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Vibration Based Damage Assessment of a Cantilever using a Neural Network

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Publication date:
1993

Document Version
Early version, also known as pre-print

[Link to publication from Aalborg University](#)

Citation for published version (APA):
Kirkegaard, P. H., & Rytter, A. (1993). *Vibration Based Damage Assessment of a Cantilever using a Neural Network*. Dept. of Building Technology and Structural Engineering, Aalborg University. Fracture and Dynamics Vol. R9345 No. 50

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FRACTURE & DYNAMICS
PAPER NO. 50

To be presented at the 10th International Conference on Experimental Me-
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P. H. KIRKEGAARD & A. RYTTER
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Vibration based damage assessment of a cantilever using a neural network

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ABSTRACT: In this paper the possibility of using a Multilayer Perceptron (MLP) network trained with the Backpropagation Algorithm as a non-destructive damage assessment technique to locate and quantify a damage in structures is investigated. Since artificial neural networks are proving to be an effective tool for pattern recognition, the basic idea is to train a neural network with simulated values of modal parameters in order to recognize the behaviour of the damaged as well as the undamaged structure. Subjecting this trained neural network to measured modal parameters should imply information about damage states and locations. During the training phase the estimates of the modal parameters are obtained by an FEM of the structure. A damage in the structure is modelled by a fracture mechanical model. The basic idea of this model is to model the crack zone of a structure by means of a local flexibility matrix found from fracture mechanics. By investigating a cracked hollow section straight steel cantilever beam the capability of the neural network approach is demonstrated. The results show that a neural network trained with simulated data is available for detecting location and size of a damage in the cantilevered beam when the network is subjected to experimental data.

1 INTRODUCTION

A sudden failure in a structure can be very costly and may be catastrophic in terms of human life and property damage. Many techniques of non-destructive evaluation are currently available to detect damage in structures. Such techniques include visual inspections, ultrasonic testing, acoustic emission, etc. However, most of these techniques are inconvenient in many situations due to the need for the investigator to have access to the structure. This inconvenience can be avoided through the use of vibration-monitoring techniques. During the last years major research efforts have been directed toward developing techniques for damage assessment based on changes in vibration characteristics. One of the consequences of the development of a crack is a decrease in local stiffness which in turn results in a decrease in some of the natural frequencies. The most commonly applied vibration based inspection damage assessment technique is based on changes of natural frequencies only. This is attractive since natural frequencies can be obtained from measurements at a single point on the structure. If measurements at several points are carried out the mode shapes in discrete points of the structure corresponding to the different natural frequencies may be established. Recently, mode shape information has

also been used for damage assessment. However, in order to be able to evaluate the deterioration state of a structure by vibration based inspection it is also necessary to estimate size and location of the damage, damage assessment. A throughout review of vibration based damage assessment techniques can be found in Rytter [1].

The problem of damage assessment on the basis of measured data is essentially one of the pattern recognition. Measured data from an undamaged structure must be distinguishable from measured data from a damaged structure. Different pattern recognition approaches have been proposed in the literature, see e.g. Yin et al. [2], Samman et al. [3]. In these papers pattern recognition techniques are presented to estimate the damage presence and location but not the magnitude of the damage. However, recently, artificial neural networks are proving to be an effective tool for pattern recognition in a variety of applications, see e.g. Hertz et al. [4] and among these also for damage assessment. The basic idea is to train a neural network in order to recognize the behaviour of the damaged as well as the undamaged structure. Subjecting this trained neural network to information from vibration tests should imply information about damage state and location. During the last years this neural network based damage assessment approach has been proposed

in different papers. In Thomsen et al. [5] a neural network was implemented and trained to classify measured ultrasonic power spectra of composite laminates, according to fabrication quality. Kudva et al. [6] used a neural network to determine location and size of a damage in a structure from measured strain values at discrete locations. The neural network is trained by finite-element data. Strain patterns are used as inputs and the damage location and size as outputs to train the neural network to a desired level of accuracy. The trained network can then be used to determine the location, size and effect of any unknown damage using measured strain values (at the same locations as before) as inputs to the neural network. The damage assessment approach proposed in Kudva et al. [6] has also been used in Worden et al. [7]. In an analytical study, Wu et al. [8] demonstrate a damage detecting network trained on measured frequency response functions from the system. Elkordy et al. [9] trained a neural network with analytically generated mode shape ratios in order to diagnose damage states obtained experimentally from series of shaking-table tests of a five-story steel frame. In Kirkegaard et al. [10] a neural network is trained with the relative change in natural frequencies obtained by an FEM. The network is then used to estimate location and size of a damage in a beam from measured natural frequencies. In all the papers, mentioned above, dealing with neural network based damages assessment a basic Multi-Layer Perceptron network trained with the back-propagation algorithm was used.

In this paper the aim is to investigate use of artificial neural networks for damage assessment of a straight hollow section steel cantilever beam. In this application, training of the network is performed with patterns of the relative changes of the natural frequencies that occur due to a damage. This implies that each pattern represents the computed changes of the natural frequencies due to a crack of a particular size at a particular location. The changes are estimated by using a finite-element model. A damage is modelled by a fracture mechanical model. The basic idea of this fracture mechanical model is to model the crack zone by means of a local flexibility matrix found from fracture mechanics.

In section 2 a short description of artificial neural networks is given and a neural network based damage assessment approach is proposed. In section 3 the proposed damage assessment approach is used in an example with a straight hollow section steel cantilever beam. The damage assessment approach is investigated with numerical data as well as experimental data. At last in section 4 conclusions are given.

2 NEURAL NETWORKS

The past decade has seen an explosive growth in the studies of artificial neural networks. In part this was the result of technological advances in personal and main-frame computing, enabling neural network investigators to simulate and test ideas in ways not readily available before 1980.

Artificial neural networks are computational models loosely inspired by the neuron architecture and operation of the human brain. The pioneering work in this field is usually attributed to McCulloch and Pitts in 1943. They developed a simplified model of a neuron. The brain is composed of neurons of many different types, see McCulloch et al. [11].

An artificial neural network is an assembly (network) of a large number of highly connected processing units, the so-called nodes or neurons. The neurons are connected by unidirectional communication channels ("connections"). The strength of the connections between the nodes is represented by numerical values which normally are called weights. Knowledge is stored in the form of a collection of weights. Each node has an activation value that is a function of the sum of inputs received from other nodes through the weighted connections. The neural networks are capable of self-organization and knowledge acquisition, i.e. learning. One of the characteristics of neural networks is the capability of producing correct, or nearly correct, outputs when presented with partially incorrect or incomplete inputs. Further, neural networks are capable of performing an amount of generalization from the patterns on which they are trained. Most neural networks have some sort of "training" rule whereby the weight of connections are adjusted on the basis of presented patterns. In other words neural networks "learns" from examples, just like children learn to recognize dogs from examples of dogs, and exhibit some structural capability for generalization. Training consists of providing a set of known input-output pairs, patterns, to the network. The network iteratively adjusts the weights of each of the nodes so as to obtain the desired outputs (for each input set) within a requested level of accuracy. Error is defined as a measure of the difference between the computed pattern and the expected output pattern.

For a more detailed description of neural networks, see e.g. Hertz et al. [4] and Hush et al. [12].

2.1 Multilayer Perceptron

Since McCulloch-Pitts in 1943 there have been many studies of mathematical models of neural networks. Many different types of neural net-

works have been proposed by changing the network topology, node characteristics and learning procedures. Examples of those are e.g. the Hopfield network, Hopfield [13], the Kohonen network, Kohonen [14] and the so-called multilayered perceptron (MLP) network trained by means of the back-propagation algorithm. The MLP trained by the back-propagation algorithm is currently given the greatest attention by application developers, see e.g. Kohonen [14]. The multilayered perceptron network belongs to the class of layered feed-forward nets with supervised learning. A multilayered neural network is made up of one or more hidden layers placed between the input and output layers, see fig. 1. Each layer consists of a number of nodes connected in the structure of a layered network. The typical architecture is fully interconnected, i.e. each node in a lower level is connected to every node in the higher level. Output units cannot receive signals directly from the input layer. During the training phase activation flows are only allowed in one direction, a feed-forward process, from the input layer to the output layer through the hidden layers. The input vector feeds each of the first layer nodes, the outputs of this layer feed into each of the second layer nodes and so on.

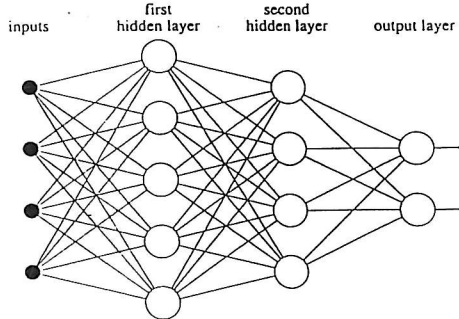


Fig. 1: Principle of a multilayer perceptron neural network.

Associated with each connection between node i and node j in the preceding layer $l - 1$ and following layer l is a numerical value $w_{lj,i}$ which is the strength or the weight of the connection. At the start of the training process these weights are initialized by random values. Signal pass through the network and the j th node in layer l computes its output according to

$$x_{lj} = f\left(\sum_{i=1}^{N_{l-1}} w_{lj,i} x_{l-1,i} + \theta_{lj}\right) \quad (1)$$

for $j = 1, \dots, N_l$ and $l = 1, \dots, k$, where x_{lj} is the output of the j th node in the l th layer. θ_{lj} is

a bias term or a threshold of the j th neuron in the l th layer. The k th layer is the output layer and the input layer must be labelled as layer zero. Thus N_0 and N_k refer to the numbers of network inputs and outputs, respectively. The function $f(\cdot)$ is called the node activation function and is assumed to be differentiable and to have a strictly positive first derivative. For the nodes in the hidden layers, the activation function is often chosen to be a so-called sigmoidal function

$$f(\beta) = \frac{1}{1 + e^{-\beta}} \quad \beta > 0 \quad (2)$$

The activation function for the nodes in the input and output layers are often chosen as linear.

During the training phase, representative examples of input-output patterns are presented to the network. Each presentation is followed by small adjustments of weights and thresholds if the computed output is not correct. If there is any systematical relationship between input and output and the training examples are representative of this, and if the network topology is properly chosen, then the trained network will often be able to generalize beyond learned examples. Generalization is a measure of how well the network performs on the actual problem once training is complete. It is usually tested by evaluating the performance of the network on new data outside the training set. Generalization is most heavily influenced by three parameters: the number of data samples, the complexity of the underlying problem and the network architecture. Currently, there are no reliable rules for determining the capacity of a feed-forward multilayer neural network. Generally, the capacity of a neural network is a function of the number of hidden layers, the number of processing units in each layer, and the pattern of connectivity between layers. However, it is shown in Cybenko [16] and Funahshi [17] that one hidden layer is sufficient to approximate all continuous functions.

2.2 Back-Propagation

The first stage of creating an artificial neural network to model an input-output system is to establish the appropriate values of the connection weights $w_{lj,i}$ and thresholds θ_{lj} by using a learning algorithm. A learning algorithm is a systematic procedure for adjusting the weights in the network to achieve a desired input/output relationship, i.e. supervised learning. The most popular and successful learning algorithm used to train multilayer neural networks is currently the Back-propagation routine, see Rumelhart [15]. The so-called Back-propagation algorithm employs a gradient descent search technique for minimizing an error normally defined as the mean square difference between desired y_j and actual outputs \hat{y}_j .

I.e. the error E is given as

$$E = 0.5 \sum_{j=1}^{N_k} (y_j - \hat{y}_j)^2 \quad (3)$$

If the error is considered small enough, the weights and thresholds are not adjusted. If however, a significant error is obtained the weights and thresholds are adjusted in the negative gradient direction, so that the error criterion E is reduced. A typical weight $w_{l,j,i}$, which could belong to any layer, is adjusted from its old value $w_{l,j,i}^{old}$ to its new value $w_{l,j,i}^{new}$ according to

$$w_{l,j,i}^{new} = w_{l,j,i}^{old} + \Delta w_{l,j,i} \quad (4)$$

where $\Delta w_{l,j,i}$ is given by, see e.g. Billings [18]

$$\Delta w_{l,j,i} = \eta \delta_{l,i} x_{l-1,i} \quad (5)$$

$\delta_{l,i}$ is the error in the output of the i th node in layer l and η is termed a "learning rate". The error $\delta_{l,i}$ is not known a priori but must be constructed from the known errors $\delta_{k,i}$ at the output layer. The errors are passed backwards through the net and a training algorithm uses the error to adjust the connection weights moving backwards from the output layer, layer by layer, hence the name "Back-propagation". In practice the "learning rate" η is chosen as large as possible (0.01-0.9) without leading to intolerable oscillations. To overcome this problem, a momentum term α is usually introduced into the update rule implying

$$w_{l,j,i}^{new} = w_{l,j,i}^{old} + \eta \delta_{l,i} x_{l-1,i} + \alpha \Delta w_{l,j,i} \quad (6)$$

The thresholds are adjusted in the same way as the weights. The process of computing the gradient and adjusting the weights and thresholds is repeated until a minimum of the error E (or a point sufficiently close to the minimum) is found. However it is generally true that the convergence of the Back-propagation algorithm is fairly slow. Attempts to speed learning include variations on simple gradient search, line search techniques and second order techniques, see e.g. Hertz [4] and Billings [18].

2.3 Use of Neural Networks for Damage Assessment

The problem of damage assessment on the basis of measured dynamic data is essentially a pattern recognition problem. Since artificial neural networks are proving to be an effective tool for pattern recognition the basic idea in a neural based

damage assessment approach is to train a network with patterns of the changes in quantities describing the dynamic behaviour that occur due to a damage. This implies that each pattern represents the computed changes of e.g. the response spectrum, natural frequencies, mode shapes etc. due to a damage of a particular size at a particular location. The patterns of the quantities describing the dynamic behaviour are used as inputs and the damage location and size as outputs to train the neural network. Then the trained network subjected to measured patterns of the quantities describing the dynamic behaviour can be used to determine the location, size and of a damage. A hierarchical, two step approach can also be used. This implies that the patterns of the quantities describing the dynamic behaviour are used as inputs and the location of the crack is used as output in one network and size of the crack as output in an other network, see Kirkegaard et al. [10].

The training of a neural network with appropriate data containing the information about the cause and effect is a key requirement of a neural based damage assessment technique. This means that the first step is to establish the training sets which can be used to train a network in a way that the network can recognize the behaviour of the damaged as well as the undamaged structure from measured quantities. Therefore, ideally, the training sets should contain data of the undamaged as well as the damaged structure in various damage states. These data can be obtained by measurements, model tests or through numerical simulation, or through a combination of all three types of data. This possibility of using all obtained information, or only a part, in a neural network based damage assessment technique is a capability which is not available in traditional damage assessment techniques.

In order to verify how well a trained network has learned the training cases the trained network is tested by subjecting it to the training sets. The important generalization capability of a neural network damage assessment technique is tested by subjecting the trained network to data not included into the training sets. How well a trained network is to generalize depends on the adequacy of the selected network architecture and the information about the damage as well as undamaged structure included in the training sets.

3 EXAMPLE

In this example the proposed neural network approach for damage assessment is applied to a hollow section steel cantilever, see fig 2. The neural based damage assessment technique is investigated by training a neural network with the relative change of the bending natural frequencies of the 3 lowest modes.

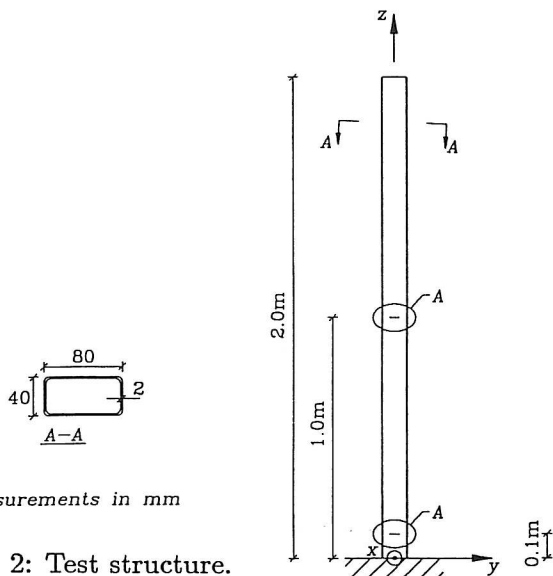


Fig. 2: Test structure.

3.1 Analytical and Experimental Results

The experimental data are estimates of the lowest three bending natural frequencies. Real line cracks were obtained in the test beams by attaching a sinus-varying load to the beams by means of a shaker. The frequency of the sinus was either the actual first or second bending natural frequency. The cracks lengths were measured by means of a microscope mounted on an electrical measurement rail. Two different crack locations were considered. In one beam a crack was initiated at $z=1.0$ m and in another beam at $z=0.1$ m. The cracks were initiated as small narrow laser cuts (width ≈ 0.15 mm). The experimental determination of the bending natural frequencies was obtained from free decays. The free decays were introduced by removing a well-defined static load from the beam momentary. The accelerations were measured at $z=0.5$ m, $z = 1.4$ and $z=2.0$ m. A more throughout description of the experimental procedure and the experimental results can be found in Rytter et al. [1].

The neural network is trained by the relative change of the bending natural frequencies due to a damage. These changes are estimated by an FEM of the beam. A damage in the beam is modelled by a fracture mechanical model. The development of a crack at a certain location of a beam corresponds to a sudden reduction of its bending stiffness at the same location. The crack divides the original non-cracked beam into two shorter beams, connected, at the crack location, by a very infinitesimal portion of beam with different characteristics. The characteristics in bending modes can be modelled by a torsion spring. Estimation of the spring stiffness by fracture mechanics has been used by several authors, see e.g. Okamura[19] and Gomes et al. [20]. The FEM was calibrated by using experimental data from the non-cracked

beam. This calibration was performed to secure that the FEM describes the beam in the best possible way. The quality of the predictions from any technique of damage assessment is critically dependent on the accuracy of the damage model, see e.g. Rytter [1].

3.2 Training and Testing of Neural Network

The following results were obtained using a hierarchical, two step approach. This implies that the patterns of the quantities describing the dynamic behaviour and the location of the crack are used as input and output, respectively, in one network. In an other network the crack location and the quantities describing the dynamic behaviour are used as inputs and the size of the crack as output. A hierarchical approach was used since it was found in Kirkegaard et al. [10] that better results could be obtained instead of using only one big network. The training sets consisted of the relative change of the bending natural frequencies of the 3 lowest modes. The training sets were generated for cracks located in intervals of 0.04 m between $z = 0$ m and $z = 1.2$ m, respectively. The cracks depths, see figure 2, were in intervals of 0.004 m between 0.02 m and 0.120 m. By a trial-and-error approach it was found for the first net that a 4 layers neural network with 3 input nodes, 8 nodes in each of the two hidden layers and 1 output nodes gave the network with the smallest output error. For the second network it was found that a 4 layers neural network with 4 input nodes, 8 nodes in each of the two hidden layers and 1 output nodes gave the smallest output error. The input and output nodes were chosen as linear while the nodes used in the hidden layers were of the sigmoidal type.

Figures 3 and 4 show results from the networks trained with analytical data and subjected to experimental data obtained from two beams. One beam with a crack located at $z = 0.1$ m and an other beam with a crack located at $z = 1.0$ m. Figure 3 shows the estimated locations while figure 4 shows the estimated crack sizes. The figures show that it is possible to get reasonable estimates of location and size of a crack in a hollow section beam by subjecting experimental data to a network trained with simulated data.

4 CONCLUSIONS

Results from an example with a hollow section steel cantilever demonstrate that a diagnostic technique based on neural networks for detecting, locating and quantifying damages based on vibration measurements works. The damage assessment technique relies on the measurements of small

changes in natural frequencies and upon adequate theoretical prediction of these frequency changes. By using the neural network based damage assessment approach an on-line technique is established. In the following work the neural based technique will be used for detecting and locating damages in civil engineering structures based on vibration measurements.

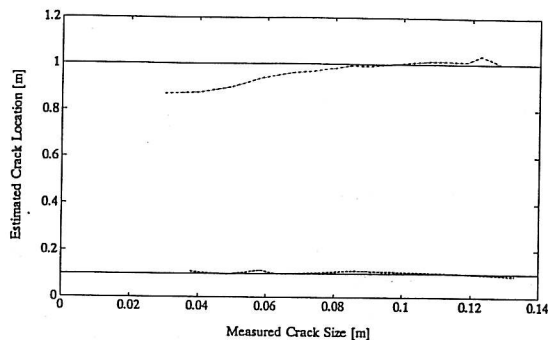


Fig. 3: Estimated crack locations.

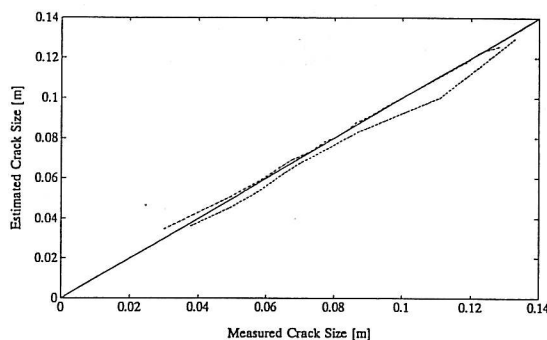


Fig. 4: Estimated crack length.

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