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THE USE OF NEURAL NETWORKS FOR DAMAGE DETECTION AND LOCATION IN A STEEL MEMBER

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This paper explores the potential of using a Multilayer Perceptron (MLP) network trained with the Back-propagation algorithm for damage assessment of a free-free cracked straight steel beam based on vibration measurements. The problem of damage assessment, i.e. detecting, locating and quantifying a damage, is essentially a pattern recognition problem. Since artificial neural networks are proving to be an effective tool for pattern recognition the basic idea is to train a neural network in order to recognize the behaviour of the damaged as well as the undamaged structure. Subjecting this trained neural network to information from vibration tests should imply information about damages states, locations and sizes. The inputs to the network are estimates of the relative changes of the lowest five bending natural frequencies due to damage. During the training these estimates are obtained by an FEM of the beam. A damage in the beam is modelled by a fracture mechanical model. The basic idea of this model is to model the crack zone of a beam by means of a local flexibility matrix found from fracture mechanics. The utility of the neural network approach is demonstrated by a simulation study as well as laboratory tests. The results show that a neural network trained with simulated data is capable for detecting location and size of a damage in a free-free beam when the network is subjected to experimental data.

INTRODUCTION

Structural diagnosis (health monitoring) by measuring and analysing vibrational signals of civil engineering structures is a subject of research investigated with increasing interest during the last decades. The main impetus for doing vibration based inspection is caused of a wish to establish an alternative damage assessment method to the more traditionally methods. The most common of the traditionally methods is visual inspection. However, damage assessment by visual inspection can be costly, risky and difficult when civil engineering structures are considered. Besides a reduction of inspection cost capable vibration based inspection techniques can lead to a cheaper, less risky and a quick means of assessing structural damage. Further, by using vibrational inspection it is not necessary for the investigator to have access to the structure, and the damage assessment is not restricted to a local area.

The basic approach for damage assessment of structures is to detect changes in the dynamic behaviour of the structure that may be characterized by e.g. the natural frequencies and corresponding mode shapes. One of the consequences of the development of a damage is a decrease in local stiffness which in turn results in a change in some of the natural frequencies. The most commonly applied vibration based inspection damage assessment technique is based on changes of natural frequencies only. This is attractive since natural frequencies can be obtained from measurements at a single point on the structure. Recently, computed changes of e.g. the response spectrum, mode shapes etc. have also been used for damage assessment. A throughout review of vibration based damage assessment techniques can be found in Ref. 1.

Most of the methods proposed for damage assessment are based on a mathematical model of the structure, established to give information about the correlation between damage and change in dynamic behaviour. Therefore, the damage assessment results are depending on how well the mathematical model describes the dynamic behaviour of the damaged as well as the undamaged structure. Many papers have considered the problem of establishing such models, see e.g. Refs. 2, 3. However, damage assessment from measured changes in dynamic behaviour is the inverse problem. This means that the damage assessment problem can be solved by a model updating procedure. A very used approach is to estimate the elements in the stiffness matrix for all the potential damage locations, see e.g. Refs. 4, 5. The largest reduction in stiffness, compared with the stiffness of the undamaged structure, is giving the most likely damage location. If the stiffness matrix is given as a function of damage size and location it is also possible to estimate the magnitude of the damage, see e.g. Refs. 6, 1. However, such a procedure based on minimization of a measure of the difference between measured data and the corresponding predictions obtained from a mathematical model implies a comprehensive search which is computationally expensive and nearly impossible for complex structures. Therefore, in any realistic health monitoring situation, a pattern recognition scheme could be needed to decipher the complex pattern of dynamic behaviour changes that occurs due to a damage. Different pattern recognition approaches have been proposed in the literature, see e.g. Refs 7, 8. In these papers pattern recognition methods are presented to estimate the damage presence and location but not magnitude of the damage.

In this paper the aim is to investigate use of artificial neural networks for damage assessment of civil engineering structures. Recently, artificial neural networks are proving to be an effective tool for pattern recognition in a variety of applications, see e.g. Ref. 9. A key requirement of the use of a neural network is that it should be "trained" beforehand. Here, the basic idea is to train a neural network in order to recognize the behaviour of the damaged as well as the undamaged structure. Subjecting this trained neural network to information from vibration tests should imply information about damage state and location. In this application, training of the network is performed with patterns of the relative changes of the natural frequencies that occur due to a damage. This implies that each pattern represents the computed changes of the natural frequencies due to a crack of a particular size at a particular location. The changes are estimated by using a finite-element model. A damage is modelled by a fracture mechanical model. The basic idea of this fracture mechanical model is to model the crack zone by means of a local flexibility matrix found from fracture mechanics.

In the following a short description of artificial neural networks is given and a neural network based damage assessment approach is proposed. Next, the proposed damage assessment approach is used in an example with a free-free straight steel beam. The approach is investigated with numerical as well as experimental data. At last, conclusions are given.

NEURAL NETWORKS

The past decade has seen an explosive growth in the studies of artificial neural networks. In part this was the result of technological advances in personal and main-frame computing, enabling neural network investigators to simulate and test ideas in ways not readily available before 1980.

Artificial neural networks are computational models loosely inspired by the neuron architecture and operation of the human brain. The pioneering work in this field is usually attributed to McCulloch and Pitts in 1943. They developed a simplified model of a neuron. The brain is composed of neurons of many different types, see Ref. 10.

An artificial neural network is an assembly (network) of a large number of highly connected processing units, the so-called nodes or neurons. The neurons are connected by unidirectional communication channels ("connections"). The strength of the connections between the nodes is represented by numerical values which normally are called weights. Knowledge is stored in the form of a collection of weights. Each node has an activation value that is a function of the sum of inputs received from other nodes through the weighted connections. The neural networks are capable of self-organization and knowledge acquisition, i.e learning. One of the characteristics of neural networks is the capability of producing correct, or nearly correct, outputs when presented with partially incorrect or incomplete inputs. Further, neural networks are capable of performing an amount of generalization from the patterns on which they are trained. Most neural networks have some sort of "training" rule whereby the weight of connections are adjusted on

the basis of presented patterns. In other words neural networks "learns" from examples, just like children learn to recognize dogs from examples of dogs, and exhibit some structural capability for generalization. Training consists of providing a set of known input-output pairs, patterns, to the network. The network iteratively adjusts the weights of each of the nodes so as to obtain the desired outputs (for each input set) within a requested level of accuracy. Error is defined as a measure of the difference between the computed pattern and the expected output pattern.

For a more detailed description of neural networks, see e.g. Refs. 9, 11.

Multilayer Perceptron

Since McCulloch-Pitts in 1943 there have been many studies of mathematical models of neural networks. Many different types of neural networks have been proposed by changing the network topology, node characteristics and learning procedures. Examples of those are e.g. the Hopfield network , Ref. 12, the Kohonen network, Ref. 13 and the so-called multilayered perceptron (MLP) network trained by means of the back-propagation algorithm. The MLP trained by the back-propagation algorithm is currently given the greatest attention by application developers, see e.g. Ref 14.

The multilayered perceptron network belongs to the class of layered feed-forward nets with supervised learning. A multilayered neural network is made up of one or more hidden layers placed between the input and output layers, see fig. 1. Each layer consists of a number of nodes connected in the structure of a layered network. The typical architecture is fully interconnected, i.e. each node in a lower level is connected to every node in the higher level. Output units cannot receive signals directly from the input layer. During the training phase activation flows are only allowed in one direction, a feed-forward process, from the input layer to the output layer through the hidden layers. The input vector feeds each of the first layer nodes, the outputs of this layer feed into each of the second layer nodes and so on.
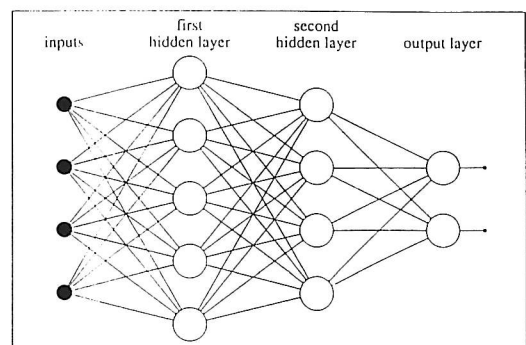
Fig. 1: Principle of a multilayer perceptron neural network.

Associated with each connection between node i and node j in the preceding layer $l-1$ and following layer l is a numerical value $w_{lj,i}$ which is the strength or the weight of the connection. At the start of the training process these weights are initialized by random values. Signal pass through the network and the jth node in layer l computes its output according to

$$x_{lj} = f(\sum_{i=1}^{N_{l-1}} w_{lj,i} x_{l-1,i} + \theta_{lj}) \qquad (1)$$

for $j = 1,..,N_l$ and $l = 1,..,k$, where x_{lj} is the output of the jth node in the lth layer. θ_{lj} is a bias term or a threshold of the jth neuron in the lth layer. The kth layer is the output layer and the input layer must be labelled as layer zero. Thus N_0 and N_k refer to the numbers of network inputs and outputs, respectively. The function $f(\cdot)$ is called the node activation function and is assumed to be differentiable and to have a strictly positive first derivative. For the nodes in the hidden layers, the activation function is often chosen to be a so-called sigmoidal function

$$f(\beta) = \frac{1}{1+e^{-\beta}} \qquad \beta > 0 \qquad (2)$$

The activation function for the nodes in the input and output layers are often chosen as linear.

During the training phase, representative examples of input-output patterns are presented to the network. Each presentation is followed by small adjustments of weights and thresholds if the computed output is not correct. If there is any systematical relationship between input and output and the training examples are representative of this, and if the network topology is properly chosen, then the trained network will often be able to generalize beyond learned examples. Generalization is a measure of how well the network performs on the actual problem once training is complete. It is usually tested by evaluating the performance of the network on new data outside the training set. Generalization is most heavily influenced by three parameters: the number of data samples, the complexity of the underlying problem and the network architecture. Currently, there are no reliable rules for determining the capacity of a feed-forward multilayer neural network. Generally, the capacity of a neural network is a function of the number of hidden layers, the number of processing units in each layer, and the pattern of connectivity between layers. However, it is shown in Refs. 15, 16 that one hidden layer is sufficient to approximate all continuous functions.

Back-Propagation

The first stage of creating an artificial neural network to model an input-output system is to establish the appropriate values of the connection weights $w_{lj,i}$ and thresholds θ_{lj} by using a learning algorithm. A learning algorithm is a systematic procedure for adjusting the weights in the network to achieve a desired input/output relationship, i.e. supervised learning. The most popular and successful learning algorithm used to train multilayer neural networks is currently the Back-propagation routine, see Ref. 14.

The so-called Back-propagation algorithm employs a gradient descent search technique for minimizing an error normally defined as the mean square difference between desired y_j and actual outputs $\hat{y}_j$. I.e. the error E is given as

$$E = 0.5 \sum_{j=1}^{N_k} (y_j - \hat{y}_j)^2 \qquad (3)$$

If the error is considered small enough, the weights and thresholds are not adjusted. If however, a significant error is obtained the weights and thresholds are adjusted in the negative gradient direction, so that the error criterion E is reduced. A typical weight $w_{lj,i}$, which could belong to any layer, is adjusted from its old value $w_{lj,i}^{old}$ to its new value $w_{lj,i}^{new}$ according to

$$w_{lj,i}^{new} = w_{lj,i}^{old} + \Delta w_{lj,i} \qquad (4)$$

where $\Delta w_{lj,i}$ is given by, see e.g. Ref. 17

$$\Delta w_{lj,i} = \eta \delta_{li} x_{l-1,i} \qquad (5)$$

δ_{li} is the error in the output of the ith node in layer l and η is termed a "learning rate". The error δ_{li} is not known a priori but must be constructed from the known errors δ_{ki} at the output layer. The errors are passed backwards through the net and a training algorithm uses the error to adjust the connection weights moving backwards from the output layer, layer by layer, hence the name "Back-propagation". In practice the "learning rate" η is chosen as large as possible (0.01-0.9) without leading to intolerable oscillations. To overcome this problem, a momentum term α is usually introduced into the update rule implying

$$w_{lj,i}^{new} = w_{lj,i}^{old} + \eta \delta_{li} x_{l-1,i} + \alpha \Delta w_{lj,i}^{old} \qquad (6)$$

The thresholds are adjusted in the same way as the weights. The process of computing the gradient and adjusting the weights and thresholds is repeated until a minimum of the error E (or a point sufficiently close to the minimum) is found. However it is generally true that the convergence of the Back-propagation algorithm is fairly slow. Attempts to speed learning include variations on simple gradient search, line search methods and second order methods, see e.g. Refs. 9, 17, 18.

Use of Neural Networks for Damage Assessment

The problem of damage assessment on the basis of measured dynamic data is essentially a pattern recognition problem. Since artificial neural networks are proving to be an effective tool for pattern recognition the basic idea in a neural based damage assessment approach is to train a network with patterns of the changes in quantities describing the dynamic behaviour that occur due to a damage. This implies that each pattern represents the computed changes of e.g. the response spectrum, natural frequencies, mode shapes etc. due to a damage of a particular size at a particular location. The patterns of the quantities describing the dynamic behaviour are used as inputs and the damage location and size as outputs to train the neural network.

Then the trained network subjected to measured patterns of the quantities describing the dynamic behaviour can be used to determine the location, size and of a damage. A hierarchical, two step approach can also be used. This implies that the patterns of the quantities describing the dynamic behaviour are used as inputs and the location of the crack is used as output in one network and size of the crack as output in an other network.

The training of a neural network with appropriate data containing the information about the cause and effect is a key requirement of a neural based damage assessment method. This means that the first step is to establish the training sets which can be used to train a network in a way that the network can recognize the behaviour of the damaged as well as the undamaged structure from measured quantities. Therefore, ideally, the training sets should contain data of the undamaged as well as the damaged structure in various damage states. These data can be obtained by measurements, model tests or through numerical simulation, or through a combination of all three types of data. This possibility of using all obtained information, or only a part, in a neural network based damage assessment method is a capability which is not available in traditional damage assessment methods.

In order to verify how well a trained network has learned the training cases the trained network is tested by subjecting it to the training sets. The important generalization capability of a neural network damage assessment method is tested by subjecting the trained network to data not included into the training sets. How well a trained network is to generalize depends on the adequacy of the selected network architecture and the information about the damage as well as undamaged structure included in the training sets.

EXAMPLE

In this example the proposed neural network approach for damage assessment is applied to two free-free straight steel beams. The beams are 0.8 m long with a 0.02×0.02 m and 0.025×0.025 m square cross section, respectively. An MLP network trained by the Back-propagation algorithm is used.

Analytical and Experimental Results

The applicability of the neural based damage assessment method is investigated by training a neural network with the relative changes of the bending natural frequencies of the 5 lowest modes. A relative change is defined as the change in a natural bending frequency divided by the same natural bending frequency of the undamaged beam. The relative changes of the bending natural frequencies due to a damage were estimated by an FEM of the beam. A damage in the beam is modelled by a fracture mechanical model. The development of a crack at a certain location of a beam corresponds to a sudden reduction of its bending stiffness at the same location. The crack divides the original non-cracked beam into two shorter beams, connected,

at the crack location, by a very infinitesimal portion of beam with different characteristics. The characteristics in bending modes can be modelled by a torsion spring. Estimation of the spring stiffness by fracture mechanics has been used by several authors, see e.g. Refs. 19, 20, 21. The fracture mechanical model used in this paper is reliable for a maximal crack depth at 60 % of the beam height. The FEM was calibrated by using experimental data from the non-cracked beam. This calibration was performed to secure that the FEM describes the beam in the best possible way. The quality of the predictions from any method of damage assessment is critically dependent on the accuracy of the damage model, see e.g. Ref.1.

The experimental data are estimates of the lowest five bending natural frequencies given in Ref. 19. Real line cracks were obtained in the test beams by using a servo-hydraulic testing machine. The locations of the crack were located in intervals of 0.08 m between 0.08 m and 0.4 m, i.e. half of the beam length. The crack depths were 1/6, 1/3, 1/2 and 2/3 of the beam height. The experimental determination of the bending natural frequencies was performed using a mobility measuring setup based on a stepped-sine steady-state technique where both a sinusoidally varying exciting force imposed to the beam and the similarly sinusoidal acceleration response were measured. A more throughout description of the experimental procedure is found in Ref. 19. In fig. 2 and 3 the experimental results are shown together with finite-element results for the beam with a 0.02×0.02 m cross section and 0.025×0.025 m cross section, respectively.

In general, fig. 2 and 3 show that there is a good agreement between theoretical predictions and experimental results. However, most of the discrepancies, such as the ones more easily visible in the graph for the first and second natural frequencies, are mainly due to irregularities of the propagation of the real line cracks, see Ref. 19. Further, it may be noticed that the fracture mechanical model is reliable for a maximal crack depth at 60 % of the beam height.

Training and Testing of Neural Network

First a neural network was trained with simulated estimates of the relative changes of the lowest five natural bending frequencies assuming that the crack can be located in intervals of 0.04 m between 0.04 m and 0.4 m. The crack depths were in intervals of 5 % of of the beam height between 5 % and 60 %. Further, the frequencies also were estimated for the undamaged beam. This means that the input to the network was 121 training sets. By a trial-and-error approach it is found that a 4 layers neural network with 5 input nodes, 7 nodes in each of the two hidden layers and 2 output nodes were giving the network with smallest output error. The two output nodes give the crack location and size, respectively. The input and output nodes were chosen as linear while the nodes used in the hidden layers were of the sigmoidal type.
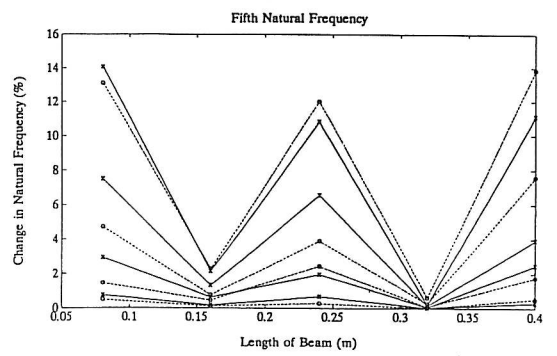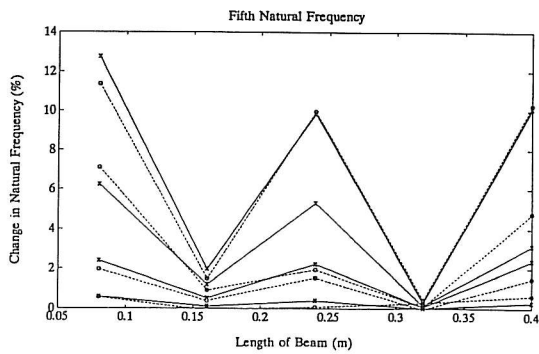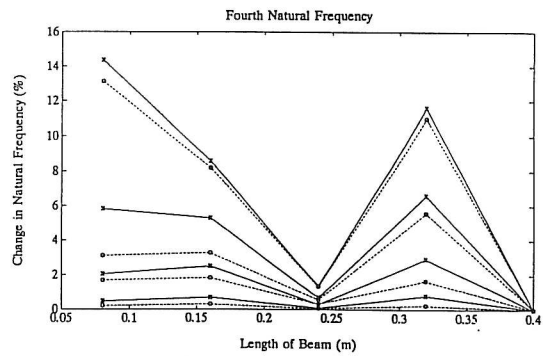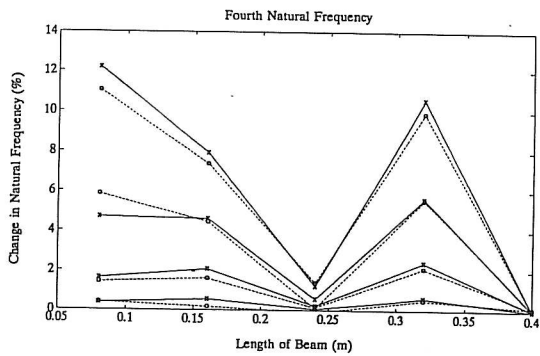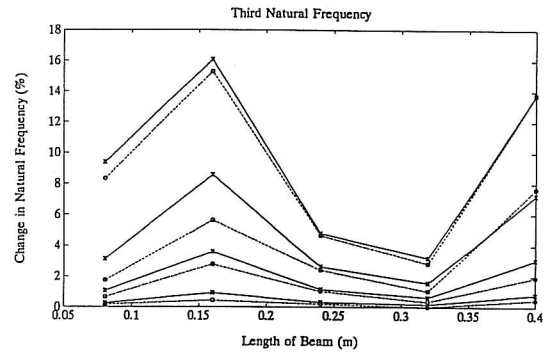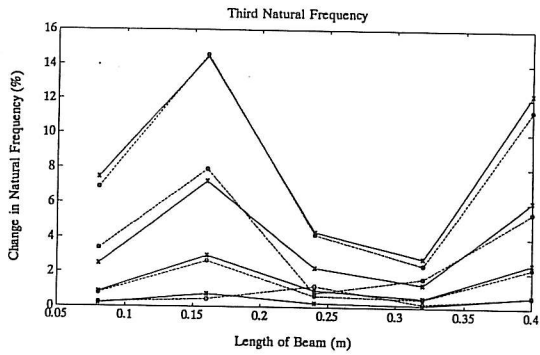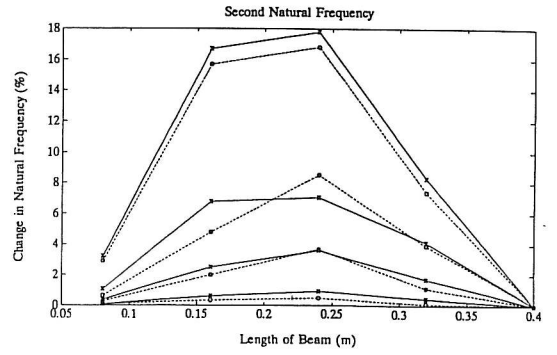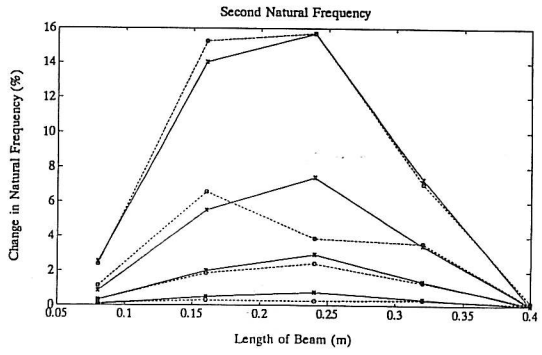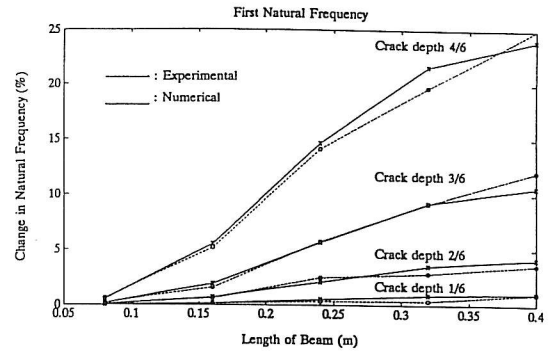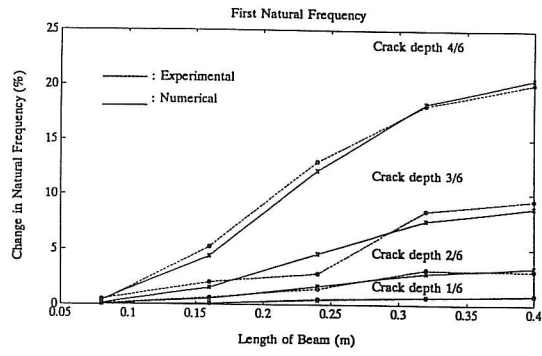
Fig. 2: Experimental versus finite-element results, beam with 0.02 x 0.02 m cross section.

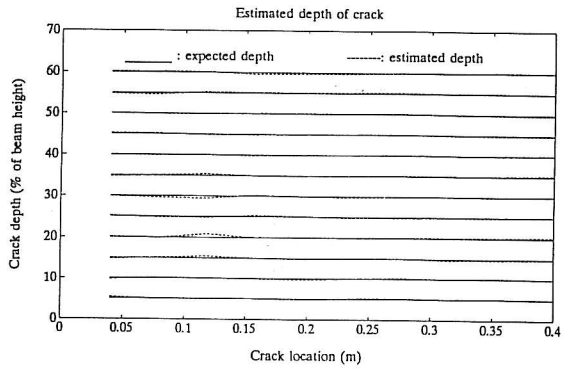Fig. 3: Experimental versus finite-element results, beam with 0.025 x 0.025 m cross section.

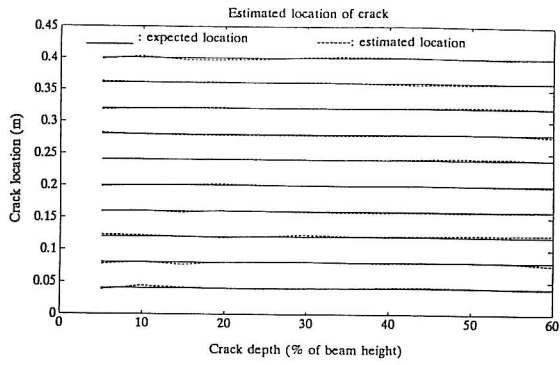Fig. 4: Outputs from network subjected to training sets, beam with 0.02 x 0.02 m cross section.

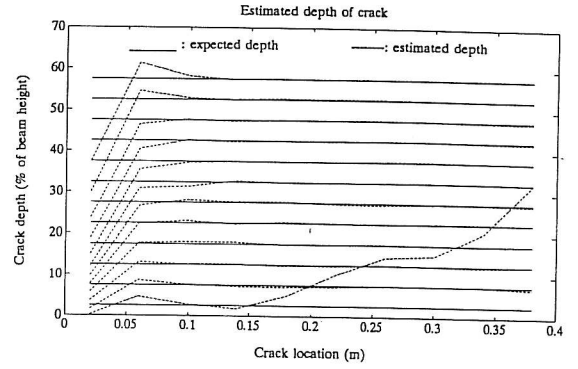Fig. 5: Outputs from network subjected to training sets, beam with 0.025 x 0.025 m cross section.

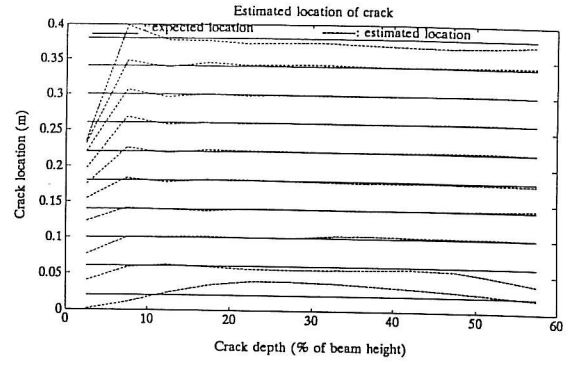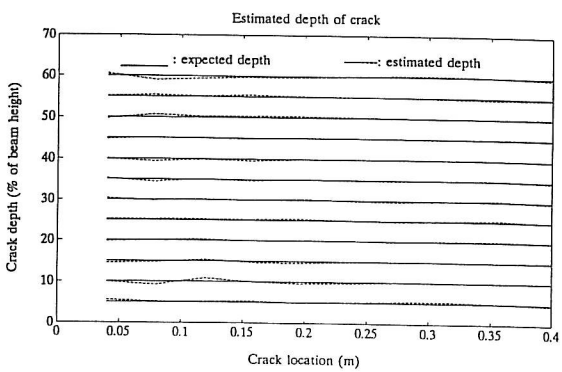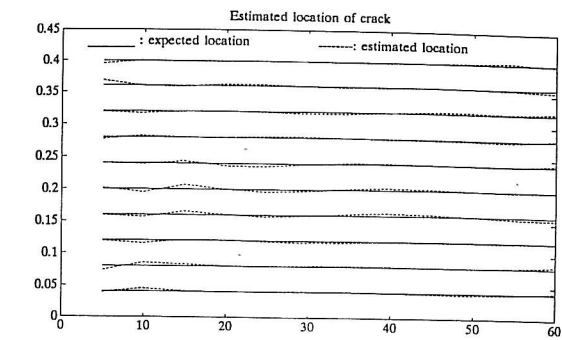Fig. 6: Outputs from the network subjected to data not include in the training sets, beam with 0.02 x 0.02 m cross section.
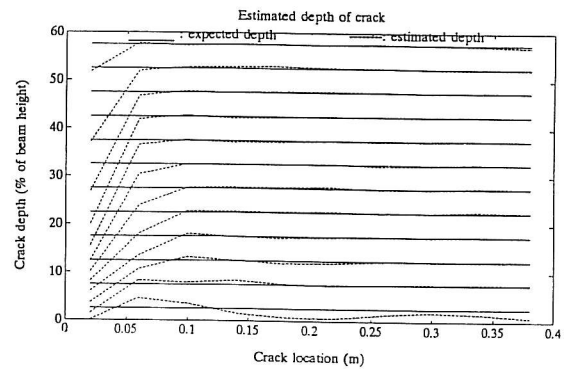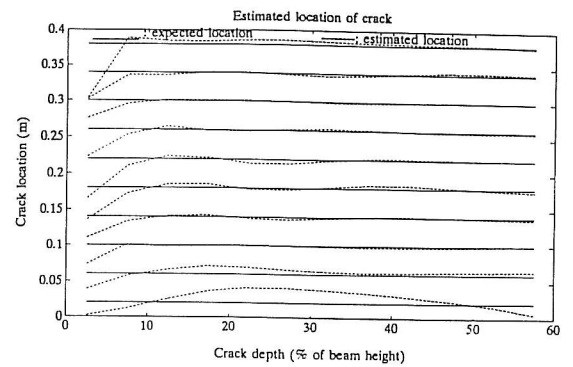

Fig. 7: Outputs from the network subjected to data not include in the training sets, beam with 0.025 x 0.025 m cross section.
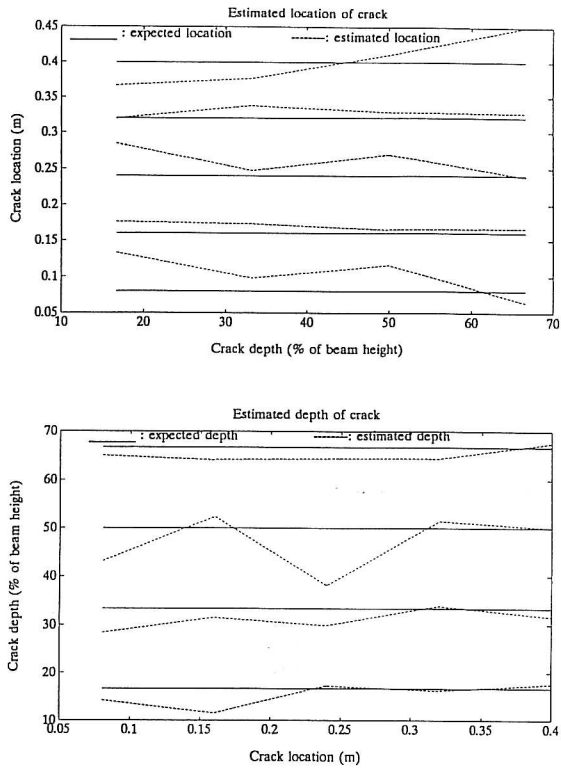
Fig. 8: Outputs from a network trained with simulated data subjected to experimental data, beam with 0.02 x0.02 m cross section.
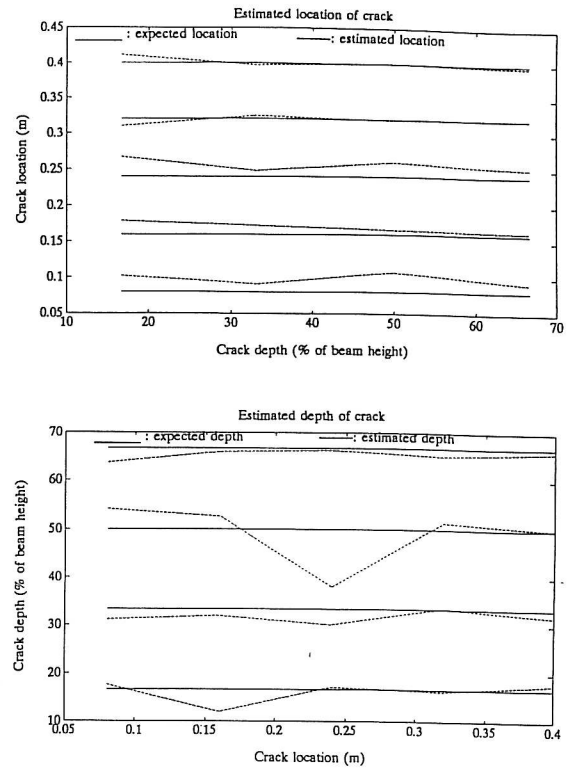
Fig. 10: Outputs from a network trained with simulated data subjected to experimental data, beam with 0.02 x 0.02 m cross section.
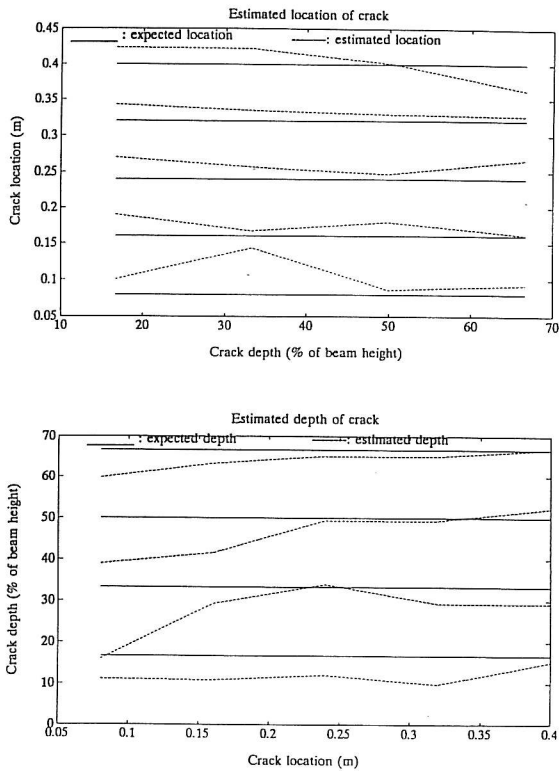
Fig. 9: Outputs from a network trained with simulated data subjected to experimental data, beam with 0.025 x 0.025 m cross section.
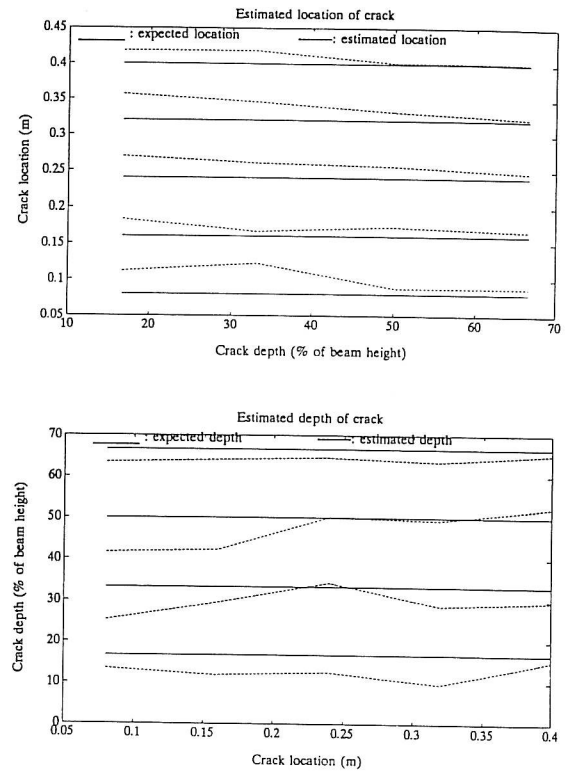
Fig. 11: Outputs from a network trained with simulated data subjected to experimental data, beam with 0.025 x 0.025 m cross section.

The network was tested by subjecting the training sets to the network. The outputs from the network trained with data for the beam with a 0.02×0.02 m and 0.025×0.025 m cross-section are shown in fig. 4 and 5, respectively. Fig. 4 and fig. 5 show that the neural network is capable of reproducing the location and size of the cracks used in training. However, the network also has to be able to give reasonable outputs for input data not included in the training sets.

In fig. 6 and fig. 7 the outputs from the network subjected to input data not included in the training sets are shown. The data are estimated by assuming that the crack can be located in intervals of 0.04 m between 0.06 m and 0.38 m of the beam length, respectively. The crack depths are in intervals of 5 % of the beam height between 7.5 % and 57.5 %. Fig. 6 and fig. 7 show that the network is capable of making a generalization based on the training sets.

Next, the trained networks are subjected to experimental data. In fig. 8 and fig. 9 outputs are shown from the networks subjected to experimental data. Fig. 8 and 9 show that it is possible to estimate location and size of a damage by subjecting experimental data to a neural network trained with simulated data.

In the following the possibility of using a hierarchical, two step neural network training approach is investigated. A neural network with 5 linear input nodes, two hidden layers with 12 sigmoidal nodes and an output layer with one linear node has been trained with the same data as the neural network considered above. The output node gives the location of the damage. An other neural network with 6 linear input nodes, two hidden layers with 8 sigmoidal nodes and an output layer with one linear node has also been trained with the same data. The output node gives the size of the damage. Further, the input to five of the input nodes is the relative change of the five lowest natural bending frequencies. The last input node is the location of the damage. In fig. 10 and fig. 11 outputs are shown from the networks subjected to experimental data. Fig. 10 and 11 show that the hierarchical approach seems to give better estimates of location and size of the crack.

CONCLUSIONS

Results from an example with a free-free straight steel beam demonstrate that a diagnostic technique based on neural networks for detecting, locating and quantifying damages based on vibration measurements works. The damage assessment technique relies on the measurements of small relative changes in natural bending frequencies and upon adequate theoretical prediction of these frequency changes. It is also found that it seems to be more efficient to use a hierarchy of neural networks rather than one big network. By using the neural network based damage assessment approach an on-line techniques is established.

ACKNOWLEDGEMENT

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