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# **A Bi-level Model for Strategic Bidding of a Price-Maker Retailer with Flexible Demands in Day-Ahead Electricity Market**

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## **Abstract**

In this paper, a bi-level Stackelberg-based model between an electricity retailer and consumers is presented, in which the upper-level consists of a price-maker retailer (PMR) modeled as the leader who seeks to maximize its own profit by adopting optimal pricing strategies for a pool-based electricity market. At the same time, PMR reduces its risks by encouraging consumers to actively participate in demand response (DR) programs. The lower-level of the model consists of 4 followers, three of them represent customer groups with distinct reactions to DR programs, and their objective function is defined as minimizing the cost of purchased electricity while preserving the welfare level. The fourth follower is the electricity pool, which is responsible for implementation of market mechanism and determination of market clearing price (MCP) with the aim of increasing the consumers' welfare. In the proposed framework, the reaction of consumers to prices and DR programs are also organized and studied in form of several scenarios. The outputs of the proposed model will be enhancing the retailer's profit and determination of the effect of DR programs on power consumption during peak hours as well as consumers' welfare.

## **Keywords:**

Demand response (DR), Bidding strategy, Bi-level optimization, Stackelberg-based model, Consumer utility function, Mathematical programming.

## Nomenclature

### Indices

H	Index for busses of High-Flexibility consumers
S	Index for busses of Semi-Flexibility consumers
L	Index for busses of Low-Flexibility consumers
n	Index for bus number from 1 to N
m	Index for bus number from 1 to M
$\omega$	Index for scenarios running from 1 to $N_\omega$

### Constants

$P_{H,t}$	Marginal utility of High-Flexibility consumers in period t
$P_{S,t}$	Marginal utility of Semi-Flexibility consumers in period t
$P_{L,t}$	Marginal utility of Low-Flexibility consumers in period t
$d_{H,t}$	Demanded power of High-Flexibility consumers in period t
$d_{S,t}$	Demanded power of Semi-Flexibility consumers in period t
$d_{L,t}$	Demanded power of Low-Flexibility consumers in period t
$\gamma$	Minimum electricity required to purchase from day-ahead market
$M_{O,t}$	Marginal utility of other consumers in period t
$B_{nm}$	Susceptance of line (n, m)
$F_{nm}$	Capacity of transmission line (n, m)

### Variables

$D_{H,\omega,t}$	Power purchased of High-Flexibility consumers in period t and scenario $\omega$ from day-ahead market
$D_{S,\omega,t}$	Power purchased of Semi-Flexibility consumers in period t and scenario $\omega$ from day-ahead market
$D_{L,\omega,t}$	Power purchased of Low-Flexibility consumers in period t and scenario $\omega$ from day-ahead market
$\lambda_{\omega,t}$	Market clearing price in period t and scenario $\omega$
$\pi_{\omega,t}$	Real-time price in period t and scenario $\omega$
$\Delta_{H,\omega,t}$	Price bid by High-Flexibility consumers in period t and scenario $\omega$ from day-ahead market
$\Delta_{S,\omega,t}$	Price bid by Semi-Flexibility consumers in period t and scenario $\omega$ from day-ahead market
$\Delta_{L,\omega,t}$	Price bid by Low-Flexibility consumers in period t and scenario $\omega$ from day-ahead market
$P_{j,b,\omega,t}$	Power produced by of block b of generating unit j in period t and scenario $\omega$
$\Lambda_{j,b,\omega,t}$	Price offer by block b of generating unit j in period t and scenario $\omega$
$D_{O,\omega,t}$	Power purchased of other consumers in period t and scenario $\omega$ from day-ahead market
$\theta_{t,n,\omega}$	Voltage angle of bus n in period t and scenario $\omega$

$B_{nm}$	Susceptance of line (n, m)
$F_{nm}$	Capacity of transmission line (n, m)

## 1. Introduction

All markets, including the electricity market, are structurally divided into two categories: 1) perfect competition and, 2) incomplete competition markets [1]. A well-accepted definition of the market with perfect competition is described by the French economist François Perroux as a market where the number of suppliers and customers is so high that every supplier/ customer is like a droplet in the sea and none of the parties is able to affect the market price by changing the supply /demand [2]. In contrast to a market with perfect competition, there is a market with imperfect competition [3]. In this market, prices can be affected by some suppliers and/or customers. In economics, the power to affect prices is regarded as "market power". In fact, market power is often defined as "the ability to divert prices from competitive levels"[4]. In electricity markets, absence of different elements such as flexibility of demand, can lead to the emergence market power in the supply side that subsequently results in phenomena such as a sharp jump in electricity prices [5]-[6].

Empirical evidences show that market power exists in most of the world's electricity markets [7]. Hence, the adoption of strategic behavior aimed at gaining more profits in the market is a logical routine. In order to relieve market power from the demand side, there are two solutions as in the supply side [8]: 1) Physical withholding (demand flexibility) and 2) Financial withholding (Bidding strategy). An example for physical withholding in electricity market is implementing demand response (DR) programs where a part of demand especially during the peak times can be reduced and/or shifted to off-peak times. Typically, in a market with complete competition, the prices offered by participants are equal to marginal costs. The financial withholding is, in fact, seeking mechanisms for making strategic bids in a different way than the marginal costs. Therefore, it is logical to take physical withholding and financial withholding for the demand side to counter the market power of the supply side. Authors in [9]-[10] show that how demand-side bidding can prevent price spikes in electricity market. In [11], a stochastic model is proposed to create bidding curves for offering to power pools. In this model, a strategic retailer is responsible for supplying electricity to consumers. In addition, consumers are assumed to be responsive to the price fluctuations. The main goal of this model is to minimize the purchase price of energy from the day-ahead electricity market and the regulation market. In [12], a procedure to help big consumers to derive bidding curve in a day-ahead market is proposed based on the Information Gap Decision Theory (IGDT). Similarly, a Monte Carlo-based algorithm is proposed in [13], to solve the problem of consumers' coalition equipped with DR actions. Authors in [14] develop a stochastic model to explain the strategic behavior of a big consumer for making bidding curve.

In the electricity market, the retailer is responsible for supplying customer demand from the wholesale market. In spite of the many uncertainties that exist in the wholesale market, all risks remain with the retailer. In [15], an optimized bidding strategy is presented that implements a DR program that reduces the risk to the retailer. Although the work provides a mechanism that forces customers to participate in DR programs, it does not address the two important properties of a good DR model namely adaptability and adjustability features as reported in [16]. The adaptability feature is defined as the adaptation of different consumers with different tendencies to the DR programs. Different consumers may behave differently to the DR programs and can be classified into at least three groups (high-/semi-/low-flexible consumers) depending on their willingness to participate in DR actions. The ability of a DR model to adapt to and accept all consumer

groups is an essential feature. The second feature of an effective DR model is defined as being adjustable to consumers' time preferences. The concept of adjustability demonstrates the ability to integrate consumers' desire (according to their habits and lifestyles) with the DR model to regulate consumption over time. These are points where current DR models show obvious weaknesses.

In [17], a new mathematical model is presented to provide an optimal bidding strategy for a large industrial consumer. In that model, several uncertainties have been considered using robust-stochastic hybrid method. The results show that the greater the number of uncertainties is considered, the greater the purchase price would be for large consumers. Residential consumers with flexibility in consuming electricity throughout the day can be considered as a great potential in DR programs. Since residential consumer capacity is typically not sufficient to participate in the day-ahead electricity market, retailers act as intermediaries between consumers and the electricity market, benefiting through the implementation of DR programs from the flexibility of consumers. In [18], the strategic interaction between the retailer, consumers and the day-ahead electricity market has been examined using a two-way optimization framework. The results show that the interaction of the retailer and the consumers has significant monetary benefits. Also, in [19], a game theory and Nash equilibrium model is proposed to obtain the optimal bidding strategy in the electricity market. The proposed scheme uses an economically responsive load model for the DR approach based on customer benefit function and price elasticity. The network operator receives the DR Service from the DR aggregators and the share of each aggregators in DR programs is determined. Retail companies that represent a number of small electricity consumers or large electricity consumers such as a large industrial consumer can participate directly in the wholesale electricity market. Large consumers face various uncertainties due to the use of different sources of energy such as renewables, self-generating units, forward contracts and the pool market. These uncertainties can lead to many financial risks for them. In [20], a new risk assessment approach is used to analyze the major consumer risks in the electricity supply process. The risk measurement method is referred to as the downside risk constraints method, which is used to model the financial risk imposed by uncertain parameters along with random problems. According to the results, large consumers can experience low-risk strategies in power supply. Also, using this method the total cost of the large consumer is independent of different scenarios, which results in less risk experience by the large consumer.

Authors in [14] clearly state that there are a few number of papers in the strategic bidding for demand side management. On the other hand, it is explicitly stated in [21] that most of the existing effort have given the focus to the price-taker retailers. This assumption can be true when the considered market is a market with perfect competition. As it was stated, empirical evidences show that market power exists in most of the world's electricity markets and so the assumption cannot be always held true[7]. Hence, above all, research gaps exist in price-maker retailer whose customers are sensitive to spot price. The next gap of the research is to understand how a price-maker retailer can model their customers' sensitivity to spot price. In view of the discussion presented, an efficient model for consumers (as followers of this bi-level model) has both adaptability and adjustability features. The existing models in this field do not include these features, and at best have been considered in several scenarios. This paper use an efficient model in the discussion of consumers. Following the mentioned procedure, the proposed model is able to enhance the retailer's profit while taking consumer's satisfaction into consideration

In this paper, a bi-level model of stochastic optimization based on single-leader and multi-follower Stackelberg model between a price-maker retailer and consumers in the residential sector is presented. In this model, a strategic retailer whose customers have enthusiasm to participate in DR programs is also considered. The responsive loads could expand

the retailer's flexibility in making efficient bidding strategies. In the proposed bi-level Stackelberg-based optimization model, the upper-level consists of a price-maker retailer that is modeled as a leader. This retailer seeks to maximize its own profit by optimal pricing strategies for the pool-based electricity market, but at the same time, it aims at reducing its risks by encouraging active participation of consumers in DR programs. The lower-level of the model consists of four followers. Three of these followers are customer groups with unique responses to DR programs. The objective function of these groups is to minimize the cost of purchased electricity. The fourth follower is the power pool, who is responsible for implementation of market mechanism and determination of market clearing price (MCP) with the aim of achieving maximum social welfare.

As a whole, contributions of this study can be summarized as:

- A bi-level Stackelberg-based framework is presented to capture interactions between retailer and end-use customers with different utility functions,
- A model is proposed based on the mathematical methods and economics theories to enable consumer response to price signals for utility maximization,
- A stochastic optimization model is proposed for strategic bidding of a price-maker retailer with flexible demands in day-ahead electricity market,
- A decision-making tool is provided for the interactions of a retailer with its customers and the market which could be applicable in real-world practices.

The organization of this paper is as follows. Section 3, the problem will be introduced and described. Sections 4 and 5, describe the bi-level Stackelberg-based optimization model and explain the mathematical formulation of the proposed model. Implementation of the model is explained in Section 6 and its validity and effectiveness is shown thereafter. Finally, Section 7 closes the paper with drawing conclusion from the provided discussions and results.

## **2. Problem Definition and Description**

This paper offers a model for determining the optimal bidding strategy of a retail company (i.e., retailer), taking into account DR programs. Indeed, in this model, the retailer seeks to determine its optimal bidding strategy by applying its market power through both the physical and financial withholdings. This paper is actually equipping and developing reference [14] with a novel DR model, although this has led to increased complexity, but has also added to its efficiency. The retail company acts as an intermediary between the wholesale market and the customers, meaning that the retailer does not have a direct relationship with the energy generation companies (GenCos) [22]. Instead, the pool market, on a higher level, makes the bridge between GenCos and the retailer while determining the MCP [23].

### **3.1. Strategic Retailer Company**

The goal of the retail company is to make profit as any business company. Given that the retailer is the intermediary between the day-ahead market and retail markets (consumers), its utility function is affected by the price changes in these two markets. In accordance with the third stage, Retail Company can compensate shortage of electrical power purchased in day-ahead market from the intraday market. However, according to a study conducted in [24], the average energy purchase price in these markets is on the average 1.19 times the market price of day-ahead market. We will also make this assumption in the retailer's utility function.

### 3.2. Power pool

In fact, the power pool is a state-owned or semi-public entity. A power pool operator takes bids from suppliers and consumers and dispatches generation and load in an economic manner based on the bids.

### 3.3. Consumers

As stated, in the first stage of the model, the retailer sends electricity prices for each hour of the next day to consumers and, consequently, consumers will modify their energy consumption based on price signals. Hence, it is necessary to provide a DR model that can simulate the impact of price signal on load profile of each consumer. A proper DR model must have two key features: adaptability and adjustability [25]. Adaptability means that the model must be compatible with each consumer with any level of response to DR programs. As shown in [26], consumers in the residential sector are divided into three categories based on their response to prices: High-Flexibility (HF), Semi-Flexibility (SF) and Low-Flexibility (LF). On the other hand, consumers have different lifestyles which result in a widespread tendency in hours of electricity consumption. This implies the concept of adjustability in DR model meaning that each consumer can reapportion his/her demand from the peak-times to the desired time periods according to his/her lifestyle. This tendency is distinct for each consumer and cannot be elucidated solely by price elasticity of demand. In this section, with the help of consumer choice theory, a DR model which enables the mentioned features, is presented.

Consumer choice theory in economics examines this issue that how each consumer spends his/her money according to his/her preferences and budget constraint[16]-[27]. Utility function is also one of the most useful concepts in the economy that measures preferences over a set of goods and services. There is no specific way to find this function, and the economists extract it empirically. In the microeconomics, various utility functions are provided, but one of the most popular is the Constant Elasticity of Substitution (CES) utility function [28]-[29] which applicable especially in cases with several different goods for consumption:

$$U(d_1, d_2, \dots, d_n) = \left[ \sum_{i=1}^n \alpha_i^{1-\rho} d_i^\rho \right]^{\frac{1}{\rho}} \quad 0 \neq \rho < 1; \sum_{i=1}^n \alpha_i = 1; \alpha_i > 0 \quad (1)$$

The extension of this function in electricity market is that we assume that the electrical energy with a specific price  $P_1$  is considered as a commodity  $d_1$  and the electrical energy at the specified price  $P_2$  is as the commodity  $d_2$ . Thus, assuming  $n$  electricity price levels for a 24-hour period, we will have  $n$  commodities. Therefore, the utility function according to the budget constraint is as follows:

$$\text{Max} \left\{ \begin{aligned} U(d_1, d_2, \dots, d_{24}) &= \left[ \sum_{i=1}^{24} \alpha_i^{1-\rho} d_i^\rho \right]^{\frac{1}{\rho}} \\ &= (\alpha_1^{1-\rho} d_1^\rho + \alpha_2^{1-\rho} d_2^\rho + \dots + \alpha_{24}^{1-\rho} d_{24}^\rho)^{\frac{1}{\rho}} \end{aligned} \right\} \quad 0 \neq \rho < 1; \sum_{i=1}^{24} \alpha_i = 1; \alpha_i > 0 \quad (2)$$

$$\text{s.t.} \quad B = d_1.P_1 + d_2.P_2 + d_3.P_3 + \dots + d_{24}.P_{24}$$

In the above formulation, inverse of  $(1 - \rho)$  is the elasticity of substitution and  $\alpha_i$  denotes the share factors. The parameter  $\rho$  actually represents the adaptability feature (i.e., the higher the value of  $\rho$ , the higher the consumer participation in DR programs). The second characteristic of a proper model is its adjustability feature. With the help of  $\alpha_i$ , it's possible to exactly model this feature. In the above formulation, for every hour,  $\alpha_i$  can be assigned a value, and the higher value refers to the higher willingness of the consumer to demand electrical power at that hour. Determination of these values is a



difficult task but not impossible as a retailer in smart grid can estimate these parameters with the help of data mining methods and studying the history of each consumer.

### 3.4. Model Description

As shown in Fig.1, the retailer acts as the intermediary for the wholesale market and the consumers. In the wholesale market, the retailer however has no direct relationship with the energy producers but through the power pool, which is responsible for the market mechanism, and determining the market clearing price and energy settlement. Generally, in each market, each player's goal is to maximize its profits according to a defined utility function. The goal of a retailer is also to increase its profits by influencing market prices as well as providing DR programs. Likewise, the goal of consumers is to reduce the cost of purchasing electricity through the modification of load profiles, while the power pool aims at increasing public welfare. The issue that will be addressed in the proposed model of this study is to suitably capture reactions and interactions of these electricity market players with the aim of maximizing their profit functions.

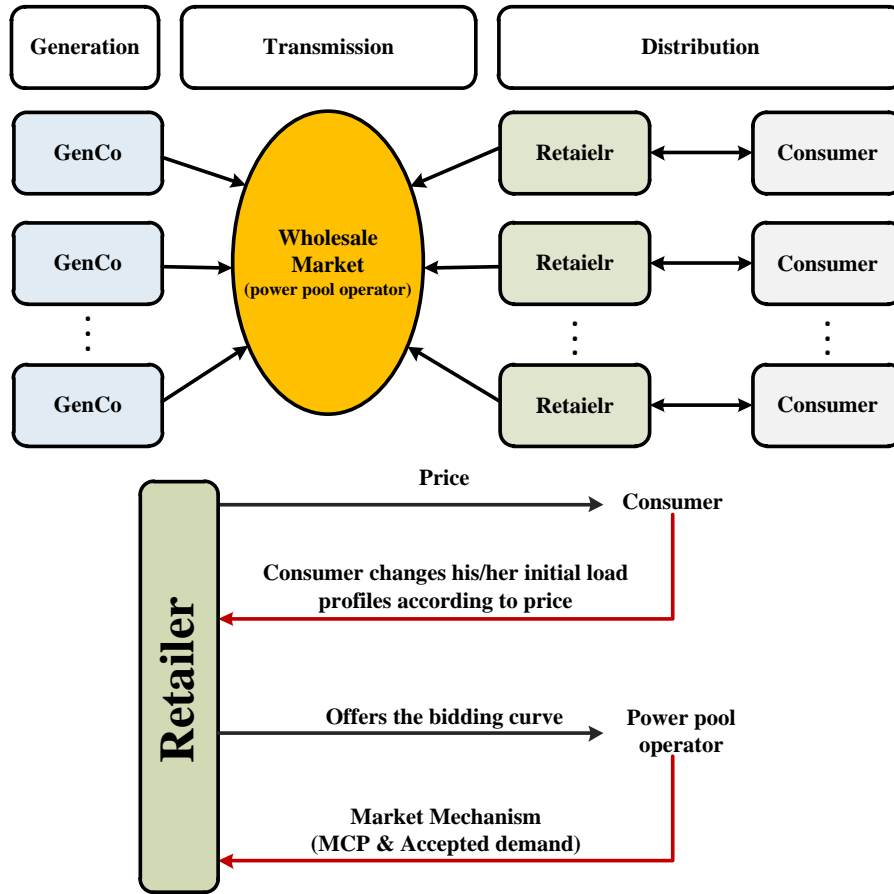


Fig. 1. Considered stages of proposed model

As shown in Fig.1, the proposed model is shaped in following stages:

- First stage: The retailer sends the price offers to the consumers for every hour of the next day in real-time pricing (RTP) format. At this stage, bids are made based on wholesale market price predictions. Then, consumers optimize their utility function based on these prices and report the results to the retailer. According

to the discussion presented, an efficient model of utility function for consumers has both adaptability and adjustability features which are considered in the proposed model of this study.

- Second stage: After determining the quantity of energy required and the proposed price in the previous stage, the retailer offers the bidding curve (including both price and quantity) to the power pool operator for every hour of the next day. The operator then adjusts the supply and demand curves for each hour of the next day and determines the MCP and the approved quantity.
- Third stage: This stage is related to compensating the shortage of electrical power purchased in the previous stage according to the amount of electrical power needed. In other words, this action is in line with the intraday market which supplements the day-ahead market and helps secure the necessary balance between supply and demand in the power market. The trade is closed in this market an hour before the physical delivery time. Of course, in real-time physical delivery of electrical, there is also a real-time market that is responsible for regulating electrical power balance.

Considering different stages as mentioned above, it can be observed that the utility function of each player in the system is somehow affected by other's utilities. In other words, the utility function of the retail company is constrained by the consumers and power pool utility functions which are independent decision-makers. This outlines a multi-level optimization process where an optimization problem constrained by other optimization problems. Multi-level optimization was introduced for the first time in 1934 by German economist Heinrich Freiherr von Stackelberg in the field of game theory which described the market and equilibrium structure as a hierarchical problem. The model that he presented in his book is known as the stackelberg game model [30].

However, in most real-world optimization problems, there are many uncertain factors that could negatively affect the optimal solutions. Therefore, proper modelling and formulation should be presented to capture those uncertainties. Stochastic programming provides a tool for modeling optimization problems that include stochastic and non-deterministic data. One of the common ways of solving such problems is to consider the input information as a set of possible data with different probabilities [31].

Bearing all this in mind, the proposed model in this study is a bi-level optimization model based on one-leader and multiple-follower Stackelberg game with some stochastic input data. At upper-level of this model, the utility function of the leader or the strategic retail company is considered and at the lower-level, the utility functions of the followers (consumers and the power pool) are taken into account. Moreover, each market player aims to maximize its profits and reach its own goal as described.

### **3. Bi-level Problem with One Leader and Multiple Followers**

With regard to the utility function defined for each player and the three stages described earlier, the bi-level model based on the Stackelberg game [32] can be presented as follows: first, the upper-level player (leader-retailer company) moves (decides), and then the lower-level player(s) (follower-consist of consumers and power pool) moves on the basis of this decision. According to the Stackelberg game model, the best lower-level player(s) moves is the action that predicts the stackelberg equilibrium. The lower-level player(s) will maximize/minimize the utility function based on the movement that the Stackelberg model predicts, and then on that basis the upper-level player utility function is maximized/minimized.

Relations (3) to (7) represent a bi-level Stackelberg-based model. Equation (3) relates to upper-level player (Retailer

Company). Each of the equations (4) to (6) relates the lower-level player(s) respectively to HF, SF and LF consumers. Formulation (7) also applies to another lower-level player (power pool). Equations (3) and (7) are extracted from [14] and modified accordingly to meet the requirement of this study. Likewise, equations (4) to (6) are derived from (2) for each group of consumers according to HF/SF/LF classification.

**Upper Level: Retailer Company**

$$\begin{aligned}
 & \text{Maximize } \sum_{\omega} \phi_{\omega} \times \left[ \sum_H \left[ D_{H,\omega,t} [P_{H,t} - \lambda_{\omega,t}] + (d_{H,t} - D_{H,\omega,t}) \times [P_{H,t} - \pi_{\omega,t}] \right] + \sum_S \left[ D_{S,\omega,t} [P_{S,t} - \lambda_{\omega,t}] + (d_{S,t} - D_{S,\omega,t}) \times [P_{S,t} - \pi_{\omega,t}] \right] \right. \\
 & \quad \left. + \sum_L \left[ D_{L,\omega,t} [P_{L,t} - \lambda_{\omega,t}] + (d_{L,t} - D_{L,\omega,t}) \times [P_{L,t} - \pi_{\omega,t}] \right] \right] ; \forall t \\
 & \text{s.t. :} \\
 & 0 \leq \Delta_{H,\omega,t} \leq P_{H,t} ; \forall t, \forall \omega \\
 & 0 \leq \Delta_{S,\omega,t} \leq P_{S,t} ; \forall t, \forall \omega \\
 & 0 \leq \Delta_{L,\omega,t} \leq P_{L,t} ; \forall t, \forall \omega \\
 & \Delta_{H,\omega,t} \leq \Delta_{S,\omega,t} ; \forall t, \forall \omega \\
 & \Delta_{S,\omega,t} \leq \Delta_{L,\omega,t} ; \forall t, \forall \omega \\
 & D_{H,\omega,t} + D_{S,\omega,t} + D_{L,\omega,t} \geq Y(d_{H,t} + d_{S,t} + d_{L,t}) ; \forall t, \forall \omega
 \end{aligned} \tag{3}$$

**Lower Level : High – Flexibility (HF) Consumers**

$$\begin{aligned}
 & d_{H,t} \in \text{Max} \left\{ U(d_{H,1}, d_{H,2}, \dots, d_{H,24}) = (\alpha_{H,1}^{1-\rho_H} d_{H,1}^{\rho_H} + \alpha_{H,2}^{1-\rho_H} d_{H,2}^{\rho_H} + \dots + \alpha_{H,24}^{1-\rho_H} d_{H,24}^{\rho_H})^{\frac{1}{\rho_H}} \right\} \\
 & \text{s.t.} \quad B = d_{H,1} \cdot P_{H,1} + d_{H,2} \cdot P_{H,2} + \dots + d_{H,24} \cdot P_{H,24}
 \end{aligned} \tag{4}$$

**Lower Level : Semi – Flexibility (SF) Consumers**

$$\begin{aligned}
 & d_{S,t} \in \text{Max} \left\{ U(d_{S,1}, d_{S,2}, \dots, d_{S,24}) = (\alpha_{S,1}^{1-\rho_S} d_{S,1}^{\rho_S} + \alpha_{S,2}^{1-\rho_S} d_{S,2}^{\rho_S} + \dots + \alpha_{S,24}^{1-\rho_S} d_{S,24}^{\rho_S})^{\frac{1}{\rho_S}} \right\} \\
 & \text{s.t.} \quad B = d_{S,1} \cdot P_{S,1} + d_{S,2} \cdot P_{S,2} + \dots + d_{S,24} \cdot P_{S,24}
 \end{aligned} \tag{5}$$

**Lower-level: Low-Flexibility (LF) Consumers**

$$\begin{aligned}
 & d_{L,t} \in \text{Max} \left\{ U(d_{L,1}, d_{L,2}, \dots, d_{L,24}) = (\alpha_{L,1}^{1-\rho_L} d_{L,1}^{\rho_L} + \alpha_{L,2}^{1-\rho_L} d_{L,2}^{\rho_L} + \dots + \alpha_{L,24}^{1-\rho_L} d_{L,24}^{\rho_L})^{\frac{1}{\rho_L}} \right\} \\
 & \text{s.t.} \quad B = d_{L,1} \cdot P_{L,1} + d_{L,2} \cdot P_{L,2} + \dots + d_{L,24} \cdot P_{L,24}
 \end{aligned} \tag{6}$$

### Lower Level: Power Pool

Lower Level : Power Pool

$$\begin{aligned}
 & D_{H,\omega,t}, D_{S,\omega,t}, D_{L,\omega,t}, \lambda_{\omega,t} \in \\
 & \text{Min} \left\{ \begin{aligned} & \left[ \sum_{j,b} P_{j,b,\omega,t} \cdot \Delta_{j,b,\omega,t} \right] - \left[ \sum_O D_{O,\omega,t} \cdot M_{O,t} \right] - \\ & \left[ \sum_H D_{H,\omega,t} \cdot \Delta_{H,\omega,t} \right] - \left[ \sum_S D_{S,\omega,t} \cdot \Delta_{S,\omega,t} \right] - \\ & \left[ \sum_L D_{L,\omega,t} \cdot \Delta_{L,\omega,t} \right] \end{aligned} \right\} \\
 & s.t. : \left\{ \begin{aligned} & \left[ \sum_{j,b} P_{j,b,\omega,t} \right] - \left[ \sum_O D_{O,\omega,t} \right] - \left[ \sum_H D_{H,\omega,t} \right] \\ & - \left[ \sum_S D_{S,\omega,t} \right] - \left[ \sum_L D_{L,\omega,t} \right] - \\ & \left[ \sum_{m \text{ connect to } n} B_{nm} (\theta_{t,n,\omega} - \theta_{t,m,\omega}) \right] = 0 \end{aligned} \right\} \quad \lambda_{t,n,\omega} \quad \forall t, \forall \omega \\
 & B_{nm} (\theta_{t,n,\omega} - \theta_{t,m,\omega}) \leq F_{nm} \quad \varepsilon_{t,n,m,\omega} \quad \forall n, \forall m \in \text{nodes connected to node } n \\
 & \theta_{t,(n=1),\omega} = 0: \quad \delta_{t,\omega}^1 \quad \forall t, \forall \omega \\
 & -\pi \leq \theta_{t,n,\omega} \leq \pi: \quad \delta_{t,n,\omega}^{\min}, \delta_{t,n,\omega}^{\max} \quad \forall n \\
 & 0 \leq D_{H,\omega,t} \leq d_{H,t}: \quad \eta_{\omega,t}^{H \min}, \eta_{\omega,t}^{H \max} \\
 & 0 \leq D_{S,\omega,t} \leq d_{S,t}: \quad \eta_{\omega,t}^{S \min}, \eta_{\omega,t}^{S \max} \\
 & 0 \leq D_{L,\omega,t} \leq d_{L,t}: \quad \eta_{\omega,t}^{L \min}, \eta_{\omega,t}^{L \max} \\
 & 0 \leq D_{O,\omega,t} \leq D_{O,t}: \quad \eta_{\omega,t}^{O \min}, \eta_{\omega,t}^{O \max} \\
 & 0 \leq P_{j,b,\omega,t} \leq P_{j,b}^{\max}: \quad \mu_{j,b,\omega,t}^{P \min}, \mu_{j,b,\omega,t}^{P \max}
 \end{aligned} \tag{7}$$

In this model, it is assumed that there are three groups of consumers with different levels of participation (different  $\rho$ ) in DR programs which is presented in three formats with high-flexibility (HF), semi-flexibility (SF) and low-flexibility (LF). Also, the value of the adjustability parameter ( $\alpha_i$ ) is considered similar at all hours. This means that there is no difference among the hours from consumers' point of view. The parameter  $\varphi_\omega$  is the probability of the scenario  $\omega$ . For the upper-level player, it is very important to consider that the price offered to the power pool operator should be less than or equal to the proposed price to the consumers. Also, the proposed prices to the consumer based on the amount of participation in the DR programs are different. Also, according to the last constraint of equation (3), the retail company should gain at least  $Y\%$  of its electrical power from day-ahead market. In the case of consumer utility function (the first three followers), the necessary information was given in equations (4) – (6). In connection with the fourth follower or operator of the power pool, the constraints include balance in production and consumption, the network constraints, the

electrical power range of each load and the electrical power produced by each power plant were given in equation (8).

The duality variables in the proposed optimization model are the  $\delta_{t,n,\omega}^{\theta \min}$ ,  $\delta_{t,n,\omega}^{\theta \max}$ ,  $\eta_{\omega,t}^{H \min}$ ,  $\eta_{\omega,t}^{H \max}$ ,  $\eta_{\omega,t}^{S \min}$ ,  $\eta_{\omega,t}^{S \max}$ ,  $\eta_{\omega,t}^{L \min}$ ,  $\eta_{\omega,t}^{L \max}$ ,  $\eta_{\omega,t}^{O \min}$ ,  $\eta_{\omega,t}^{O \max}$ ,  $\mu_{j,b,\omega,t}^{P \min}$ , and  $\mu_{j,b,\omega,t}^{P \max}$ .

Duality is the principle that provides viewing an optimization problem from either of the two perspectives. The solution to the dual problem provides a lower bound to the solution of the problem.

#### 4. KKT Formulation

As mentioned above, the functions of the upper- and the lower-levels are interdependent and therefore, they must be all considered in one shot. Lower-level terms are linear and continuous; hence they are convex, and can be replaced by the Karush–Kuhn–Tucker (KKT) conditions [33]. The problem of single-level optimization is known as Mathematical Programming with Equilibrium Constraints (MPEC) [34], which is one of the standard formats and includes:

- 1- The objective function and constraints of the upper-level problem.
- 2- The KKT conditions of the lower-level problems.

##### 5.1. Consumers

Because the first three followers (consumers) have the same problems but with different adaptability parameters, the KKT condition can be generalized from one to another. For example, according to the objective function of HF consumers (Equation (3)), Lagrange function can be expressed as follows:

$$L = (\alpha_{H,1}^{1-\rho_H} d_{H,1}^{\rho_H} + \alpha_{H,2}^{1-\rho_H} d_{H,2}^{\rho_H} + \dots + \alpha_{H,24}^{1-\rho_H} d_{H,24}^{\rho_H})^{\frac{1}{\rho_H}} + \Psi_H * [B - d_{H,1} \cdot P_{H,1} - d_{H,2} \cdot P_{H,22} - \dots - d_{H,24} \cdot P_{H,24}] \quad (8)$$

The partial derivatives of Equation (8) with respect to any  $d_{H,t}$  and  $\Psi_H$  are given by:

$$\begin{aligned} \frac{\partial L}{\partial d_{H,t}} &= \alpha_{H,t}^{1-\rho_H} d_{H,t}^{\rho_H-1} * (\alpha_{H,1}^{1-\rho_H} d_{H,1}^{\rho_H} + \dots + \alpha_{H,24}^{1-\rho_H} d_{H,24}^{\rho_H})^{\frac{1-\rho_H}{\rho_H}} - \Psi_H \cdot P_{H,t} = 0 \\ \Rightarrow \Psi_H \cdot P_{H,t} &= \alpha_{H,t}^{1-\rho_H} d_{H,t}^{\rho_H-1} * (\alpha_{H,1}^{1-\rho_H} d_{H,1}^{\rho_H} + \dots + \alpha_{H,24}^{1-\rho_H} d_{H,24}^{\rho_H})^{\frac{1-\rho_H}{\rho_H}} \end{aligned} \quad (9)$$

and;

$$\frac{\partial L}{\partial \Psi_H} = [B - d_{H,1} \cdot P_{H,1} - d_{H,2} \cdot P_{H,22} - \dots - d_{H,24} \cdot P_{H,24}] = 0 \quad (10)$$

Taking (8) - (9) into account, one can easily conclude that:

$$\begin{aligned}
d_{H,2} &= \frac{\alpha_{H,2}}{\alpha_{H,1}} \left( \frac{P_{H,2}}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}} d_{H,1} \\
d_{H,3} &= \frac{\alpha_{H,3}}{\alpha_{H,1}} \left( \frac{P_{H,3}}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}} d_{H,1} \\
&\vdots \\
d_{H,24} &= \frac{\alpha_{H,24}}{\alpha_{H,1}} \left( \frac{P_{H,24}}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}} d_{H,1}
\end{aligned} \tag{11}$$

By substituting the relations extracted from (11) into (10), we arrive at:

$$\begin{aligned}
B_H &= \sum_{t=1}^{24} d_{H,t} P_{H,t} = \sum_{t=1}^{24} \frac{\alpha_{H,t}}{\alpha_{H,1}} \left( \frac{P_{H,t}}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}} d_{H,1} P_{H,t} \\
\Rightarrow d_{H,1} &= \frac{B_H}{\sum_{t=1}^{24} \frac{\alpha_{H,t}}{\alpha_{H,1}} \left( \frac{P_{H,t}}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}}}
\end{aligned} \tag{12}$$

Therefore, by substituting the relations extracted from (12) into (11):

$$d_{H,t} = \frac{\alpha_t}{\alpha_{H,1}} \left( \frac{P_t}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}} \frac{B}{\sum_{t=1}^{24} \frac{\alpha_{H,t}}{\alpha_{H,1}} \left( \frac{P_{H,t}}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}}} \tag{13}$$

It is supposed that there is no difference between the total electricity consumed over a 24-hour period before and after the implementation of DR program. In other words, the change will be in the pattern (hours) of consumption, not in the total energy consumed meaning that:

$$D_{H,primary} = \sum_{t=1}^{24} d_{H,t} = \frac{\alpha_t}{\alpha_{H,1}} \left( \frac{P_t}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}} \frac{B}{\sum_{t=1}^{24} \frac{\alpha_{H,t}}{\alpha_{H,1}} \left( \frac{P_{H,t}}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}}} \tag{14}$$

$D_{H,primary}$  is total electricity consumed before DR program.

$$B = \frac{\sum_{t=1}^{24} \frac{\alpha_{H,t}}{\alpha_{H,1}} \left( \frac{P_{H,t}}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}}}{\frac{\alpha_t}{\alpha_{H,1}} \left( \frac{P_t}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}}} \tag{15}$$

By substituting (15) into (13), it can be deduced that:

$$d_{H,t} = \frac{\alpha_{H,t}}{\alpha_{H,1}} \left( \frac{P_{H,t}}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}} \cdot \frac{D_{H,primary}}{\sum_{t=1}^{24} \frac{\alpha_{H,t}}{\alpha_{H,1}} \left( \frac{P_{H,t}}{P_{H,1}} \right)^{\frac{1}{\rho_H - 1}}} \quad \forall t \quad (16)$$

According to (16), the following equations can be extracted for the second and third followers, respectively:

$$d_{S,t} = \frac{\alpha_{S,t}}{\alpha_{S,1}} \left( \frac{P_{S,t}}{P_{S,1}} \right)^{\frac{1}{\rho_S - 1}} \cdot \frac{D_{S,primary}}{\sum_{t=1}^{24} \frac{\alpha_{S,t}}{\alpha_{S,1}} \left( \frac{P_{S,t}}{P_{S,1}} \right)^{\frac{1}{\rho_S - 1}}} \quad \forall t \quad (17)$$

And:

$$d_{L,t} = \frac{\alpha_{L,t}}{\alpha_{L,1}} \left( \frac{P_{L,t}}{P_{L,1}} \right)^{\frac{1}{\rho_L - 1}} \cdot \frac{D_{L,primary}}{\sum_{t=1}^{24} \frac{\alpha_{L,t}}{\alpha_{L,1}} \left( \frac{P_{L,t}}{P_{L,1}} \right)^{\frac{1}{\rho_L - 1}}} \quad \forall t \quad (18)$$

In relation to the first three followers, there is only Lagrange's equal constraint, and there are no complementarity conditions due to the lack of inequality constraints.

## 5.2. Power pool

For the last follower, the KKT conditions include the constraints of the equation derived from the Lagrangian derivative, as well as the complementary conditions. Equality constraints derived from the derivation of Lagrange function for the last follower (Equation (4)):

$$\begin{cases} \frac{\partial \ell_{\omega,t}}{\partial P_{j,b,\omega,t}} = \Lambda_{j,b,\omega,t} + \lambda_{t,n:\text{connected to } n,\omega} + \mu_{j,b,\omega,t}^{p^{\max}} - \mu_{j,b,\omega,t}^{p^{\min}} = 0 \\ \frac{\partial \ell_{\omega,t}}{\partial D_{H,\omega,t}} = -\Delta_{H,\omega,t} + \lambda_{t,n:H \text{ connected to } n,\omega} + \eta_{\omega,t}^{H^{\max}} - \eta_{\omega,t}^{H^{\min}} = 0 \\ \frac{\partial \ell_{\omega,t}}{\partial D_{S,\omega,t}} = -\Delta_{S,\omega,t} + \lambda_{t,n:S \text{ connected to } n,\omega} + \eta_{\omega,t}^{S^{\max}} - \eta_{\omega,t}^{S^{\min}} = 0 \\ \frac{\partial \ell_{\omega,t}}{\partial D_{L,\omega,t}} = -\Delta_{L,\omega,t} + \lambda_{t,n:L \text{ connected to } n,\omega} + \eta_{\omega,t}^{L^{\max}} - \eta_{\omega,t}^{L^{\min}} = 0 \\ \frac{\partial \ell_{\omega,t}}{\partial D_{O,\omega,t}} = -M_{O,t} + \lambda_{t,n:O \text{ connected to } n,\omega} + \eta_{O,\omega,t}^{O^{\max}} - \eta_{O,\omega,t}^{O^{\min}} = 0 \\ \frac{\partial \ell_{\omega,t}}{\partial \theta_{t,n,\omega}} = \sum_{m=1, m \text{ connected to } n} B_{nm} (\lambda_{t,n,\omega} - \lambda_{t,m,\omega}) \\ + \sum_{m=1, m \text{ connected to } n} B_{nm} (\varepsilon_{t,n,\omega} - \varepsilon_{t,m,\omega}) + (\delta_{t,n,\omega}^{\theta^{\max}} - \delta_{t,n,\omega}^{\theta^{\min}}) + (\delta_{t,\omega}^1)_{n=1} \end{cases} \quad (19)$$

And the complementary conditions for the last follower:

$$\begin{aligned}
0 \leq P_{j,b,\omega,t} \perp \mu_{j,b,\omega,t}^{p,\min} &\geq 0 && ; \forall j, \forall t, \forall \omega, \forall b \\
0 \leq P_{j,b}^{\max} - P_{j,b,\omega,t} \perp \mu_{j,b,\omega,t}^{p,\max} &\geq 0 && ; \forall j, \forall t, \forall \omega, \forall b \\
0 \leq D_{H,\omega,t} \perp \eta_{\omega,t}^{H,\min} &\geq 0 && ; \forall t, \forall \omega \\
0 \leq d_{H,t} - D_{H,\omega,t} \perp \eta_{\omega,t}^{H,\max} &\geq 0 && ; \forall t, \forall \omega \\
0 \leq D_{S,\omega,t} \perp \eta_{\omega,t}^{S,\min} &\geq 0 && ; \forall t, \forall \omega \\
0 \leq d_{S,t} - D_{S,\omega,t} \perp \eta_{\omega,t}^{S,\max} &\geq 0 && ; \forall t, \forall \omega \\
0 \leq D_{L,\omega,t} \perp \eta_{\omega,t}^{L,\min} &\geq 0 && ; \forall t, \forall \omega \\
0 \leq d_{L,t} - D_{L,\omega,t} \perp \eta_{\omega,t}^{L,\max} &\geq 0 && ; \forall t, \forall \omega \\
0 \leq D_{O,\omega,t} \perp \eta_{\omega,t}^{O,\min} &\geq 0 && ; \forall t, \forall \omega \\
0 \leq D_{O,t} - D_{O,\omega,t} \perp \eta_{\omega,t}^{O,\max} &\geq 0 && ; \forall t, \forall \omega \\
\left\{ \begin{aligned} 0 \leq [F_{nm} - B_{nm}(\theta_{t,n,\omega} - \theta_{t,m,\omega})] \perp \varepsilon_{t,n,m,\omega} &\geq 0 \\ \forall t, \forall \omega, \forall n, \forall m \in \text{connected to } n \end{aligned} \right. &&& \\
0 \leq (\pi + \theta_{t,n,\omega}) \perp \delta_{t,n,\omega}^{\theta,\min} &\geq 0 && ; \forall t, \forall \omega, \forall n \\
0 \leq (\pi - \theta_{t,n,\omega}) \perp \delta_{t,n,\omega}^{\theta,\max} &\geq 0 && ; \forall t, \forall \omega, \forall n
\end{aligned} \tag{20}$$

### 5.3. Model Extracting from MPEC

By replacing the obtained equations (16)-(20) for lower- level problems, the bi-level problem is transformed into a one-level problem within the framework of the MPEC. The MPEC model consists of two nonlinear sections:

- 1- All components of (20).
- 2- The utility function of upper-level has a nonlinear term:

$$\left\{ \begin{aligned} \Gamma_{t,\omega} = \sum_H [D_{H,\omega,t} \pi_{\omega,t} - D_{H,\omega,t} \lambda_{\omega,t}] \\ + \sum_S [D_{S,\omega,t} \pi_{\omega,t} - D_{S,\omega,t} \lambda_{\omega,t}] + \sum_L [D_{L,\omega,t} \pi_{\omega,t} - D_{L,\omega,t} \lambda_{\omega,t}] \quad \forall t, \forall \omega \end{aligned} \right\}. \tag{21}$$

After the linearization of these two terms, the Mixed-Integer Linear Programming (MILP) model is extracted from the MPEC model.

## 5. Performance Evaluation

In this section, we examine the performance of the proposed model. In the lower level of the proposed bi-level model, consumers' utility functions are considered. These utility functions are in fact a DR model that examines the effect of price change on consumers' behavior and showing changes on their initial load profiles.

In this section, we first evaluate the performance of DR model independently and separate from the bi-level model. Finally, the bi-level model will be evaluated.

### 6.1. Demand Response Model

The electricity price profile used in this study is shown in the Fig.2 [35]. The load profile shown in the Fig.3 also refers to demand of residential area extracted from [36]. In the same figure, it can be observed that with the increase of  $\rho$ , the



consumer's willingness to participate in DR programs increases especially during hours when the electricity price is high (8am to 12am). Moreover, as shown in Fig.4, the required budget will be reduced by increasing the level of participation  $\rho$ .

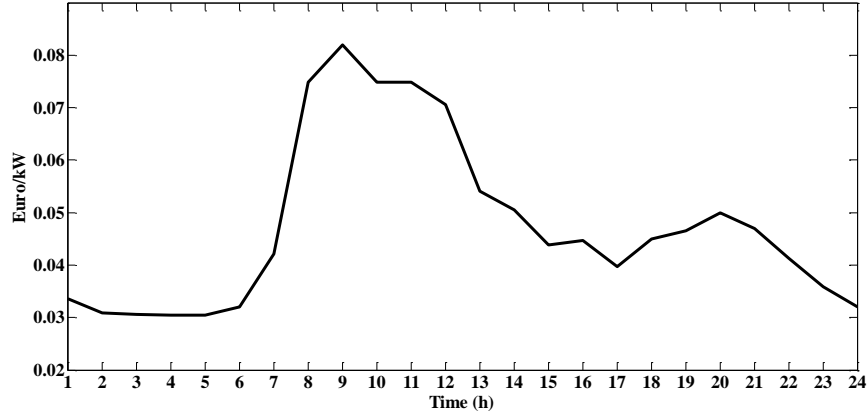


Fig. 2. Electricity prices according to RTP pricing

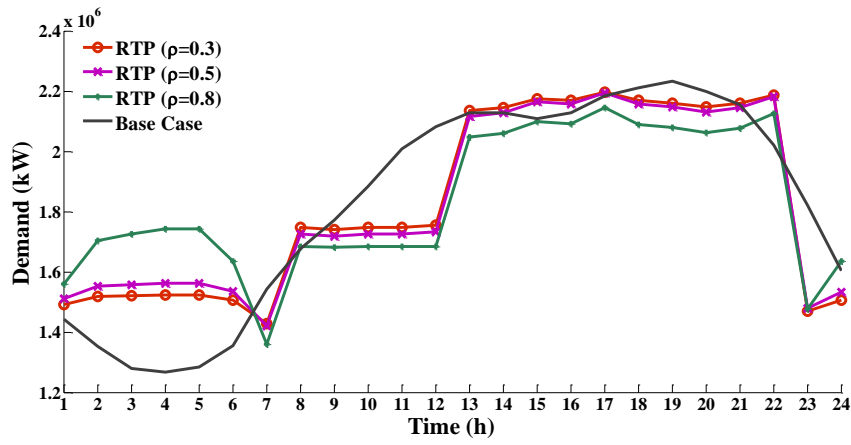


Fig. 3. The impact of DR programs on load profile

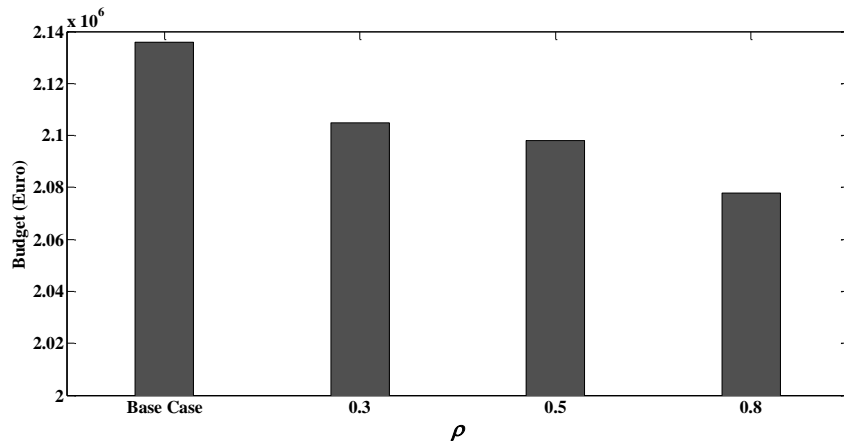


Fig. 4. Changes in the budget for different values of adaptability parameter ( $\rho$ )

Fig.5 shows the effect of adjustability parameter ( $\alpha_i$ ) on the load profile in different scenarios described in Table.1. In all the mentioned scenario, it is assumed that the adaptability parameter ( $\rho$ ) remains unchanged ( $\rho=0.5$ ).

Table 1- Considered scenarios

Scenario	Adjustability parameters
1	$\alpha_{day} = \alpha_{evening} = \alpha_{night}$
2	$\alpha_{day} = 2\alpha_{evening} = 2\alpha_{night}$
3	$\alpha_{day} = 0.6\alpha_{evening} = 0.6\alpha_{night}$

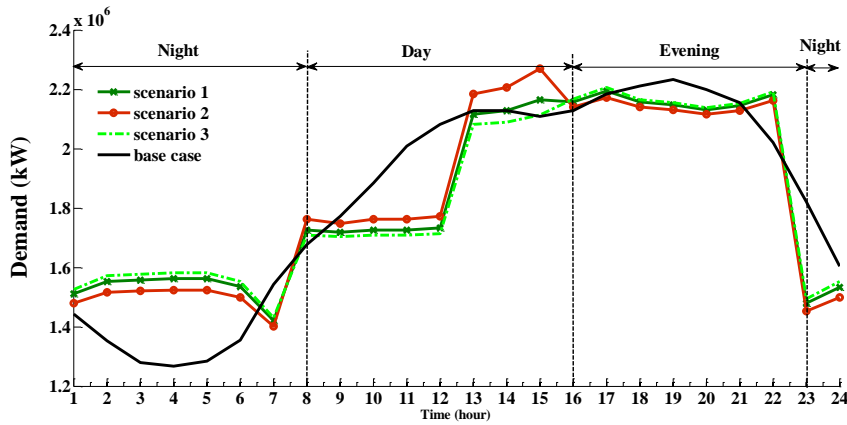


Fig. 5. The effect of the adjustability parameter ( $\alpha$ ) on the load profile

The first scenario,  $\alpha_{day} = \alpha_{evening} = \alpha_{night}$ , describes a situation where consumers do not have a time preference, and their sole purpose is to reduce the cost of purchasing electrical energy.

The second scenario,  $\alpha_{day} = 2\alpha_{evening} = 2\alpha_{night}$ , shows that the adjustability parameter for the day time is weighted more than those in the evening and the night. This means that consumers have more tendency to consume energy in the day than the evening and the night. The impact of this preference change can be observed in Fig.5 with respect to the other scenarios. As shown in this scenario, the consumption increases in the day and decreases in the evening and the night compared to the first scenario.

The third scenario,  $\alpha_{day} = 0.6\alpha_{evening} = 0.6\alpha_{night}$ , represents an opposite condition where the adjustability parameter in the day has declined, meaning that the consumers prefer to have more energy consumption during the evening and the night times rather than the day time.

These results also show that each consumer can reapportion his/her demand to the desired time periods according to his/her lifestyle, what is not seen in the existing models.

## 6.2. Bi-level model

In this case-study, one-area IEEE reliability test system (RTS) as shown in Fig.6 is considered to examine the performance of the proposed model. This case study is composed of 24 buses, 34 power lines, 11 generation stations, and 17 load centers (points) each includes three blocks. The total generating capacity in this area is 3405MW. Daily electricity consumption profile at this network is shown in Fig.7. Technical specifications of the generation units are available in

[37]-[38]. The load centers are powered by 11 retailers which one of them is our strategic retailer company. This strategic retailer company provides electric power for load centers in nodes 1, 2, 3, 4, 5, 18 and 19 which provides nearly 37% of demand of the network while the rest is met by other retailers. More detailed information about these loads can be found in [39]. Demand data of strategic retailer company at hour  $t_{20}$  (which is randomly selected) is presented in Tables 2-4. As can be seen in these tables, strategic retailer company supplies three groups of consumers with different levels of participation in DR programs (different  $\rho$ ).

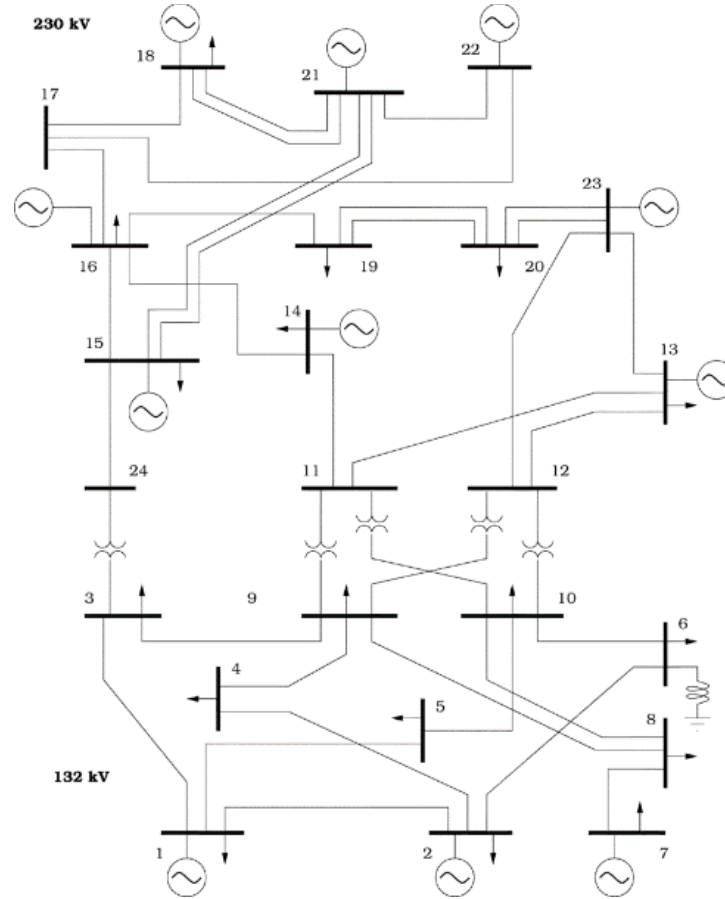


Fig. 6. One-area IEEE 24-bus reliability test system [40]

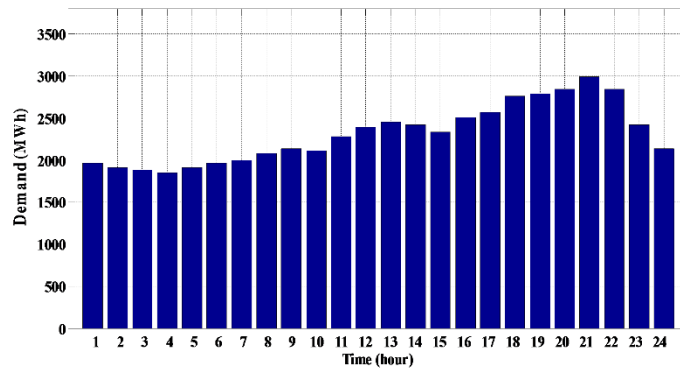


Fig.7. Daily electricity consumption profile at case study network

Table 2- High-flexibility demands served by the strategic retailer company

High-flexibility							
Adaptability parameter	$\rho=0.8$						
Location (Node)	1	19	1	2	3	18	19
Block of Demand	B <sub>2</sub>	B <sub>20</sub>	B <sub>3</sub>	B <sub>6</sub>	B <sub>9</sub>	B <sub>18</sub>	B <sub>21</sub>
Load (MW) - (without DR)	32	58	35	15	15	51	28

Table 3- Semi-flexibility demands served by the strategic retailer company

Semi-flexibility							
Adaptability parameter	$\rho=0.6$						
Location (Node)	2	3	4	5	18	4	5
Block of Demand	B <sub>5</sub>	B <sub>8</sub>	B <sub>11</sub>	B <sub>14</sub>	B <sub>17</sub>	B <sub>12</sub>	B <sub>15</sub>
Load (MW) - (without DR)	17	45	25	17	95	9	15

Table 4- Low-flexibility demands served by the strategic retailer company

Low -flexibility							
Adaptability parameter	$\rho=0.4$						
Location (Node)	1	2	1	4	5	18	19
Block of Demand	B <sub>1</sub>	B <sub>4</sub>	B <sub>7</sub>	B <sub>10</sub>	B <sub>13</sub>	B <sub>16</sub>	B <sub>19</sub>
Load (MW) - (without DR)	65	65	100	40	39	180	98

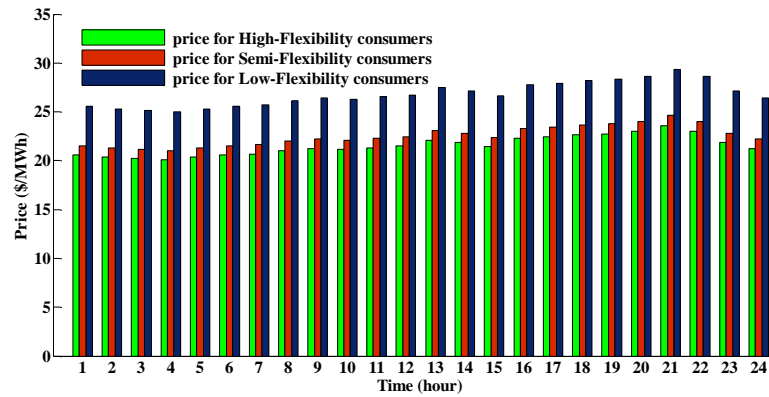


Fig.8. Price offered by retailer to consumers for the next day

The strategic retailer company offers its price to the consumers for each hour of the next day based on price prediction

and marginal costs for each category of consumers. Here, as shown in Fig.8, it is assumed that in addition to real-time pricing (a price-based program), the price offers are also adjusted by increasing the flexibility of consumers, which represents an incentive-based program. These price adjustments are also considered as the marginal prices of the retailer company. All other required data are available in [39].

For our stochastic models, as shown in Table 5, a three-step process is used as follows:

- First step: similar to [39], we assume that the prices offered by the GenCos are stochastic and are presented in eight different scenarios with the equal probability of occurrence. These prices are obtained through the multiplication of the uncertainty coefficients of each hour at the marginal cost of the GenCo. Table 5 shows eight different scenarios with equal probability according to this step for presented case study.
- Second step: the bi-level model is implemented and the outputs are extracted, which are the amount of accepted demand of the strategic retailer and the MCP for each scenario. According to this step by implementing proposed model in all scenario, the results (MCP & Accepted demand) are extracted as tabulated in Table 5.
- Third step: The main purpose of the implementation of the bi-level model by the retailer is to make the strategic bidding curve as shown in Fig.9. Hence, in order to derive a strategic bidding curve, the extraction of  $(Q_p, PB_p)$  is necessary. To this end, we use "worst-case scenario" to determine  $(Q_p, PB_p)$  [41]. According to the "worst-case scenario", the novel algorithm presented in Fig.10 is customized for our bi-level model. This algorithm employs the data from the second step to extract the  $(Q_p, PB_p)$ . For our case study, the results of this step are tabulated in Table 5.

Finally, to construct the strategic bidding curve similar to Fig.9, the Table 6 is formed and the  $(q_n, PB_n)$  are acquired. With this data, the strategic bidding curve is obtained as shown in Fig.11. Also, in this figure, another bidding curve has been shown to indicate a situation in which the retailer has no strategy, in other words, retailer acts as a price taker.

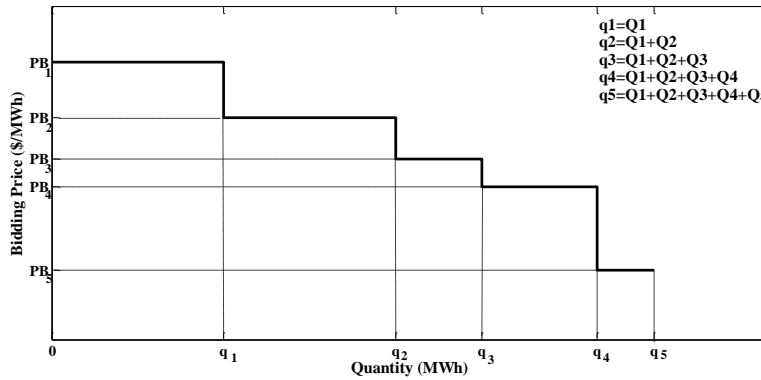


Fig. 9. Strategic bidding curve

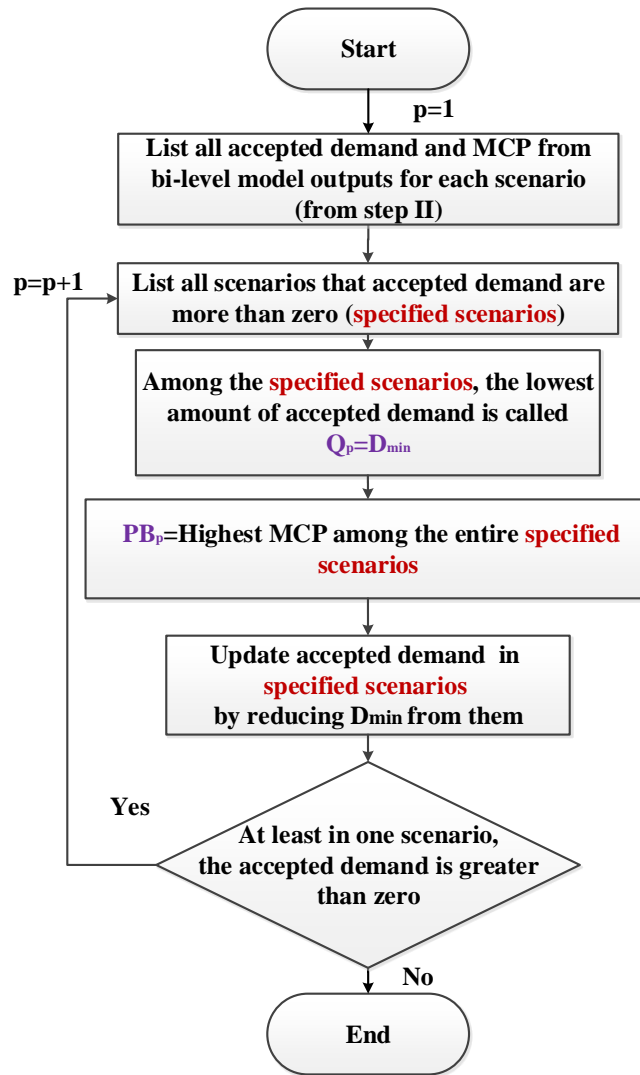


Fig. 10. Third step: algorithm according to the "worst-case scenario" plan

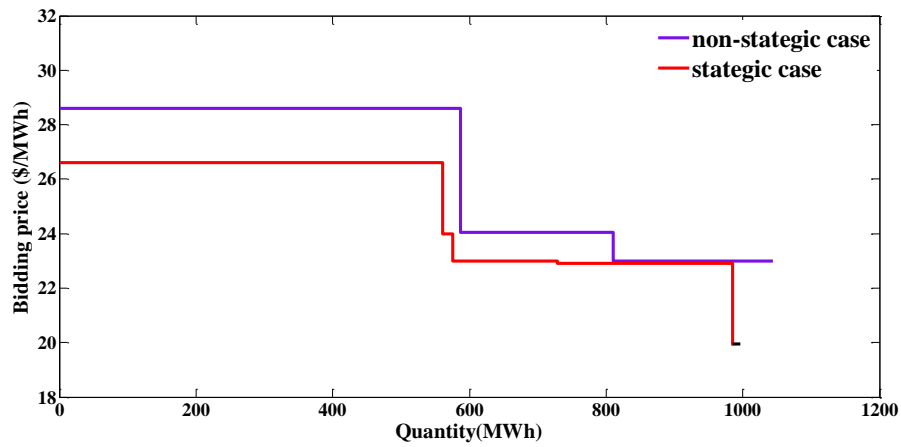


Fig. 11. Comparison between the non-strategic curve and strategic curve

Table 5- Three-step process to extract strategic bidding data

Step 1	scenario	1	2	3	4	5	6	7	8
	Uncertainty coefficients	1	1.1	1.2	1.3	1.5	1.4	1.7	1.9
Step 2	MCP	19.90	21.50	22.92	23	24.45	24	25.04	26.60
	Accepted demand	985.3	971.8	985	729	560.5	576	301	560.5
Step 3: Start Algorithm									
Step 3: run I	Accepted demand I	985.3	971.8	985	729	560.5	576	301	560.5
	Considered scenario I	■	■	■	■	■	■	■	■
	$Q_1$	-	-	-	-	-	-	301	-
	$PB_1$	-	-	-	-	-	-	-	26.60
Step 3: run II	Accepted demand II	684.3	670.8	684	428	259.5	275	-	259.5
	Considered scenario II	■	■	■	■	■	■	-	■
	$Q_2$	-	-	-	-	259.5	-	-	-
	$PB_2$	-	-	-	-	-	-	-	26.60
Step 3: run III	Accepted demand III	424.8	411.3	424.5	168.5	-	15.5	-	-
	Considered scenario III	■	■	■	■	-	■	-	-
	$Q_3$	-	-	-	-	-	15.5	-	-
	$PB_3$	-	-	-	-	-	24	-	-
Step 3: run IV	Accepted demand IV	409.3	395.8	409	153	-	-	-	-
	Considered scenario IV	■	■	■	■	-	-	-	-
	$Q_4$	-	-	-	153	-	-	-	-
	$PB_4$	-	-	-	23	-	-	-	-
Step 3: run V	Accepted demand V	256.3	242.8	256	-	-	-	-	-
	Considered scenario V	■	■	■	-	-	-	-	-
	$Q_5$	-	242.8	-	-	-	-	-	-
	$PB_5$	-	22.92	-	-	-	-	-	-
Step 3: run VI	Accepted demand VI	13.5	-	13.2	-	-	-	-	-
	Considered scenario VI	■	-	■	-	-	-	-	-
	$Q_6$	-	-	13.2	-	-	-	-	-
	$PB_6$	-	-	22.92	-	-	-	-	-
Step 3: run VII	Accepted demand VII	0.3	-	-	-	-	-	-	-
	Considered scenario VII	■	-	-	-	-	-	-	-
	$Q_7$	0.3	-	-	-	-	-	-	-
	$PB_7$	19.9	-	-	-	-	-	-	-
End	Accepted demand VIII	-	-	-	-	-	-	-	-

Table 6- Extract strategic bidding curve by using obtained data in Table V

Step & Run	$q_n$	$PB_n$
From Step 3: run 1 & 2	$Q_1 + Q_2 = 560.5$	26.6
From Step 3: run 3	$Q_1 + Q_2 + Q_3 = 576$	24
From Step 3: run 4	$Q_1 + Q_2 + Q_3 + Q_4 = 729$	23
From Step 3: run 5	$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 = 985$	22.92
From Step 3: run 6	$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 = 985.3$	19.9

Comparison of extracted results between strategic and non-strategic behavior are given in Table 7. According to the first row, strategic behavior could reduce demand by about 5.6% because of DR programs. Moreover, as can be seen in the second row of the same table, the profit of the strategic retailers increases relatively by 20%. According to the third

row, the profit of GenCos is declined due to the strategic behavior of the retailer. The volume of electrical power purchased from the day-ahead market is dropped a little in strategic case (as can be understood by the fourth row). Finally, the expected cost per megawatt-hour electricity for the retail company is decreased from the day-ahead market according to the last row.

Table 7- Comparison of extracted results between strategic and non-strategic behavior ( $t_{20}$ )

Description	Strategic Case	Non-strategic case	Changes %
Demand (MW)	985.4	1044	-5.6
Strategic retail company profits	1944.3	1612.8	20.5
Profit of all generation companies from the day-ahead market	18417	19583	-5.9
Percentage of power purchased from the day-ahead market	75.8	77.1	-1.6
Expected cost of per megawatt-hour electricity for the retail company from the day-ahead market	23.04	23.38	-1.45

In the following, we assumed that no consumers are included in the DR program except HF consumers. The results of this comparison are shown in Table 8. As can be seen in the second row of the same table, the profit of the strategic retailer increases relatively by 6.4%. The amount of electrical power purchased from the day-ahead market is decreased according to the fourth row. Finally, the expected cost per megawatt-hour electricity for the retail company is dropped from the day-ahead market according to the last row. It can be concluded that, firstly, the lack of participation of SF and LF consumers has had an impact, and secondly, this impact is minor because most participation in DR programs has been undertaken by HF consumers.

Table 8- Comparison of extracted results in strategic behaviors with different DR actions ( $t_{20}$ )

Description	Strategic Case (All HF/SF/LF consumers)	Strategic Case (Only HF consumers)	Changes%
Demand (MW)	985.4	1025.12	- 3.8
Strategic retail company profits	1944.3	1826.41	6.4
Percentage of power purchased from the day-ahead market	75.8	76.1	-0.4
Expected cost of per megawatt-hour electricity for the retail company from the day-ahead market	23.04	23.21	-0.7

It is obvious in the Fig.11 that the amount of electrical power demand in strategic case is less than the one in non-strategic case, which is due to the implementation of the DR program (physical withholding). Moreover, the bidding curve of the strategic case takes a lower position compared to the non-strategic case denoting a financial withholding policy enforced by the strategic retailer. Fig.12 illustrates changes in the MCP in considered scenarios between strategic and non-strategic case.



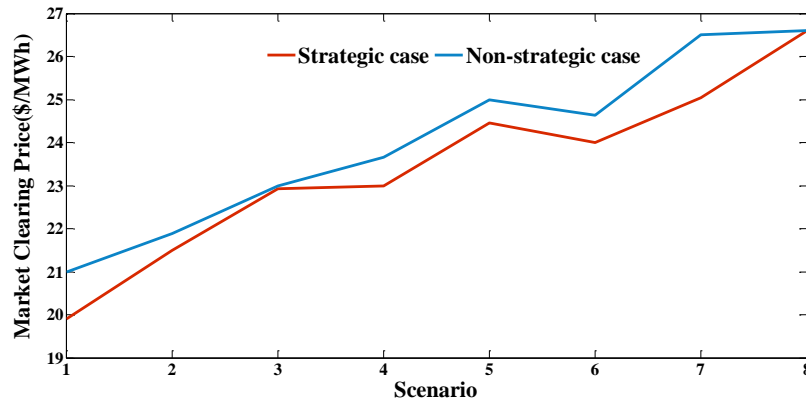


Fig. 12 Comparison market clearing price between non-strategic case and strategic case

## 6. Conclusions

In this paper, a bi-level stochastic optimization model based on a leader-follower Stackelberg game was presented. The purpose of this model was to create strategic bidding curves by a retailer company to increase its profits. A novel DR model was also introduced to simulate different consumers' behavior against price signal offered by the retailer. To show the effectiveness and applicability of the proposed model, first several studies were conducted on DR performance. It was observed that the proposed model is able to adapt to different consumers with different flexibilities against prices, and allows the consumption levels to be adjusted over different time intervals. In the next step, the bi-level model was evaluated. The results of this model were used to construct optimal strategic price curves for a retailer company with the help of the "worst-case scenario" action plan. The results indicated that the retailer is able to affect the MCP through its action and to make profit by motivating consumers to actively participate in different DR programs.

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