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Abstract: Taking its point of departure in the origin of the notions of "diagram" and "iconicity" in Peirce's philosophy of logic, this paper reviews and discusses a series of dimensions along which such diagrams may be compared, measured and subdivided: diagrams versus images and metaphors, operational versus optimal iconicity in diagrams, diagram tokens versus diagram types, diagrams as general signs; corollarial versus theorematic diagram reasoning; pure versus applied diagrams; logic diagrams versus diagrams facilitating logical inferences; continuous versus discontinuous diagrams; diagrams in non-deductive reasoning. Most of these developments occur in the mature Peirce after the turn of the century and thus form an important part of his mature semiotics – yet, they do not relate in any simple or straight-forward manner to his attempts at enlarging his combinatorial semiotics from its bases in the three-trichotomy theory of the 1903 *Syllabus* over the six-trichotomy theory of 1904–1906 to the sketchlike ten-trichotomy version of 1908, where diagrams rarely figure in the names of sign-types discussed – why?

Keywords: semiotics, Peirce, diagrams

1 From the 1885 "algebra of logic" to the 1903 image-diagram-metaphor trichotomy

The very notions of diagram and iconicity grows out of Peirce's life-long strive for the analysis and formalization of logic and reasoning, only to reach center stage in his mature logic and semiotics after the turn of the century. The first thorough reflection upon diagram reasoning thus occurs in the connection with his linear formalization of first-order predicate logic in the second "Algebra of Logic" paper in 1885. In that paper, he constructs the first version of modern formal logic with quantifiers, what is now called the prefix-matrix distinction and the norm of prenex normal form of expressions (isolating the quantifiers in the left side of the formula and the relational proposition in its right, "Boolean" side). Via Schröder, Peano, Russell, etc., that formalism, with

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minor changes in single sign conventionsgns but essentially no changes in syntax and meaning, developed into modern “symbolic” logic. Peirce, however, in the principal semiotic introduction to the paper, strongly emphasized that such a formalism could not be exclusively symbolic but was forced to make use of both iconic, indexical, and symbolic signs. The single conventional signs with a general meaning were symbolic; quantifiers pointing out how to determine the objects referred to were indexical; while icons appeared at two important levels: that of predicates which, though symbols, must imply some sort of iconic description of objects, on the one hand, and that of the overall arrangement of symbols and indices in a spatial syntax which cannot be generally described nor indicated but must be *shown*, in order for the reader to observe the relations between signs and become able to reason by permuting those signs. As to the latter issue, the iconicity of the syntactic relations between symbols and indices in assertions, Peirce wrote:

> With these two kinds of signs alone [indices and symbols, fs] any proposition can be expressed; but it cannot be reasoned upon, for reasoning consists in the observation that where certain relations subsist certain others are found, and it accordingly requires the exhibition of the relations reasoned within an icon. (CP 3.363)

The overall structure of logic formalisms, then, must be iconic. Peirce develops the notion of how to reason with such formalisms as follows:

> … all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those of the parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts. (CP 3.363)

This is probably his first close connection of diagram experiments with deduction. It is only, however, in connection with Peirce’s development of his mature semiotics after the turn of the century that this germ of a theory of diagrammatical reasoning unfolds, partially in connection to Peirce’s development of the alternative logic formalism of Existential Graphs (EGs) from around 1897 where the selection of each single convention is delicately tested for its degree of iconicity – and partially as a generalization from the experience with the EGs to a general doctrine of diagrammatical reasoning, covering mathematics and deduction as such.

In the same period, Peirce undertook the final development of his theory of signs, beginning in the *Minute Logic* (1902) and the various drafts of the *Syllabus* and the Pragmatism and Lowell Lectures (1903). It seemed natural to embed the flowering notion of diagrams in that endeavor, and a much-quoted attempt of that theoretical fusion occurred with the image-diagram-metaphor trichotomy (as a small section of the attempt of integration of logic and semiotics addressed in Pietarinen, this volume). The central quote regarding this trichotomy stems from the early versions of the *Syllabus* manuscript on which Peirce worked in 1903 and which furnishes the first drafts of his mature sign theory and its principle of defining signs in a combinatory of sign aspects:

> Hypoicons may be roughly divided according to the mode of Firstness of which they partake. Those which partake of simple qualities, or First Firstnesses, are *images*; those which represent the relations, mainly dyadic, or so regarded, of the parts of one thing by analogous relations in their own parts, are *diagrams*; those which represent the representative character of a representamen by representing a parallelism in something else, are *metaphors*. (CP 2.277; EP 2: 274)

This brief quote is the sole argued presentation of the image-diagram-metaphor trichotomy, and numerous interpreters have attempted to further develop that promising distinction. The notion of diagram is the only one among the three terms of the trichotomy to receive thorough discussion in Peirce's work – so the determination of "image" and "metaphor" as technical terms in the classification of signs has not much more than this brief quote to build upon. The strange thing is that two conflicting criteria seem to compete in the short quote. Images and diagrams seem to be classified as First and Second among icons after the adicity of the relations they depict: images use "simple qualities, or First Firstnesses," that is, monadic relations such as "_is red," "_is round," an idea which fits nicely with the brief description of diagrams as representing by means of "relations, mainly dyadic." This principle would obviously lead one to expect, then, that the third term of the trichotomy, metaphors, would be characterized by representing *triadic* (or maybe polyadic) relations. This is not the case, however, and it would indeed be a strange description of metaphors. They, by contrast, are described by means of the existence of an intermediary object, involving a parallelism, between the sign and its object. This parallelism supposedly pertains to the relational structure which is mapped from object to sign via the intermediary object, thus resulting in a triad in the sign-object representational structure which is normally conceived of as dyadic. But that is a completely different issue than triadic relations *within* the object and its depiction. So the first criterion seems to point to the adicity of relations involved, that is, concerning which aspects of the object are depicted in the icon, while the second criterion rather concerns the degree of complexity of the sign-object relation, and it is difficult to see how it could be generalized to cover the definition of "image" and "diagram" which, both of them, accord to the standard dyadic sign-object relation.

The problem seems to be that none of the two competing criteria are really generalizable to all three cases which is probably why they change along the triad. As mentioned, it would seem strange if icons depicting triadic relations should form a special category, and it also does not seem really correct that diagrams depict “relations, mainly dyadic, or so regarded” only, as indicated in the famous quote. Rather, triadic relations seem firmly within the scope of diagrammatic depiction – in another part of the *Syllabus*, Peirce explicitly defines a graphical representation of triadic relations: “… a point upon which three lines of identity abut is a graph expressing the relation of *teridentity*” (CP 4.406).

The triadic structure of the metaphor, of course, would have a nice dyadic sign-object counterpart in the diagram and its analogous mirroring of relational structure between the part-whole structure of the sign and that of its object – but here images would suffer, for they could not work with one entity only, they must also display the basically dual sign-object structure; if not, they would cease to be signs at all.[1] You might say that the sign-object duality is more *pronounced* in diagrams while the sharing of simple qualities in images may tend to make a confusion or even a merging or indistinguishability between the sign and its object, which share basic qualities, more probable, cf. Peirce’s early claim that “Icons are so completely substituted for their objects as hardly to be distinguished from them” (“On the Algebra of Logic,”, 1885; CP 3.362, EP 1: 226 – but here, his example of such hard-to-distinguish icons are not images and their supposed simplicity, but the “diagrams of geometry”). Thus, a clear 1–2-3 structure in terms of complexity of the icon-object sign relation also does not really seem a possibility. The fact that metaphorical mappings may take all adicities of relation as their basis of the structure mapped also argues against any idea of the metaphor as defined by adicity of the relation depicted (cf. examples like the croissant pastry metaphorically named after its shape in common with the crescent moon [adicity 1]; the father-children metaphor for the relation between king and people [adicity 2]; the nuptial gift from groom to bride as a metaphor for love [adicity 3], etc.).

Not only is the proposed tripartition not developed any further in the semiotically fertile years after the *Syllabus*, it also comes in variant versions in discarded drafts of the *Syllabus*:

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1 A possible way might be to insist that images are those icons which particularly realizes the fact that “Icons are so completely substituted for their objects as hardly to be distinguished from them.” (“On the Algebra of Logic,”, 1885, W 5: 163; CP 3.362), so that in *images*, icon and object almost merge, in *diagrams* the two are distinct, and in *metaphors*, they connect via an intermediary. That would remain approximate only; Peirce does not go in this direction, and it also does not seem to grasp the essence of diagram signs.

> Icons may be distinguished, though only roughly, into three which are icons in respect to the qualities of sense, being *images*, and those which are icons in respect to the dyadic relations of their parts to one another, being *diagrams*, or dyadic analogues, and these which are icons in respect to their intellectual characters, being *examples*. (MS 478, Alt. version 44, ISP 174)

Here, the third member of the trichotomy is no longer metaphors, but "examples." No further explanation of the category is given, but the idea seems to be that general conceptions are thirds, and icons functioning as examples of such conceptions are then general pictures (maybe like the typicalized mushroom drawing illustrating the general description of a mushroom species in a field guidebook for mycologists or collectors of fungi – such a drawing must take care to include all of the typical general feature of the species in question and is, in that sense, a general depiction). Again, such an explanation will lack the expected triadic relations in the third category after the monadic qualities and dyadic relations defining its first two members – maybe that was why metaphors were substituted for it in the final version.

So, something fishy remains about the attempts of constructing a tripartition of the hypoicon, and it is probably no coincidence that this attempt at a taxonomic determination of the diagram never turns up again in Peirce's many discussions of definition and description of the essentials of diagrams and diagrammatical reasoning in the following years.

Rather, it seems like three different issues are confused in the image-diagram-metaphor triad. One is the degree of skeletal part-whole analysis of the object in the icon sign – which does indeed come in a range of very different shades from images to diagrams. Quite another is the iconic depiction of relations of different adicities. And a third is the issue of direct versus indirect, mediated sign-object reference. So the conclusion seems to be that Peirce saw three important issues relating to icons and tried – in a quick but unsuccessful attempt, in his hasty, intensive work on the *Syllabus* in the fall of 1903 – to synthesize them.

A related issue here is that the trichotomy of image-diagram-metaphor is not presented as a subdivision of icons as such, but rather of "hypoicons," that is, actual, mixed signs which are primarily iconic by nature. This implies two things: that hypoicons are actual, concrete functioning signs, that which only a bit later in Peirce's elaboration of his combinatorial semiotics in the *Syllabus* is covered by the "sinsign" or "token" category of the qualisign-sinsign-legisign trichotomy, developed only in the course of his work on the *Syllabus*. The other thing is that hypoicons are *mixed* signs, with both indexical and symbolical aspects in addition to their central, iconic aspect. The former issue may be dealt with by regarding "hypoicon" as a preliminary term for what, in the 10-sign taxonomy of the later *Syllabus*, would appear as “rhematic iconic sinsigns.” That is the road taken by Farias and Queiroz (2006) who go on to try and trace the different subtypes of that category in Peirce’s more developed (but also much more tentative and unfinished) 66-sign taxonomy of 1906–1908. The lack of any definitive definition nor sequence of the 10 trichotomies whose combination would yield the 66 sign types, however, gives rather different combination results depending upon which definitions and sequence are chosen. Even if this road may eventually prove fruitful, once the character and sequence of the 10 triads are finally determined, it still would not address the second issue, the fact that hypoicons seem to cover not only iconic sinsigns – a pretty narrow category – but also mixed signs with an emphatic aspect of iconicity in general. The diagram type of the topographic map, to take an example, is indeed predominantly iconic but at the same time, it typically is a dicisign claiming to be true of a particular landscape indicated in the map by means of the indices of proper names added to the diagram structure; moreover, the single, token map on paper or screen can only be understood as a token of the related diagram type, and this type, again, has a *general* object, namely the general features of the landscape which are depicted in the sign, and thus functions as a symbol. Other examples include arguments completed by means of diagram transformation where iconicity plays a prominent role – but where the sign use involved vastly transgresses the confines of iconic sinsigns only. Such connections are developed in detail in Peirce’s mature doctrine of diagrams, but they seem to fall under the table if we simply stay content with taking diagrams to form a subset under iconic sinsigns only.

So, my conclusion here is that the famous and much-quoted introduction of the concept of “hypoicon” and its associated trichotomy “image-diagram-metaphor” in the early sketches of the *Syllabus* not only forms a *hapax* in Peirce’s work, but also that they form a first attempt at solving issues which are better and more convincingly dealt with later – some of them already in the later versions of the *Syllabus* from the same year, where the first trichotomy, *quali-sin-legisign* (or *tone-token-type*), is introduced.

2 Operational versus optimal iconicity

The trichotomy of icon, index, symbol is not only one of the most well-known features of Peirce’s sign classifications, it is also early and constant already from the 1860s (even if the exact definitions and terminological notions used may vary). As to the hypostatic abstraction of the term “iconicity” as a property, which signs may possess to different degrees, it appears only around 1900 – in the same period where Peirce undertakes what Bellucci (2017) calls the "first reform" of the basic principles of Speculative Grammar, of semiotics, which is to pass from the classification of signs to the classification of semiotic parameters which combine, in turn, to give types of signs (Bellucci 2017: 211, 285). That reform finally reached clarity in the "Minute Logic" of 1902, and "iconicity" and its antonym "aniconicity" appear in Peirce papers beginning in 1897 – all in the context of discussing the degree of iconicity of different logic representations (Stjernfelt 2015: 36). In that respect, Peirce certainly took EGs to be superior to his earlier Algebra of Logic – even if he himself from now on continued to use both, each of them seeming preferable over the other in certain contexts and for certain purposes.

Simultaneously, however, Peirce developed a definition of iconicity based on *operations* using an icon. Thus, the "Algebra of Logic" of 1885 and the Beta system of EGs were equivalent because the same set of theorems could be proved by operating with them.[2] In that sense, they were iconically equivalent. In another sense, however, EGs were taken to be the iconically superior system – an example being that in the EGs, one and the same variable is referred to by means of *one* continuous "Line of Identity" (however branched it may be), while that same reference necessitated *several* different occurrences of the same sign in the Algebra of Logic. Take the proposition "Everybody loves somebody," which, expressed in the 1885 Algebra of Logic, would be:

$$\Pi_{x}\Sigma_{y}l_{x,\,y}$$

Here, Π is the universal quantifier, Σ the existential quantifier, l the two-place predicate "_loves_" and *x* and *y* two variables. It consists of an indexical and an iconical part – the "Hopkinsian" part of the quantifiers provide an index of which objects are spoken about,[3] while the "Boolean" part of the predicate

2 This is due to an important reformulation of what it means for a sign to be iconic, namely, the following: "For a great distinguishing property of the icon is that by the direct observation of it other truths concerning its object can be discovered than those which suce to determine its construction" ("That Categorical and Hypothetical Propositions are one in essence, with some connected matters" c. 1895, CP 2.279) – which I have called the "non-trivial icon definition." Cf. Pietarinen's discussion of semantic completeness in his contribution to this volume.

3 After Peirce's Johns Hopkins student O. H. Mitchell who was the first to propose the sifting of quantifiers and predicate expressions into two separate parts of the linear formalism.

with variables provide a symbolic icon of the two-place relation of loving. Finally, the syntax uniting the two parts by putting them together on the line is, as quoted in the beginning of this paper, iconic. At the same time, all of the single signs are conventional symbols. In the EGs, the same proposition would be represented as in Figure 1.

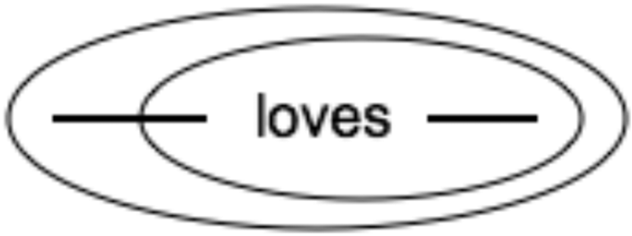

Figure 1: Beta Graph for "Everybody loves somebody".

The enclosures formalize negation, the two variables are represented by the two Identity Lines at each end of the predicate, and so the graph expresses that it is not the case that there exists someone who does not love someone. This, now, is more iconic, so Peirce, because each variable is here represented *once*, each by its Ligature (the two horizontal lines), while in the algebraic expression, *x* and *y* occur *twice* each. So, the representation *one variable – one occurrence* is more iconic than several occurrences of the same variable. Even if operationally equivalent, then, the latter of the two representations is more iconic than the former. The optimality principle may be expressed as follows: "A diagram ought to be as iconic as possible; that is, it should represent relations by visible relations analogous to them" ("Logical Tracts. No. 2. On Existential Graphs, Euler's Diagrams, and Logical Algebra," 1903, MS 492). Thus, I argued in Stjernfelt (2011a, 2014) that two different notions of iconicity are, implicitly, at stake in the mature Peirce – *operational* iconicity which focuses upon which implicit propositions may be made explicit by the manipulations of a certain diagram – and *optimal* iconicity, distinguishing even between formalisms which are otherwise operationally equivalent. Both iconicity concepts are important, each for their purpose, so it is perfectly reasonable that Peirce applies both of them. I argue, however, that it is preferable to provide them with explicit terminology so as to be able to make clear which of them you appeal to in a particular case or argument.

3 Diagram tokens versus diagram types

A very elementary distinction is that between the individual physical diagram token on a piece of paper, on the screen, on the blackboard, etc. – and the *type* of diagram which is instantiated in those different tokens. This, of course, is based on the *type-token* or *legisign-sinsign* distinction of Peirce's first trichotomy, introduced during the work on the *Syllabus* in 1903, which holds for all signs which are identically repeatable. As no two physical objects are exactly identical, the type comes in tokens, which necessarily have a lot of individual, different, superfluous properties that have to be bracketed by the observer in order to grasp the type. Given a particular spoken word, e.g., dialect, prosody, voice pitch, speed of pronunciation, etc. may have to be bracketed in order to grasp the word type instantiated. The same goes for diagrams: color, thickness of lines, insufficient rectilinearity of lines, imprecision of drawing and much more may have to be bracketed as accidental token qualities (Peirce: "prescinded") in order for the simpler, ideal type of diagram to be reached. In many cases, the information of which properties are essential to the type and which ones are accidental and thus only belong to certain tokens of that type, comes quasi-automatically, from tradition, context, learning, etc. – in other cases it requires specification by means of explicit symbolic legends, axioms, rules, etc., accompanying the diagram:

> Accordingly, every diagram must be supplemented by certain general understandings or explicit rules, which shall warrant the substitution for one diagram of any other conforming to certain rules. These will be rules of permissible substitution, partly limited to the special proposition, partly extending to an entire class of diagrams to which this one belong. ("An Appraisal of the Faculty of Reasoning," around 1906, MS 616)

Very often, such rules, implicit or explicit, will be co-motivated by the general type of *object* which is depicted by the diagram. The diagram as type or legisign, namely, furnishes the precondition for diagrams to function symbolically, that is, being not only general in themselves, but also possessing a general meaning, as when we take a type of a parabola as the general (hence, symbolic) diagram for the trajectory of all falling bodies in a field of gravity without friction.

4 Diagrams as general signs and as conclusions of arguments

A diagram is a type that is instantiated in tokens – but how does it come about that the diagram type is, in turn, capable of referring to a general object? The reference to general objects is the very definition of the Peircean sign type of symbols[4] – does this mean that the diagram has to be accompanied by explicitly added symbols in order to accomplish the reference to general objects, or may the diagram perform, in itself, the symbolic task of general object reference?

Sometimes, Peirce expresses himself as if the former were the case, as if diagrams invariably need the assistance of explicit symbols in order for them to perform general reference. Take for instance the following:

> A diagram appeals to the eye like a picture, while it differs from a picture in that it obtrusively involves conventional signs. A conventional sign has, since Aristotle and earlier, received the name of symbol; but besides conventional symbols there are signs of the same nature except that instead of being based on express conventions they depend on natural dispositions. They are natural symbols. All thought takes place by means of natural symbols and of conventional symbols that have become naturalized. (Lowell Lecture II A, 1903, MS 450: 6)

The detailed exposition of the general reasoning process, which Peirce puts forward in the 1906 MS known as "PAP," may also be interpreted in that direction:

> Meantime, the Diagram remains in the field of perception or imagination; and so the Iconic Diagram and its Initial Symbolic Interpretant taken together constitute what we shall not too much wrench Kant's term in calling a Schema, which is on the one side an object capable of being observed while on the other side it is General. ("PAP," 1906, NEM 4: 318)

This idea makes of the diagram a descendant to Kant's famous notion of the Schema, uniting the generality of understanding with the particular exposition of intuition, by coupling the Diagram with its Symbolic Interpretant, the two furnishing the observable and the general aspect of the Schema, respectively. Similarly, in the other end of the reasoning process, Peirce concludes:

> The transformate Diagram is the Eventual, or Rational, Interpretant of the transformand Diagram, at the same time being a new Diagram of which the Initial Interpretant, or signification, is the Symbolic statement, or statement in general terms, of the Conclusion. By this labyrinthine path, and by no other, is it possible to attain to Evidence; and Evidence belongs to every Necessary Conclusion. ("PAP," 1906, NEM 4: 318)

4 Often, the erroneous idea is met that Peircean symbols should be defined by being conventional signs. That is not the case. Peirce explicitly, many times, says that symbols may be conventional or natural – the superordinate concept of the two subtypes being "habit." Bellucci has insistently pointed to the fact that symbols are defined by being general signs – in the double sense that they are signs general in themselves (legisigns), which, moreover, have a general meaning and refer to general objects (e.g. Bellucci 2017: 65).

After having been transformed, the final diagram presents a new symbolic interpretant which is the conclusion and its statement expressed in general terms. Thus, we may get the idea that there are symbols in the beginning and end of the diagram transformation process, while the intermediary manipulation phases dispense with generality, such as Peirce noted 20 years earlier in his first presentation of diagram reasoning in connection to the Algebra of Logic:

> A diagram, indeed, so far as it has a general signification, is not a pure icon; but in the middle part of our reasonings we forget that abstractness in great measure, and the diagram is for us the very thing. So in contemplating a painting, there is a moment when we lose the consciousness that it is not the thing, the distinction of the real and the copy disappears, and it is for the moment a pure dream, – not any particular existence, and yet not general. At that moment we are contemplating an icon. ("On the Algebra of Logic," Peirce 1885: 180–181)

During the reasoning phase, we bracket the abstractness and treats the diagram icon as if it were the object itself. Still, I think the important analogy with Kant's Schema may lead us astray in presupposing that in order to make a statement – that is, assert a proposition – the diagram must necessarily involve the support of an *additional* symbol. In the two PAP quotes, symbolicity was not something which was *added* by means of a further sign to the diagram, rather, both in the initial and final phase, symbolicity lay in the initial *interpretant* of the diagram sign, that is, in the immediate, obvious meaning of the sign. But the meaning of a sign is not an additional entity, but part and parcel of the sign itself.[5]

This has to do with the fact that diagrams are, themselves, "iconic legisigns," they are types which may appear in many different individual, material tokens. As signs, they are general, and this combined with the insight that icons are often difficult to distinguish clearly from their object, and even in certain phases of reasoning are *identified* with their object, makes it easy to see that they

5 You may indeed wonder how those two, initial and final symbolic interpretant come to be with diagrammatic bases. Peirce was happy to cite, working on "How to Reason" in 1893, the quote "Omne symbolum de symbol" ("Symbols," CP 2.302), all symbols come from symbols, which seems not to cover the two cases mentioned where two diagrams have a symbolic interpretant, so that symbols in these cases come from diagrams. In his mature period, after the development of the EGs, it seems to have dawned on Peirce that diagrams may themselves appear in propositions making general claims, so emphasis shifts toward propositions, cf. "... no sign of a thing or kind of thing – the ideas of signs to which concepts belong – can arise except in a proposition; and no logical operation upon a proposition can result in anything but a proposition; so that non-propositional signs can only exist as constituents of propositions" ("An Improvement on the Gamma graphs," 1906, CP 4.583). To function as propositions, of course, diagrams are unsaturated predicates which need to be saturated by the addition of subjects in the shape of indexical signs pointing out which object they refer to, but that has nothing necessarily to do with symbols.

may transfer that generality to their object and hence function symbolically (cf. symbols as sign with general objects).

So, let us instead investigate the possibility that the symbolic generality is something in or about the diagram sign *itself*. It is interesting that in several of his definition of symbolicity, Peirce explicit mentions images or diagrams which may function as symbols, e.g. when he once more defines the icon-index-symbol trichotomy and comes to the symbol:

> … it may, as a "Symbol," be related to its object only because it will be represented in its interpretant as so related, as is the case with any word or other conventional sign, or any general type of image regarded as a schema of a concept … (1904?, MS 914: 8)[6]

Sometimes, Peirce even *identifies* a general idea as such with a diagram able to be applied to a manifold of objects:

> It is from that experience that we draw our conception of existence, – if it can be called a conception. Properly it is not a conception; because a conception is general idea, – a sort of picture or diagram which we think of as variously applicable. (Lowell Lecture III(b), 1903, MS 462)

Variable applicability – generality of meaning – is the central defining property of symbols. Peirce also entertains the idea that the diagram and its conclusion may be so complicated that ordinary languge is unable to express it, which implies that we should not expect that the symbolicity of the conclusion of a diagram transformation necessarily lies in its translation from diagram back to language:

> It will also be seen that the reasoning must in all cases be at bottom schematic, or diagrammatic, or else it will not be reasoning at all, but the mere practice of the rule of thumb; and furthermore, the diagram must be made very prominent by alterations being made in it, in the course of the demonstration, or else no vigorous advance in knowledge will be made. As a general rule the diagram will be so complicated that ordinary language is put to a severe strain to express it at all, even though facile perspicuity be not attempted; and naturally the clear mental representation of the problem, and then the invention of the proper alteration of the diagram, call for the closest of thought. ("Minute Logic," 1902, MS 430, 431a. Chapter III of Minute Logic)

Regarding Peirce's obsession with the development of formalisms representing elementary logic, we must ask: how is the conclusion of an argument expressed in those formalisms? Is it sufficient to present the transformate diagram which

6 MS 914 is undated in the Robin catalogue, but as it speaks of a combinatory of six sign trichotimies, it must be from around 1904–1906 when Peirce went from three to six trichotomies, only to expand to ten in 1908.

states the theorem reached in the system of the formalism – like the two formalizations of the proposition "Everybody loves someone" above taken as conclusions to preceding arguments? That is indeed not sufficient, but not because the conclusion is in need of further symbolic elaboration. It is rather because the conclusion, taken *in nuce*, does not represent which previous reasoning process lead to accept that conclusion as a result. Here, again Peirce speaks of the Beta version of the EGs formalizing first order predicate logic:

> I will now only say that, which this system does present Semes, yet it would not be incorrect to say that everything scribed according to this system, down to its smallest parts, is a Pheme; and is not only a Pheme, but is a Proposition. Delomes (dee'loamz) also are brought to view. Yet no Delome (dee'loam) is ever on the diagram. A Graph in this system is a type which expresses a single proposition. Without just not troubling you with an adequate description of the Delome (dee'loam), I may point out that it represents no statical determination of thought, but a process of change from one state of belief to another. So, let us agree that this new system of diagrammatizing the course of thought, which you and I, Reader, are to build up again from the foundation together, as a new thing to you, and to me a critical review and reading of what I have done before by myself, – let us, I say, make it our very first germ of the plan of this system that the diagram, in any one state of it, shall represent a state of belief, usually a pretended state merely, upon some point or points of some subject. A Delome (dee'loam) shall be represented by a series of such diagrams imagined to be phenakistoscopically combined. ("Prolegomena to an Apology for Pragmaticism," 1906, MS 295)[7]

In his newly adopted Seme-Pheme-Delome terminology for the generalization of the Term-Proposition-Argument trichotomy (see Bellucci's contribution to this volume) which already lay in the observation that Dicisigns may use proposition structure in many different speech acts, Peirce sees that a single sheet of graphs cannot present anything but propositions. Inferences are made by series of manipulations with the figures on the sheet so as to constitute Delomes – Arguments. But that implies that the Argument may be presented as a *cartoon of a series of diagrams*. The "moving picture of thought" may be represented, in the absence of actually performing the manipulation with a token diagram, by a series of snapshots of the changing conformation of diagram structure during the phases of reasoning, maybe brought to life by the technical representation of as a movement. Had Peirce known Disney or computer graphics, the depiction of the actual diagram movements and changes – with a replay button for control – might have constituted an alternative. But only when you have access to the

7 The "phenakistoscope" was a nineteenth-century technology to animate a simple sequence of pictures of a movement by means of displaying them on a turning wheel seen through a slit.

whole of the transformation history – like the steps of a mathematical proof – you are able to synthetically grasp the conclusion *as* a conclusion. It also presupposes, of course, that the general meaning of the diagrams used is kept in mind (even if maybe bracketed during phases of diagram manipulation). But it does not require *additional* symbolic signs to make the conclusion evident.

This is obviously an issue of actually mastering the formalism. Early phases of learning will include instruction and thus the use of symbols in the shape of linguistic symbols. In case you have practiced with the formalism and have become a skilled user, you are able to appreciate the conclusion directly from the diagrammatic formalism without necessitating a previous translation-back into ordinary language or other additional symbols. The very cartoon sequence of diagram phase snapshots may be, in that case, sufficient. But *all* symbols, being based on "rules of thumb" as they are, are habits that require learning, onto- or phylogenetically, so there is nothing strange if learning by practicing the rules of a diagram system provides the condition for the direct understanding of the general meaning of a transformate diagram. Skilled map readers – scientific diagrammers, military campaign commanders or orienteering athletes – similarly may turn directly to action in the field after consulting their detailed map diagram, without any necessity of translating their conclusion into supplementary, non-diagrammatical signs before understanding it. Of course, such translation may be very useful, even indispensable, for communication purposes: "Follow the small ditch in NNE direction until you reach a water mill, everybody!" But it is not necessary in order to grasp the general conclusion from the map manipulation undertaken. Cf. how Peirce presents the appreciation of "evident consequences" of his two types of diagram reasoning without any appeal to accompanying signs (more about those two types below):

> Here I will tell you a secret about necessary consequences. It is a very useful thing to know, although most logicians are entirely ignorant of it. It is that not even the simplest necessary consequence can be drawn except by the aid of *Observation*, namely, the observation of some feature of something of the nature of a diagram, whether on paper or in the imagination. I draw a distinction between *Corollarial* consequences and *Theorematic* consequences. A corollarial consequence is one the truth of which will become evident simply upon attentive observation of a diagram constructed so as to represent the conditions stated in the conclusion. A *theorematic* consequence is one which only becomes evident after some experiment has been performed upon the diagram, such as the addition to it of parts not necessarily referred to in the statement of the conclusion. (Lowell Lecture II, 1903, MS 455, MS 456, NEM 3: 419)

We shall return to the corollarial-theorematic distinction below. Here, the important thing is evidence provided directly by diagram observation in each of the two cases. So, our conclusion is that the skilled diagram user may be able directly to appreciate the meaning of the final, transformate diagram in a general, that is, symbolic conclusion – without the aid or support from further symbolic signs. In that sense, diagrams function as symbols – having a general meaning and object.

5 Levels of generality in diagrams

An important issue here is that the type reading of the diagram gives a special sort of *observational* access to general features of the sign, so that any sterile dualism between perception and conception is avoided and those general features are subject to acts of perception involving a special, generalizing attitude. And the generality of the diagram sign itself is what makes it possible for diagrams also to perform the symbolic act of referring to general objects. Maps, e.g. may refer to general features of the landscape; algebraical expressions, e.g. may refer to general regularities of arithmetics, empirical or a priori. Such generality, however, comes in many different degrees. A topographic map is general because, unlike an aerial photograph of the same area, it does not depict a particular moment's snapshot information of the object territory, but an information covering an indefinite timespan, typically in the range of years (until, that is, landscape changes affecting the features mapped necessitate a redrawing of the map). Such generality, of course, has a much narrower span than, say, diagrams of the structure of the solar system which may be valid for millennia or more, not to speak about the eternal diagrams of geometry, supposedly valid at any time. The generality of diagrams, thus, comes in many different degrees and may refer to empirical regularities of different timescales as well as to atemporal a priori structures.

6 Diagram experiments versus real experiments

Logical inferences in diagrams are reached by a process which Peirce very often describes as experimentation with the diagram. The idea is that something must be *done* – the very collection and synthetization of the premises into one overarching diagram sometimes directly gives the conclusion; in other cases, that synthetic diagram must be further processed by manipulation, moving, deleting or substituting parts of it, adding further parts to it, etc. These procedures are undertaken as an open trial-and-error process, attempting to reach the intended conclusion, which is why the analogy with the open-endedness of empirical lab experiments suggested itself to Peirce:

> Indeed, just there is the advantage of diagrams on general. Namely, one can make exact experiments upon diagrams, and look out for unintended changes thereby brought about in the relations of different parts of the diagram of one another. These operations in reasoning take the place of the experiments upon real things that one performs in chemical and physical research. Chemists have sometimes described experimentation as putting questions to "Nature." Experiments upon diagrams are question put to the Nature of the relations concerned. ("Prolegomena to an Apology for Pragmaticism," 1906, MS 292a, March 1906)

The analogy between empirical lab experiments and experiments with diagrams thus turns on two things: 1) their openness in the sense that it is not beforehand given what will be the result of the experiment, whether it will show up to support the hypothesis tested or not; 2) a wide conception of "Nature," which may mean both Nature in the sense of the empirical world studied by the special sciences and the structures, patterns and laws that it involves, one the one hand, and Nature in the sense of the structure of hypothetical, ideal relations studied by mathematics, supposedly a priori and subject to deduction, on the other. It is this which makes it possible for Peirce to include mathematics and philosophy under the classic empiricist requirement that all knowledge is by observation. This idea is not, however, connected to empiricist reduction attempts of general or abstract knowledge to results from secondary mental processings of empirical data. Rather, the idea is that observation may take several kinds of objects, both empirical, concrete particular objects in lab experiments (even if such objects are typically subjected to experiments as token examples of some type of object to which the knowledge collected pertains) – and general, ideal objects addressable due to their representation in diagrams.

7 Explicit versus implicit diagrams

Diagrams most often appear in externalized, physical media, like paper, blackboard, computer, etc. Simple diagrams, of course, may be imagined by the user without external support. But an even more indirect case is that where the diagram itself is only implicitly present, be it ex- or internally. Peirce, in MS 1214 (mid–1890s) described the process of synthezising two propositions into one, made possible by the fact that they share subjects. He takes the two propositions of "A loves B" and "B hates A" as an example and concludes that it is the co-localization of the two, which furnishes the relevant diagram:

> The student will ask: where is that diagram we have heard so much about? The reply is that in this case the diagram is so elementary, and the observation of it is so easy, that it escapes notice.
>
> Yet it is there. It consists of the two lines of writing
>
> A loves B
>
> B hates A
>
> We observe the sentence. We see that the B that ends the first begins the second. In imagination, we fit the second upon the end of the first, and look upon the loving of a hater of oneself as a character of A.
>
> The peculiarity of these conjunctive signs is not so much that they join two things as that they actually display a relation the like of what they signify. It is they that are entitled to be called demonstrative signs; although the word is by the usage of grammarians applied to signs that are merely indicative or finger-pointing. (MS 1214: 97–98, ISP 18–19)

Thus, here it is the very action of co-localizing[8] the two propositions on one and the same sheet, which suffices to make it clear that their subjects are the same and that they may be taken as one, complex proposition in which A is the lover of his own enemy. Thus, simple diagrammatic representations and manipulations may be so ingrained and obvious that their diagrammatical character may pass unnoticed by the users. This possibility may be closely connected to the proficiency of the user in practical diagram use where the *logica utens* interest lies in reaching and considering the conclusion as quickly and easily as possible, rather than dwell on the single steps taken to reach it. In many applied diagrams where the interest of the reasoner lies firmly with the subject matter considered and with the ease of reaching a conclusion, we may expect that the diagrammatical means used, its structure and possibilities, may be, to some degree, only implicitly present.

8 Co-localization

The realization that the simple co-localization of two propositions in order to connect them into one, complex proposition, forms a simple and widespread

8 Peirce does not use the concept of "co-localization" but I find it apt to describe the elementary procedure he is considering; cf. Stjernfelt (2014: ch. 4) and below. When discussing the co-localization of linguistic units on the page, Peirce uses the standard English notion "collocation" (meaning the idiomatic connection of words): "It is impossible for a word to have much of the iconic element. Syntax alone, and the collocation of words and sentences, can build up this sort of sign" (1893, MS 409: 106, from the "How to Reason" project. See also Stjernfelt in press b). "Co-localization" seem to me to generalize that idea also to non-linguistic signs including diagrams.

form of diagram reasoning was used by Peirce in one of his basic convention in the Existential Graph system – namely, that the scribing of two proposition symbols on the same area of the sheet represents the logical conjunction of the two. But this explicit convention of the EG system is a formalization of a practice used implicitly in a wide variety of diagrams in the wild.

This goes for the conjunction of several propositional signs (as in a road sign combining the sign meaning "parking prohibited" with the sign "roadworks ahead"), but it also appears in the coupling of subject and predicate to constitute a proposition. This is addressed by Peirce in

> Every proposition relates to something which can only be pointed out or designated but cannot be specified in general. "No admittance, except on business." over a door is a general proposition, but it relates to that door which may have no qualities different from these of some other door in some other planet … But the hanging of the sign over this door indicates that this is the one referred to. (MS 789, no date)

The subject of the general proposition is indicated by placing the predicate rheme *close* to the relevant door. This elementary spatial syntax is widespread in diagrams. When, in a topographical map, the name of a city is given *near* the sign indicating that city on the map, this co-localization is what provides the partial proposition of that map that this city bears the name indicated. When, in a geometrical diagram, the letter *b* is positioned close to the vertex of an angle, it is a sign that this angle will, in the proof attempt to follow, be referred to by the proper name of "b." When, in a matrix structure like a timetable, the slot indicated by the intersection of the column of Tuesday with the row of 8–9 a. m. may be filled in by the determination "Math," then the localization of that noun in that structural location gives the partial proposition that math will be taught during that Tuesday morning hour. We are so used to reading diagrams like the ones mentioned, that the peculiar properties of this co-localization very easily escapes us. In a certain sense, co-localization forms a primitive, diagrammatic ur-syntax – implying that the indexical closeness in space-time of two semiotic devices is iconically interpreted as indicating some form of connection between them. This is not to say that such co-localization may not be the subject of further conventionalization – in Western painting since the Renaissance, for instance, the convention has become widespread that if a name is added in the corner of the painted surface, it refers to the painter, while a name added on the frame refers to the person portrayed in the painting. Both of these, of course, are cases of co-localization, with a simple syntactical convention added, a convention which might have been different. But co-localization seems to form the basis of such conventions, not vice versa, and the combination could arguably not be subject to convention only, without regard to co-localization. We might imagine a convention informing us that the name of the man portrayed was not given on the picture frame, but rather in some specified region of Saturn. Such a convention, however, would have to be described symbolically, and that description, in turn, would have to be indexically connected both to the painting and the Saturnian location, e.g. by means of a telescope. So the user of that dispersed set of semiotic devices would have to stand in contact with both locations of the parts, in order to be able to undertake the co-localization required. Rather, all sorts of syntactic conventionalizations would be understandable as secondary, analytical elaborations and sophistications, for different purposes, of the basic, spatio-temporal, diagrammatical co-localization of sign parts. Linguistic syntax, e.g. in its many variants, will form elaborated conventions of how to co-localize signs in the close diagram structure of one single sentence or period. Simple, pre-grammatical co-localization abundantly appear in diagrams as the self-evident means of piecing diagram parts, e.g. diagram structure and names of diagram parts, together. Co-localization pertains not simply to the metric distance between the sign parts involved, rather, it rests upon a basic notion of topological connectedness (which may, of course, in many cases be realized metrically). But the basic status of co-localization in propositions seems to indicate that semiotic space is intrinsically diagrammatic: connecting things spatially may immediately imply their semiotic connectedness. Peirce formalized such an idea in his Existential Graphs in the notion of the blank sheet as the "Sheet of Assertion," containing *in potentia* all true propositions of the universe of discourse referred to – but this step seems to formalize a widespread use of local wild such sheets in ordinary sign use where the co-localization of terms is immediately interpreted as meaningful. This elementary and often-overlooked phenomenon of diagrammatic co-localization seems to be in need of further study: which possible types of co-localization are there, which types of conventionalizations of them?

9 Corollarial versus theorematic diagram reasoning – explicit versus implicit meanings of diagrams

Sometimes, like in the action of drawing near the two sign tokens of "A loves B" and "B hates A," the very co-localizing of the representation of two premisses is sufficient not only to form a synthetical proposition, but also for the conclusion to be directly read off of that proposition. In other cases, of course, like in

Euclid's major theorems, an ingenious manipulation with the diagram must be undertaken in order to reach that conclusion. This observation lies behind Peirce's distinction, after 1900, of two different form of diagram reasoning, named "corollarial" and "theorematic," respectively, after two inference types in Euclid, the easy "corollaries" and the more difficult "theorems" (cf. Stjernfelt 2011b, 2014). They now form two basic varieties of deduction in Peirce's system, and they correspond to what is explicitly, versus what is implicitly represented by a proposition: "The "meaning" of a proposition is what it is intended to convey. But when a mathematician lays down the premises of the Theory of Numbers, it cannot be said that he then intends to convey all the propositions of that theory of which the great majority will occasion him much surprise when he comes to learn them. If to avoid this objection a distinction be drawn between what is explicitly intended and what is implicitly intended, I submit that this manifestly makes a vicious circle; for what can it be implicitly to intend anything, except to intend whatever may be a necessary consequence of what it explicitly intended?" ("Logical Tracts. No. 2. On Existential Graphs, Euler's Diagrams, and Logical Algebra," 1903, MS 492) He may also say that the immediate interpretant of a proposition is all the obvious inferences from it, while the final interpretant is the sum of all possible inferences from it (which we might never be able to grasp in its totality).

Peirce was very proud of that distinction, in the Carnegie Application of 1902 he famously called it "my first real discovery," and this is how he presented it to James some years later:

> There are two kinds of Deduction; and it is truly significant that it should have been left for me to discover this. I first found, and subsequently proved, that every Deduction involves the observation of a Diagram (whether Optical, Tactical or Acoustic) and having drawn the diagram (for I myself always work with Optical Diagrams) one finds the conclusion to be represented by it. Of course, a diagram is required to comprehend any assertion. My two genera of Deductions are, first, those in which any Diagram of a state of things in which the premisses are true represents the conclusion to be true and such reasoning I call Corollarial because all the corollaries that different editors have added to Euclid's Elements are of this nature. Second kind. To the Diagram of the truth of the Premisses something else has to be added, which is usually a mere May-be and then the conclusion appears. I call this Theorematic reasoning because all the most important theorems are of this nature. (Letter to William James, Dec 28 1909)

Much can be said about the nature and varieties of Theorematic Deduction, suffice it here to point out that theorematic experiment with diagrams constitutes an important de-trivialization of deduction. Famously, Kant claimed, based on the simple example of the unmarried bachelor, that all information in the conclusion of a deduction was already given in the premises – from which it follows that deduction is little more than a banal repeating of what is already known, an idea which were taking further in logicism and its triumphant claim that mathematics was but a sum of logic tautologies. Theorematic reasoning, rather, points to the fact the process of deductively proving something may involve the work of making information explicitly which was only, maybe in remote and convoluted ways, *implicitly* given in the premises. Furthermore, it entails that there is not in all cases any algorithm of how to do disentangle that information. In many cases, auxiliary lines or other additional material must be added to the diagram. Such addition, of course, is subjected to the rules governing that specific type of diagram – but that is not the same as to know how those rules should be applied. So, an important trial-and-error phase here may concern which series of diagram experiments to undertake in order to reach the desired theorem proof. Famously, in mathematics, such processes may take centuries and involve many vain attempts and the effort of many generations of scholars. Only when the proof is actually reached, the clear chain of deductions may be isolated to stand out. Thus, Peirce's notion of theorematic diagram experiment importantly implies that the deductive experimentation with the idealized structures of diagrams may be hugely non-trivial.

10 Logic diagrams versus diagrams facilitating logical inferences

Discussing diagrams in general is something which Peirce predominantly does in the context of introducing and discussing formalizations of logic – the Algebras of Logic of the 1880s and the alternative formalizations of his Existential Graphs around 1900. Those formal representation systems, of course, have their aim in formalizing *logical* structure and relations like those normally associated with logical connectors ("second-intention icons" like *and, or, implies, not*, etc.), as in propositional logic and Alpha Graphs, as well as the addition of quantifiers and predicate structure in predicate logic and Beta Graphs. The doctrine of diagrams, however, is not restricted to the diagrammatization of logical relations which rather appears as a special concern of logicians.

Here, an important distinction is that between icons used for explicitly representing logical relations (icons of second intention) holding between symbols, indices and icons of first intention on the one hand versus icons appearing as predicates in propositions (icons of first intention) on the other (cf. Pietarinen and Stjernfelt 2015). The special icons of second intention are those concerned with the connectors and syntax of logical representation systems – the linear syntax of standard formal logic and the 2-D syntax of the Existential Graphs. Such icons serve to present logical structure in a way so that it is, simultaneously, observable and intelligible. In formal logic, the first-order icons are typically underdeveloped; predicate logic famously does not say much about the content of predicates except for their relational structure (is it a 0, 1, 2, 3, 4 … place predicate). Most often, the content of the predicate is represented by some natural language expression of it or its abbreviation (like “Red(x)” or “R(x)” meaning “x is red”). This in contradistinction to the vast amount of diagrams in mathematics, the special sciences, media, communication, etc., where it is structural aspects of the *subject matter* discussed which are predicated to play center stage and where the content of the relevant predicates used to describe that subject matter is diagrammatically depicted. Peirce often referred to such diagrams even if he rarely went far into analyzing them:

> For there is no way of representing complicated facts that begins to be so expressive as the way a diagram represents it. Compare for example all one could possibly carry away from a half-hour’s oral description of a tract of country with the idea one could gain of it from three minutes study of a good map, – supposing of course, that one is habituated to the use of maps. The latter idea would far surpass the former; and yet it would not nearly so much surpass the description as the knowledge one would gain from a halfhour’s study of the map would surpass that which could be gained from a five hours’ oral account; – to say nothing of the relative fatigue. (“Diversions of Definition,” MS 650: 8, 1910)

Here, the topographical map of an area is given as example. The focus in this quote lies upon the synthezising and information-condensing effect of landscape information in the map. But that should not give us the idea that diagrams of non-logical matter do not offer inference possibilities, quite on the contrary. One of the chief virtues of topographical maps is that they, just like logic diagrams, make inferences possible that makes explicit relations which were only implicitly given in the diagram. So even if maps do not represent logical relations (they obviously represent relations present in their object, in this case the landscape), they offer rich possibilities for making explicit relations which are not given as such in the diagram (e.g. the number of possible routes between two locations on the map, the relative sizes of woodland, agriculture and lake areas in a given district, the density of roads in an area, the height profiles of the landscape, etc.). So here, inferences may be made directly in the material of the predicate. In a map, the spatial structure given by coastlines, contours, roads, categorization of landscape parts, etc., forms the predicate aspect of the map diagram, while the proper names connecting that predicate to an existing object landscape form the index subjects of the diagram. The two co-constitute the whole of the map as one huge proposition – so in this case, the logical inference-drawing takes place by the manipulation of parts of the predicate (tagged, of course, with subject indices) describing landscape structure. In the vast majority of diagrams which are non-logical in this sense, it is still the possibility offered of drawing inferences by manipulating diagram parts that gives them their epistemological strength.[9] So, the conclusion is that the diagrams of logic formalizations constitute a very special subset of diagrams – characterized by selecting logical relations and structure as the particular object to be depicted – while the more general genus of diagrams may take any structured field, a priori or empirical or combinations of both, as its object of depiction. Peirce even made the argument that such manipulation of diagrams invariably contains a mathematical core (just like topographical maps involve the mathematical issue of 3D -> 2D projections mentioned). This lies in his mature identification of mathematics with ideal reasoning, which is possible only by making deductions which, again, are possible only in diagram manipulation. So, all diagrams are mathematical, and all of mathematics is diagrammatical. This entails that applied diagrams in different subject fields contain mathematical structure – even if, in simple cases, they are so easily readable that this may escape notice. Here, logic diagrams form that special case which uses mathematical structure to chart logic regularities (apparent in the name of *algebra* of logic for Peirce's 1885 system).

The distinction between logic diagrams and diagrams facilitating reasoning is connected to Peirce's distinction between "logica utens" and "logica docens." The former is characterized by logic in use, while the second is characterized by taking logic as its object of study. Logic diagrams form a central tool of the latter, and, as Peirce says, the best diagrammatic representation in this respect is that which provides as differentiated and detailed picture of the single steps of the logical reasoning process as possible. That is not the purpose, however, for other users of diagrams, e.g. by

9 I have often discussed this issue with my friend Ahti Pietarinen (see Pietarinen and Stjernfelt 2015) who maintains that as second-order Existential Graphs may be used to express virtually any content whatsoever, there is no need to consider other diagrams. I claim this is wrong, for two reasons. One is that a semiotic theory should be able to study how people actually reason, in the sciences as well as in everyday life – and they do that, to a large extent, by using diagrams which do not depict logical relations but rather the object field of interest. Another is that even if a the use of a topographical map might be translated somehow (even if I cannot really see how) into a second-order EG representation, my guess would be that actually finding your way in an area will be helped more by a good topographical map rather than by an EG representation.

mathematicians, who must be expert reasoners in the sense that they know how to make and manipulate diagrammatical formalizations of the mathematical structure they investigate – but who need not at all be expert reasoners in the sense that they are able to make explicit the logical principles put to use in their reasoning. They practice "logica utens" – like all other non-logicians using diagrams – while leaving it to the logicians to investigate logical structure by means of making explicit logical structure and process in "logica docens." So the distinction between logic diagrams versus diagrams facilitating inferences correspond to the *docens-utens* dichotomy, and the diagrams used correspondingly differ: from the *docens* requirement of diagrams best fit to make explicit the minutest detail of reasoning versus the *utens* requirement of diagrams best fit to quick and easy inference-drawing in the relevant universe of discourse and its subject field. This does not imply, of course, that diagrams of *utens* may not require highly skilled expertise for their construction, ongoing perfection and use, cf. the centuries-old process of perfection of mapmaking, involving the nontrivial issue of the geometry of projections from a 3-D sphere to a 2-D plane map, the determination of the "geoid," the precise shape of the earth in geodesy (a business to which Peirce was himself contributing during a large part of his life as a gravitation measurer for the American Coast Survey), the triangulation of landscape survey measurements from selected stations in the area, the rule-bound stylization of aerial photography, and much more, an expertise which involves both mathematical, physical and practical abilities. So, non-logical diagrams in this sense of diagrams not depicting logical structure but facilitating the drawing of inferences directly in the predicative structure of the sign forms a vast field of signs in the wild, from the complexity and variety of diagrams in the special sciences to elementary diagrams of media and everyday communication.

11 Pure versus applied diagrams

A closely related, but not identical distinction is that between pure and applied diagrams. It concerns whether a diagrams is used in application to addressing and formalizing a given empirical subject matter. Take again the example of a topographical map of a given area. If we bracket the reference of that diagram by deleting all proper names and place names from it, we can treat the resulting diagram as an unsaturated diagram rheme which no longer refers to any particular empirical area, but only to some possible, imaginary territory. In that sense, it is now a pure diagram icon as it, like all pure icons, only refers to possible areas and no longer functions with any accompanying indices making truth claims pertaining to an object. That is one level of purification where we isolate the diagram predicate from the propositional function of map. But we may go further and delete also the *legend* of the map (its scale, and all the standard explanations of how to connect map features with particular natural kinds of the ontology of geography (blue for lakes, green for woodland, contour lines for height, parallel lines for roads, etc.).[10] Now the map predicate not even refers to any possible geographical object preferentially – rather, more generally, it refers to any spatial object that happens somehow to share the shape of the naked line structure remaining in the map predicate. This structure, of course, may still be the object of geometrical and topological study in order to determine pecularities about its metrics, shape, topology, connectedness, etc. It will still be a geometrical diagram, a pure diagram which, given the addition of general, geographical ontology on the one hand and particular subject references to some concrete landscape on the other, may come to function as the core of an applied diagram of topography. But it is a corollary of Peirce's doctrine of diagrammatical reasoning, deduction and mathematics being co-extensive, that such a pure, mathematical object, simple or complex, explicit or implicit, must constitute the core of *every* diagram. So, under the regional ontological and referential cloak of any applied diagram, there must lie the pure, relational-skeletal part-whole core structure which makes the application possible.

12 Continuous versus discontinuos diagrams (are parts of the diagram also diagrams?)

The rational subdivision of diagram types forms one of the great challenges of diagram research, and Peirce only vaguely gestures at a taxonomic proposal, that of *maps, algebras*, and *graphs*, in that sequence. He describes them as composed from lines, from arrays of signs, or from both, respectively – without further giving many reasons for that proposal.[11] So the following interpretation attempt at that trichotomy is my responsibility. In Peirce's life-long obsession

10 Legend indications like the ones mentioned are given using the special *Syllabus* sign category of Dicent Indexical Legisigns, cf. Stjernfelt in press a).

11 In MS 15 "On Quantity" (ca. 1895, NEM 4: 275): "… a diagram, or visual image, whether composed of lines, like a geometrical figure, or an array of signs, like an algebraical formula, or of a mixed nature, like a graph …"

with the continuum, in metaphysics as well as mathematics, he reaches the idea that a continuum is a whole whose parts are homogeneous with the whole, are of the same kind as the whole. This is, of course, a recursive determination, holding also for the parts of parts, etc. Based on this definition, we may be able to distinguish continuous from discontinuous diagrams, depending on whether parts of a given diagram of the same dimensionality as the whole, are themselves diagrams. It is obvious that a part of a map is also a map, albeit of a smaller territory, as is a part of a part of a map, and so on (up to granularity, of course). It is equally obvious that an algebraic diagram does not in general have parts which are also algebraic diagrams: partial algebraic sign complexes may easily cease to satisfy grammatical norms, be deemed ungrammatical and refer to nothing, in the absence of the whole of the equation or expression they form part of. So, maps and algebras seem to be clearly distinguished by being continuous and discontinuous representations in this sense, respectively. It even fits well with their positions as firsts and seconds of Peirce's proposed triad.

What then about *graphs*, the supposed third member of the trichotomy? "Graphs" were also chosen by Peirce to be the technical term for the Existential Graph systems of his mature years, so we should expect that they exemplify the third category of the trichotomy. It is well-known that Peirce's trichotomies are not equal distinctions between independent phenomena – rather, the higher levels of the trichotomies typically involve specimens of the lower ones: algebras, albeit discontinuous, involve continua as represented in variables, e.g. Graphs being mixed signs involving both lines and arrays of signs, then, might more generally mean diagrams mixing continuous and discontinuous representations of their content. The EGs actually do that: they are basically a continuous topological formalism adding a strongly discontinuous interpretation convention: systems of closed, nonintersecting curves subdivide the plane into "even" and "uneven" regions, interpreted as regions of positive versus negative propositions. In that sense, they would be a candidate for a Graph mixing continuos and discontinuous diagram features.

Candidates for the Graphs category may also, literally, be continuous diagrams supported by algebraic expressions – the most well-known example of course being the Cartesian plane where all arithmetic equations may find expression in curves, and all curves may be seen as the expression of equations algebraically expressable. In that case, by contradistinction, maps would be continuous diagrams where the shapes are not analyzed while graphs would be continuous diagrams whose shapes are, to some degree, analyzed or conventionalized by algebraic means. Graphs would then differ from maps by explicitly furnishing analyses of spatial structure – while maps are simpler icons which may be recorded by some feature-preserving procedure but without the addition of any explicit analysis of their content. The process from an aerial photo to a topographical map via a series of stylizations and categorizations would, in that case, count as a move from a simple map in the direction of a graph in this sense?

13 Diagrams in non-necessary inferences

As we have seen, in Peirce's mature semiotics, diagrammatic reasoning form the defining feature of deduction, which, in turn, is identified with mathematics. But as the non-necessary inferences abduction and induction are defined as inversions of deduction, they are also involved with diagrams, although in different manners: "All necessary reasoning is diagrammatic; and the assurance furnished by all other reasoning must be based upon necessary reasoning. In this sense, all reasoning depends directly or indirectly upon diagrams" ("PAP," 1906, NEM 4: 314).

It is a major issue in the development of Peirce's mature semiotics that he vacillates in the ordering of abduction, deduction, and induction after the three categories. After proposing the abduction-induction-deduction ordering in the 1860's where the three follows a sequence after inferential strength from possible over probable to necessary, and their definition is based on whether their inference is based on likeness, indexicality or symbolicity, respectively, the standard view of the mature Peirce inverts the places of induction and deduction to the abduction-deduction-induction sequence. This is elaborated after the turn of the century where the combinatorial schema now demands there must be two subtypes of deduction and three of induction – giving rise to the corollarial/theorematic deduction and crude/quantitative/qualitative induction distinctions, respectively. This ordering also has the benefit that it follows the standard sequence of reasoning so that abduction, based on a surprising information, proposes an explanatory diagram as a hypothesis; this hypothetical diagram is then investigated in deduction, and various implications of it are laid bare; these hypothetical implications are then tested in induction where collections of samples of empirical material after different techniques check the validity of the deductive proposals. While the explanatory hypothesis in abduction is general and of a diagrammatic nature, the inductive testing seems to follow a principle where the ideal diagram results of deduction are measured on a sort of diagrams emerging from the empirical material:

> It is true that a distinctively mathematical reasoning is one that is so intricate that we need some kind of a diagram to follow it out. But something of the nature of a diagram, be it only an imaginary skeleton proposition, or even a mere noun with the ideas of its application and signification is needed in all necessary reasoning. Indeed one may say that something of this kind is needed in all reasoning whatsoever, although in induction it is the real experiences that serve as diagrams. (Lowell Lecture V(a) 1903: Multitude MS 459, MS 459[s])

Even if strangely matching, however, there seems to be no theoretical principle dictating that the phase sequence of inferences in inquiry should follow the order of their categorial structure – there are no other Peircean trichotomies where their 1–2-3 structure is made the basis for a corresponding 1–2-3 temporal process unfolding. The strange thing is that after having developed this abduction-deduction-induction ordering in some detail over the intensely creative years after the turn of the century, Peirce again gives it up in 1905 to return to the original abduction-induction-deduction categorial ordering. As Bellucci says:

> In Peirce's classification of signs of the years 1905 to 1908 … induction is connected with the category of secondness, while deduction is connected with the category of thirdness. Peirce thus finally reverted to the 1867 scheme, as announced in the draft of the Harvard lecture quoted above. As I noticed, the 1867 scheme does not fit with the principle of categorial subdivisibility and with the three-stages conception of scientific inquiry. Yet, it is a fact that Peirce reverted to his original position and maintained it in his subsequent classifications of signs … (Bellucci 2017: 211)

There are several quotes indicating that the mature, post-1900 Peirce vacillated with respect to this issue which is ironic given the fact that it was the issue of analyzing and classifying the three inference types which prompted his investigation of semiotics as a means to this classification in the 1860s in the first place. This classificatory enigma does not seem, however, to shake the relation of the three types to diagrams: the role of abduction is to propose an explanatory diagram which has the fact to be explained as its necessary implication; the role of deduction is to scrutinize and perform experiments on the diagram in order to trace and make explicit hitherto unseen implications of it; the role of induction is to make and analyze an empirical sample from the field in question, structuring the data in a diagram comparable to the ideal diagram results of deduction.

14 Conclusion

Diagrams and diagrammatical reasoning loom large in the mature Peirce's sign theory. His interest in these issues is constantly fueled by the development of the

Existential Graphs, and thus logic graphs naturally forms the particular examples occupying him the most. But simultaneously, important developments holding for diagrams in general take place again and again – some of which I have addressed in this paper, attempting to provide an overview over distinctions relevant for a Peircean diagrammatology. It may appear strange that diagrams, on the surface, may seem to take a peripheral role in Peirce's simultaneous attempts at further developing his three-trichotomy combinatorial semiotics into a six- and later ten-trichotomy version which is addressed by many papers in this special issue. The only taxonomical attempt at defining diagrams is the 1903 image-diagram-metaphor distinction which was given up and must be considered a failure. The reason for this seeming lack of connection between two of the important semiotic developments in the mature Peirce, however, is probably pretty simple. The reason is that, increasingly, diagrams came to be the central species of icons, becoming responsible for most if not all that is analyzed as iconicity in the Existential Graphs – and responsible for the ambitious, overarching identification of mathematics and deduction more generally. So, the diagram concept has grown so as to swallow most of the iconicity concept – thus when the classic icon-index-symbol triad is involved in the various combinatorial attempts of 1904–1908, diagrams generally come as contraband under the "icon" headline. Another important place where diagrams increasingly play center stage is in the ongoing issues with categorizing Peirce's three inference types, which sparked the whole development of his semiotics in the 1860's in the first place – *abduction, deduction*, and *induction*. With the identification of deduction with diagram experiment (and the less evolved insistence that even the other two inference types are dependent upon diagrams), this trichotomy of arguments places diagrams centrally in the various versions of the term-proposition-argument trichotomies.

If we take a look at Peirce's classic articulation of his mature semiotics – in the three-trichotomies, ten-signs combinatorial taxonomy developed in the 1903 *Syllabus*, it is true that diagrams appear nowhere as species name under the sign genus (nor under the icon genus). But that is probably because diagrams are involved, again and again, at so many places that they could not serve as single sign category. Thus, the Second sign category of "Iconic Sinsigns" give as examples diagram tokens, that is, individual, concrete diagram signs, which form Replicas of the diagram types mentioned as the typical example of the Fourth sign category of "Iconic Legisigns" – general iconic signs. Simple diagrams may appear already in the Fifth category of "Dicent Sinsigns," because such signs function by means of an index involving an icon – thus the simple diagram of a pointing arrow provides the icon giving the weathercock its predicative meaning. Again, diagrams may appear as predicates in propositions which is why they are there in the Eight category of Symbolic Rhemes – general, unsaturated predicates – as well as when those predicates may be saturated by the addition of index subjects to form full-blown Dicent Symbols, ordinary propositions which are now generalized to embrace non-linguistic (or only partially linguistic) examples of propositions such as those appearing in the EG formalizations. Finally, the crowning tenth category of Arguments are based on the stepwise experimental modification of Diagrams from premises to conclusion. The articulation of the proposition syntax is, in itself, diagrammatical, as is the level-higher structure of inference-drawing. In a certain sense, there are diagrams all over the place in the 1903 sign taxonomy, which forms, of course, the core of the more tentative six- and ten-trichotomies experiments.

My contention, in any case, is that there is a rich vein of important insights to be found in the mature Peirce's doctrine of diagrams – and that they lie at the core of also of the shifting immediate object trichotomies (and their attempt at grasping the interface between the diagrammatic predicate of propositions and their dynamical objects), as well as at the core the extension attempts in the direction of speech acts (where the triads addressing illocutionary acts and perlocutionary effects generalize the diagrammatical propositions and arguments from the 1903 doctrine). So, the reason that diagrams are not visible on the surface of the many different taxonomical proposals of the 1904–1908 period is rather that it is them, their structure, use, activities, purposes and results which those taxonomies aim to explain.

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