with greater conduction. Thus, this factor augments with increase of $Ha$ and it decreases with augment of $Da$ and $Ra$.

6. Conclusions

Magnetic force role on treatment of Ferrofluid flow and entropy generation through a permeable space was reported by employing CVFEM. In current research, environment-friendly magnetic fluid namely $Fe_3O_4$-water ferrofluid has been studied which is useful in magnetic nanostructured materials have been found to be very efficient in wastewater decontamination. The impact of iron oxide–water nanofluid, as working fluid, was employed to evaluate entropy generation in an enclosure in existence of magnetic force. Different parts
of entropy generation are reported as separate contours. Variation of Bejan number and exergy loss are depicted due to changing $Da$, $Ha$ and $Ra$. Bejan number reduces with rise of conduction mode. As these variables augments, magnetic entropy generation enhances. As magnetic forces enhance, exergy loss augments.

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Fig. 1. Porous enclosure under the effect of magnetic field
Fig. 2. Validation for (a) natural convection (Calcagni et al. (2003)); (b) nanofluid flow (Khanafer et al. (2005))
Fig. 3. Exergy and entropy contours for various $Ra$ at $\phi = 0.04, Ha = 1, Da = 0.01$
Fig. 4. Exergy and entropy contours for various $Ra$ at $\phi=0.04, Ha = 20, Da = 0.01$
Fig. 5. Exergy and entropy contours for various $Ra$ at $\phi=0.04, Ha=1, Da=100$
Fig. 6. Exergy and entropy contours for various Ra at $\phi=0.04$, $Ha = 20$, $Da = 100$
Fig. 7. Variation of $N_{ave}$ due to change of permeability, buoyancy and Lorentz forces at $\phi = 0.04$.  

$Da = 50$
$Ha = 5$

$\log(Ra) = 3.5$

$Da = 50$

$Ha = 5$

$\log(Ra) = 3.5$
Fig. 8. Variation of $Be$ due to change of permeability, buoyancy and Lorentz forces at $\phi = 0.04$. 

$Da = 50$
\( Ha = 5 \)

\( \log(Ra) = 3.5 \)

\( Da = 50 \)
Fig. 9. Variation of $X_d$ due to change of permeability, buoyancy and Lorentz forces at $\phi = 0.04$. 

$Da = 50$
$Ha = 5$

$log(Ra) = 3.5$

$Da = 50$
$Da = 50$

Fig. 10. Variation of $En$ due to change of permeability, buoyancy and Lorentz forces at $\phi = 0.04$. 
Table 1. Properties of H$_2$O and nanoparticles [30]

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$\beta \times 10^{3}$ (K$^{-1}$)</th>
<th>$\sigma (\Omega \cdot m)^{-1}$</th>
<th>$C_p (j/kgK)$</th>
<th>$k$(W/m.k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure water</td>
<td>997.1</td>
<td>21</td>
<td>0.05</td>
<td>4179</td>
<td>0.613</td>
</tr>
<tr>
<td>Fe$_3$O$_4$</td>
<td>5200</td>
<td>1.3</td>
<td>25000</td>
<td>670</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2. Various meshes' presentation at $Ra = 10^4$, $Ha = 20$, $Da = 100$ and $\phi = 0.04$.

<table>
<thead>
<tr>
<th>Mesh size in radial direction $\times$ angular direction</th>
<th>51$\times$151</th>
<th>61$\times$181</th>
<th>71$\times$211</th>
<th>81$\times$241</th>
<th>91$\times$271</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.91505</td>
<td>1.91671</td>
<td>1.91772</td>
<td>1.91781</td>
<td>1.91796</td>
</tr>
</tbody>
</table>

Table 3. Validation for MHD flow when $Pr=0.733$ and $Gr = 2 \times 10^4$.

<table>
<thead>
<tr>
<th>Ha =10</th>
<th>Ha=50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
<td>2.26626</td>
</tr>
<tr>
<td>Rudraiah et al. (1995)</td>
<td>2.2234</td>
</tr>
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</table>