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# Energy Efficient Control of a Boosting System with Multiple Variable-Speed Pumps in Parallel

Zhenyu Yang and Hakon Børsting

**Abstract**—The control objective of a water boosting system equipped with multiple variable-speed pumps in parallel is to minimize the pump system energy consumption by control the number of running pumps and their corresponding speeds in a real-time manner, subject to potential changes of (system head) set-points and operating conditions. After a number of static models for different pump combinations are derived, a number of optimal scheduling algorithms are proposed from a formulated Mixed Integer Non-Linear Program (MINLP) problem. The Branch and Bound method is employed to cope with the considered MINLP problem and the Lagrangian Multipliers method is used to handle the corresponding nonlinear programming within each iteration. In order to cope with potential modeling errors, a feedback control mechanism is introduced into the proposed framework. In case of unknown operating conditions, an identification algorithm is proposed to estimate unknown system coefficients in an online manner. The experimental results show a huge potential to improve the energy efficiency of multi-pump systems using the proposed method and algorithms.

## I. INTRODUCTION

Pump optimization is always challenging in many application areas, such as pump network systems for irrigation [15], infrastructure water supply [13], and pump boosting systems for air conditioning systems [8], refrigeration [10], oil and gas pipeline transportation [1] etc.. An energy efficient pump scheduling strategy has the huge potential to significantly reduce pump system's operational and maintenance costs, and hence to reduce the CO<sub>2</sub> emission. For instance, up to 90% of electricity used in UK and USA water industry is consumed by pumps [1].

A major study of pumps commissioned by the European Commission in 2001 [7] recommended that the largest energy saving of pumps can be made through the better design and control of pump systems. In general, the control of pump systems involves the scheduling of pumps so that they can operate close to their best efficient points [5]. In order to cope with versatile applications, normally a number of pumps are connected into a group for many large scale applications. As a consequence, the real-time optimal pump scheduling problem consists of determination of which and when available pump should be put into operation or pull out from the operation, and the corresponding speeds of all operating Variable Speed Pumps (VSPs), with the objective to maximize the entire system's operating energy efficiency,

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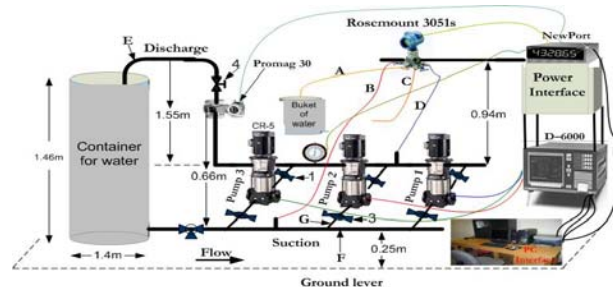


Fig. 1. Schematic diagram of the Considered Multi-Pump Setup

subject to the conditions that the operating system should keep the required performances.

Extensive study can be found about pump scheduling and optimization in recent decades [1]-[3]; [4]-[8]; [9]-[15]. For instance, Westerlund *et al* studied the configuration optimization of multiple pumps, and proposed a Mixer Integer NonLinear Programming (MINLP) solution to this structural process optimization problem in [12]. Due to the concerned problem is not convex [2], thereby a binary separable programming method is developed in [6] for getting the global optimum. These methods provide the best configuration of pumps in series and/or parallel, in terms of minimum total cost for some given required pump head and flow. However, these methods are not oriented for real-time dynamic pump scheduling [1]. An optimal pump scheduling algorithm for water distribution systems is proposed in [15]. A number of water reservoirs and pump stations are considered and modeled as nodes in a networked system model. Each node has its dynamic constraints in terms of hydraulic and/or electric characteristics. The generalized reduced gradient method is used to derive the optimal control of pump flows and valve positions so as to minimize the entire system operation costs. [4] studied energy efficient control of VSPs in central air-conditioning systems for a complex building. Using the efficiency prediction based on VSP's pump characteristics and models of pressure drops over the entire water network, an optimal pump sequence control is proposed to determine both the operating order and point that pumps should be brought online and off-line. Within these methods, each pump group or pump station was simply modeled as a network node with some specific hydraulic and/or electric constraints. The structural configuration of pumps within the group/station and the speed control of each running VSP are not considered. In recent decades, Evolutionary Algorithms (EAs) have been investigated to handle the pump

optimization problem [9], [13]. The EA methods can cope with non-convex optimization problem which is often faced in the pump optimization [6]. However, the development of these methods can be time-consuming, due to the fact that it often requires a huge amount of extensive experiments and data for training purpose. Moreover, the computation load can not be ignored if concerned for real-time implementation. Nevertheless, few of these methods discussed in the above have been implemented in the real-life applications [1].

Motivated by model-based control approaches and concern of the potential industrial application, here we focus on a simplified multi-pump boosting system to study the real-time pump optimization problem. As shown in Fig.1, three Grundfos CRE-5 VSP are configured in parallel and connected with a simple water circular loop and a storage tank. the differential pressure and flow rates of all pumps are measured. A NI-PCI-6229 is used as the DAC to bridge the hardware setup and a PC which has LabVIEW program installed. A power measurement instrument named Power Analyzer D-6000 is used to monitor the power consumption of each pump. Our task is to investigate some optimal algorithm for real-time scheduling and control of this pump system, so that the controlled system can follow some expected (head) load demand in a satisfactory and best energy efficient manner, subject to possible changes in demands and/or operating conditions.

## II. PROBLEM FORMULATION

### A. Modeling a Group of VSPs

In general, For a fixed-speed pump, a pump model can be expressed as set of static relationships among the pump head  $H$  (m), flow rate  $Q$  ( $m^3/h$ ) and Brake Horse Power (BHP)  $P$  (W) as [8]:

$$\begin{aligned} H &= \bar{a}_0 + \bar{a}_1 Q + \bar{a}_2 Q^2, \\ P &= \bar{p}_0 + \bar{p}_1 Q + \bar{p}_2 Q^2 + \bar{p}_3 Q^3. \end{aligned} \quad (1)$$

System parameters  $\bar{a}_i$ ,  $\bar{p}_j$  for  $i = 0, 1, 2$ ,  $j = 0, \dots, 3$  are determined by specific pump characteristics and they can be identified through experiment [8].

Assume the pump model (1) for a VSP at a specific speed  $\bar{\omega}_0$  is obtained, and its parameters are  $(\bar{a}_0, \bar{a}_1, \bar{a}_2)$  and  $(\bar{p}_0, \bar{p}_1, \bar{p}_2, \bar{p}_3)$ . There is

**Lemma 1** [14]: The pump model of a VSP for any given speed  $\omega$  has the property:

$$\begin{aligned} H(\omega) &= a_0 \omega^2 + a_1 \omega Q(\omega) + a_2 (Q(\omega))^2, \\ P(\omega) &= p_0 \omega^3 + p_1 \omega^2 Q(\omega) + p_2 \omega (Q(\omega))^2 + p_3 (Q(\omega))^3, \end{aligned} \quad (2)$$

where  $H(\omega)/Q(\omega)/P(\omega)$  represents the head/flow-rate/BHP of the considered pump at speed  $\omega$ , and

$$\begin{aligned} a_0 &= \frac{\bar{a}_0}{\bar{\omega}_0^2}, \quad a_1 = \frac{\bar{a}_1}{\bar{\omega}_0}, \quad a_2 = \bar{a}_2, \\ p_0 &= \frac{\bar{p}_0}{\bar{\omega}_0^3}, \quad p_1 = \frac{\bar{p}_1}{\bar{\omega}_0^2}, \quad p_2 = \frac{\bar{p}_2}{\bar{\omega}_0}, \quad p_3 = \bar{p}_3. \end{aligned}$$

**Lemma 2** [14]: The pump model for  $N$  identical pumps in parallel with a common speed  $\omega$  is

$$\begin{aligned} H_s(\omega) &= a_0^s \omega^2 + a_1^s \omega Q_s(\omega) + a_2^s (Q_s(\omega))^2, \\ P_s(\omega) &= p_0^s \omega^3 + p_1^s \omega^2 Q_s(\omega) + p_2^s \omega (Q_s(\omega))^2 + p_3^s (Q_s(\omega))^3, \end{aligned} \quad (3)$$

where  $H_s(\omega)/Q_s(\omega)/P_s(\omega)$  represents the head/flow-rate/BHP of the entire pump group at speed  $\omega$  and system parameters are

$$\begin{aligned} a_0^s &= a_0, \quad a_1^s = \frac{a_1}{N}, \quad a_2^s = \frac{a_2}{N^2}, \\ p_0^s &= N p_0, \quad p_1^s = p_1, \quad p_2^s = \frac{p_2}{N}, \quad p_3^s = \frac{p_3}{N^3}. \end{aligned}$$

In the following we assume there are  $N$  pumps with different pump features.  $H_i(\omega_i)/Q_i(\omega_i)/P_i(\omega_i)$  represent the  $i$ th pump's head/flow-rate/BHP. Instead of using model  $H - Q - \omega$ , the  $Q - H - \omega$  model is adopted here. Thereby, the  $i$ th pump model is represented by

$$\begin{aligned} Q_i(\omega_i) &= W_H^i T B^i W_H^i, \\ P_i(\omega_i) &= p_0^i \omega_i^3 + (W_Q^i T P^i W_Q^i) Q_i, \end{aligned} \quad (4)$$

where

$$\begin{aligned} W_H^i &= [\omega_i \quad H_i]^T, \quad W_Q^i = [\omega_i \quad Q_i]^T, \\ B^i &= \frac{1}{\omega_i^3} \begin{bmatrix} b_0^i & \frac{b_1^i}{2} \\ \frac{b_1^i}{2} & b_2^i \end{bmatrix}, \quad P^i = \begin{bmatrix} p_1^i & \frac{p_2^i}{2} \\ \frac{p_2^i}{2} & p_3^i \end{bmatrix}, \end{aligned}$$

with parameters

$$\begin{aligned} a_0 &= \frac{\bar{a}_0}{\bar{\omega}_0^2}, \quad a_1 = \frac{\bar{a}_1}{\bar{\omega}_0}, \quad a_2 = \bar{a}_2, \\ p_0 &= \frac{\bar{p}_0}{\bar{\omega}_0^3}, \quad p_1 = \frac{\bar{p}_1}{\bar{\omega}_0^2}, \quad p_2 = \frac{\bar{p}_2}{\bar{\omega}_0}, \quad p_3 = \bar{p}_3. \end{aligned}$$

**Lemma 3** [14]: Assume the considered  $N$  pumps have the property

$$H_1^{max}(\omega_1) > H_2^{max}(\omega_2) > \dots > H_N^{max}(\omega_N), \quad (5)$$

Then, the model of  $N$  pumps in terms of one pump system can be described as

$$\begin{aligned} Q_s(\bar{\omega}) &= \bar{W}_H^T \bar{B}_\omega \bar{W}_H, \\ P_s^N(\bar{\omega}) &= \bar{P}_0 \bar{\omega} + \bar{W}_Q^T \bar{P}_Q \bar{W}_Q, \end{aligned} \quad (6)$$

where

$$\begin{aligned} \bar{\omega} &= [\omega_1 \quad \dots \quad \omega_N]^T, \quad \bar{P}_0 = [p_0^1 \quad \dots \quad p_0^N]^T, \\ \bar{W}_H &= [W_H^1 \quad \dots \quad W_H^N], \quad \bar{W}_Q = [W_Q^1 \quad \dots \quad W_Q^N], \\ \bar{B}_\omega &= \text{diag}\{B^i\}_{N \times N}, \quad \bar{P}_Q = \text{diag}\{P^i Q_i\}_{N \times N}. \end{aligned}$$

We refer to [14] for more details about modeling and validation of this multi-pump systems.

### B. Pump Operating Point and Efficiency

The pump operating point is the cross-point of a pump curve with the system curve. The system curve models the terminal impedance that the pump system has to face to and it can be simply modeled as

$$H_s = k_0 + k_1 Q_s^2, \quad (7)$$

where  $k_0$  is the static head that the pump system needs to lift.  $Q_s$  is the system flow rate and  $k_1$  is the head loss coefficient.

The system efficiency, denoted as  $\eta_s$ , is defined as the ratio of the pump's hydraulic power to the BHP, under assumption that the motor's efficiency and motor driver's efficiency are constants, there is

$$\eta_s = \frac{\rho g H_s Q_s}{P_s}. \quad (8)$$

### C. Optimal Pump Scheduling Problem

Assume the expected head for the considered pump system is  $H^*$ . Once the  $i$ th pump is determined to be put into operation with a specific speed  $\omega_i$ , there should be

$$\sqrt{\frac{H^*}{a_0^i}} \leq \omega_i \leq \omega_i^{max}, \text{ for } i = 1, \dots, N. \quad (9)$$

The left inequality is due to the fact that  $H_i(\omega_i)|_{Q_i(\omega_i)=0} \geq H^*$ , i.e., the shut-off head should be over the given head in order to have a safe operation [10].

If system coefficients in (7) are known beforehand, the system flow rate at the operating point can be estimated as

$$H^* = k_0 + k_1 Q_s^{*2}, \quad (10)$$

this implies to

$$Q_s^* = \sqrt{\frac{H^* - k_0}{k_1}}. \quad (11)$$

With respect to the parallel configuration characteristics, there is

$$Q_s^*(\bar{\omega}) \triangleq \sum_{i=1}^N Q_i(\omega_i) = Q_s^*, \quad (12)$$

where  $Q_i(\omega_i)$  can be obtained from (4) for  $i = 1, \dots, N$ .

Similarly, the total system power consumption  $P_s(\bar{\omega})$  regarding to this specific head demand  $H^*$  can be estimated by

$$P_s(\bar{\omega}) = \sum_{i=1}^N P_i(\omega_i), \quad (13)$$

where  $P_i(\omega_i)$  can be obtained from (4) for  $i = 1, \dots, N$ .

The optimization problem for  $N$  available pumps is defined as: *For a given demanded head and known system curve coefficients, find a number of available pumps, denoted as  $i_1, \dots, i_k$ , which combination leads to*

$$\begin{aligned} & \max \quad \eta_s(\bar{\omega}), \\ & i_1, \dots, i_k, \\ & i_1, \dots, i_k \in \{1, 2, \dots, N\} \\ & \omega_{i_1}, \dots, \omega_{i_k} \\ & 0 \leq k \leq N \end{aligned} \quad (14)$$

subject to constraints (9) and (12) for selected pump  $i_1, \dots, i_k$ .

## III. OPTIMAL SCHEDULING AND CONTROL

### A. Equivalent MINLP Formulation

The system hydraulic power is constant for a given system head  $H_s^*$  w.r.t. the relationship (11). Then, the maximal optimization problem (14) can be formulated as a equivalent MINLP minimal problem.

Define a set of binary variable  $r_i$  for  $i = 1, \dots, N$ .  $r_i = 1$  (0) means the  $i$ th pump is (not) selected to be put into operation. The equivalent MINLP problem is formulated as

$$\begin{aligned} & \min \quad P_s(\bar{\omega}_N), \\ & r_1, \dots, r_N \in \{0, 1\} \\ & \omega_1, \dots, \omega_N \end{aligned} \quad (15)$$

subject to constraints (9) and (12), where  $\bar{\omega}_N = [r_1 \omega_1 \ \dots \ r_N \omega_N]^T$ .

### B. Classification of MINLP Solutions

Several methods and algorithms have been proposed to handle convex/non-convex MINLP problem in recent decades [2], [3], [6], [12]. With respect to fact that integers considered in problem (15) are binary (0-1) problem [11], the BB method along with Lagrangian Multipliers method is adopted to attack our problem. The proposed solutions are classified into two categories, namely (a) a group of identical pumps in parallel; (b) a group of different pumps in parallel. This classification is motivated by the following Theorem.

**Theorem 4:** The optimal solution to MINLP problem for a group of  $N$  identical pumps in parallel is to operate all selected pumps at a common speed. The number of pumps to be put into operation and the corresponding speed  $\omega$  are the solution to the following MINLP problem:

$$\begin{aligned} & \min \quad nP(\omega), \\ & 0 < n \leq N \\ & \sqrt{\frac{H^*}{a_0}} \leq \omega \leq \omega^{max}, \end{aligned} \quad (16)$$

subject to constraint  $nQ(\omega) = \sqrt{\frac{H^* - k_0}{k_1}}$ , where  $P(\omega)/Q(\omega)$  is one pump's head/capability at speed  $\omega$ , which are characterized by (2).

### C. Optimal Scheduling for $N$ Identical Pumps

There are total  $N$  different combinations for  $N$  identical pumps, if they are only allowed to operate at common speed. We use  $n$  to represent that  $n$  pumps are selected for operation, and the corresponding speed is  $\omega_n$ . List all different combinations for  $n = 1, \dots, N$ , and take the following procedure for each combination:

- *Step 1:* according to the head demand  $H^*$  and system curve (7), determine the demand system's flow rate  $Q_s^* = \sqrt{\frac{H^* - k_0}{k_1}}$ ;
- *Step 2:* determine each pump's demand flow,  $Q^* = \frac{Q_s^*}{n}$ ;
- *Step 3:* according to  $H - Q - \omega$  model (2), determine the demand pump speed  $\omega_n^*$  w.r.t.  $H^*$  and  $Q^*$ ;
- *Step 4:* calculate the expected power consumption  $P_n(\omega_n^*) \triangleq nP(\omega_n^*)$  by substituting  $Q^*$  and  $\omega_n^*$  into  $P - Q - \omega$  equation in (2).
- *Step 5:* the optimal solution, denoted as a pair of  $(n^*, \omega_{n^*}^*)$ , is the combination and the corresponding speed which lead to a minimal value of  $P_n(\omega_n^*)$  for  $n = 1, \dots, N$ .

### D. Optimal Scheduling for $N$ Different Pumps

In our previous work [14], an algorithm based on  $Q - H - \omega$  and  $P - Q - \omega$  model is proposed. Here we will propose another algorithm which uses the  $H - Q - \omega$  model instead of  $Q - H - \omega$  model. Compared with the previous work where the cost function and constraints of the formulated MINLP problem are type of rational-like functions, here the proposed new algorithm has both the cost function and constraints of polynomial-like functions, so that we needn't to worry about the potential non-convex problem.

Without loss of generality, we assume the  $i$ th pump has the  $H - Q - \omega$  and  $P - Q - \omega$  models as

$$\begin{aligned} H_i(\omega_i) &= a_0^i \omega_i^2 + a_1^i \omega_i Q_i(\omega_i) + a_2^i (Q_i(\omega_i))^2, \\ P_i(\omega_i) &= p_0^i \omega_i^3 + p_1^i \omega_i^2 Q_i(\omega_i) + p_2^i \omega_i (Q_i(\omega_i))^2 + p_3^i (Q_i(\omega_i))^3. \end{aligned} \quad (17)$$

For a given head demand  $H^*$ , there is  $H_i(\omega_i) = H^*$  if the  $i$ th pump is chosen to be put into operation. Thereby, the  $i$ th pump flow rate  $Q_i^*(\omega_i)$  corresponding to head  $H^*$  can be determined from (17) (positive solution), i.e.,

$$Q_i^*(\omega_i) = \frac{-a_1^i \omega_i \pm \sqrt{(a_1^i)^2 - 4a_2^i a_0^i} \omega_i^2 + 4a_2^i H^*}{2a_2^i}. \quad (18)$$

The precondition for  $i$ th pump to be able to operate is that its shutoff head should be greater or equal to  $H^*$ . This leads to constraint (9). The total system flow rate follows the constraint (12), but computation of  $Q_i(\omega_i)$  will follow (18). According to the BB method, the following steps are proposed to solve the general problem:

- *Step 1:* Have all possible pump combinations on the candidate list for the best efficiency;
- *Step 2:* Regarding to each combination, e.g., suppose  $i_1, i_2, \dots, i_k$  pumps are combined for operation, the following nonlinear programming problem need to be solved

$$P_s^* \triangleq \min_{\omega_{i_1}, \dots, \omega_{i_k}} P_s^k(\bar{\omega}_k), \quad (19)$$

subject to constraints (9) and (12). This optimization problem can be solved using some standard nonlinear optimization methods [2]. Here the Lagrangian Multipliers method is used to handle the above problem.

- *Step 3:* Compare the obtained (local minimal)  $P_s^{k*}$  with the so-far obtained global minimal record, which is denoted as  $P_s^{*glob}$ . If  $P_s^{k*} > P_s^{*glob}$ , erase this considered combination from the potential best candidate list; If  $P_s^{k*} \leq P_s^{*glob}$ , set the new global minimal record  $P_s^{*glob} = P_s^{k*}$  and move this considered combination to the top position at the candidate list;
- *Step 4:* After the enumeration of all possibilities, the combination at the top of the candidate list is the optimal solution. The optimal operating speed(s) is the solution to problem (19) corresponding to this best combination.

#### E. Feed-forward and Feedback Control Structure

From the control point of view, the solution to (14) acts as a kind of feed-forward control of the entire pump system. The correctness of the obtained solution is purely determined by pump model's precision. Nevertheless, some modeling errors can be observed in the model validation [14]. Thereby, some feedback control mechanism needs to be introduced so as to enhance the controlled system's robustness. For instance, a feedback PI controller is used in our proposed framework as illustrated in Fig.2.

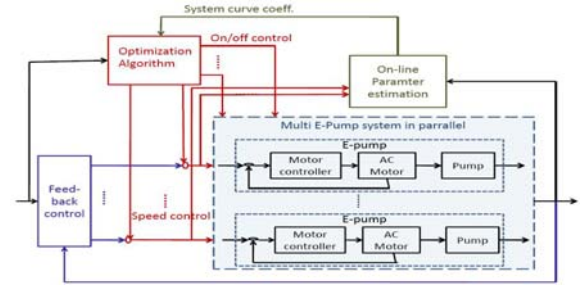


Fig. 2. Combined Configuration of Feed-forward, Feedback Control and Online Parameter

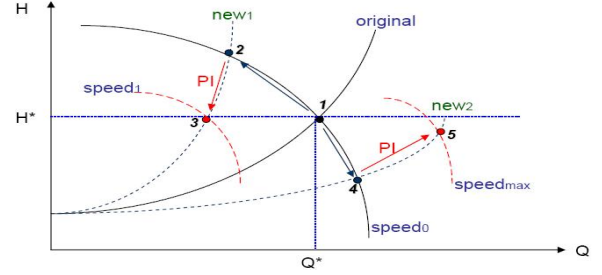


Fig. 3. Effects of changed operating conditions and feedback control

#### IV. ON-LINE PARAMETER ESTIMATION

The control algorithms proposed in the previous section assume that the system operating condition described by system curve (7) is known or can be informed beforehand. However, this kind of assumption is not true in many practical applications. In general, the controlled system needs to estimate some system coefficients representing the current operating condition before making any decision.

##### A. Effects of Changed Operating Conditions

The effects of changed operation conditions to pump system operation is illustrated in Fig.3. Suppose the considered system initially stays at point 1. At some time point, the system curve changes from the *original* curve to the *new1* curve. Suppose the feedback control is at the *off* status and the information about changed  $k_1$  is not informed to the optimization algorithm either. Then, each pump's internal speed control system will control pump's speed as usual. Consequently the system operating point will change from point 1 to point 2 after a short while. The system head will be away from the demanded  $H^*$ . When the feedback control is switched on, it will regulate each pump speed so that the demanded head could be recovered. This means the system operating point will change from point 2 to point 3. During this considered period, the operating pump combination won't change.

Another typical situation is that the system operating condition changes from *original* curve to be the *new2* curve. correspondingly, the system operating point will change from point 1 to point 4 if the feedback control is off. When

the feedback control is switched on, it will try to regulate each pump speed to catch up with the demand system head. In some case, system may not be able to recover the system head, since pumps already run at their maximal speeds. Under this kind of situation, the system operator or supervisor needs to be informed immediately that the current pump combination can not accomplish the expected task.

### B. On-line Estimation of Operating Condition

In case of an unknown operating condition, the system coefficients  $k_0$ ,  $k_1$  in (7) need to be estimated by the following online parameter estimation algorithm.

*Estimation algorithm A -  $k_0$  is known and  $k_1$  is unknown:*

- *Initialization:* the current pump combination, each pump's operating speed, demanded system head and the operating system head need to be informed;
- *Step 1:* if the system operating head starts to deviate from the demanded head and after a while stabilize at a new head value (e.g., from *point 1* to *point 2*), or the system operating head still stays at the demanded value, but some (or all) pump running speeds deviate from the previous values and stabilize at some new values (e.g., from *point 1* to *point 3*), start the estimation algorithm by obtaining each pump's running speed  $\omega_i$  and the new operating head value denoted as  $H_{new}$ .
- *Step 2:* substitute these  $\omega_i$  and  $H_{new}$  into each pump's  $H - q - \omega$  model (17) and calculate the corresponding flow rate  $Q_i(\omega_i)$  by following (18);
- *Step 3:* accumulate all  $Q_i(\omega_i)$  to be  $Q_s^{new} = \sum_i Q_i(\omega_i)$ ;
- *Step 4:*  $k_1$  can be estimated by

$$k_1 = \frac{H_{new} - k_0}{(Q_s^{new})^2}; \quad (20)$$

*Estimation algorithm B - both  $k_0$  and  $k_1$  are unknown:*

- Repeat the *Initialization* and *Step 1-3* listed in Algorithm A;
- *Step 4:* try to manage the system to a slightly different operating point (but within safety range) by some deliberative way for a short while. For example, to switch off the feedback control if it is originally on, then the system operating point will move from *point 3* to *point 2*; or, to switch on the feedback control if it is originally off, then the system operating point will move from *point 2* to *point 3*. Once the pump speeds, denoted as  $\omega_i^{new2}$ , and the system head, denoted as  $H_{new2}$ , are measured, return the system immediately to its original operation.
- *Step 5:* substitute these  $\omega_i^{new2}$  and  $H_{new2}$  into each pump's  $H - q - \omega$  model (17) and calculate the corresponding flow rate  $Q_i^{new2}(\omega_i^{new2})$  by following (18); estimate both  $k_0$  and  $k_1$  using both two set of data, i.e.,  $\{Q_s^{new}, H_{new}\}$  and  $\{Q_s^{new2}, H_{new2}\}$ , from

$$k_1 = \frac{H_{new} - H_{new2}}{(Q_s^{new})^2 - (Q_s^{new2})^2}, \quad k_0 = H_{new} - k_1(Q_s^{new})^2. \quad (21)$$

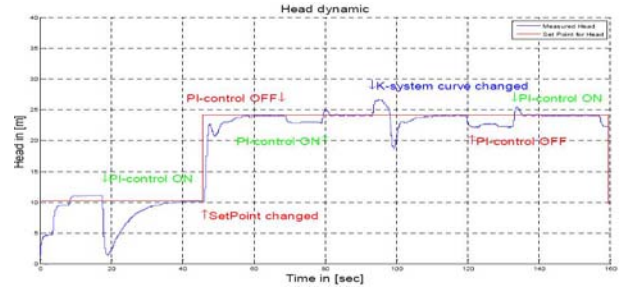


Fig. 4. Identical Pumps with Same Speeds: Head Dynamics

### C. Control Integrated with On-line Parameter Estimation

The parameter estimation algorithm can be integrated into the feed-forward and feedback control structure as shown in Fig.2. In general, the parameter estimation algorithm only needs to be "woken up" when some steady-state deviations of system head and/or pumps' running speeds are observed, otherwise it just stays at standby status. However, practically this algorithm needs to be operated with some specific frequency in order to make a tradeoff between the detectability and time-delay caused by this algorithm.

## V. TEST RESULTS AND DISCUSSIONS

The entire control system is implemented in LabVIEW and Matlab Optimization Toolbox.

### A. Control of Multiple Identical Pumps

There are only three possible combinations. The system head dynamic is plotted in Fig.4 for one scenario. At the beginning, PI control was off and system started with only one pump. The optimization algorithm initialized the process for given initial values of  $k_0$ ,  $k_1$ . After a short while, the algorithm decided to switch on the second pump. After the operating condition ( $k_1$ ) changed at 92sec., at 97sec. the optimization algorithm decided to turn off one pump. It's clear that the controlled system had a good tracking capability to a given reference when the feedback PI control was on. If the PI controller was switched off, some offset was observed which is mainly caused by pump modeling errors. Some oscillations can be observed when the feedback control was switched on or off, or the operating condition was changed (at 92 sec), or the pump combination was changed (at 97 sec), or the demanded head was changed (at 47 sec). They are mainly caused by the dynamic characteristics of the entire system.

The estimated power consumptions for all three possible solutions are plotted in Fig.5. Ideally, all these estimations should be constants and only possibly change when (a) the demanded system head is changed; or (b) the change of operating conditions is informed. However, the fluctuations of the head measurement and speed measurements sometimes make the judgement difficult to say whether the system is already at steady-state or not. Thereby for this test scenario, we decided to let the parameter estimation algorithm (A)



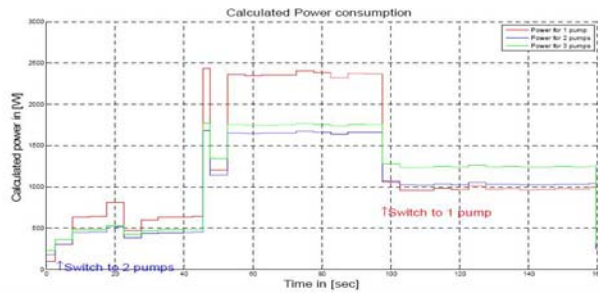


Fig. 5. Identical Pumps with Same Speeds: Power Prediction

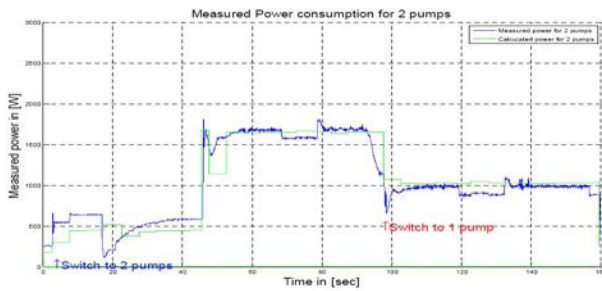


Fig. 6. Identical Pumps with Same Speeds: Measured P- Consumption

automatically run one time for every five seconds. It can be noticed that the change of operating condition at 92sec. can be tracked quickly and the optimization algorithm decided to change pump combination at 97sec. However, this periodic estimation of  $k_1$  could cause some deviations of estimated power consumptions under the influence of measurement noises and transient dynamic behaviors. Nevertheless, some tradeoff has to be compromised between the quick detection of changed operating condition and the robustness of the optimal control system.

The comparison of measured and calculated power consumptions is illustrated in Fig.6. There are some visible deviations between the measured and estimated ones for low power consumption period (until 47 sec). The feedback control helped reduce these deviations slightly. The measured and estimated ones match very well for the higher power consumption period (of two pumps: (52 sec - 92 sec). It is quite obvious that after 97 sec, the real power consumption (of only one pump) is indeed below the predicted two pump consumption.

### B. Control of Multiple Different Pumps

It is more challenging to implement the optimization algorithm for different pumps. Due to the current limitation, we have to manually switch computations between LabVIEW and MatLab/Optimization Toolbox, thereby the testing results lost "real" real-time sense. Nevertheless, these tests still show some clear consistency of the developed algorithm to the reality. We refer to [14] for some preliminary results.

## VI. CONCLUSIONS

The optimal control of a water boosting system with multiple VSP in parallel is discussed. a framework with combination of feed-forward and feedback control is proposed. The BB method is used to handle a formulated MINLP optimization problem along with the Lagrange multipliers method for internal nonlinear programming problem. This optimization algorithm acts as a feed-forward controller. A feedback control is also introduced in order to eliminate the offsets caused by potential modeling errors. An on-line parameter estimation algorithm is integrated within the proposed framework in order to cope with an unknown system operating condition. The proposed method and algorithms are tested on a physical setup. The preliminary testing results already showed a huge potential to significantly improve the energy efficiency of multiple pump systems.

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