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# Simplified frequency-dependent formulae for series-impedance matrices of single-core HVAC cables

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**Abstract**—The installation of HVAC underground cables became more common in recent years, a trend expected to continue in the future. Underground cables are more complex than overhead lines and the calculation of their resistance and reactance can be challenging and time consuming for frequencies that are not power frequency.

Different methods exist for estimating the impedance of single-core cables. The IEC60287-1-1 standard presents equations that may be use for power frequencies, whereas the loop-method, based on Bessel equations, is typically used for higher frequencies.

Simplifications of the reference formula using Bessel equations exist, but they still have some complexity, e.g., hyperbolic cotangents and hyperbolic cosecants are used. Moreover, the series impedance matrix that is designed with these equations and used to estimate the resistance and reactance, requires the summation of at least eight different terms, all calculated in a different way.

This paper simplifies the equations used in the loop method, by means of expansions in Taylor series, with other further simplifications that can be made for typical cable thicknesses and materials.

**Index Terms**—Underground Cables, Series Impedance Matrix, Taylor Series

## I. NOMENCLATURE

$A$ : impedance of the conductor-ground loop  
 $B$ : mutual impedance between the conductor-ground and the screen-ground loops  
 $C$ : impedance of the screen-ground loop  
 $D$ : mutual impedance between cables  
 $Z_{Couter}$ : conductor outer series impedance  
 $Z_{CSinsul}$ : main insulation series impedance  
 $Z_{Sinner}$ : screen inner-series impedance  
 $Z_{Souter}$ : screen outer-series impedance  
 $Z_{SEinsul}$ : outer insulation series impedances  
 $Z_{Smutual}$ : mutual impedance of two loops  
 $Z_{Earth}$ : earth-return impedance  
 $Z_{Earth\_mutual}$ : mutual impedance between cables  
 $R_1$ : radius over the conductor  
 $R_2$ : radius over the insulation  
 $R_3$ : radius over the screen  
 $R_4$ : outer radius of the cable  
 $\mu_C$ : permeability of the conductor  
 $\mu_S$ : permeability of the screen  
 $\rho_C$ : conductor resistivity  
 $\rho_S$ : screen resistivity

$\rho_E$ : earth resistivity

$h_{ij}$ : depth of the cable conductor

$d_{ij}$ : distance between two conductors

$CS$ : cross-section area of the conductor

$T$ : Taylor series

## II. INTRODUCTION

Several power system phenomena in underground cables are frequency dependent and require the accurate calculation of different parameters at a target frequency. Simplified formulae exist for estimating the resistance and reactance at power frequency [1], but calculations for higher frequencies may be particularly complicated, as the existing reference formulae are based on Bessel equations [1] and require several steps.

A simplification of the formulae was derived in the past [2], whereas [3] did a generalisation for different types of cables. These formulae are excellent for obtaining accurate descriptions of a cable different modes or the distribution of the current by conductor and screen, for example. However, they are time consuming and too complex if one only wishes to estimate the dominant frequency of a transient [4] or to know the positive-sequence impedance of a cable for a given frequency [5]. As an example, the calculation of the positive-sequence impedances of a three-phase single-core cable installed in trefoil formation, for a frequency different from power frequency, requires to solve a minimum of eight equations [2], which have to be assembled into a 6x6 matrix with four different terms [2], followed by a transformation using the Fortescue matrix and four 3x3 sub-matrices of the 6x6 matrix [6].

Software packages capable of calculating the 6x6 matrix directly (e.g., PSCAD or ATP) exist. However, they are typically only familiar to engineers working with electromagnetic transients, being normally not used by those working in other topics.

The objective of this paper is to simplify the expressions used to estimate the loop impedances. These results will then be used in a second paper for the development and simplification of the references equations into two formulae, a first for estimating the resistance and a second for the reactance, with a complexity level alike the formulae used for estimating these parameters at power frequency, but also able to correctly estimate these two quantities for a large range of frequencies.

### III. ASSUMPTIONS AND IMPEDANCE MATRIX

The equations are developed for a cable with conductor and screen and use the theoretical principles of [2], [3] and [7]. Semi-conductive layers are not considered and are seen as part of the insulation as recommended in [8]. The core and the screen are solid and the corrections proposed in [1] and [8] can be used for stranded conductors, composite screens or other types. The installation formation can be trefoil or flat.

The reference formulae have been validated up to 100 kHz by field measurements [9], but they also have limitations. For example, they are inaccurate for very high and very low frequencies, they do not take into consideration stratified soils, among other cases described in [1] and [10].

Yet, they are valid for the majority of the transient and resonance phenomena [1] and the simplifications done in this research assume that the frequencies of interest are those of typical transients (temporary overvoltages and slow-front overvoltages [8],[11]) and resonance phenomena.

The impedance matrix of a three-phase single-core cable is a 6x6 matrix [2], [3], which can be simplified to (1) for trefoil formations.

$$[Z] = \begin{bmatrix} A & B & D & D & D & D \\ B & C & D & D & D & D \\ D & D & A & B & D & D \\ D & D & B & C & D & D \\ D & D & D & D & A & B \\ D & D & D & D & B & C \end{bmatrix} \quad (1)$$

$$\begin{aligned} A &= Z_{Couter} + Z_{CSinsul} + Z_{Sinner} + Z_{Souter} + Z_{SEinsul} + Z_{Earth} - 2Z_{Smutual} \\ B &= Z_{Souter} + Z_{SEinsul} + Z_{Earth} - Z_{Smutual} \\ C &= Z_{Souter} + Z_{SEinsul} + Z_{Earth} \\ D &= Z_{Earth\_mutual} \end{aligned} \quad (2)$$

### IV. SIMPLIFICATION OF THE LOOP IMPEDANCE FORMULAE

The simplification of the impedances indicated in (2) is made in this chapter. The exceptions are  $Z_{CSinsul}$  and  $Z_{SEinsul}$  whose formulae do not require further simplifications.

The relative permeability is equal to one for all materials and ground. This simplification is valid, because the variation of the ground permeability is usually very small [12] and only single-core cables, which typically do not contain steel, are considered.

The simplifications are made using a 3<sup>rd</sup> order Taylor series centred at 0Hz. The frequency of the majority of the overvoltages involving cables are in the order of kHz [8]. The exceptions are phenomena involving GIS or restrike of shunt reactors. Thus, the Taylor series is centred at 0Hz, in order to be accurate at the frequencies of interest; moreover, to centre the series at 0Hz equals some of the terms to zero, as  $\sin(0)=0$ , simplifying the equations. The use of a 3<sup>rd</sup> order series is a compromise between accuracy and simplicity. Lower orders are too inaccurate and higher orders bring little improvements to the accuracy while complicating the equations.

#### A. $Z_{Sinner}$ , $Z_{Souter}$ and $Z_{Smutual}$

The reference formulae of  $Z_{Sinner}$ ,  $Z_{Souter}$  and  $Z_{Smutual}$  are very similar, and thus, the simplified formulae are also similar. The demonstration is done for  $Z_{Sinner}$  and it is also valid for the other two impedances. Equation (3) shows the reference formula from [2].

$$Z_{Sinner} = \frac{\rho_s m_s}{2\pi R_2} \coth(m_s(R_3 - R_2)) - \frac{\rho_s}{2\pi R_2(R_2 + R_3)}, \text{ where } m_s = \sqrt{\frac{j\omega\mu_s}{\rho_s}} \quad (3)$$

The main objective of the simplification is to eliminate the  $\coth$ , which can be replaced by exponential terms, and to separate the real and imaginary parts of (3). Equation (4) shows a variation of (3), where  $M_s$  is given by (5).

$$Z_{Sinner} = \frac{\sqrt{\frac{\rho_s\mu_0\omega}{2}}}{2\pi R_2} \cdot \frac{(1 - e^{2M_s} - 2e^{M_s} \sin(M_s)) + j(1 - e^{2M_s} + 2e^{M_s} \sin(M_s))}{1 + e^{2M_s} - 2e^{M_s} \cos(M_s)} - \frac{\rho_s}{2\pi R_2(R_2 + R_3)} \quad (4)$$

$$M_s = -2(R_3 - R_2) \sqrt{\frac{\mu_0\omega}{2\rho_s}} \quad (5)$$

A 3<sup>rd</sup> order Taylor expansion is applied to (4) and results in simplified expressions for the real (6) and imaginary (7) parts of  $Z_{Sinner}$ . Other minor simplifications, besides the Taylor expansion, exist in (6) and (7). The analysis of the equations shows that some terms are negligible up to several kHz; this is more noticeable for high order Taylor expansions. Hence, these terms are not included in the equations, since the cases of interest do not have so high frequencies.

$$\text{Re}(Z_{Sinner}) = \frac{\rho_s}{\pi(R_3^2 - R_2^2)} \quad (6)$$

$$\text{Im}(Z_{Sinner}) = \frac{\omega \cdot \mu_0(R_3 - R_2)}{6\pi R_2} \quad (7)$$

#### B. $Z_{Couter}$

The next step is to simplify  $Z_{Couter}$ , whose reference formula is given by (8) [2]. The method previously used for  $Z_{Sinner}$  is applied, resulting in (9), where  $M_c$  is given by (10).

$$Z_{Couter} = \frac{\rho_c m_c}{2\pi R_1} \coth(m_c R_1 \cdot 0.777) + \frac{\rho_c}{\pi R_1^2} \cdot \frac{277}{777}, \text{ where } m_c = \sqrt{\frac{j\omega\mu_c}{\rho_c}} \quad (8)$$

$$Z_{Couter} = \frac{\sqrt{\frac{\rho_c\mu_0\omega}{2}}}{2\pi R_1} \cdot \frac{(1 - e^{2M_c} - 2e^{M_c} \sin(M_c)) + j(1 - e^{2M_c} + 2e^{M_c} \sin(M_c))}{1 + e^{2M_c} - 2e^{M_c} \cos(M_c)} + \frac{277\rho_c}{777\pi R_1^2} \quad (9)$$

$$M_c = -2R_1 \cdot 0.777 \sqrt{\frac{\mu_0\omega}{2\rho_c}} \quad (10)$$

The variable  $M_c$  of  $Z_{Couter}$  (10) is dozens of times larger than the variable  $M_s$  in (5) and results in a steep variation of (9) for low frequencies. As a result, a Taylor series centred in zero requires an expansion of high order in order to be accurate at the frequencies of interest, something that derails the objective of this paper. Instead, a different approach is applied. Equation (11) is separated from (9), and a Taylor

series of 3<sup>rd</sup> order centred at 0Hz (12) is calculated for both the real and imaginary parts of (11).

$$G(M_C) = \frac{(1 - e^{2M_C} - 2e^{M_C} \sin(M_C)) + j(1 - e^{2M_C} + 2e^{2M_C} \sin(M_C))}{1 + e^{2M_C} - 2e^{M_C} \cos(M_C)} \quad (11)$$

$$\text{Re}(T) = \frac{-2}{M_C} - 2 \quad \text{and} \quad \text{Im}(T) = \frac{-M_C}{3} \quad (12)$$

Fig. 1 compares (11) with the respective Taylor expansions. The expansion of the imaginary part matches (11) very well, but the same is not true for the real part, which diverges from (11) (see dashed-dotted line in Fig. 1). However, the real part can be altered to (13), which matches (11) much better, by eliminating the highest order term in high order Taylor expansions.

$$\text{Re}(T) = \frac{-2}{M_C} \quad (13)$$

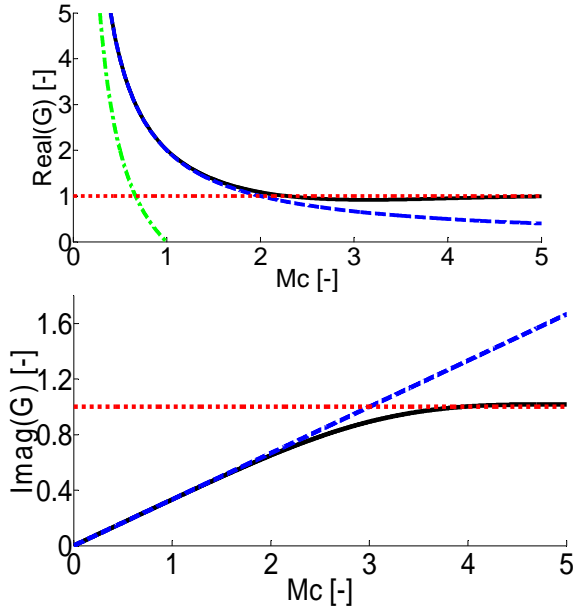


Fig. 1. Comparison of (11) with the respective Taylor approximation (MC in absolute value). Top: Real part; Bottom: Imaginary part. Solid Line: Reference formula, Dashed line: Taylor approximation (Dashed-dotted line on real part is (12)); Dotted line: Asymptote of the final value

The analysis of Fig. 1 shows that the approximation is accurate up to  $M_C=2$  for the real part and up to  $M_C=3$  for the imaginary part. It also shows that (11) tends to one as  $M_C$  increases, for both real and imaginary parts. Thus, it can be proposed that (11) is approximated by the Taylor expansion up to  $M_C=2$  and  $M_C=3$  for the real and imaginary parts respectively, and equal to  $1+j$  after that.

Considering the reactance, the mathematical development (14) is made by replacing  $M_C$  by  $-3$  in (10) (Fig. 1 shows absolute values, thus, the minus). The second step in (14) is the correction of the resistivity for stranded conductors. The value 1150 is obtained for  $M_C=-2$ , i.e., the resistance.

$$3 = 2R_1 \cdot 0.777 \sqrt{\frac{\mu_0 \omega}{2\rho_C}} \Leftrightarrow 3 = 1.554R_1 \sqrt{\frac{\mu_0 \omega}{2\rho_C \frac{\pi R_1^2}{CS}}} \Leftrightarrow 1725 \approx \sqrt{\frac{CS \cdot f}{\rho_C}} \quad (14)$$

Equation (14) says that the Taylor approximation is used for the imaginary part of (11) if the calculated value is inferior to 1725 and for the real part of (11) if the calculated value is inferior to 1150; else (11) is equal to  $1+j$ .

With the exception of low cross-section areas and/or low frequencies, the calculated value is typically larger than 1725, because of the low resistivities of copper and aluminium. Thus, (11) may be considered equal to  $1+j$  for the majority of the cases. For example, if 100Hz is the minimum frequency of interest, (11) needs to be approximated by the Taylor expansion only for cross-sections inferior to 513mm<sup>2</sup> and 841mm<sup>2</sup> for respectively copper and aluminium conductors.

In summary, (11) is approximated by one of two functions depending on the cross-section and frequency. The real part of (11) is replaced by (15), whereas the imaginary part is replaced by (16).

$$\text{Re} = \begin{cases} \frac{1}{R_1 \cdot 0.777 \sqrt{\frac{\mu_0 \omega}{2\rho_C}}}, & \text{if } \sqrt{\frac{CS \cdot f}{\rho_C}} < 1150 \\ 1, & \text{if } \sqrt{\frac{CS \cdot f}{\rho_C}} \geq 1150 \end{cases} \quad (15)$$

$$\text{Im} = \begin{cases} \frac{2}{3} R_1 \cdot 0.777 \sqrt{\frac{\mu_0 \omega}{2\rho_C}}, & \text{if } \sqrt{\frac{CS \cdot f}{\rho_C}} < 1725 \\ 1, & \text{if } \sqrt{\frac{CS \cdot f}{\rho_C}} \geq 1725 \end{cases} \quad (16)$$

Finally, the values in the reference equation (9) are substituted and the simplified expressions for the real (17) and imaginary (18) parts of  $Z_{Counter}$  are obtained.

$$\text{Re}(Z_{Counter}) = \begin{cases} \frac{\rho_c}{\pi R_1^2}, & \text{if } \sqrt{\frac{CS \cdot f}{\rho_C}} < 1150 \\ R_1 \sqrt{\frac{\omega \cdot \mu_0 \cdot \rho_c}{8}} + \rho_c \frac{277}{777}, & \text{if } \sqrt{\frac{CS \cdot f}{\rho_C}} \geq 1150 \end{cases} \quad (17)$$

$$\text{Im}(Z_{Counter}) = \begin{cases} \frac{259 \cdot \omega \cdot \mu_0}{2000\pi}, & \text{if } \sqrt{\frac{CS \cdot f}{\rho_C}} < 1725 \\ \frac{\sqrt{\omega \cdot \mu_0 \cdot \rho_c}}{\sqrt{8}\pi R_1}, & \text{if } \sqrt{\frac{CS \cdot f}{\rho_C}} \geq 1725 \end{cases} \quad (18)$$

### C. Z<sub>Earth</sub> and Z<sub>Earth\_mutual</sub>

The simplification is made for  $Z_{Earth}$  and the conclusions can also be applied to  $Z_{Earth\_mutual}$ . The reference formula for  $Z_{Earth}$  is given by (19) [1], [2]. The first step is to separate (19) into real (20) and imaginary (21) parts.

$$Z_{Earth} = \frac{j\omega \cdot \mu_0}{2\pi} \ln \left( \frac{2e^{0.5}}{1.7811m_E} \frac{R_4}{R_4} \right) - \frac{4}{3} m_E h \quad (19)$$

$$\text{Re}(Z_{Earth}) = \frac{\omega \cdot \mu_0}{8} \left( 1 + \frac{16}{3\pi} h \sqrt{\frac{\omega \cdot \mu_0}{2\rho_E}} \right) \quad (20)$$

$$\text{Im}(Z_{Earth}) = \frac{\omega \cdot \mu_0}{8} \ln \left( \frac{1.85}{R_4 \sqrt{\frac{\omega \cdot \mu_0}{\rho_E}}} \right) - \frac{4}{3} h \sqrt{\frac{\omega \cdot \mu_0}{2\rho_E}} \quad (21)$$

The outmost right parcels of (20) and (21) are very small for typical installation depths and frequencies associated to the phenomena of interest. Thus, their elimination can possibly simplify the system without affecting the accuracy.

Table I shows for 5kHz, the error present if the parcels are eliminated for different ground resistivities and installation depths. The error of the imaginary part is small and the simplified formula can be considered a good approximation for typical values. The same is not true for the real part, where the error can be large for low earth resistivities and/or big installation depths. However, the error is acceptable in the calculation of the total resistance and inductance, as it is low for typical parameters ( $\rho_E \sim 100\Omega \cdot m$  and  $h \sim 1m$ ) and  $Z_{Earth}$  is only one of several terms used in the calculations. Moreover,  $\rho_E$  does not appear in the final formulae for positive-sequence impedances. Similar conclusion can be drawn for  $Z_{Earth\_mutual}$ .

TABLE I. RELATIVE ERROR AT 5KHz DUE TO THE SIMPLIFICATION MADE IN (20) (LEFT) AND (21) (RIGHT)

	h=0.1[m]	h=1[m]	h=3[m]		h=0.1[m]	h=1[m]	h=3[m]
$\rho_E=30[\Omega \cdot m]$	0.4%	4.4%	13%		0.04%	0.4%	1.2%
$\rho_E=100[\Omega \cdot m]$	0.2%	2.4%	7.2%		0.02%	0.2%	0.6%
$\rho_E=150[\Omega \cdot m]$	0.2%	1.9%	5.8%		0.02%	0.2%	0.5%

#### D. Summary

Table II shows the simplified equations for all variables, summarising the work made in chapter IV.

TABLE II. SUMMARY OF THE SIMPLIFICATIONS PROPOSED FOR THE DIFFERENT LOOP IMPEDANCES

	Real	Imaginary
$Z_{Couter}^*$	$\frac{\rho_c}{\pi R_1^2}$	$\frac{259 \cdot \omega \cdot \mu_0}{2000\pi}$
$Z_{Couter}$	$\frac{R_1 \sqrt{\frac{\omega \cdot \mu_0 \cdot \rho_c}{8}} + \rho_c \frac{277}{777}}{\pi R_1^2}$	$\frac{\sqrt{\omega \cdot \mu_0 \cdot \rho_c}}{\sqrt{8\pi R_1}}$
$Z_{CSinsul}$	0	$\frac{\omega \cdot \mu_0}{2\pi} \ln \left( \frac{R_2}{R_1} \right)$
$Z_{Sinner}$	$\frac{\rho_s}{\pi (R_3^2 - R_2^2)}$	$\frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi R_2}$
$Z_{Souter}$	$\frac{\rho_s}{\pi (R_3^2 - R_2^2)}$	$\frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi R_3}$
$Z_{SEinsul}$	0	$\frac{\omega \cdot \mu_0}{2\pi} \ln \left( \frac{R_4}{R_3} \right)$

$Z_{Smutual}$	$\frac{\rho_s}{\pi (R_3^2 - R_2^2)}$	$-\frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi \left( \frac{R_2 + R_3}{2} \right)}$
$Z_{Earth}$	$\frac{\omega \cdot \mu_0}{8}$	$\frac{\omega \cdot \mu_0}{2\pi} \ln \left( \frac{659 \sqrt{\frac{\rho_e}{f}}}{R_4} \right)$
$Z_{Earth\_mutual}$	$\frac{\omega \cdot \mu_0}{8}$	$\frac{\omega \cdot \mu_0}{2\pi} \ln \left( \frac{659 \sqrt{\frac{\rho_e}{f}}}{d_{ij}} \right)$

#### V. DEFINITION OF A, B, C, AND D

The simplifications can be combined and the variables of the series impedance matrix (1) can be written using the results previous developed.

Equations (23) to (27) shows the equations correspondent to  $A$ ,  $B$ ,  $C$  and  $D$  after substituting the loop impedances defined in (2) with the values from Table II.

##### A. Variable A

$$\text{Real} \begin{cases} A^*, \text{ if } \sqrt{\frac{CS \cdot f}{\rho_c}} < 1150 \\ A, \text{ if } \sqrt{\frac{CS \cdot f}{\rho_c}} \geq 1150 \end{cases} \quad \text{Imaginary} \begin{cases} A^*, \text{ if } \sqrt{\frac{CS \cdot f}{\rho_c}} < 1725 \\ A, \text{ if } \sqrt{\frac{CS \cdot f}{\rho_c}} \geq 1725 \end{cases} \quad (22)$$

$$\text{Re}(A^*) = \frac{\rho_c}{\pi R_1^2} + \frac{\omega \cdot \mu_0}{8} \quad \text{Im}(A^*) = \frac{259\omega \cdot \mu_0}{2000\pi} + \frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi} \frac{8}{R_2 + R_3} + \frac{\omega \cdot \mu_0}{2\pi} \ln \left( \frac{R_2 \cdot 659 \sqrt{\frac{\rho_e}{f}}}{R_1 \cdot R_3} \right) \quad (23)$$

$$\text{Re}(A) = \frac{R_1 \sqrt{\frac{\omega \cdot \mu_0 \cdot \rho_c}{8}} + \rho_c \frac{277}{777}}{\pi R_1^2} + \frac{\omega \cdot \mu_0}{8} \quad \text{Im}(A) = \frac{259\omega \cdot \mu_0}{2000\pi} + \frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi} \frac{8}{R_2 + R_3} + \frac{\omega \cdot \mu_0}{2\pi} \ln \left( \frac{R_2 \cdot 659 \sqrt{\frac{\rho_e}{f}}}{R_1 \cdot R_3} \right) \quad (24)$$

##### B. Variable B

$$\text{Re}(B) = \frac{\omega \cdot \mu_0}{8} \quad \text{Im}(B) = \frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi} \left( \frac{1}{R_3} + \frac{1}{R_2 + R_3} \right) + \frac{\omega \cdot \mu_0}{2\pi} \ln \left( \frac{659 \sqrt{\frac{\rho_e}{f}}}{R_3} \right) \quad (25)$$

##### C. Variable C

$$\text{Re}(C) = \frac{\omega \cdot \mu_0}{8} + \frac{\rho_s}{\pi (R_3^2 - R_2^2)} \quad \text{Im}(C) = \frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi R_3} + \frac{\omega \cdot \mu_0}{2\pi} \ln \left( \frac{659 \sqrt{\frac{\rho_e}{f}}}{R_3} \right) \quad (26)$$

### D. Variable D

$$\begin{aligned} \text{Re}(D) &= \frac{\omega \cdot \mu_0}{8} \\ \text{Im}(D) &= \frac{\omega \cdot \mu_0}{2\pi} \ln \left( \frac{659 \sqrt{\frac{\rho_e}{f}}}{d_{ij}} \right) \end{aligned} \quad (27)$$

There is an extra simplification in  $\text{Im}(A)$  and  $\text{Im}(A^*)$ . The thickness of a screen is typically small and  $R_2$  and  $R_3$  have similar values, allowing (28).

$$\begin{aligned} Z_{CSinul} + Z_{Sinner} - 2Z_{Smual} &= \frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi R_2} + \frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi R_3} + 2 \frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi \left( \frac{R_2 + R_3}{2} \right)} \\ &= \frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi} \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{2}{R_{2,3av}} \right) \approx \frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi} \left( \frac{1}{R_{2,3av}} + \frac{1}{R_{2,3av}} + \frac{2}{R_{2,3av}} \right) \\ &\approx \frac{\omega \cdot \mu_0 (R_3 - R_2)}{6\pi} \left( \frac{8}{R_2 + R_3} \right) \end{aligned} \quad (28)$$

## VI. CONCLUSIONS

This paper presents a simplification of the different formulae used for estimating the loop impedances of three-phase single core cables.

These results will be used in a second paper [13] for the simplification of the formulae used to calculate the positive-sequence resistance and reactance of a cable.

The validation of the simplifications proposed in this paper will be made in this second paper together with the new proposed formulae.

## VII. REFERENCES

- [1] CIGRE WG B1.30, "Cable Systems Electrical Characteristics", 2013
- [2] L. M. Wedepohl, D. J. Wilcox, "Transient analysis of underground power-transmission systems: System-model and wave-propagation characteristics", Proceedings of the Institution of Electrical Engineers, Vol. 120, No. 2, 1973
- [3] A. Ametani, "A General Formulation of Impedance and Admittance of Cables", IEEE Transactions on Power Apparatus and Systems, 1980
- [4] Teruo Ohno, Claus Leth Bak, Ametani Akihiro, Wojciech Wiechowski, Thomas K. Sørensen, "Derivation of Theoretical Formulas of the Frequency Component Contained in the Overvoltage related to Long EHV Cables", IEEE Transactions on Power Delivery, 2012
- [5] F. Faria da Silva, C. L. Bak, "Energization of long HVAC cables in parallel – Analysis and estimation formulas", Electric Power Systems Research, 2013
- [6] Nassler Tleis, "Power Systems Modelling and Fault Analysis", Elsevier-Newnes, 2008
- [7] S. A. Shelkunoff, "The electromagnetic theory of coaxial transmission lines and cylindrical shields", Bell Systems Technical Journal, 1934
- [8] CIGRE WG C4.502, "Power System Technical Performance Issues Related to the Application of Long HVAC Cables", October 2013
- [9] Unnur Stella Gudmundsdóttir, "Modelling of long High Voltage AC cables in Transmission Systems", PhD Thesis, Aalborg University, 2010
- [10] CIGRE WG C4.501, "Guideline for Numerical Electromagnetic Analysis Method and its Application to Surge Phenomena", June 2013
- [11] IEC 60071-2, "Insulation co-ordination – Part 2: Application guide", 3<sup>rd</sup> edition, 1996
- [12] C. M. Portela, J. B. Gertrudes, M. C. Tavares, J. Pissolato, "Earth conductivity and permittivity data measurements: Influence in transmission line transient performance", Electric Power Systems Research, 2006

- [13] F. Faria da Silva, "Simplified formulae for the estimation of the positive-sequence resistance and reactance of three-phase cables for different frequencies", UPEC 2015