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Controller Reconfiguration through Terminal Connections based on Closed-loop System Identification¹

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Abstract: Often, when a controlled plant is modified, e.g. if a new sensor or actuator becomes available, it is desirable to retain the existing controllers and apply the new control capabilities in a gradual, online fashion rather than decommissioning the entire existing system and replacing it with the new system. This paper combines recently developed methods for gain scheduling based on closed loop identification and controller reconfiguration. The resulting scheme allows for introducing new control components in a smooth manner, providing stability guarantees during the transition phase, while retaining the original control structure.

Keywords: Closed-loop system identification, Gain scheduling control, Controller Reconfiguration, Multi-variable systems, Youla-Kucera parameterisation

1. INTRODUCTION

All medium- to large-scale automation systems, such as power plants, refineries, factories, supermarkets or even large ships, invariably have control systems to handle the automated processes, such as production facilities, chemical batch processing, climate control or steam production. These control systems are often designed at the time of commissioning of the plant and tend to rely on PLCs or similar hardware to implement classically designed (and often conservatively tuned) control loops. However, as time goes by and new technology and knowledge becomes available, it may become desirable to introduce new sensor and/or actuator hardware.

There can be various reasons for this: wear and tear on the existing devices; new technology that can supplement with better or cheaper measurements or actuation becomes available; better knowledge about the process dynamics invites more precise control; etc.

In (Bendtsen and Trangbæk [2004]), it is demonstrated how to modify the control system and perform a smooth transition to a new controller based on a closed loop system identification of the new dynamics. By using Youla parameterisations, it is possible to ensure that stability is maintained during the transition. The Youla parameterisation requires that a signal is added inside the original controller.

On the other hand, there may be also be a strong argument for keeping the existing control system intact, since it has a proven track record, and designing an entirely new control system from the bottom up is likely to be very costly both in terms of commissioning and operation stop. Furthermore, in addition to a linear control dynamic,

the original controller may be part of a safety critical interlocking circuit as well.

In (Trangbaek et al. [2008]) a method for introducing new control components in a smooth manner is presented, which provides stability guarantees during the transition phase, and which retains the original control structure intact.

In the present paper, these results are combined to provide a method with the following desirable characteristics:

- An additional model of the new dynamics is identified in closed loop with the original controller.
- An additional controller is designed using mainly the additional model.
- This additional controller is attached at the terminals of the original controller.
- The transition is performed smoothly and stability is maintained.

We begin by providing some basics on Youla parameterisations and the Hansen scheme in Sections 2-3. Then we introduce the previous results on gain scheduling and controller reconfiguration by Youla parameterisations in Sections 4-5. Then, the new design method is presented in Section 6 and demonstrated on a simulation of a district heating pressure control system in Section 7.

The notation is mostly standard, \star denotes the Redheffer star product (Zhou et al. [1996]), i.e. system interconnection.

2. YOULA PARAMETERISATION

This section gives a short introduction to some basic concepts of coprime factorisation and the Youla-Jabr-Bongiorno-Kucera (Youla from now) parameterisation of stabilising controllers. See (Youla et al. [1976], Kucera

¹ This work was supported by The Danish Research Council for Technology and Production Sciences.

[1975], Anderson [1998], Niemann [2006]) for further details.

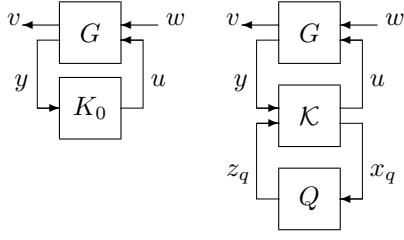


Fig. 1. Left: The interconnection of the system G and the controller K_0 . Right: Controller implemented as $K(Q) = K \star Q$.

Consider the control loop in the left part of Figure 1 and assume that the controller K_0 stabilises the system G . Factorise the lower right part of G as

$$G_{yu} = NM^{-1} = \tilde{M}^{-1}\tilde{N} \quad (1)$$

with $N, M, \tilde{M}, \tilde{N} \in \mathcal{RH}_\infty$, and K_0 as

$$K_0 = UV^{-1} = \tilde{V}^{-1}\tilde{U} \quad (2)$$

where $U, V, \tilde{U}, \tilde{V} \in \mathcal{RH}_\infty$, with the factors chosen to satisfy the double Bezout identity

$$\begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U \\ N & V \end{bmatrix} = \begin{bmatrix} M & U \\ N & V \end{bmatrix} \begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \quad (3)$$

All stabilising controllers for G can now be parameterised according to the Youla parameterisation

$$K(Q) = K \star Q = K_0 + \tilde{V}^{-1}Q(I + V^{-1}NQ)^{-1}V^{-1},$$

with $Q \in \mathcal{RH}_\infty$, i.e., $G \star K(Q)$ is stable for any stable Q and for any stabilising controller K_i , a stable Q exists so that $K(Q) = K_i$. This linear fractional transformation setup is depicted in the right part of Figure 1, and, due to the Bezout identity, can also be implemented as in Figure 2.

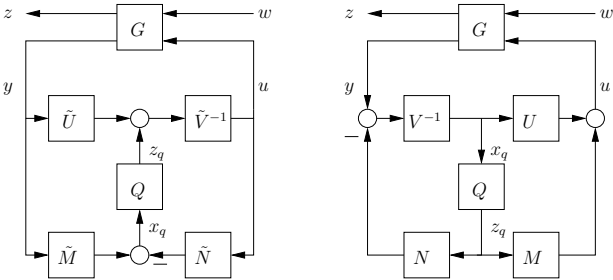


Fig. 2. Left and right coprime factorisation-based Youla parameterisation of all stabilising controllers.

Thus, it is possible to implement a given controller as a function of a stable parameter system Q based on another stabilising controller, as depicted in the right part of Figure 1. As stated in (Niemann and Stoustrup [1999]) this implies that it is possible to change between two controllers online, say, from a nominal controller K_0 to another controller K_1 , in a smooth fashion *without losing stability*, by scaling the Q parameter by a factor $\gamma \in [0; 1]$.

One interesting feature of the parameterisation is that the performance transfer function from w to z is affine in Q , i.e.

$$T_{zw} = T_1 + T_2QT_3, \quad (4)$$

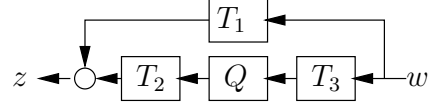


Fig. 3. Classical model matching setup.

also illustrated in Figure 3, where T_1, T_2 , and T_3 are stable transfer functions. Thus, a control design can be carried out by finding a stable Q that minimises T_{zw} in some sense. This is known as a model matching problem (Francis [1987]).

Alternatively, if a desired transfer function for the a new stabilising controller K_1 has been obtained, $K(Q) = K_1$ can be realised by factoring $K_1 = \tilde{V}_1^{-1}\tilde{U}_1$ with

$$\begin{bmatrix} \tilde{V}_1 & -\tilde{U}_1 \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U_1 \\ N & V_1 \end{bmatrix} = \begin{bmatrix} M & U_1 \\ N & V_1 \end{bmatrix} \begin{bmatrix} \tilde{V}_1 & -\tilde{U}_1 \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix},$$

and setting (Bendtsen et al. [2005])

$$Q = \tilde{U}_1V - \tilde{V}_1U = \tilde{V}_1(K_1 - K_0)V. \quad (5)$$

Once a Q has been designed, the affine dependence also means that if Q is scaled by γ as mentioned above, then the performance will change in a predictable way for values of γ between 0 and 1. (In fact, stability will be preserved even for quickly time-varying γ (Hespanha and Morse [2002]), but that is not essential here.)

3. THE HANSEN SCHEME

To motivate the usage of the Youla parameterisation in system identification, we first consider normal open-loop identification of the system G . Some input u is applied to the system, and corresponding output measurements y affected by noise n_y are obtained. These measurements are related through

$$y = Gu + n_y$$

and an unbiased estimate of G can be obtained if u and n_y are uncorrelated. Unfortunately, in a closed-loop setting u is *not* uncorrelated with n_y , since the noise is fed back through the controller. To alleviate this, we employ the dual Youla factorisation to recast the closed-loop system identification problem into an ‘open-loop-like’ problem (Hansen et al. [1989]).

Assume that a controller K stabilises the plant we wish to identify, and that some nominal plant estimate G is known, factorised as in (2) and (1), respectively. Then the set of all plants stabilised by K can be represented as shown in Figure 4. Here, $n' = (\tilde{M} + S\tilde{U})n_y$ is the measurement noise that would normally affect the measurements y , relocated in the block diagram to affect the output of the dual Youla parameter, S , instead, and r_1 and r_2 are external excitation signals.

By manipulating the block diagram and using (3), it is possible to check that $y = G(S)u + n_y$. From Figure 4 it is then possible to deduce (see e.g., (Anderson [1998]), but please note that here we are using positive feedback control) that

$$x_s = \tilde{U}r_1 + \tilde{V}r_2 \quad (6)$$

$$z_s = \tilde{M}y - \tilde{N}u \quad (7)$$

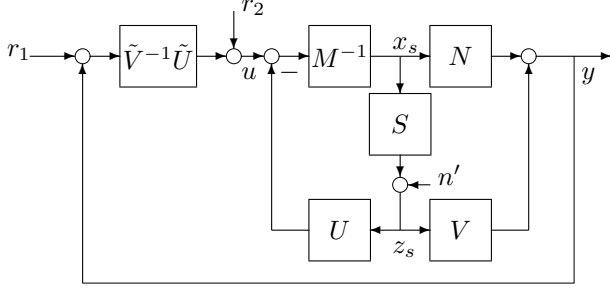


Fig. 4. Dual Youla parameterisation used for closed-loop system identification

and, obviously, $z_s = Sx_s + n'$. x_s and z_s are available from filtered measurements. Furthermore, if n_y is independent of r_1 and r_2 , then x_s is independent of n' as well. Also, S is known to be stable due to the dual Youla theory (cf. the previous section). Thus, it can be seen that although u and y are measured in closed-loop, the identification of S becomes equivalent to an open-loop identification problem.

4. RECONFIGURING CONTROLLERS THROUGH THE TERMINALS

We now turn our attention to a situation, where we wish to modify the controller behaviour but without removing the original controller. The reasons for the desired change can be numerous. The system may have changed due to equipment being added or replaced or simply due to wear and tear, or maybe a better understanding of the system has been obtained.

As mentioned in the introduction, the reasons for desiring to keep the original controller in the loop can also be numerous. It may for instance contain supervisory logic that we do not wish to replicate. Also, the operator will often be wary of replacing a functioning controller with an entirely new one. Instead, adding a controller to the original one and slowly turning it on would be much more appealing.

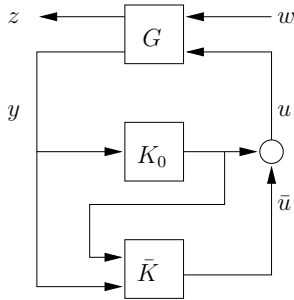


Fig. 5. Modifying a controller through the control signal terminal.

We assume that the original controller still stabilises the system, but we cannot access the inside of it, as Figure 2 would suggest is needed to use a Youla parameterisation. Rather, the additional controller, \tilde{K} , must be applied at the terminals of the existing controller as shown in Figure 5.

In (Trangbaek et al. [2008]) a method is presented for designing an additional controller to be applied at the

terminals of the original controller, in order to improve the performance. It is possible to perform the switch gradually while maintaining stability, so that the process can be monitored.

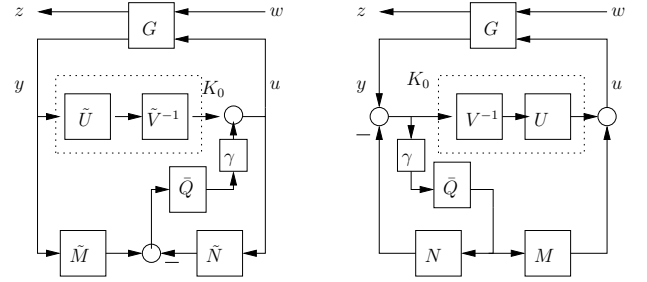


Fig. 6. Controller parameterisation modified for connection to terminals of existing controller.

By modifying the Youla parameterisation in Figure 2, we arrive at the two possible setups in Figure 6. Here, the original controller, K_0 is kept in place and is only accessed at the terminals. Stability of \tilde{Q} still implies stability of the closed loop, but not all stabilising controllers are parameterised by a stable \tilde{Q} :

Theorem 1. Let $G_{yu} = \tilde{M}^{-1}\tilde{N}$ be a coprime factorisation of a system, and assume that $K_0 = \tilde{V}_0^{-1}\tilde{U}_0 = U_0V_0^{-1}$, is a stabilising controller, i.e. $G \star K_0 \in \mathcal{RH}_\infty$. Consider a second controller $K_1 = \tilde{V}_1^{-1}\tilde{U}_1 = U_1V_1^{-1}$. Then

$$G \star K_1 \in \mathcal{RH}_\infty \wedge V_0^{-1}V_1 \in \mathcal{RH}_\infty \quad (8)$$

$$\Updownarrow$$

$$\exists \tilde{Q} \in \mathcal{RH}_\infty : K_1 = (I + \tilde{Q}\tilde{N})^{-1} [I \quad \tilde{Q}\tilde{M}] \begin{bmatrix} K_0 \\ I \end{bmatrix} \quad (9)$$

i.e., (9) is a parameterisation of all stabilising controllers that include the right half plane (RHP) pole structure of K_0 .

Proof: See (Trangbaek et al. [2008]) ◁

Thus, the setup in the left part of Figure 6 corresponding to (9) parameterises all stabilising controllers containing the same unstable poles as K_0 , i.e. we cannot move these unstable poles, but we can introduce new ones.

Note that the implementation in Figure 6 only requires the factorised plant model, although the model of the original controller is of course needed for the design of \tilde{Q} .

5. GAIN SCHEDULING IN THE YOULA FRAMEWORK

In (Bendtsen and Trangbaek [2004]) a method is proposed for designing a gain scheduled controller for moving a plant to a new operating point. This method can also be applied for plants where the dynamics change due to equipment being added etc.

Consider the following problem: A controller K_0 has been designed for a plant in operating point p_0 , yielding stability and good performance in this operating point. The controller may have been tuned by hand if no system model is available. We now wish to obtain a controller for the

entire range between p_0 and some other operating point p_1 . K_0 stabilises the plant at both operating points (and in between), but the performance is not satisfactory due to the different system dynamics caused by the nonlinearities. (This situation occurs, for instance, in power plants that have traditionally been working at full load most of the time and have only passed through low load as a startup procedure.) To address this problem, the following systematic method for designing a gain scheduling controller is suggested:

(1) Obtain a model G_0 in p_0 . Since a satisfying controller K_0 has already been found for this operating point, G_0 does not have to describe the system very accurately. Thus, a low order model can be obtained, for instance using standard system identification methods based on measurements of u and y .

(2) Perform a coprime factorisation. Explicit factors for state space controllers with integral action can be found in (Trangbæk et al. [2006]).

(3) Perform a closed-loop identification in p_1 obtaining a dual Youla parameter S_1 . Using the obtained parameterisation, the Hansen scheme described in Section 3 is now used to perform a closed-loop identification in the operating point p_1 using the stabilising controller K_0 . This will result in the identification of a dual Youla parameter S_1 describing the difference between the plant dynamics in operating points p_0 and p_1 . That is, if $S = 0$ we have $G = G_0$, whereas for $S = S_1$ we have $G = G_1$, the linearised model in p_1 .

(4) Design a gain scheduling controller exploiting the complimentary structures of the Youla and dual Youla parameterisations. At this point, rather than recovering a plant model G_1 in p_1 , we will instead exploit the structure of the Youla parameterisations to design a gain scheduling controller.

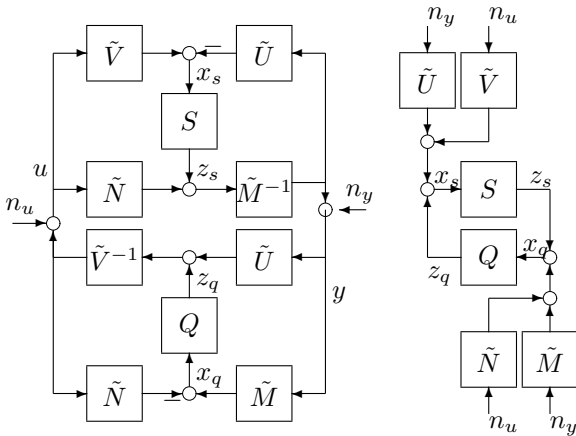


Fig. 7. Left: system-controller interconnection with noise inputs. Right: transformed system-controller interconnection used for design.

The system-controller interconnection is illustrated in the left part of Figure 7, where n_u represents input noise. Using the Bezout identity (3) and some block manipulation, this block diagram can be transformed to the right-hand block diagram in the same figure. At this point, we construct a controller for the plant in p_1 , by determining a Youla parameter Q_1 to yield stability and

good performance. In order to obtain stability we need to assure simultaneous stability of Q_1 and $S_1 \star Q_1$, which is a strong stabilisation problem. To ensure stability during gain scheduling we need unconditional stability of the loop, i.e. $S_1 \star (\gamma Q_1)$ must be stable for $\gamma \in [0; 1]$. Although the strong stabilisation problem is not readily solved, it is easy to analyse the stability a posteriori. The performance problem can be cast as a standard \mathcal{H}_∞ problem, minimising the gain from n_u and n_y to the signals u and y . Motivated by the structure of the reduced block diagram in Figure 7, however, we can instead focus on the signals x_q and z_q , considering the gains from n_u and n_y to x_q and z_q as internal sensitivities. Minimising these gains will in some sense minimise the effect of the change of operating point. Therefore, we can cast the problem in Figure 7 as a standard \mathcal{H}_∞ problem and minimise the gain from the external inputs to z_q and x_q – see Figure 8.

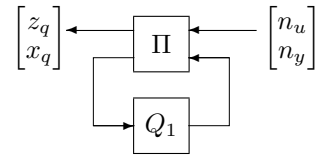


Fig. 8. Setup for controller synthesis.

Once a Youla parameter Q_1 has been found for the operating point p_1 yielding appropriate performance, we can then construct a gain-scheduling controller for the entire range between p_0 and p_1 . The gain scheduling is introduced in the control law by scaling the input from the Youla parameter to the controller \mathcal{K} as $Q = \gamma Q_1$, where $\gamma \in [0; 1]$ is a scheduling variable, according to an appropriate measurement obtained from the plant. In this way, the resulting controller becomes equal to K_0 when the plant state is in p_0 and equal to K_1 when the plant state is in p_1 .

6. ADDITIONAL CONTROLLER DESIGN

The above procedure can of course also be used for situations where the dynamics change for some other reason than moving to a new operating point, for instance if new equipment is introduced in the plant.

If we wish to retain the original controller as discussed in Section 4, the design method only needs to be altered slightly. Instead of the right hand side of Figure 7, we will have the setup in Figure 9.

Again, the design problem can be cast in the standard form, focusing on the actual outputs and control inputs or on the internal signals x_q and z_q in order to get a problem of smaller order. Depending on the situation at hand, we may also choose some mixture of real and virtual signals. For instance, if the controller is to be used in a gain scheduling context, where the scheduling parameter γ changes quickly, then a small low frequency gain of Q is recommended. This can be achieved by focusing on y and z_q , and penalising low frequency content of the latter.

On the other hand, if the objective is to handle additional actuators or sensors, we may be interested in adding integral action. Note that it is possible to do this, assuming that the extra integrator can be stabilised by the plant, by

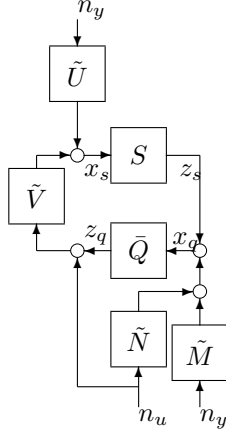


Fig. 9. Modified system-controller interconnection used for terminal connection design.

placing a pole in $s = 0$ or $z = 1$ in the loop containing \bar{Q} and N (or \tilde{N}) in Figure 6.

Alternative procedure: Modifying the factors

The method in Section 4 assumes that the coprime factors describe the new plant, i.e. that $S = 0$, whereas the design method above uses the interconnection between Q and S for the design. Note that when inserting an identified S ,

$$\begin{aligned} G(S) &= (N + VS)(M + US)^{-1} \\ &= (\tilde{M} + S\tilde{U})^{-1}(\tilde{N} + S\tilde{V}), \end{aligned}$$

we again have a coprime factorisation, and that the new factors

$$N_1 = (N + VS)$$

$$M_1 = (M + US)$$

$$\tilde{M}_1 = (\tilde{M} + S\tilde{U})$$

$$\tilde{N}_1 = (\tilde{N} + S\tilde{V}),$$

also fulfill the Bezout equation (3) with the original controller factors U, V . Thus we can implement the scheme in Section 4 using N_1, M_1 or \tilde{N}_1, \tilde{M}_1 , guaranteeing stability.

It is a bit unclear how to design Q then, but one possibility is to use a model matching method. The implementation will be of significantly higher order.

7. DISTRICT HEATING SIMULATION

In this section, the presented method is applied to a simulation model of the district heating system shown in Figure 10. Heat is supplied from a heat source to two building complexes through a network of pipes.

The aim of the control system is to maintain the differential pressures dp_1 and dp_2 (the only measurements) at the end-users by controlling the pumps.

A schematic view of the system is shown in Figure 11. Pump 3 is running at constant speed, so the only control inputs are pump 1 and pump 2. The pumps are modelled as static gains from control signal to rotational speed, making the model a second order system, since there are two independent flows, with time constants in the range of

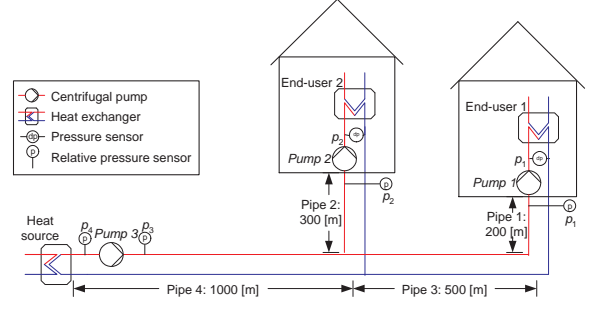


Fig. 10. A sketch of the very small district heating system.

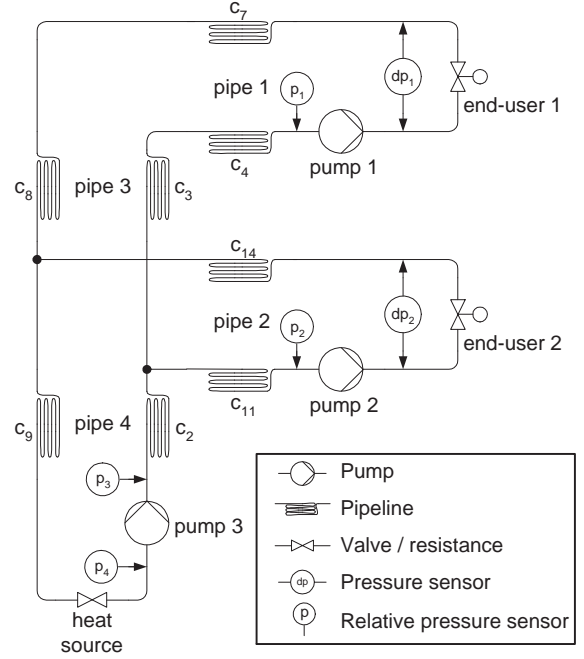


Fig. 11. The hydraulic network of the district heating system depicted in Fig. 10.

10 seconds. The system is nonlinear, since pressure drops are proportional to the flow squared, but can be considered approximately linear near an operating point. However, the two subsystems interact heavily, and a change of one valve setting affects the dynamics of the other loop. (For a more thorough description of the system and simulation model, see [Knudsen et al. [2008]].)

We now consider the situation, where only end-user 1 is attached. A controller K_0 tries to keep dp_1 at 0.5 bar by controlling pump 1. End-user 1 acts as a disturbance by varying the consumption and consequently the valve setting. This is the situation shown in interval 1 of the simulation in Figure 12. We see that the controller manages to suppress the disturbances nicely.

Planners now want to attach the second loop containing end-user 2 to the network. This is what happens in interval 2 in Figure 12. The valve at end-user 2 is slowly opened and pump 2 is turned on at an appropriate constant speed. Consumption starts immediately resulting in varying valve settings and resulting disturbances to dp_2 . Because of the strong couplings in the network, the first loop is also affected. The pressure drop in the main pipes leads

to a drop at dp_1 as well, but the controller is able to compensate by increasing the speed of pump 1, although the disturbances at end-user 2 also affect dp_1 .

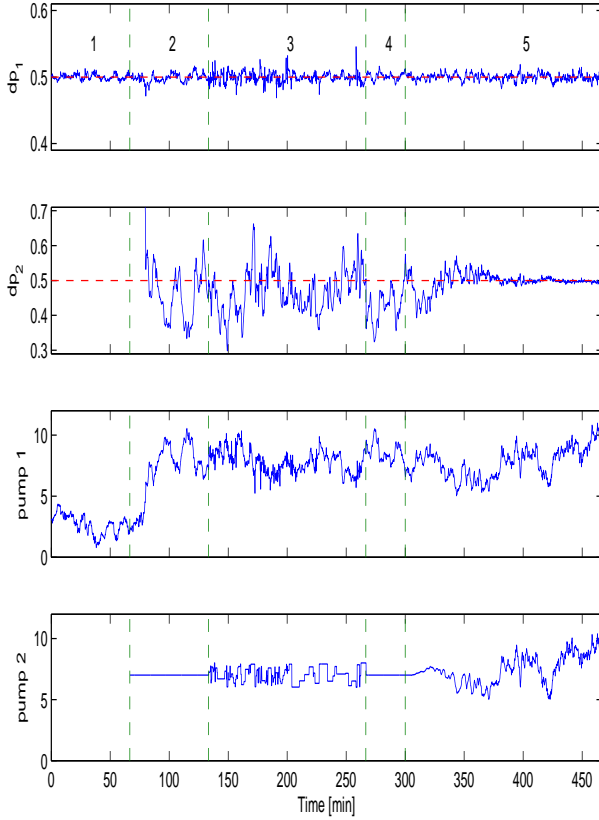


Fig. 12. Adding a loop to the district heating system. Intervals are separated by vertical dashed lines.

In order to build a new controller, a model is identified using the closed loop method in Section 3. In interval 3, excitation signals are added to both pump speeds. In interval 4, the system returns to normal operation and a system identification is performed using the data from interval 3. Note that the resulting dual Youla parameter, S_1 , reflects not only the dynamics of the additional loop but also the changes arising from moving to a new operating point. A noise model is included in the system identification.

A Youla parameter Q_1 is then designed for $S_1\tilde{V}$, in this case using an LQG design for the identified stochastic model, penalising z_q and the estimated states of S_1 .

In interval 5, the new controller is slowly added by increasing γ from 0 to 1, bringing dp_2 close to the desired pressure without disrupting the control of dp_1 .

At no point in the process was the original controller taken out of operation. The new controller takes measurements from both sensors and controls pump 2 while also providing an additive control signal to pump 1.

8. CONCLUSIONS

This paper combines recent results on closed loop identification and controller reconfiguration to present a method for adapting a control system to changes in the process without removing or altering the original controller. By

exploiting properties of the Youla-Kucera parameterisation, the design can be based mainly on the changes in the process dynamics, and the adaption can be made smoothly, maintaining stability.

The method was demonstrated on a simulation model of a district heating system. The next step will be to test it on a recently constructed scaled laboratory setup of the same system.

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