Abstract—Recently, Finite Control Set Model Predictive Control (FCS-MPC) has been successfully applied in the grid-tied inverter with LCL filter. However, to achieve active damping and grid synchronization, many sensors are required, increasing cost and complexity. In addition, a considerable computational delay should be addressed when it is experimentally implemented, which may degrade the performance of overall system. In order to reduce the number of sensors, eliminate the computational delay, and enhance the control reliability of system, a novel FCS-MPC strategy with merely grid-injected current sensors is proposed, which contains four compositions: virtual flux observer, state observer, delay compensation and FCS-MPC algorithm based on estimations. A 3-kW/3-phase/110V experimental platform is established to validate that utilizing the proposed observations-based control method with only grid-injected current sensors is capable to obtain satisfactory performance of grid synchronization and high quality grid-injected current both under balanced and unbalanced grid voltage condition.

Index Terms—Control reliability, delay compensation, FCS-MPC, grid synchronization, LCL filter, state observer, virtual flux observer.

I. INTRODUCTION

As interfaces between DC sources and grids, grid-tied inverters are core components of distributed generation systems, like roof-top photovoltaics [1]-[3]. In order to achieve great performance of them, besides the classical linear control strategies [4], [5], a large amount of nonlinear control schemes, such as sliding mode control (SMC), passivity-based control (PBC) [6], [7] and so on, have been proposed. As one of the nonlinear control schemes, model predictive control (MPC) has also attracted many attentions in recent years, since the problem of computational burden has been preliminary solved [8]-[10].

Generally, MPC can be classified into two categories: Continuous Control Set Model Predictive Control (CCS-MPC) and Finite Control Set Model Predictive Control (FCS-MPC), depending on how the switching signals that control the operation of grid-tied inverter are generated [11], [12]. For CCS-MPC, the control scheme yields a continuous output which is needed to be processed by a modulator to obtain the gate signals [13]. For FCS-MPC, it does not adopt a modulation stage but relies on a finite number of output states of inverters, which is employed to evaluate the cost function [14]. In contrast to CCS-MPC, FCS-MPC is the best known due to the technique advantages include: straightforward handling of nonlinearities and constraints, good dynamic response, and simple implementation [15]-[17].

Recently, FCS-MPC strategy was proposed for the LCL-filtered grid-tied inverter [11], where it requires two kinds of current sensors and two types of voltage sensors, increasing the cost and complexity. In order to reduce the number of sensors for measuring the state variables [18]-[23] and the grid voltage [24]-[27], many control methods with fewer sensors had been utilized, respectively.

An approach, which decreases the sensors of state variables, was described in [18], where the capacitor current loop was merged with the grid-injected current loop by transforming the block diagram. However, it saves only two capacitor current sensors. For further sensors saving, the state observer [19], [20] and Kalman filter (KF) [21], [22] based on state-space model were investigated, which were both utilized to estimate the state variables of filter via the closed-loop system. Compared with the state observer, the issues of higher computational burden and more complex parameter adjustment process are existed for KF [23]. Therefore, the state observer is adopted for estimating the all three state variables in this paper. Although the total number of current sensors can be reduced by adopting the state observer, the grid voltage still needs to be measured, which leads to an extra sensor.

Many grid voltage sensorless schemes have been reported in the literature [24], [25], where the concept of virtual flux (VF) is well-known, and it has been universally investigated. However, due to problems of DC drift and initial bias caused by pure integrator, some necessary solutions for addressing these
issues are required. A first order low pass filter (LPF) is generally used owing to its simple implementation, and the magnitude and phase errors introduced by this filter can be handled by a compensation gain [26]. For enhancing theVF performance, many complex solutions were proposed, e.g., reference [27] utilizes a virtual flux observer with negative feedback resonant filter and a delay compensation algorithm to alleviate the steady-state error and the delay. However, although these strategies offer a better performance, the complexity of algorithms should be taken into account.

In this paper, a novel observations-based FCS-MPC strategy for LCL-filtered grid-tied inverter is proposed using only grid-injected current sensors. The control method is based on all four observations instead of measurements, where inverter-side current, capacitor voltage and grid-injected current are estimated via state observer. For the grid voltage, a VF-based grid voltage sequence extraction strategy is adopted to separate the positive- and negative-sequence components, so that the satisfactory performance of grid-injected current is obtained under unbalanced grid voltage condition. The performance of the control strategy is verified by theoretical analysis and experimental tests. The rest of this paper is organized as follows. The conventional FCS-MPC strategy for LCL filtered grid-tied inverter is first presented in Section II. Then, the proposed FCS-MPC method with full status estimations based on virtual flux and state observer is described in Section III. Next, experimental results are documented in Section IV, where the performance of grid synchronization and the high quality grid injected current both under balanced and unbalanced grid voltage condition are analyzed. Finally, conclusions are drawn in Section V.

II. CONVENTIONAL FCS-MPC STRATEGY FOR LCL-FILTER-BASED GRID-TIED INVERTER

A. Grid-Connected Inverter Model

![Fig. 1 Proposed FCS-MPC strategy for grid-tied inverter with LCL filter.](image)

The structure of three phase grid-tied inverter with LCL filter is depicted in Fig. 1. As shown in Fig. 1, \( U_d \), \( v_i \), \( \mu_c \) and \( \nu_g \) represent the DC bus voltage, the inverter output voltage, the capacitor voltage and the grid voltage, respectively, \( i_1 \) and \( i_2 \) denote the inverter-side current and the grid-injected current respectively. The inverter output voltage is combined by eight switching states and it can be described as a complex space vector in \( \alpha \beta \) stationary coordinate system,

\[
\nu_{smp}(n) = \begin{cases} 0, & n = \{0, 7\} \\
\frac{2}{3} U_{dc} e^{\frac{j(\alpha-1)}{3}}, & n = \{1, 2, \ldots, 6\} 
\end{cases}
\]

Adopting the Clarke’s transformation, the inductor currents and the capacitor voltage dynamic model for the three phase grid-tied inverter in \( \alpha \beta \) stationary reference frame are given as,

\[
\frac{dx}{dt} = Ax + Bu_{smp} + B_1 v_{smp}
\]

where

\[
x = \begin{bmatrix} i_{smp} \\ i_{smp} \\ u_{smp} \end{bmatrix}, A = \begin{bmatrix} 0 & 0 & -1/L_i \\
\end{bmatrix}, B = \begin{bmatrix} 1/L_i & 0 \\
\end{bmatrix}
\]

Next, according to the state-space model described in (2), the discrete time model of LCL filter based on the sampling time \( T_s \) can be obtained as,

\[
x(k+1) = A_1 x(k) + B_1 v_1(k) + B_2 v_2(k)
\]

where matrices \( A_1 \), \( B_1 \), and \( B_2 \) are

\[
A_1 = e^{A T_s}, B_1 = \int_0^{T_s} e^{A T_s} B_1 d T, B_2 = \int_0^{T_s} e^{A T_s} B_2 d T
\]

The detailed expressions of (6) can be found in the appendix.

B. Conventional FCS-MPC strategy

Unlike other common control algorithm applied in the grid-tied inverter, the FCS-MPC strategy does not adopt a modulation stage, but depends on a finite number of output voltage vectors of inverter, which can be obtained by utilizing a traversal method. These voltage vectors are used to minimize the selected cost function online, and then the optimal inverter output voltage vector can be captured.

The conventional FCS-MPC scheme is implemented in following steps [11].

1) Measure the inverter-side current, the grid-injected current, the capacitor voltage, and the grid voltage;

2) Calculate the reference of the capacitor voltage and the inverter-side current by utilizing the reference of grid-injected current;

3) Deduce the references of three state variables at next step using the Lagrangian Extrapolation;

4) Predict the values of three state variables in next sampling instant for all possible inverter output voltage vectors;

5) Construct the cost function;

6) Select the optimal inverter output voltage vector.

The conventional FCS-MPC strategy requires two kinds of current sensors for inductor current measurements and two types of voltage sensors for capacitor voltage and grid voltage measurements. Additionally, the computational delay was not taken into account, which may degrade the system performance and increase the ripples of grid-injected current. For the purpose of reducing the number of sensors and eliminating this
considerable computational delay, a novel control scheme with full status observations based on virtual flux and state observer is proposed in the following section, which only two grid-injected current sensors are used to generate the high quality grid-injected currents and achieve the grid synchronization.

III. PROPOSED FCS-MPC STRATEGY WITH FULL STATUS ESTIMATIONS BASED ON VIRTUAL FLUX AND STATE OBSERVER

The proposed FCS-MPC strategy with full status estimation based on virtual flux and state observer is depicted in Fig. 1. As shown in Fig. 1, only the grid-injected current sensors are utilized for measurement, and the other variables, including the inverter-side current, the capacitor voltage and the grid voltage, are estimated by the state observer and the virtual flux observer, respectively. However, it should be noticed that although the grid-injected currents are measured, the proposed FCS-MPC strategy does not utilize these measured variables for the control, but relies on all four observations.

The design process of proposed FCS-MPC method can be realized as follows:

Firstly, estimate the grid voltage and the reference value of grid-injected current by utilizing the virtual flux observer, based on the measured grid-injected currents, active power, reactive power and the switching state $S(k)$ at the $(k+1)^{th}$ instant, where this switching state is calculated in the last period.

Secondly, estimate the inverter-side inductor currents, the grid-injected currents, and the capacitor voltages by adopting the state observer, which is based on the observed grid voltages and the measured grid-injected currents.

Thirdly, predict the three observed state variables at the $(k+2)^{th}$ instant, based on the discrete time model of LCL filter described in (5).

Next, obtain the references of inverter-side inductor currents and capacitor voltages as:

$$
\begin{align*}
\mathbf{u}_{c_{ref}} &= \hat{\mathbf{v}}_{c_{ref}} - \omega L_c (i_{c_{ref}}^g - j i_{c_{ref}}^s) \\
i_{c_{ref}} &= (1 - \omega^2 L_c C) i_{c_{ref}}
\end{align*}
$$

Then, calculate the reference $x$ at the $(k+2)^{th}$ instant utilizing the Lagrangian Extrapolation, and it can be expressed as,

$$
x_{c_{ref}}(k+2) = 6x_{c_{ref}}(k) - 8x_{c_{ref}}(k-1) + 3x_{c_{ref}}(k-2)
$$

According to the analysis mentioned above, the cost function is constructed, which includes the errors between three estimated state variables and their references in this paper. However, due to the different nature (including different units and magnitudes) for three state variables, a suitable weighting factor is necessary, which can regulate the influences of variables on the cost function. Therefore, the modified cost function based on full status estimates and delay compensation can be expressed as,

$$
J = \sum (e_{\alpha}^2 (k+2) + e_{\beta}^2 (k+2) + e_{\psi}^2 (k+2) + e_{\alpha}^2 (k+2) + e_{\beta}^2 (k+2) + e_{\psi}^2 (k+2))
$$

where $e$ is the error of controlled variables between reference value and estimated value. The weighting factor $\lambda_{\omega}$ is set to achieve active damping [28], and $\lambda_{\alpha}$ is tuned to obtain a high quality grid-injected current [11]. And the weighting factors included in Table I are designed according to [29].

Finally, the optimal switching state, which is employed at the $(k+1)^{th}$ instant, is selected based on the modified cost function.

A. Virtual Flux Observer

1) The Concept of Virtual Flux

Generally, the grid voltage of grid-tied inverter connecting with L filter and resistance $R$ can be considered as a virtual AC electric motor [30]. Furthermore, $R$ and $L$ in the inverter represent the corresponding the stator resistance and stator inductance in an AC motor, respectively. For the LCL-filter-based system, owing to the high impedance of filtering capacitor in low frequency, the fundament current of this capacitor can be treated as zero. Therefore, the currents of the grid-side and inverter-side inductors can be regarded as equal to each other, and we can use the sum of their inductances as the equivalent inductance. Consequently, neglecting $R$, the grid virtual flux vector in $\alpha\beta$ coordinate system for grid-tied inverter with LCL filter can be deduced as,

$$
\begin{align*}
\psi_{\alpha} &= \int v_{\alpha} dt + \left( (v_{alpha} - (L_\alpha + L_\beta) \frac{di_{alpha}}{dt}) dt \right) \\
\psi_{\beta} &= \int v_{\beta} dt + \left( (v_{beta} - (L_\alpha + L_\beta) \frac{di_{beta}}{dt}) dt \right)
\end{align*}
$$

where $v_{\alpha}, v_{\alpha}, v_{\alpha}, v_{\beta}, v_{\beta}, v_{\beta}, i_{\alpha}$ and $i_{\beta}$ are the component values of the virtual grid voltage flux vectors, the grid voltages, the optimal inverter output voltage vectors and the grid-injected currents in $\alpha\beta$ coordinate system, respectively.

2) Amplitude and Phase Compensations

To eliminate undesired DC offsets associated with the integration, the flux observer in AC electric motor generally employs the first order low-pass filter to substitute the pure integrator. Thus, the same strategy is utilized for obtaining the virtual grid voltage flux in this paper. However, utilizing LPF will result in the errors of phase and amplitude, which deteriorates the performance of system. Therefore, in order to enhance the performance of grid-connected inverter, the phase and amplitude compensations are required to achieve the same effect of pure integrator, when utilizing the low-pass filter.

Assuming that $\psi_{\alpha}$ is the virtual grid voltage flux described as complex space vector in $\alpha\beta$ stationary reference frame,

$$
\psi_{\alpha} = \int v_{\alpha} dt = \psi_{\alpha} + j \psi_{\beta} = |\psi| \angle \theta
$$

Adopting the first order low-pass filter, virtual grid voltage flux can be obtained as,

$$
\psi_{\alpha} = \int v_{s} dt = \psi_{\alpha} + j \psi_{\beta} = |\psi| \angle \theta
$$

$$
\psi_{\alpha} = \int v_{s} dt = \psi_{\alpha} + j \psi_{\beta} = \frac{\psi_{\alpha}}{\frac{\psi_{\alpha}}{\psi_{\beta}} + \frac{\psi_{\alpha}}{\psi_{\beta}}} \angle \theta
$$

where $\phi = \pi/2 - \arctan(\omega_{c}/\omega_{s})$, $\omega_{c}$ is the fundamental angular frequency and $\omega_{s}$ is the cut-off frequency, which usually can be set to $0.1\omega_{c} - 0.5\omega_{c}$.

According to the analysis above, the magnitude and the phase of the compensation gain is respectively $\sqrt{\frac{\omega_{c}}{\omega_{s}} + \frac{\omega_{c}}{\omega_{s}}}$ and $\arctan(\omega_{c}/\omega_{s}) - \pi/2$. 
3) The PLL Based on Virtual Flux

In order to realize the grid synchronization, the grid voltage phase angle or frequency should be detected. Since the satisfied results can be obtained by adjusting the bandwidth of PLL when grid voltage fluctuates, PLL is universally utilized as a common method to receive the information of the phase angle and frequency in power electronics. However, the voltage sensor, which provides the grid voltage phase angle and frequency, is not used in this paper. So, an approach based on VF is presented to estimate the phase angle and frequency, which has a great suppression on interference signal.

By adopting Clarke’s transformation (equal amplitude transformation), the virtual flux in αβ coordinate system can be expressed as,

\[
\begin{bmatrix}
\psi_\alpha \\
\psi_\beta
\end{bmatrix} = \begin{bmatrix}
\cos \theta & \sin \theta \
-\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
\psi_d \\
\psi_q
\end{bmatrix}
\]

where \( \psi \) is the amplitude of virtual flux. And then, by utilizing the estimated grid voltage flux phase angle \( \hat{\theta} \) of PLL output for synchronous rotating coordinate transformation, the virtual grid voltage flux in dq reference frame can be expressed as,

\[
\begin{bmatrix}
\psi_{\alpha_d} \\
\psi_{\beta_d}
\end{bmatrix} = \begin{bmatrix}
\cos(\hat{\theta} - \theta) & \sin(\hat{\theta} - \theta) \\
-\sin(\hat{\theta} - \theta) & \cos(\hat{\theta} - \theta)
\end{bmatrix} \begin{bmatrix}
\psi_{\alpha} \\
\psi_{\beta}
\end{bmatrix}
\]

(15).

Substituting (14) into (15), the flux components in dq axis can be represented in another form,

\[
\begin{bmatrix}
\psi_{\alpha_d} \\
\psi_{\beta_d}
\end{bmatrix} = \begin{bmatrix}
\cos(\hat{\theta} - \theta) & \sin(\hat{\theta} - \theta) \\
-\sin(\hat{\theta} - \theta) & \cos(\hat{\theta} - \theta)
\end{bmatrix} \begin{bmatrix}
\psi_{\alpha} \\
\psi_{\beta}
\end{bmatrix} = \begin{bmatrix}
\cos(\Delta \theta) & \sin(\Delta \theta) \\
-\sin(\Delta \theta) & \cos(\Delta \theta)
\end{bmatrix} \begin{bmatrix}
\psi_{\alpha} \\
\psi_{\beta}
\end{bmatrix}
\]

(16).

It can be seen from the (16), if the error \( \Delta \theta \) between the estimated grid flux phase angle and the actual grid flux phase angle is equal to zero, i.e. \( \psi_{\alpha_d} = \psi_{\alpha} \) and \( \psi_{\beta_d} = \psi_{\beta} \). Therefore, we can know that the actual grid flux can be locked immediately by adjusting \( \psi_{\alpha_d} \) to zero, which has the advantage that no information is required about the magnitude of the actual grid flux. The virtual grid voltage flux observer based on PLL is shown in Fig. 2.

4) VF-based Grid Voltage Sequence Extraction Strategy

Under the balanced grid voltage condition, the grid voltage can be simply estimated by the expression of \( \hat{\psi}_{\alpha} = j \omega \hat{\psi}_{\beta} \) and \( \hat{\theta} = \hat{\theta} + \pi/2 \). However, for the unbalanced grid voltage, the relationship between the grid voltage and the virtual flux cannot be simply expressed as the expression mentioned above, due to the existence of negative sequence components of the unbalanced grid voltage. Thus, a VF-based grid voltage sequence extraction strategy is proposed to eliminate the adverse effects caused by the negative sequence components.

Under the unbalanced grid voltage condition, by neglecting the zero sequence components, the grid voltage can be described as the sum of positive- and negative-sequence components in αβ reference frame as

\[
\begin{align*}
v_{\alpha} &= v_{\alpha} + v_{\alpha}^n = V_{\alpha}^p \cos(\hat{\theta} + \phi_p) + V_{\alpha}^n \cos(-\hat{\theta} + \phi_n) \\
v_{\beta} &= v_{\beta} + v_{\beta}^n = V_{\beta}^p \sin(\hat{\theta} + \phi_p) + V_{\beta}^n \sin(-\hat{\theta} + \phi_n)
\end{align*}
\]

(17),

where \( v_{\alpha} \), \( v_{\beta} \) and \( v_{\alpha}^n \), \( v_{\beta}^n \) are the positive- and negative-sequence components of the grid voltage in αβ reference frame, \( V_{\alpha}^p \) is the amplitude of the grid voltage, and \( \phi_p \) and \( \phi_n \) are the initial phase angle, \( \hat{\omega} \) is the grid angular frequency.

Considering the negative-sequence components of the grid voltage are not zero under unbalanced grid voltage, the virtual grid voltage flux can be expressed as follows:

\[
\begin{align*}
v_{\alpha} &= \int (v_{\alpha}^p \sin(\hat{\theta} + \phi_p) - v_{\alpha}^n \sin(-\hat{\theta} + \phi_n)) \, dt = \frac{1}{\omega} (v_{\alpha}^p - v_{\alpha}^n) \\
v_{\beta} &= \int (v_{\beta}^p \cos(\hat{\theta} + \phi_p) + v_{\beta}^n \cos(-\hat{\theta} + \phi_n)) \, dt = \frac{1}{\omega} (v_{\beta}^p + v_{\beta}^n)
\end{align*}
\]

(18).

Combining (17) with (18), the positive- and negative-sequence components of the grid voltage can be obtained as

\[
\begin{align*}
v_{\alpha}^p &= (v_{\alpha} - \hat{\omega}v_{\beta})/2 \\
v_{\alpha}^n &= (v_{\alpha} + \hat{\omega}v_{\beta})/2 \\
v_{\beta}^p &= (v_{\beta} + \hat{\omega}v_{\alpha})/2 \\
v_{\beta}^n &= (v_{\beta} - \hat{\omega}v_{\alpha})/2
\end{align*}
\]

where the grid voltage in αβ reference frame is estimated by utilizing the second order generalized integrator-based quadrature signal generator (SOGI-QSG) [31], \( \hat{\omega} \) is the grid angular frequency obtained by the PLL. And the analysis process is shown in the following part.

By neglecting the current of filtering capacitor, the continue-time model of grid-tied inverter with LCL filter in αβ coordinate system can be represented as

\[
\begin{align*}
v_{\alpha} &= v_{\alpha} - (L_1 + L_2) \frac{d}{dt} [i_{\alpha}] \\
v_{\beta} &= v_{\beta} - (L_1 + L_2) \frac{d}{dt} [i_{\beta}]
\end{align*}
\]

(20).

Since the in-phase and in-quadrature signals of the input signal can be obtained by using SOGI-QSG [31], and the differential of the sinusoidal signal can be transformed into the in-phase or inverted value of its quadrature signal. Equation (20) can be written in the following form,

\[
\begin{align*}
v_{\alpha} &= v_{\alpha} - \hat{\omega} (L_1 + L_2) \frac{d}{dt} [\hat{i}_{\alpha}] \\
v_{\beta} &= v_{\beta} - \hat{\omega} (L_1 + L_2) \frac{d}{dt} [\hat{i}_{\beta}]
\end{align*}
\]

(21),

where \( \hat{v}_{\alpha} \) and \( \hat{v}_{\beta} \) are the estimated grid voltage in αβ coordinate system. \( \hat{i}_{\alpha} \) and \( \hat{i}_{\beta} \) are the in-phase and in-quadrature signals of the input inverter-side voltage and grid-injected current respectively.

Substituting (21) into (19), the positive- and negative-sequence components of the grid voltage in αβ coordinate system can be deduced. Then, in order to get the balanced and sinusoidal grid-injected current, the negative sequence components of the grid-injected current are regarded as zero and the positive sequence components of the grid-injected current should satisfy the equations of \( \hat{P} = P_{ref} \) and \( \hat{Q} = Q_{ref} \) [32]. Hence, the reference of the grid-injected current in αβ
coordinate system can be deduced \((P_{\text{ref}} = 3 \text{ kW}, Q_{\text{ref}} = 0 \text{ var})\) in this paper as
\[
\begin{bmatrix}
\hat{v}_{\alpha} \\
\hat{v}_{\beta}
\end{bmatrix} = \frac{2 P_{\text{ref}}}{3[(\hat{v}_{\alpha})^2 + (\hat{v}_{\beta})^2]} \begin{bmatrix}
\hat{v}_{\alpha} \\
\hat{v}_{\beta}
\end{bmatrix} + \frac{2 Q_{\text{ref}}}{3[(\hat{v}_{\alpha})^2 + (\hat{v}_{\beta})^2]} \begin{bmatrix}
\hat{v}_{\beta} \\
-\hat{v}_{\alpha}
\end{bmatrix}.
\]
(22).

B. State Observer

The predictions in next sampling instant need to be calculated by utilizing the predictive discrete time model expressed in (5) for the FCS-MPC algorithm. However, it is worth noting that for evaluating (5), besides the grid voltage estimated by virtual flux observer, it is necessary to measure the inverter-side inductor currents, the capacitor voltages and the grid-injected currents. These state variables can be probed by current and voltage sensors, but increasing the cost and complexity of system. Therefore, to avoid the additional sensors, a full-order observer is adopted to estimate the inverter-side inductor currents, the capacitor voltages and the grid-injected currents based on the measured grid-injected current in this paper.

\[
\Delta x(k+1) = A \Delta x(k) + B_{\text{vopt}} y(k) + B_{\text{s}} \hat{y}(k) + L(y(k) - \hat{y}(k))
\]
(23),

where \(L = [l_1 \ l_2 \ l_3]^T\) denotes the observer feedback gain vector and \(C_i = [0 \ 1 \ 0]\) is the output matrix, which represents the measured state variable (grid-injected current in this paper). As expressed in (24), it can be found that the observability matrix is full rank, which indicates that the system is observable. Consequently, the eigenvalues of state observer can be assigned arbitrarily.

\[
\text{rank } [C_i A_i] = 3
\]
(24).

The dynamics of state observation error is derived as,
\[
\Delta x(k+1) = (A - LC_i) \Delta x(k)
\]
(25).

The conclusion we can draw from (25) that if the matrix of \(A_i \cdot LC_i\) is Hurwitz, the observation error is asymptotically stable. According to pole placement of the state observer, the characteristic polynomial of the observer can be set as,
\[
\text{det}(\bar{z} - A_i + LC_i) = (z - p_1)(z - p_2)(z - p_3)
\]
(26),

where \(p_1, p_2\) and \(p_3\) are the desired poles of the state observer. In order to acquire the states of \(L\), these desired poles of the state observer need to be ensured. It is typically easier to identify the poles first in the \(s\)-domain and then map them to the \(z\)-domain via \(z = \exp(sT_i)\). In the \(s\)-domain, the closed-loop characteristic polynomial can be expressed as \((s + \alpha_{od})(s^2 + 2\zeta_{od}\alpha s + \alpha_{od}^2)\). Then, the poles \(p_1, p_2\) and \(p_3\) in the \(z\)-domain can be described in (27).

Thus, the observer gain vector \(L\) can be deduced by solving the equation (26). A simple method to solve the equation (26) is using the MATLAB function, i.e. \textit{acker}. And, \(L = [-0.4196 \ 1.1663 \ 11.9272]^T\) is adopted in this paper.

\[
\begin{aligned}
p_1 &= \exp(-\alpha_{od} T_i) \\
p_{2,3} &= \exp[(\zeta_{od} \pm j\sqrt{1 - \zeta_{od}^2}) \alpha_{od} T_i]
\end{aligned}
\]
(27)

The pair of complex-conjugate poles, determined by \(\zeta_{od}\) and \(\alpha_{od}\), are set to decide the dominant dynamics of the estimation errors. The real pole \(\alpha_{od}\) is located at a higher frequency. \(\zeta_{od}\) is the damping ratio, usually set as 0.707. The range of the natural frequency \(\omega_{od}\) with respect to the resonance frequency of LCL filter is regarded from 0.5 to 1. And the value of \(\alpha_{od}\) is 5–10 times larger than the pair of complex-conjugate poles. A rule of thumb to choose the observer poles is to make them several times faster than the open-loop system dynamics.

C. Computational Delay Compensation

For the conventional FCS-MPC algorithm mentioned in Section II, the computational time is not addressed. However, when applying a dSPACE platform or DSP to verify the control strategy in practical systems, the calculations are time consuming and it will lead to a computational delay, which may degrade the control effect. Therefore, even if this computational time is very short, it also needs to be taken into account in the control algorithm. Otherwise, the system performance will be greatly degraded.

\[
\begin{aligned}
\hat{\eta}(t+1) &= \hat{\eta}(t) + \hat{\eta}(t) \\
\eta(t+1) &= \eta(t) + \hat{\eta}(t)
\end{aligned}
\]
(28)

\[
\begin{aligned}
\eta(t) &= \hat{\eta}(t) \\
\eta(t) &= \eta(t)
\end{aligned}
\]
As displayed in Fig. 4, assuming that the computational time is not negligible compared with the sampling time, there will be a delay between the sampling and the action instant of new switching state. In order to eliminate the influence of the computational delay, the control algorithm with delay compensation is presented, and its operation process is described in Fig. 4. Firstly, the estimated variables obtained from the state observer and the inverter output voltage vector at \( t_k \) obtained at \( t_{k+1} \) are applied in (5) to predict three state variables in next sampling interval. Then, the discrete-time LCL filter model is shifted one step forward in time and those three state variables at \( t_{k+1} \) are employed as starting points for the next-step predictions. Next, the predictions at \( t_{k+2} \) can be obtained by using seven different inverter output voltage vector. Finally, these predictions are taken into the cost function for evaluating. The switching state, which minimizes the cost function, is selected and stored to be applied at next sampling instant.

D. Overcurrent protection

Generally, there is an overshoot current at the first period for the sensorless control method, since the observations do not have any information until the system starts up. It also exists in the proposed control method in this paper. The approach of solving the overcurrent phenomena as described below can be also found in IV-C of [33].

Based on the proposed cost function \( J \) in this paper, an additional term \( h_{\text{lim}} \), which is used to realize overcurrent protection, is added. It is defined in the following type:

\[
h_{\text{lim}} = \begin{cases} 
  \infty & i_2 \geq I_{\text{max}} \\
  0 & i_2 < I_{\text{max}} 
\end{cases}
\]  

(28)

where \( I_{\text{max}} \) is the current protection value.

Thus, the cost function can be further expressed as:

\[
J_1 = J + h_{\text{lim}}
\]  

(29)

It can be seen from (28), if the amplitude of the grid-injected current is larger than the current protection value \( I_{\text{max}} \), the additional term \( h_{\text{lim}} \) of the cost function \( J_1 \) will be set to \( \infty \). Therefore, the cost function \( J_1 \) will be considered to \( \infty \), and the corresponding inverter voltage vector \( v_1 \) will not be chosen. If the amplitude of the grid-injected current is smaller than the current protection value \( I_{\text{max}} \), the additional term \( h_{\text{lim}} \) of the cost function \( J_1 \) will be set to zero, and it will not affect the original cost function \( J \). Consequently, this method can successfully realize overcurrent protection.

IV. EXPERIMENTAL RESULTS

Since a large amount of sensors are used for the conventional FCS-MPC algorithms applied in the grid-tied inverter with LCL filter, a novel FCS-MPC strategy based on full status estimations is proposed to reduce the number of sensors, but not deteriorate the performance of system. The performance of the proposed FCS-MPC strategy with full status estimations based on virtual flux and state observer are evaluated under both balanced and unbalanced grid voltage condition on a laboratory test-rig. The programmable ac source (Chroma 61830) is used to simulate the grid. The power stage consists of a two-level voltage-source inverter (Danfoss-FC320) with a dc-link voltage provided by Chroma 62150H-600S DC power supply. The digital control algorithm is implemented in dSPACE 1202 platform, where a controldesk project is developed to tune control parameter and reference value, as well as display the observation results which cannot be captured by Yokogawa DL 1640 digital oscilloscope. It should be noticed that the following waveforms are all based on pure observations throughout the experiment. In all measured signals, including the grid voltage, the grid-injected current, the capacitor voltage and the inverter-side inductor current, only the grid-injected current is applied for VF observer and state observer estimation. The measured signals are only utilized for comparisons with the observation results. The experimental parameters are listed in Table I.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_1 )</td>
<td>Grid voltage</td>
<td>110V_{\text{max}}</td>
</tr>
<tr>
<td>( U_{\text{dc}} )</td>
<td>DC voltage</td>
<td>350V</td>
</tr>
<tr>
<td>( L_1 )</td>
<td>Inverter-side inductance</td>
<td>3.6mH</td>
</tr>
<tr>
<td>( L_2 )</td>
<td>Grid-side inductance</td>
<td>2.8mH</td>
</tr>
<tr>
<td>( C )</td>
<td>LCL filter capacitance</td>
<td>12uF</td>
</tr>
<tr>
<td>( T_s )</td>
<td>Sampling time</td>
<td>40\mu s</td>
</tr>
<tr>
<td>( f_{\text{sw}} )</td>
<td>Average switching frequency</td>
<td>5.8 kHz</td>
</tr>
<tr>
<td>( \lambda_u )</td>
<td>Weighting factor of capacitor voltage</td>
<td>0.0826</td>
</tr>
<tr>
<td>( \lambda_3 )</td>
<td>Weighting factor of grid-injected current</td>
<td>87</td>
</tr>
<tr>
<td>( L )</td>
<td>Gain vector</td>
<td>([-0.4196 1.1663 11.9272]^T]</td>
</tr>
</tbody>
</table>

A. Estimated Results of State Variables and Grid Voltage

Fig. 5 shows the steady-state estimation waveforms of state variables and grid voltage captured by the controldesk project. It can be found that the estimations are consistent with the measurements, and the errors of them caused by uncertain disturbance are relatively small, which can be acceptable.

The performance of the state observer and virtual flux observer also should be evaluated during a transient response for assessing the performance of the overall system. Fig. 6 displays the transient observation waveforms of the state variables and the grid voltage. The reference of active power is set at a value of 1.5 kW and then stepped up to 3 kW, i.e. the reference of grid-injected current is set at a value of 6.28 A (peak) and then stepped up to 12.86 A (peak). The experimental results demonstrate that, the state observer and the virtual flux observer estimate the state variables and the grid voltage accurately, and the observation of grid-injected current reaches to its steady state in approximately 2 ms, performing a good dynamic response.

B. Experimental Results under Balanced Grid Voltage

1) Starting process

Unlike the conventional measurement-based control, the sensorless control cannot give as good operation in the first period, because the estimations do not have any information until the system starts up. Fig. 7 displays the transient waveforms of grid-injected current after starting up. By the pro-
proposed control strategy with overcurrent protection, the system can converge and achieve grid synchronization after one and a half grid period. It can be illustrated that the strategy of overcurrent protection is effective.

2) Steady State Waveforms

The experimental steady-state waveforms and the harmonic spectrum of grid-injected current under the stiff grid condition are depicted in Fig. 8. Due to the limitation of the experimental device, only a-phase and b-phase currents are measured. It can be seen that the grid-injected current is sinusoidal with low THD and the average switching frequency is about 5.8 kHz. And, the grid synchronization is achieved, which enhances the energy efficiency via unity power factor operation.

3) Dynamic State Performance

Fig. 9 displays the experimental transient waveforms of grid-injected current and grid voltage under the stiff grid condition when the reference of active power steps up from 1.5 kW to 3 kW, and steps down from 3 kW to 1.5 kW, respectively. (i.e. the reference of grid-injected current amplitude steps up from 6.43 A to 12.86 A, and steps down from 12.86 A to 6.43 A, respectively). The experimental results demonstrate that the grid-injected current is able to accurately track the reference within approximately 2 ms, reflecting the good dynamic response and reliability of proposed control method. Fig. 10 illustrates the experimental result of the grid-injected current with sharp phase angle variation from 0° to 30°. It can be seen from Fig. 10 that the effectiveness and good dynamic response for adjusting the power factor can be achieved by using the proposed control strategy. Fig. 11 shows the experimental waveforms of grid-injected current and grid voltage for a 25% three phase grid voltages dip, and it can be concluded that grid-injected current can track the reference value accurately and quickly so that keep the active power constant.

4) Parameter Mismatch Influence

Model parameter mismatch error is unavoidable for observations due to the must-use of system model. The weak grid is one of the most common parameter mismatch situations. Due to the possible wide variation of grid impedance in the actual grid, this weak grid situation should be emulated by adding the additional grid inductance \( L_g \) in the experimental tests. As depicted in Fig. 12, it shows the experimental transient waveforms of grid-injected current and grid voltage under weak grid condition \( (L_g=4 \text{ mH}) \) when the reference of active power steps down from 3 kW to 1.5 kW. It can be seen from Fig. 12 that the proposed control strategy is robust against the grid impedance variations, and it also has a good dynamic response even under the weak grid condition. To further verify the robustness of proposed control strategy, the inverter-side indu-
Fig. 9 Experimental transient waveforms of grid-injected current and grid voltage under the stiff grid condition: (a) step-up from 1.5 kW to 3 kW, (b) step-down from 3 kW to 1.5 kW.

Fig. 10. Experimental waveforms of grid-injected current with sharp phase angle variation from 0° to 30°.

Fig. 11 Experimental waveforms of grid-injected current and grid voltage for a 25% three phase grid voltages dip.

Fig. 12 Experimental transient waveforms of grid-injected current and grid voltage under the weak grid condition ($L_1 = 4$ mH).

Fig. 13. Experimental waveforms of grid-injected current with filter parameter mismatch ($L_1 = 2.4$ mH, $C \approx 9 \mu$F, $L_2 = 2$ mH).

Fig. 14 Experimental waveforms of grid-injected current when grid voltage varies (a) from balance to unbalance, (b) from unbalance to balance.

The voltage dip is generally caused by the failure of the power grid like short circuit faults, or sudden changes in the load, such as the start of high-power equipment. It can cause many problems in control of the inverter. Fig. 14 demonstrates experimental results of grid-injected current when grid voltage varies from balance to unbalance and changes from unbalance to balance. The unbalanced grid voltage is emulated by 40% grid voltage dip in phase B. As depicted in Fig. 14, the grid-injected currents are balanced under both balanced and unbalanced grid voltage conditions, and the dynamic response is fast. It can sufficiently verify the effectiveness of proposed FCS-MPC strategy under unbalanced grid voltage condition.
2) Under Distorted Grid Voltage

Actually, in the case of grid voltage dips, often accompanied by harmonics. Thus, the proposed control algorithm should also be robust enough to handle this kind of disturbance. Fig. 15 shows the experimental waveforms of grid-injected current and grid voltage under both unbalanced and distorted grid voltage. The unbalanced grid voltage is emulated by 40% grid voltage dip in phase B. The grid voltage is distorted by the 3rd, 5th, 7th and 9th harmonics, whose magnitudes with respect to the grid fundamental voltage are 3%, 3%, 3%, 3%, respectively. The experimental result displays an almost sinusoidal current waveform, which is mainly due to the low equivalent switching frequency of 5.8 kHz.

Fig. 15 Experimental waveforms of grid-injected current and grid voltage under both unbalanced and distorted grid voltages.

V. CONCLUSION

In this paper, a novel FCS-MPC strategy with full status observations based on virtual flux and state observer is proposed for the LCL-filtered grid-tied inverter. A step-by-step design procedure of the proposed control strategy is described in detail. A 3-kW/3-phase/110V experimental platform is established to verify the performance of proposed control scheme. By theoretical analysis and experimental verification, following conclusions can be drawn:

1) The proposed control strategy can achieve the grid synchronization, the delay compensation, and the high quality grid-injected current both under balanced and unbalanced grid voltage condition. Note that only the grid-injected current is measured.

2) The proposed strategy can be applied as the backup control of system by utilizing other control method with more sensors, where the grid-tied inverter still need operate under some sensor-fault conditions. Therefore, as a backup control, the proposed strategy shall inevitably improve the control reliability and enhance the sensor fault-through capacity of original system. Certainly, it may be at the cost of performance degradation, due to the switching and sampling frequencies.

APPENDIX

\[ A_1 = \begin{bmatrix}
    L_4 + L_2 \cos(\omega_{rs} T_i) & L_2 \left(1 - \cos(\omega_{rs} T_i)\right) & \sin(\omega_{rs} T_i) \\
    L_4 + L_2 & L_2 \cos(\omega_{rs} T_i) & \omega_{rs} L_4 \\
    L_2 \left(1 - \cos(\omega_{rs} T_i)\right) & L_2 \cos(\omega_{rs} T_i) & \omega_{rs} L_2 \\
    \sin(\omega_{rs} T_i) & \sin(\omega_{rs} T_i) & \sin(\omega_{rs} T_i)
\end{bmatrix}. \]

\[ B_1 = \begin{bmatrix}
    \frac{T_i}{L_4 + L_2} & \frac{L_2}{L_4 + L_2} \sin(\omega_{rs} T_i) \\
    \frac{T_i}{L_4 + L_2} & \frac{L_2}{L_4 + L_2} \sin(\omega_{rs} T_i) \\
    -\frac{T_i}{L_4 + L_2} & -\frac{L_2}{L_4 + L_2} \sin(\omega_{rs} T_i) \\
    -\frac{T_i}{L_4 + L_2} & -\frac{L_2}{L_4 + L_2} \sin(\omega_{rs} T_i)
\end{bmatrix} \]

where \( \omega_{rs} = \sqrt{\frac{L_4 + L_2}{L_4 L_2 C}} \).

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