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Sliding Mode Control With Grid Voltage Modulated Direct Power Control for Three-Phase AC-DC Converter

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Abstract: In this study, a sliding mode control (SMC) based on the grid voltage modulated direct power control (GVM-DPC) is designed for a three-phase pulse-width modulated (PWM) AC-DC converter (rectifier) system. The GVM-DPC method makes the differential active and reactive power equations of the PWM rectifier system transform into a linear time-invariant system from a linear time-varying one. Thus, the conventional linear control techniques consisting of feedback and feedforward controllers are applicable to control active and reactive powers independently. The proposed method is guaranteed that the closed system is globally exponentially stable. In order to maintain a constant dc-link voltage, an SMC method is employed to generate the active power reference of the GVM-DPC method. From the simulation results, the proposed method improves the transient performance of the system and has a robust property to the parameter mismatch.

Key Words: AC-DC converter, grid voltage modulated direct power control, sliding mode control, transient performance.

1 Introduction

Power converters have recently gained tremendous attention due to a broad range of applications such as smart grid, flexible AC transmission systems, renewable energy sources (wind and photovoltaic), and energy storage systems [1–4]. One of the key devices of power converters is a three-phase AC-DC converter (rectifier) with the pulse-width modulation (PWM), which are widely used in various applications, e.g., distributed generation systems, drives of electrical motors, uninterruptible power supplies, etc. [5–7]. One of the most important control objectives of rectifier system is to control its dc-link voltage at a certain constant value.

Normally, two decoupled proportional-integral (PI) controller are designed for the PWM rectifier system in a synchronous rotating reference frame to control d–q axes currents independently [8]. The controller designed in the synchronous rotating reference frame has an advantage that it converts the rectifier system from ac signal to dc one. Based on this concept, various control algorithms are applied to the PWM rectifier system, e.g., sliding-mode control (SMC) [9], passivity-based control [10], and model predictive control [11], etc. In order to apply the synchronous rotating reference frame, normally, a phase lock system is used to extract the phase of the grid voltage, which suffers from a slow dynamical response.

To deal with such issue, the method based on direct power control (DPC) is designed through the calculation of active and reactive powers [12]. One of the advantages is that the DPC method eliminates the inner-loop current regulators. However, it obtains a variable switching frequency, which causes a difficulty for the filter design. To overcome such problem, the space vector modulation (SVM) based DPC method is proposed to achieve a constant switching frequency [13,14]. In addition, the SVM-DPC proposed in [15] obtains a low total harmonics distortion line currents without using positive and negative sequence components under unbalanced grid conditions. In order to obtain an active power oscillation cancellation automatically, a novel definition of reactive power based on pq theory has been applied in DPC strategy [16,17]. This method provides a constant active power and sinusoidal grid currents even under an unbalanced grid condition. Furthermore, a model predictive control (MPC) based DPC strategy has been applied to power converter applications [11]. The MPC-DPC calculates duty cycles for the rectifier system in every sampling period, i.e., it achieves a constant switching frequency [18]. One of the main problems is that an incorrect voltage sequence selection will adversely affect the converter performance [19]. Moreover, the computation time is a critical issue. Recently, Gui et al. introduced a grid voltage modulated (GVM)-DPC algorithm for voltage source converters (VSCs) firstly in [20], and later developed this into a grid-connected voltage source inverter [21]. The main advantage of the GVM-DPC is that it can transform the DPC model of VSCs to a linear time invariant system. That means the various linear technique could be used based on the GVM-DPC technique. Moreover, Gui et al. proved that the DPC model with the GVM-DPC is mythically same with the current model in the synchronous rotating reference frame [22]. That means the GVM-DPC concept could be used various applications with consideration its benefits [23–27].

Furthermore, in order to obtain an enhanced dynamic performance of the dc-link voltage, feedforward control techniques have been researched by considering the mismatched power/current disturbances [28,29]. In addition, to remove the load current sensor on the dc-link for the improvement of the reliability and decrease of the cost, disturbance observers are designed to identify the load current or power disturbance [9,30]. However, the disturbance observer is not studied in this paper.

Although, the GVM-DPC algorithm was firstly employed to the three-phase rectifier system [24], the input-output linearization (IOL) method is used to control the dc-link voltage. Compared to the work in [24], we modify the GVM-DPC for the ac-side of rectifier system in order to obtain a conventional two second-order system. Then, it is guaranteed that the closed system is globally exponentially stable; Moreover, we use an SMC method to regulate the dc-link voltage in order to enhance the regulation performance compared to the IOL method proposed in [24]. The effectiveness
of the proposed method is validated through a simulation using MATLAB/Simulink and PLECS blockset.

The rest of the paper is organized as follows. In Section 2, the model of the rectifier system combining ac and dc side is introduced. In Section 3, we present the design of the modified GVM-DPC and SMC for the dc-link voltage in detail. Section 4 shows the simulation results using MATLAB/Simulink and PLECS blockset. Finally, the conclusions are given in Section 5.

2 Modeling of PWM Rectifier System

Fig. 1 shows the configuration of a typical three-phase two-level PWM rectifier connected to the grid with an L-filter. $v_{sabc}$, $L_{sabc}$, $R_{sabc}$, $i_{sabc}$, and $u_{sabc}$ are the three-phase of grid voltage, filter inductance, filter resistance, line current, and rectifier voltage, respectively. $V_{dc}$ is the voltage at the dc-link, where $C$ is the dc-link capacitor. $I_{dc}$ is the current flowing into the load at the dc-link.

2.1 Model of AC-Side

In this study, we consider a balanced grid voltage, where the relationship between the grid voltage and converter voltage could be described as follows [31]:

$$
\begin{align*}
v_{sa} &= R_{sa}i_a + L_{sa}\frac{di_a}{dt} + u_a, \\
v_{sb} &= R_{sb}i_b + L_{sb}\frac{di_b}{dt} + u_b, \\
v_{sc} &= R_{sc}i_c + L_{sc}\frac{di_c}{dt} + u_c.
\end{align*}
$$

Assumption 1. We assume that the filter inductances and resistances of the three phase are the same in this study, i.e., $L_{sa} = L_{sb} = L_{sc} = L$ and $R_{sa} = R_{sb} = R_{sc} = R$.

Based on the Clark transformation, the system in (1) is represented in the stationary reference frame as follows:

$$
\begin{align*}
v_{sa} &= R_{sa}\alpha + L_{sa}\frac{d\alpha}{dt} + u_a, \\
v_{sb} &= R_{sb}\beta + L_{sb}\frac{d\beta}{dt} + u_b,
\end{align*}
$$

where $v_{sa}$ and $v_{sb}$ indicate the grid voltages, $R_{sa}$ and $R_{sb}$ indicate filter resistance, $L_{sa}$ and $L_{sb}$ indicate filter inductance, $i_\alpha$ and $i_\beta$ indicate the output currents, and $u_a$ and $u_b$ indicate the rectifier voltages in the stationary reference frame, respectively.

Based on the definition of instantaneous active and reactive powers in the stationary reference frame, we can use the following equations.

$$
P = \frac{3}{2}(v_{s\alpha}i_\alpha + v_{s\beta}i_\beta),
$$

$$
Q = \frac{3}{2}(v_{s\beta}i_\alpha - v_{s\alpha}i_\beta),
$$

where $P$ and $Q$ are the injected active and reactive powers of rectifier system, respectively. If we differentiate the injected active and reactive powers of rectifier system in (3) with respect to time, then their variations are expressed such as

$$
\frac{dP}{dt} = \frac{3}{2}(i_{s\alpha}\frac{dv_{s\alpha}}{dt} + v_{s\alpha}\frac{di_\alpha}{dt} + i_{s\beta}\frac{dv_{s\beta}}{dt} + v_{s\beta}\frac{di_\beta}{dt}),
$$

$$
\frac{dQ}{dt} = \frac{3}{2}(i_{s\alpha}\frac{dv_{s\beta}}{dt} + v_{s\beta}\frac{di_\alpha}{dt} - i_{s\beta}\frac{dv_{s\alpha}}{dt} - v_{s\alpha}\frac{di_\beta}{dt}).
$$

Since we consider a nondistorted grid in this study, we can define the grid voltage in the stationary reference frame as follows:

$$
v_{s\alpha} = V_s\cos(\omega t), \quad v_{s\beta} = V_s\sin(\omega t),
$$

where $V_s$ indicates the magnitude of $v_{s\alpha,\beta}$ and $\omega$ indicates angular frequency of $v_{s\alpha,\beta}$. If we differentiate the grid voltage in (5), then its variations could be expressed such as

$$
\frac{dv_{s\alpha}}{dt} = -\omega V_s\sin(\omega t) = -\omega v_{s\beta},
$$

$$
\frac{dv_{s\beta}}{dt} = \omega V_s\cos(\omega t) = \omega v_{s\alpha}.
$$

Considering from (2) to (6), we can obtain the dynamics of the injected active and reactive powers of rectifier such as [32]

$$
\frac{dP}{dt} = -\frac{R_s}{L_s}P - \omega Q + \frac{3}{2L_s}(v_{s\alpha}u_a + v_{s\beta}u_b) - \frac{3}{2L_s}V_s^2,
$$

$$
\frac{dQ}{dt} = \omega P - \frac{R_s}{L_s}Q + \frac{3}{2L_s}(v_{s\beta}u_a - v_{s\alpha}u_b),
$$

where $u_a$ and $u_b$ are the control inputs (rectifier voltages).

Remark 1. Notice that the dynamics of the injected active and reactive powers in (7) describes a time-varying system since the grid voltages multiply by the control inputs.

2.2 Model of DC-Side

In this study, we neglect the system losses. Thus, the power variation in the dc-link capacitor can be expressed as

$$
P_{\text{cap}} = CV_{dc}\frac{dV_{dc}}{dt} = P_{\text{rec}} - P_{\text{load}},
$$

where $P_{\text{cap}}$ is the active power stored in the capacitor, $P_{\text{rec}}$ is the injected active power of the rectifier system from ac to dc-side, and $P_{\text{load}}$ is the consumed power by the load connected in the dc-side. Since we consider a resistor load in this study for the sake of simplicity, $P_{\text{load}}$ can be represented such as

$$
P_{\text{load}} = V_{dc}I_{dc}.
$$

Consequely, the dynamics of the dc-link voltage could be simplified by substituting (9) into (8) such as

$$
\frac{dV_{dc}}{dt} = \frac{P_{\text{cap}}}{C} \frac{1}{V_{dc}} - \frac{1}{C}I_{dc}.
$$

Remark 2. Notice that the dynamics of the dc-link voltage in (10) is nonlinear due to the state in the denominator.
3 Controller Design for PWM Rectifier

To simplify the controller’s design, an LTI system is needed as a system described in the \( d-q \) frame. To this end, we find a relationship between DPC model and system model in the \( d-q \) frame.

3.1 Voltage Modulated DPC

In order to obtain an LTI system, we apply the grid voltage modulated (GVM) inputs such as [24]

\[
\begin{align*}
\nu_{GVM1} &= \frac{3}{2L_s}(v_{sa}u_a + v_{sb}u_b), \\
\nu_{GVM2} &= \frac{3}{2L_s}(v_{sb}u_a - v_{sa}u_b).
\end{align*}
\]

Then, the dynamics of the injected active and reactive powers of rectifier in (7) is rewritten as

\[
\begin{align*}
\frac{dP}{dt} &= -\frac{R_s}{L_s}P + \omega Q - \frac{3}{2L_s}V_s^2 + \nu_{GVM1}, \\
\frac{dQ}{dt} &= \omega P - \frac{R_s}{L_s}Q + \nu_{GVM2}.
\end{align*}
\]

Remark 3. The dynamics of the injected active and reactive powers of rectifier in (12) is changed into an LTI system.

At first, we define that \( P^* \) and \( Q^* \) are the references of injected active and reactive powers of rectifier, respectively.

Theorem 1. Consider the system in (12) and the controller such as

\[
\begin{align*}
\nu_{GVM1} &= \frac{R_s}{L_s}P + \omega Q + \frac{3}{2L_s}V_s^2 + K_{P1}(P^* - P) \\
&+ K_{I1}\int (P^* - P)dt, \\
\nu_{GVM2} &= -\omega P + K_{P2}(Q^* - Q) + K_{I2}\int (Q^* - Q)dt,
\end{align*}
\]

where \( K_{P1}, K_{I1}, K_{P2}, \) and \( K_{I2} \) are any positive values; the closed-loop interconnection of the system and controller is exponentially stable.

Proof. At the first step, we define that errors of injected active and reactive powers of rectifier as follows:

\[
\begin{align*}
e_P &= P^* - P, \\
e_Q &= Q^* - Q.
\end{align*}
\]

then, we can obtain error dynamics as such

\[
\begin{align*}
\dot{e}_P &= \dot{P}^* - \dot{P}, \\
\dot{e}_Q &= \dot{Q}^* - \dot{Q}.
\end{align*}
\]

For the simplicity, we consider the references of injected active and reactive powers of rectifier as constant in this study. Then, substituting (13) into (15), we can obtain the closed-loop system such as

\[
\begin{align*}
\dot{e}_P &= K_{P1}e_P + K_{I1}\int e_Pdt, \\
\dot{e}_Q &= K_{P2}e_Q + K_{I2}\int e_Qdt.
\end{align*}
\]

Let us define the new variables as follows:

\[
\begin{align*}
\psi_P &= P^* - P, \\
\psi_Q &= Q^* - Q.
\end{align*}
\]

Then, the whole closed-loop system with (16) and (17) could be represented as follows: Differentiating (16) with respect to time yields

\[
\begin{pmatrix}
\dot{e}_P \\
\dot{e}_Q
\end{pmatrix} =
\begin{pmatrix}
-K_{P1} & -K_{I1} \\
0 & -K_{P2} -K_{I2}
\end{pmatrix}
\begin{pmatrix}
e_P \\
e_Q
\end{pmatrix} +
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\psi_P \\
\psi_Q
\end{pmatrix},
\]

where \( x \) and \( A_{cl} \) are the state and the state-space matrix of the closed-loop system, respectively. If \( K_{P1}, K_{I1}, K_{P2}, \) and \( K_{I2} \) are positive values, then \( A_{cl} \) has all negative eigenvalues. That means the closed-loop system is exponentially stable.

\[
\Box
\]

It is obvious that the closed-loop system consists of two independent second-order systems, which corresponding to the active and reactive power, respectively. The detailed controller gain design process could be found in [24]. In order to obtain the original control inputs (rectifier voltages), we can use the inverse of (11) such as

\[
\begin{align*}
u_a &= \frac{2L_s}{3}v_{sa}u_{GVM1} - v_{sb}u_{GVM2}, \\
u_b &= \frac{2L_s}{3}v_{sb}u_{GVM1} + v_{sa}u_{GVM2}.
\end{align*}
\]

3.2 DC-Link Controller

In order to control the dc-link voltage at a constant value, we design an SMC method, which is one of best tracking solution. In this study, we neglect the losses between the injected power of the rectifier and dc power.

Let us define a sliding surface \( s \) as follows:

\[
s = K_{PVdc}(V_{dc}^* - V_{dc}) + K_{I1Vdc}\int (V_{dc}^* - V_{dc})dt,
\]

where \( K_{PVdc} \) and \( K_{I1Vdc} \) are the controller gains. \( V_{dc}^* \) is the reference of \( V_{dc} \). On the surface (i.e., \( s = 0 \)), the motion is governed by

\[
K_{PVdc}(V_{dc}^* - V_{dc}) + K_{I1Vdc}\int (V_{dc}^* - V_{dc})dt = 0
\]

Remark 4. Choosing \( K_{PVdc} > 0 \) and \( K_{I1Vdc} > 0 \) guarantees that the dc-link voltage tends to its reference as the time tends to infinity and the rate of convergence can be determined by the selection of \( K_{PVdc} \) and \( K_{I1Vdc} \).

Thus, we can obtain the following equation.

\[
s = K_{PVdc}(V_{dc}^* - V_{dc}) + K_{I1Vdc}\int (V_{dc}^* - V_{dc})dt
\]

Normally, the dc-link voltage is regulated to a constant value, hence, \( V_{dc}^* = 0 \) in this study.

Theorem 2. Consider the system in (10), if we take a control law such as

\[
P_{rec} = I_{dc}V_{dc} + \frac{K_{I1Vdc}CV_{dc}}{K_{PVdc}}(V_{dc}^* - V_{dc}) + K_{sat}(\frac{s}{\epsilon}),
\]

where \( s \) and \( \epsilon \) are the sliding surface and sliding surface width.
where

\[
\text{sat}(s) = \begin{cases} 
  s & \text{if } |s| \leq |\epsilon| \\
  +1 & \text{if } s > \epsilon \\
  -1 & \text{if } s < -\epsilon
\end{cases}
\]

and \( \epsilon \) is a positive constant value and taking the controller gain \( K_s > 0 \), then the trajectory reaches the boundary layer \( |s| \leq \epsilon \) in finite time.

**Proof.** At first, we take a Lyapunov function candidate such as

\[
V = \frac{1}{2} s^2.
\]

The derivative of Lyapunov function candidate in (24) results in

\[
\dot{V} = ss'
\]

\[
= s \left[ K_P V_d (\dot{V}_d - \dot{V}_d') + K_I V_d (V_d' - V_d) \right].
\]

If we apply a control law in (23), then in \( s \geq \epsilon \), (25) yields

\[
V = -K_s |s|
\]

It is obvious that if \( K_s > 0 \), then the trajectory reaches the boundary layer \( |s| \leq \epsilon \) in finite time.

**Remark 5.** From Theorem 2, we can conclude that the dc-link voltage reaches the boundary layer \( |s| \leq \epsilon \) in finite time. After that, the dc-link voltage will converge to its reference based on \( K_P V_d \) and \( K_I V_d \).

It should be noted that the control input, \( P_{\text{rec}} \) in (23), is generated and sent to the reference of the active power in (13). Consequently, the whole block diagram of the proposed method is shown in Fig. 2.

**4 Performance Validation**

To validate the proposed SMC of GVM-DPC method, we use MATLAB/Simulink to implement the control algorithm and PLECS blockset to conduct the electrical system. The parameters of the system used in the simulation are listed in

![Fig. 2: Block diagram of the proposed method for a two-level PWM rectifier.](image-url)
Table 2: Controller gains used in simulation

<table>
<thead>
<tr>
<th>Methods</th>
<th>$K_{P_{V_{dc}}}$</th>
<th>$K_{I_{V_{dc}}}$</th>
<th>$K_s$</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMC</td>
<td>1</td>
<td>10</td>
<td>200</td>
<td>0.2</td>
</tr>
<tr>
<td>IOL</td>
<td>100</td>
<td>1000</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1. The proposed method is compared with the IOL of GVM-DPC proposed in [24]. The controller gains of both method used in the simulation are listed in Table 2.

At first, we compare two methods when the load is connected to dc-link at 0.05 s. Fig. 3 shows the performance of the proposed method and the IOL method. The proposed method has a faster convergence time and a smaller overshoot in the dc-link voltage than the IOL method using 100 times larger gains, as shown in Fig. 3(d). We can observe that the trajectory with the proposed method reaches its sliding surface in the finite time and remains inside the boundary layer. After that, it converges to its equilibrium point smoothly, as shown in Fig. 4. In addition, we also test a case that there is parameter mismatches between the control implementation and real system. Normally, the capacitance of the dc-link capacitor will be decrease after a certain operation time. At this case, we assume that the capacitance in the control implementation has 50% of the nominal value. From Fig. 5, we can observe that the proposed method has the same performance. However, the performance of the IOL method is adversely affected by the parameter mismatch, however.

5 Conclusions

In this paper, we modified the GVM-DPC for three-phase PWM rectifier to control instantaneous active and reactive powers. The SMC is used to obtain the power reference of the ac side in order to maintain the dc-link voltage at a certain value. Simulation results show that the proposed method effectively reduces the overshoot of the dc-link voltage and be robust to the parameter mismatch of the capacitance of the capacitor at the dc-link.

In the future work, a high-order SMC strategy could be considered to avoid the chattering phenomenon. In addition, the disturbance observer of dc-link current will be designed to remove the current sensor for the improvement of the reliability and decrease of the cost.

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Fig. 3: Simulation results of (a) active and reactive powers, (b) three-phase currents with SMC method, (c) three-phase currents with IOL method, and (d) dc-link voltage.

Fig. 4: System trajectory and sliding surface when the load is connected.
Fig. 5: Dc-link voltage when the capacitance of the dc capacitor has 50% error in the control implementation.


