Modelling and Simulation of Wave Loads

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Publication date:
1985

Document Version
Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA):
Sørensen, J. D., & Thoft-Christensen, P. (1985). *Modelling and Simulation of Wave Loads*. Institute of Building Technology and Structural Engineering. Structural Reliability Theory Vol. R8503 No. 10

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STRUCTURAL RELIABILITY THEORY

PAPER NO. 10

Presented at the 13th IASTED International Conference on Modelling and Simulation, Lugano, Switzerland, June 24-26, 1985

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ABSTRACT

A simple model of the wave load on slender members of offshore structures is described. The wave elevation of the sea state is modelled by a stationary Gaussian process. A new procedure to simulate realizations of the wave loads is developed. The simulation method assumes that the wave particle velocity can be approximated by a Gaussian Markov process. Known approximate results for the first-passage density or equivalently, the distribution of the extremes of wave loads are presented and compared with rather precise simulation results. It is demonstrated that the approximate results are unconservative at least for the spectra used in this investigation.

Key words: Simulation, First-passage problems, Wave loads, Engineering modelling.

1. INTRODUCTION

Due to the non-linear relation between the wave characteristics of a sea state and the wave loading on slender members of e.g. an offshore structure it has until recently been necessary to make a number of approximations to be able to estimate the extremes of wave loads [1, 2]. Usually, the wave load is linearized and assumed to be Gaussian. Recently, Grigoriu [3] has shown how to determine exactly the mean outcrossing rate of a safe area of the wave load and how to determine approximations to the first-passage density function or equivalently, the distribution function of the extreme wave load.

In this paper a new simulation procedure to obtain estimates of the first-passage density function is described. The wave elevation in a sea state is modelled as a stationary stochastic process. Further, it is assumed that the stochastic process can be approximated by a Gaussian Markov process. It is described how this approximation can be made with high degree of precision.

2. MODELLING OF WAVE LOADS

The first step in modelling wave loads on slender cylindrical members is modelling of the sea state, Sarpkaya & Isacson [1] and Soares & Moan [2]. For convenience the water particle kinematics is here determined on the basis of a linear wave theory which connects the water depth d, the wave angular frequency ω, and the wave height H (and then the surface elevation η). The waves are assumed to be plane. The surface elevation is modelled as a Gaussian stochastic process $\{\eta(t), t \in [0, T]\}$ with a given spectrum $S_\eta(\omega)$. Two choices of $S_\eta(\omega)$ are investigated: the JONSWAP spectrum and the Pierson-Moskowitz spectrum [1].

The horizontal particle velocity u can now also be modelled as a Gaussian stochastic process with the spectrum

$$S_u(\omega) = |H_u(\omega)|^2 S_\eta(\omega) \tag{1}$$

where $H_u(\omega)$ connects u(t) and $\eta(t)$. The stochastic process $\{u(t)\}$ is modelled such that

$$E[u(t)] = u_0$$

$$\sigma_u^2 = 1$$

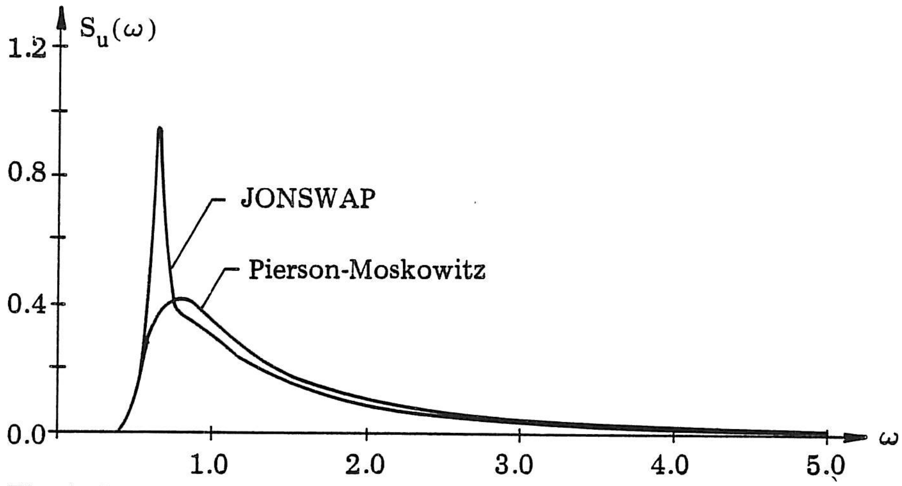

Fig. 1. Spectra for horizontal particle velocity u.

where E[ ] and σ^2 signify the expectation operation and the variance operation. u_0 can e.g. be interpreted as the current. In fig. 1 S_u is shown when S_η is the JONSWAP and the Pierson-Moskowitz spectra. The water depth d is chosen equal to 50 m and the frequency corresponding to the peak in the spectra $\omega_0 = 0.2\,\pi$.

Generally, the loads on structures composed of slender cylindrical members are calculated by using the Morison equation [1], [2] and Grigoriu [3], i.e. the loads are proportional to

$$P(t) = u(t)|u(t)| + a\dot{u}(t) \tag{2}$$

where $\dot{u}(t)$ is the wave particle acceleration and a is a coefficient which measures the relative importance of the non-linear drag forces and the linear forces of inertia.

Because of the non-linear relation (2) the stochastic process $\{P(t), t \in [0, T]\}$ which models the wave loads is not Gaussian. Therefore, P(t) is often linearized, [1], so that the wave loads become Gaussian. It has to be emphasized that there is considerable model uncertainty connected with using (2) to predict the wave loads.

3. ESTIMATES OF EXTREME WAVE LOADS AND FIRST-PASSAGE DENSITIES

The event of failure is defined as the event that the wave load exceeds some critical value B which is assumed to be time independent, i.e. the safe area is the interval

$$S =]-\infty, B[\tag{3}$$

Only the set Ω_1 of realizations of the process $\{P(t), t \in [0, T]\}$ which is in the safe area to the time t = 0 is considered. Let the first-passage density function be $\nu(t)$. The probability of failure of the process $\{P(t), t \in [0, T]\}$ in the time interval [0, t] is then

$$Q(t, B) = \int_0^t \nu(\tau)d\tau \tag{4}$$

The mean outcrossing rate of $\{P(t)\}$, $\nu_0(t) = \nu_0(t, B)$ is always greater than or equal to the first-passage density.

If the first-passage density is known, the distribution function of the stochastic variable P_{max} modelling the maximum load in the interval $[0, t]$ is

$$F_{P_{max}}(x) = P(P_{max} \leq x) = 1 - Q(t, x) \tag{5}$$

The density function of the maximum load can then be obtained by differentiation of $F_{P_{max}}(x)$.

For this modelling of the wave load it is not possible analytically to calculate the first-passage density. In section 4 a method to obtain simulation estimates of the first-passage density is described. In this section different approximations to the first-passage dentisy and the failure probability are described, Grigoriu [3].

First it is assumed that a = 0, i.e. only drag forces are assumed important. The density function f_D of P can then be found as

$$f_D(x) = \frac{1}{\sqrt{2\pi|x|}} \exp(-\frac{1}{2}(\text{sign}(x)\sqrt{|x|} - u_0)^2) \tag{6}$$

The mean $E[P_D]$ and variance $\sigma^2_{P_D}$ are then

$$E[P_D] = u_0^2(1 + u_0^{-2})(2\Phi(u_0) - 1) + 2u_0^{-1}\varphi(u_0)) \tag{7}$$

$$\sigma^2_{P_D} = u_0^4(3u_0^{-4} + 6u_0^{-2} + 1) - E[P_D]^2 \tag{8}$$

where φ and Φ are the standard normal density and distribution functions, respectively.

The mean outcrossing rate $\nu_{0,D}$ (on condition that all realizations initiate in the safe area) is

$$\nu_{0,D}(t,B) = \frac{\sigma_{\dot{u}}}{2\pi}(\Phi(\text{sign}(B)\sqrt{|B|} - u_0))^{-1} \exp(-\frac{1}{2}(\text{sign}(B)\sqrt{|B|} - u_0)^2) \tag{9}$$

where $\sigma^2_{\dot{u}}$ is the variance of the derivative process $\{\dot{u}(t)\}$.

In the literature it is often assumed that the drag force can be linearized. Then the drag force is normally distributed with mean $E[P_D]$ and variance $\sigma^2_{P_D}$. The variance of $\dot{P}_D$ is approximately

$$\sigma^2_{\dot{P}_D} = 4\sigma^2_{\dot{u}} u_0^2(1 + u_0^{-2}) \tag{10}$$

and the mean outcrossing rate $\nu^G_{0,D}$ is

$$\nu^G_{0,D}(t,B) = \frac{\sigma_{\dot{P}_D}}{2\pi\sigma_{P_D}}(\Phi(\frac{B - E[P_D]}{\sigma_{P_D}}))^{-1} \exp(-\frac{1}{2}(\frac{B - E[P_D]}{\sigma_{P_D}})^2) \tag{11}$$

If the critical value B of the force is relatively high it is reasonable to assume that the outcrossings of the safe area are independent and follow a Poisson process, Lin [4], i.e. the failure probability can be approximated by

$$Q(t, B) \cong 1 - \exp(-\nu_{0,D}(t, B)t) \tag{12}$$

and the first-passage density by

$$\nu(t) \cong \nu_{0,D}(t, B)\exp(-\nu_{0,D}(t, B)t) \tag{13}$$

Instead of $\nu_{0,D}$ the Gaussian approximation $\nu_{0,D}^G$ can be used.

If $a \neq 0$, i.e. the forces of inertia are important, the expected value and variance of P are

$$E[P] = E[P_D] \tag{14}$$

$$\sigma_P^2 = \sigma_{P_D}^2 + a^2\sigma_{\dot{u}}^2 \tag{15}$$

The variance of $\dot{P}$ is then

$$\sigma_{\dot{P}}^2 = \sigma_{\dot{P}_D}^2 + a^2\sigma_{\ddot{u}}^2 \tag{16}$$

where $\sigma_{\ddot{u}}^2$ is the variance of $\{\ddot{u}(t)\}$.

The mean outcrossing rate ν_0 of $\{P(t)\} = \{u(t)|u(t)| + a\dot{u}(t)\}$ can be approximated by, Thoft-Christensen & Baker [5],

$$\nu_0(B) \cong \int_{-\infty}^{\infty} \frac{1}{a\sigma_{\dot{u}}}\varphi(\frac{x}{a\sigma_{\dot{u}}})\nu_{0,D}(t, B-x)\,dx +$$

$$\int_{-\infty}^{\infty} f_D(x)\frac{1}{\sqrt{2\pi}}\frac{\sigma_{\ddot{u}}}{\sigma_{\dot{u}}}\varphi(\frac{B-x}{a\sigma_{\dot{u}}})dx \tag{17}$$

When the drag force is linearized the mean outcrossing rate can be approximated by

$$\nu_0^G(B) = \frac{1}{2\pi}\frac{\sigma_{\dot{P}}}{\sigma_P}\exp(-\frac{1}{2}(\frac{B-E[P_D]}{\sigma_P})^2) \tag{18}$$

If the outcrossings are rare the failure probability and the first-passage density can be approximated by (12) and (13) when ν_0 and ν_0^G are used instead of $\nu_{0,D}$ and $\nu_{0,D}^G$.

It is convenient to introduce the normalized critical value b defined by

$$b = \frac{B - E[P]}{\sigma_P} \tag{19}$$

4. SIMULATION OF WAVE LOADS

The basic simulation method used is a method described by Franklin [6, 7]. This method is used to simulate realizations of the normal stochastic process $\{u(t), t \in [0, T]\}$. In order to use Franklin's method it is necessary to approximate $\{u(t), t \in [0, T]\}$ by a normal Markov process $\{\bar{u}(t), t \in [0, T]\}$ which is characterized by the assumption that its spectral density can be written as follows

$$S_{\bar{u}}(\omega) = \frac{1}{2\pi}\left|\frac{C(i\omega)}{D(i\omega)}\right|^2 \qquad (20)$$

where

$$C(z) = c_0 z^m + c_1 z^{m-1} + \ldots + c_m z \quad , \quad c_i \in R \qquad (21)$$

$$D(z) = z^n + d_1 z^{n-1} + \ldots + d_n z \quad , \quad d_i \in R \qquad (22)$$

$S_{\bar{u}}$ is assumed to be integrable which implies $n > m \geqslant 0$. Further, the zeros of D have to fulfil $\mathrm{Re}(z) < 0$.

If $\{F(t)\}$ is a white noise stochastic process with mean zero, variance 1, and spectral density

$$S_F(\omega) = \frac{1}{2\pi} \qquad (23)$$

then $\bar{u}(t)$ is related to $F(t)$ by the following system of equations:

$$\bar{u}(t) = c_0 Z_{m+1}(t) + c_1 Z_m(t) + \ldots + c_m Z_1(t) \qquad (24)$$

$$Z_{n+1}(t) + d_1 Z_n(t) + \ldots + d_n Z_1(t) = F(t) \qquad (25)$$

where $Z_i(t)$ is the $(i-1)^{th}$ derivative of Z_1.

Franklin [6, 7] has shown that the vector process $\underset{\sim}{Z}(t) = (Z_1(t), Z_2(t), \ldots, Z_n(t))$ at the time $t + \Delta t$ can be determined by

$$\underset{\sim}{Z}(t+\Delta t) = \underset{\sim}{E}(\Delta t)\underset{\sim}{Z}(t) + \underset{\sim}{T}\underset{\sim}{W} \qquad (26)$$

where $\underset{\sim}{E}(\Delta t)$ is a matrix with constant elements which depend on Δt and $\underset{\sim}{T}$ is a matrix with constant elements and where $\underset{\sim}{W}$ is a stochastic vector with independent elements which are normally distributed with mean 0 and variance 1. Franklin has shown how $\underset{\sim}{E}$, $\underset{\sim}{T}$ and an initial realization of $\underset{\sim}{Z}$ can be determined for given coefficients $d_1, d_2, \ldots, d_n$.

Once realizations of $\underset{\sim}{Z}(t)$ at the times $0, \Delta t, 2\Delta t, \ldots$ are known realizations of $\bar{u}(t)$ at the times $0, \Delta t, 2\Delta t, \ldots$ can be determined using (24).

As seen from (2) realizations of the acceleration $\dot{\bar{u}}(t)$ are also necessary in order to determine realizations of the wave load $\{F(t)\}$. Realizations of $\dot{\bar{u}}(t)$ can be simulated, using instead of (24),

$$\dot{\bar{u}}(t) = c_0 Z_{m+2}(t) + c_1 Z_{m+1}(t) + \ldots + c_m Z_2(t) \qquad (27)$$

Simultaneous realizations of $\bar{u}(t)$ and $\dot{\bar{u}}(t)$ are calculated using the same realization of $\underset{\sim}{Z}(t)$ in (24) and (27). $\bar{u}(t)$ and $\dot{\bar{u}}(t)$ are then independent as required.

Because the spectrum of $\{\dot{\bar{u}}(t)\}$ is assumed to be integrable the degrees of C and D have to fulfil the requirement

$$n - 1 > m \geqslant 0$$

In order to estimate the first-passage density function when a realization of the stationary process $\{P(t)\}$ is known the following relation is used which is valid when the safe area is time independent, Rice [10]

$$\nu(t) = \nu_0(0, B)(1 - F_Y(t)) \tag{28}$$

where F_Y is the distribution function of the stochastic variable Y, representing the length of time between succeeding in and outcrossings of a certain realization. Further, $\nu_0(0, B)$ can be estimated by

$$\nu_0(0, B) = \frac{1}{E[Y]} \tag{29}$$

5. EXAMPLES

In the following example a sea state is considered which is modelled as shown in section 2, i.e. the spectrum of the wave particle velocity is either the JONSWAP spectrum or the Pierson-Moskowitz spectrum. These spectra are then approximated by rational spectra as required by the simulation method in section 4. This curve fitting problem is solved using an optimization algorithm developed by Schittkowski [8, 9]. In figures 2 and 3 the results are shown for the JONSWAP spectrum when (n = 4, m = 1), and (n = 6, m = 4) are used.

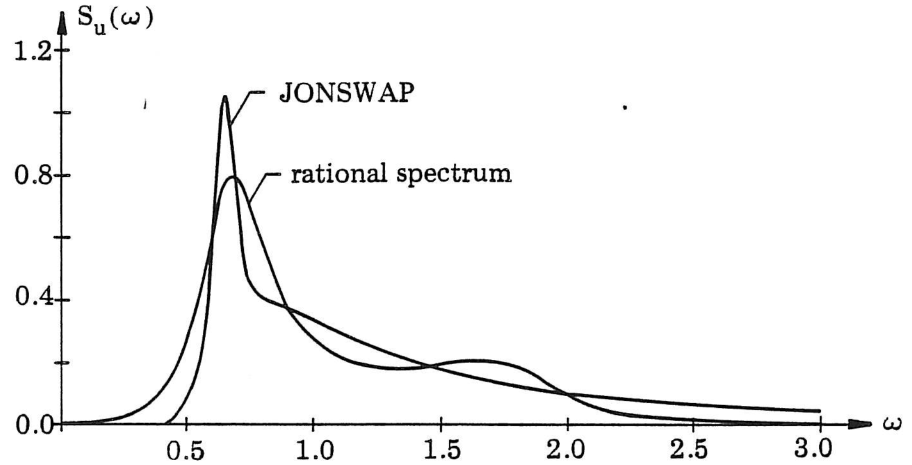

Figure 2. Approximation of JONSWAP spectrum. n = 4 and m = 1. d_1 = 0.9839, d_2 = 4.053, d_3 = 1.352, d_4 = 1.496, c_0 = 0.8274, and c_1 = 0.

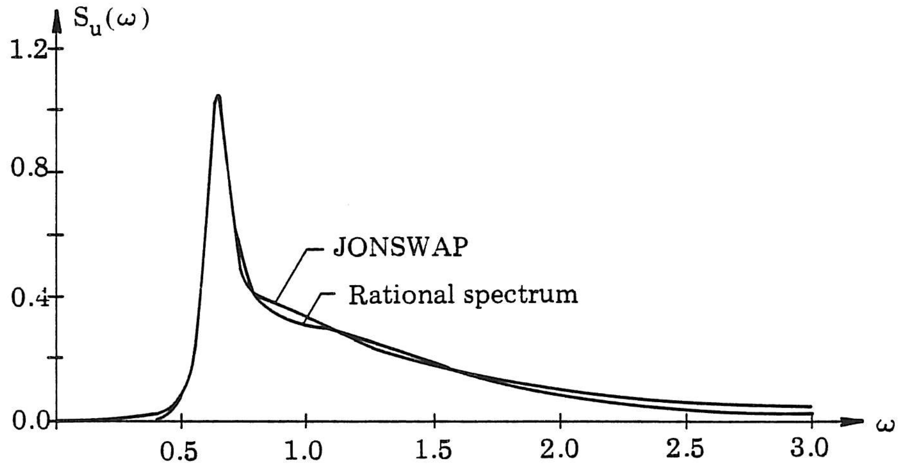

Figure 3. Approximation of JONSWAP spectrum. n = 6 and m = 3. d_1 = 10.13, d_2 = 27.54, d_3 = 38.85, d_4 = 32.44, d_5 = 14.49, d_6 = 7.266, c_0 = 10.14, c_1 = 3.063, c_2 = 2.834, and c_3 = 0.

From the figures 2 and 3 it is seen that with n = 6 and m = 3 a rather good approximation is obtained.

The approximation of the Pierson-Moskowitz spectrum is shown in figure 4. Also here a very good approximation is obtained with n = 6 and m = 3.

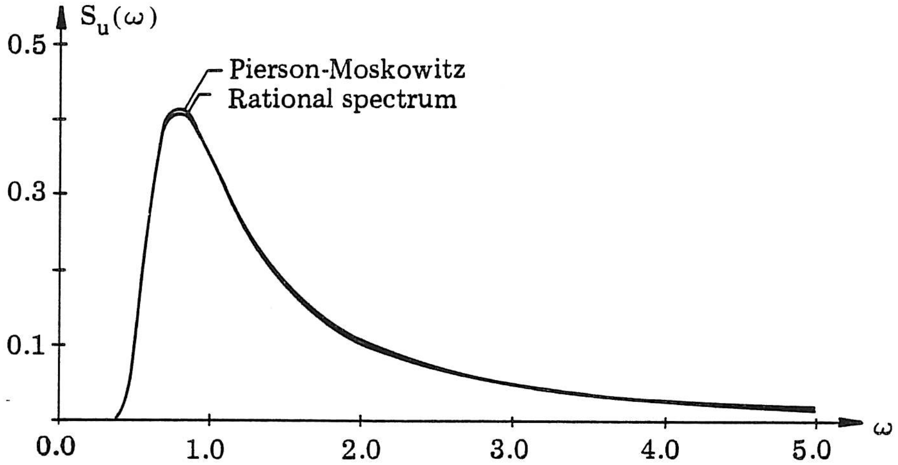

Figure 4. Approximation of Pierson-Moskowitz spectrum. n = 6 and m = 3. $d_1 = 6.428$, $d_2 = 32.27$, $d_3 = 40.95$, $d_4 = 35.68$, $d_5 = 14.43$, $d_6 = 5.100$, $c_0 = 13.43$, $c_1 = 0.01178$, $c_2 = 1.634$, $c_3 = 2.686 \cdot 10^{-4}$.

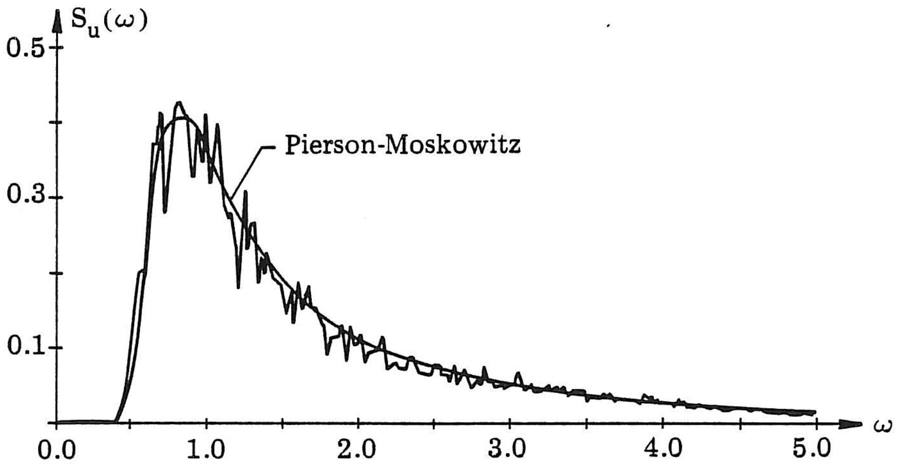

Figure 5. Spectrum of simulated velocities (n = 6, m = 3) obtained by averaging over 50 spectra with 1024 points.

In figure 5 a spectrum of the velocity u(t) obtained by the simulation method in section 4 is shown. The spectrum is seen to agree very well with the Pierson-Moskowitz spectrum.

Given the rational approximation to the velocity spectrum it is easy by using calculus of residues to calculate the standard deviations $\sigma_{\dot{u}}$ and $\sigma_{\ddot{u}}$. For the JONSWAP spectrum we obtain $\sigma_{\dot{u}}$ = 1.496 and $\sigma_{\ddot{u}}$ = 6.249. In figure 6a the ratio between the exact result $\nu_{0,D}$ (9) and $\nu^G_{0,D}$ (11) is shown. It is seen that for high values of the barrier b it is non-conservative to use the Gaussian approximation $\nu^G_{0,D}$. Simulation results obtained using (29) are seen to be close to (9). These and all the following simulation results are obtained on the basis of realizations of a length of $1.2 \cdot 10^5$ seconds. Corresponding results are also shown for a = $4/\sigma_{\dot{u}}$ = 2.674. Here (17) and (18) are used to estimate ν_0.

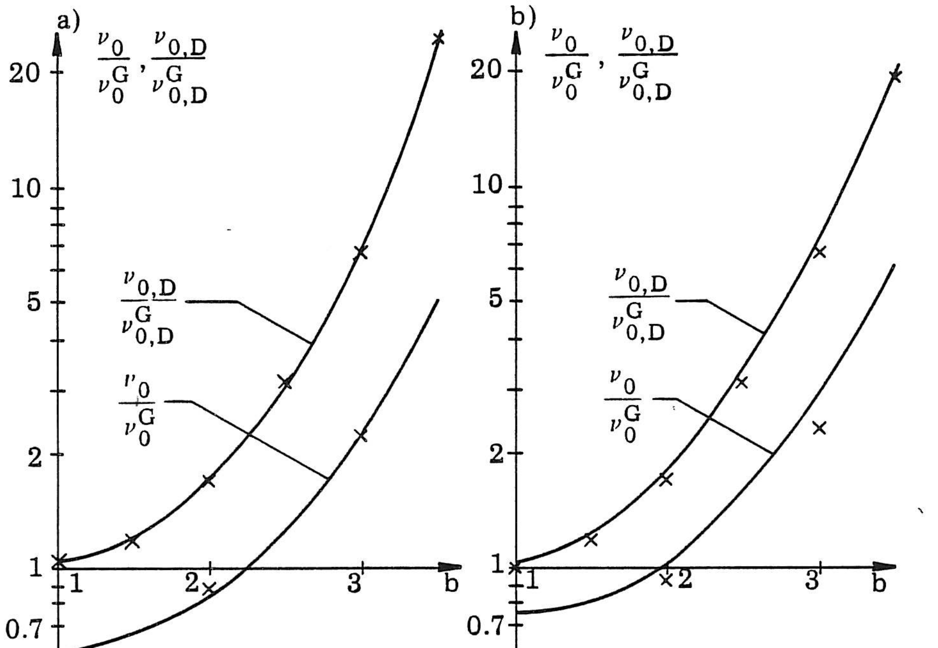

Figure 6. Mean outcrossing rates. X : simulation results, a: JONSWAP, b: Pierson-Moskowitz.

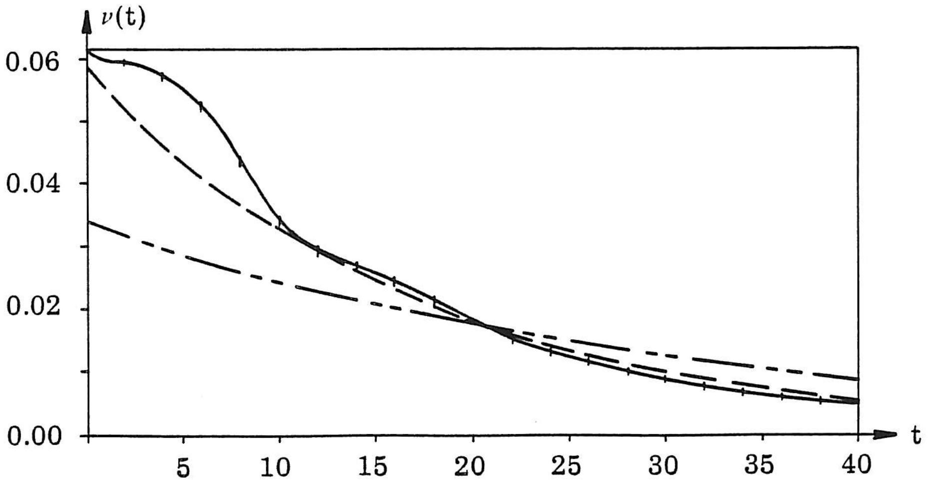

Figure 7. First-passage density for u_0 = 2, a = 0, and b = 2, JONSWAP spectrum. ———: simulation, — — —: (13) with $\nu_{0,D}$ given by the exact result (9), —·—·—·—: (13 with $\nu_{0,D} = \nu^G_{0,D}$, (11). Vertical cross lines indicate 95% confidence intervals.

For the Pierson-Moskowitz spectrum we obtain $\sigma_{\dot{u}} = 2.124$ and $\sigma_{\ddot{u}} = 10.71$, i.e. higher values than for the JONSWAP spectrum. In figure 6b results corresponding to figure 6a are shown. The same trends are seen to hold true here.

Estimates of the first-passage density based on the Poisson process approximation (13) are in the following compared with simulation results, (28). In figures 7 and 8 results for the JONSWAP spectrum with a = 0 (i.e. only drag forces), b = 2 and 3.5 are shown, and in figures 9 and 10 results for $a = 4/\sigma_{\dot{u}}$, b = 2, and the JONSWAP and Pierson-Moskowitz spectra are shown. The accuracy of the simulation estimates is indicated in figure 8 by 95% confidence intervals. It is seen that the estimates are rather precise.

As expected the Poisson approximation (13) and the simulation estimates are rather close for relatively high barriers, but for small barriers there is some difference, especially in the first seconds. The Gaussian approximation is very non-conservative for high barriers.

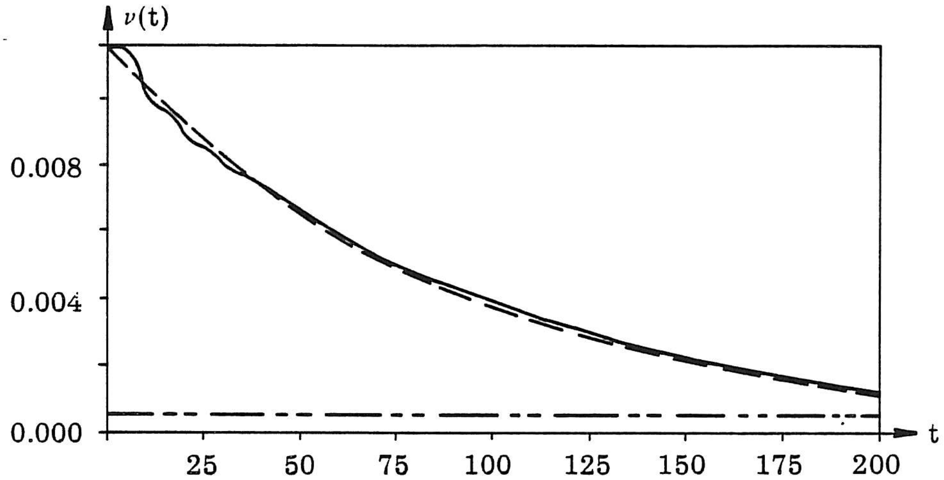

Figure 8. First-passage density for $u_0 = 2$, a = 0, and b = 2, JONSWAP spectrum. ——: simulation, – – –: (13) with $\nu_{0,D}$ given by (9), –·–·–: (13) with $\nu_{0,D} = \nu^G_{0,D}$, (11).

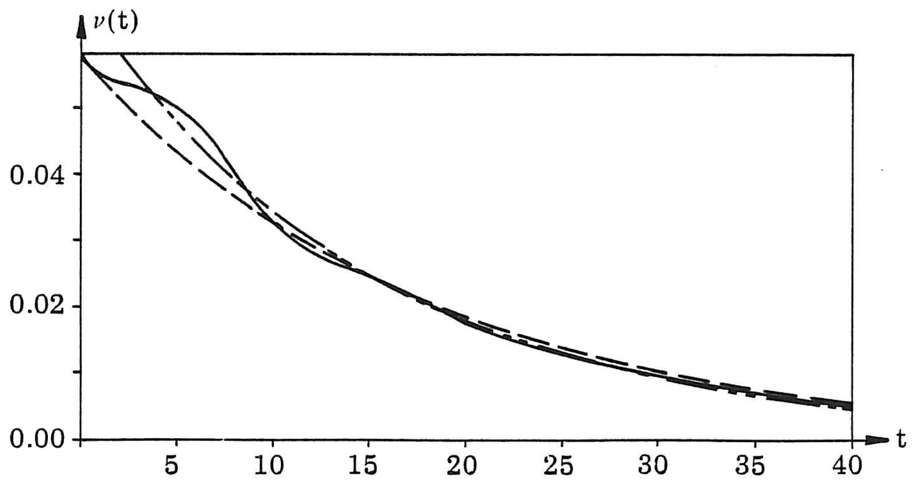

Figure 9. First-passage density for $u_0 = 2$, $a = 4/\sigma_{\dot{u}}$, b = 2, JONSWAP spectrum. ——: simulation, (28) with ν_0 estimated by (29): – – –: (13) with ν_0 given by (17), –·–·–: 13) with $\nu_0 = \nu_0^G$, (18).

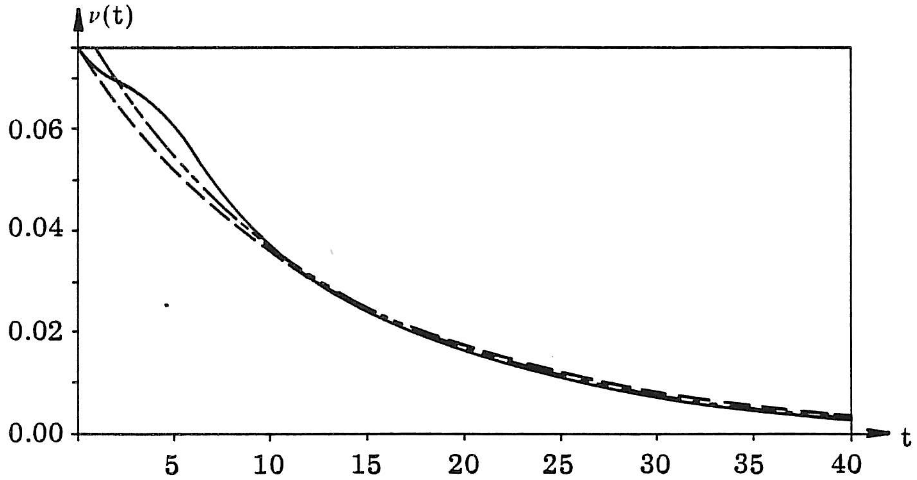

Figure 10. First-passage density for $u_0 = 2$, $a = 4/\sigma_{\dot{u}}$, $b = 2$, Pierson-Moskowitz spectrum. ——: simulation, (28) with ν_0 estimated by (29), — —: (13) with ν_0 given by (17), —·—·—: (13) with $\nu_0 = \nu^G_{0,D}$, (18).

The results in figures 9 and 10 show some difference between the results obtained by modelling the wave elevation by a JONSWAP and a Pierson-Moskowitz spectrum. The latter model gives the highest first-passage density in the first seconds.

When a increases, i.e. when the inertia term becomes more and more dominant, then the Gaussian approximation will be increasingly better, because in the limit the wave load will be Gaussian.

6. CONCLUSIONS

A model of the wave load on slender members of offshore structures is described. Exact and approximate results for the mean outcrossing rates of a time independent safe area are presented. It is assumed that the wave elevation can be modelled by a Gaussian stationary stochastic process. Further some approximate results for the first-passage density are given. These assume that the events that the wave load exceeds a critical value are rare.

A new simulation procedure to obtain rather precise estimates of the first passage density function is described. The simulation method assumes that the wave elevation can be modelled by a Gaussian Markov process. It is described how this approximation can be made quite accurate.

The simulation results show that for relatively low critical values of the wave load the approximate estimates of the failure probability are non-conservative. Further the choice of wave spectrum is seen to have some influence on the failure probability.

The results described in this paper can be used in reliability analysis of offshore platforms when Morison's equation is valid.

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