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Two theoretically consistent methods for analysing triaxial tests

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Abstract

Constitutive models for geomaterials are frequently developed and calibrated on the basis of element tests. Prior to the analysis of element tests a suitable set of work-conjugated stress and strain measures has to be selected. The paper points out that the traditional analysis of triaxial tests is theoretically inconsistent as finite and infinite strain measures are mixed in the analysis. Therefore, two theoretically consistent methods are proposed and examined for the analysis of triaxial tests. These three methods affect strain-dependent material parameters differently. The effect is analysed using key geotechnical parameters and an advanced constitutive model. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Strain measures; Triaxial tests; Constitutive modeling; Volume change

1. Introduction

Traditional triaxial tests, drained and undrained, are commonly used in the study of the stress–strain behaviour of geomaterials. Drained tests are solely considered here, but all observations presented in this paper apply to the undrained case as well. During drained triaxial tests simultaneous values of axial displacement, volume change, confining pressure and axial load are measured. Since all directional measurements coincide with the principal axes of stresses and strains, the analysis of the test data ought to be straightforward.

The stress and strain measures must, according to Malvern [1], be work-conjugated and, furthermore, refer to the same configuration (reference or current) when constitutive relations are investigated.

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From an engineering point of view, it is obvious to use the Cauchy, or true stress, as stress measure. The Cauchy stress can in simple terms be expressed as the ratio between current load and current area [2] and can with ease be calculated from the measurements carried out during a triaxial test. The Cauchy stress measure is adopted throughout this paper.

In cases where strains and displacements are assumed infinitesimal, the distinction between a description based on a reference or a current configuration becomes arbitrary as all stress and strain measures are work-conjugated in this case. Consequently, the engineering strain measure is work-conjugated with the Cauchy stress under this assumption. In situations where displacements or strains are large, another strain measure, a finite strain measure, must be introduced. This measure must be work-conjugated to the Cauchy stress measure. The natural strain increment, as stated in ABAQUS [3,4] and Crisfield [2], satisfies this requirement. Both strain measures, the natural strain (increment) and the engineering strain, are adopted in this paper.

In the traditional analysis of triaxial tests (denoted method T), the axial strain is calculated as the ratio between the measured axial displacement and the initial height of the specimen, i.e. the engineering strain measure or the infinitesimal strain measure is used. Products of displacement derivatives are neglected in the theory of infinitesimal deformations [5].

The volumetric strain is traditionally calculated as the ratio between measured volume change and the initial volume of the specimen. Squares and products of displacement derivatives are not neglected in this calculation [5]. Hence, a finite strain measure is adopted and an inconsistency arises in the assumptions as finite and infinite strain measures are mixed. The inconsistency can be eliminated by following one of two distinct methods, either by adopting the natural strain increment or simply by adopting the engineering strain consistently in the analysis. Using these two methods denoted N and E, respectively, requires a computation of an exact displacement field before the strains can be calculated. Both methods are in the following illustrated for the triaxial case and the results are compared with the traditional method T. The effect of the three methods on some key geotechnical parameters is investigated together with the effect on modelling the stress–strain behaviour using an advanced constitutive model.

2. Analysis of triaxial tests

During a drained triaxial test, simultaneous values of axial displacement, volume change, axial load and confining pressure are measured. On this basis, it is possible to obtain the radial displacement and true axial stress, thus yielding a complete stress–strain description of the soil specimen under axisymmetric conditions. The radial displacement can only be calculated under the assumption that the radial and tangential strains are equal which, as stated by Kirkpatrick and Belshaw [6], is the case for all practical purposes.

2.1. Analysis based on the exact displacement field

In a triaxial test the deformation of the soil specimen is characterised by the compression or elongation in the axial and radial directions.

The original size of a sample is fully described by the initial height, H_0, and the initial diameter, D_0, whereas the size in a deformed stage is fully described by its original size and the displacement components u_1, and u_3. Compression, as shown in Fig. 1, is considered positive. The current height, H, diameter, D, and cross-sectional area, A, are given by:

$$H = H_0 - u_1; D = D_0 - 2u_3 \tag{1}$$

$$A = \frac{\pi}{4}(D_0 - 2u_3)^2 = \frac{V_0 - \Delta V}{H_0 - u_1} \tag{2}$$

The current cross-sectional area is traditionally used in the calculation of the axial stress. Therefore, it follows that the axial stress is of the Cauchy type. The volume change, ΔV, is of great importance in geomechanics. In terms of the displacement components it may be expressed as:

$$\Delta V = \frac{\pi}{4}\left[H_0 D_0^2 - (H_0 - u_1)(D_0 - 2u_3)^2\right] \tag{3}$$

The radial displacement is traditionally not calculated in the analysis of a triaxial test, but this quantity is indispensable for a complete description of the displacement

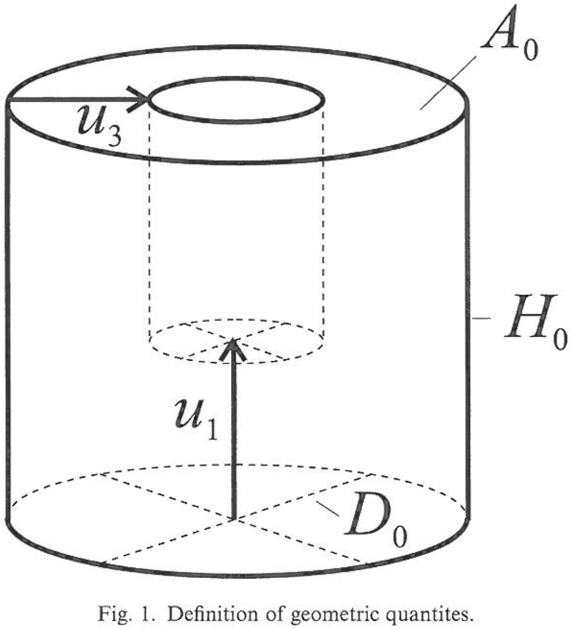

Fig. 1. Definition of geometric quantites.

field. As the axial displacement and the volume change are measured directly, the radial displacement can be expressed as:

$$u_3 = \frac{D_0}{2} - \sqrt{\frac{V_0 - \Delta V}{\pi(H_0 - u_1)}} \tag{4}$$

The radial stress or the confining pressure is measured directly as a Cauchy type of stress. Therefore, no intermediate calculations are required. An exact representation of the displacement and the Cauchy type of stress field has now been set-up.

The determination of geotechnical design parameters and the general study of material behaviour are commonly based on element tests, such as the triaxial test. As initial sample dimensions may vary from sample to sample and from apparatus to apparatus, the relevant parameters can obviously not be based on displacements. The parameters have to be based on relative deformations. As mentioned in the introduction there are two strain measures that are work-conjugated to the Cauchy type of stress measure. That is, as stated in [1], the engineering strain and the natural strain increment. Both strain measures are adopted in this paper.

2.2. *Engineering strain versus natural strain*

The linear engineering strain measure and the non-linear natural strain measure are briefly discussed in order to indicate their use and limitations. The simplest definition is the engineering strain, which is traditionally used in the theory of infinitesimal deformations:

$$\varepsilon_1^E = \frac{u_1}{H_0}; \varepsilon_3^E = 2\frac{u_3}{D_0} \tag{5}$$

The natural strain increment is often employed in the theory of finite deformations and/or in the theory of plasticity [4]. The natural strain increment is closely associated with the natural strain and based on the ratio between the initial height, H_0, and the initial diameter, D_o, and the current quantities, respectively:

$$\varepsilon_1^N = \ln\left(\frac{H_0}{H_0 - u_1}\right); \varepsilon_3^N = \ln\left(\frac{D_0}{D_0 - 2u_3}\right) \tag{6}$$

The natural strain measure makes no distinction between initial and final quantity and an interchange merely changes the sign. The difference between the natural and engineering strain measures in the one dimensional case is illustrated in Fig. 2. It is seen that $\varepsilon_1^E > \varepsilon_1^N$ and that the deviation is in the order of 4–5% for $|u_1/H_0| < 0.1$. The deviation is in general accepted and the assumption of infinitesimal deformations is commonly assumed to be valid. However, the difference becomes more pronounced when it comes to calculating the volumetric strain.

The volumetric strain is within the framework of the theory of infinitesimal deformations defined as the sum of the principal strains:

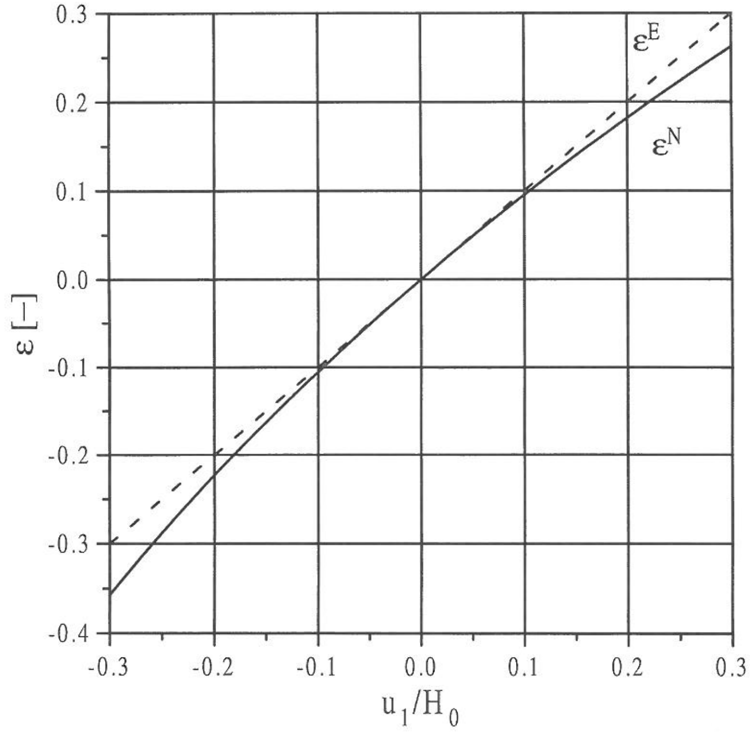

Fig. 2. The strain measures ε_1^E and ε_1^N for the one dimensional case.

$$\varepsilon_v^E = \varepsilon_1^E + 2\varepsilon_3^E \tag{7}$$

The volumetric strain is traditionally calculated as the ratio between the measured volume change and the initial volume of the specimen, i.e. a finite strain measure:

$$\varepsilon_v^T = \frac{\Delta V}{V_0} \tag{8}$$

Within the normal range of deformations, the disparity between the expressions in (7) and (8) may exceed 15–20%.

The volumetric strain, based on the natural strain definition, is found by addition of the principal strains:

$$\varepsilon_v^N = \varepsilon_1^N + 2\varepsilon_3^N = \ln\left(\frac{V_0}{V_0 - \Delta V}\right) \tag{9}$$

The disparity between the expressions in (7) and (9) may exceed 15–20% within the normal range of deformations.

Whether the engineering or the natural strain measure is chosen in the analysis of triaxial tests depends on whether finite or infinite deformations apply to the geotechnical problem under investigation. The form of the constitutive relation must, moreover, be considered [4]. The volumetric strain measures are more thoroughly discussed in the succeeding section.

2.3. Analysis based on strains

The traditional analysis of triaxial tests, denoted method T, is strain wise performed by using the expressions in (5) and (8).

An analysis based solely on the theory of infinitesimal deformations (method E) is on the other hand performed by using the expressions in (5) and (7), whereas the method denoted N uses the expressions in (6) and (9). The traditional analysis of triaxial tests leads to an inconsistent use of the theory of infinitesimal deformations, as a finite strain measure (8) is mixed with an infinitesimal measure (5).

The two proposed methods, N and E, use their theoretical background consistently and could, therefore, both be used in the analysis of triaxial tests. However, method E has some limitations as significant errors are introduced under certain conditions. The error is investigated in the following and it is shown how the use of the natural strain measure leads to an exact description of the deformations.

2.3.1. Errors produced using E and T

The radial displacement component, u_3, can by using the expressions in (5), (7) and (8) be expressed as:

$$u_3^{T,E} = \frac{4\Delta V - \pi D_0^2 u_1}{4\pi D_0 H_0} \tag{10}$$

A comparison of (4) and (10) reveals the effect of the linear approximation. The error, e, on the radial displacement component is given by (11) and shown in Fig. 3.

$$e = u_3 - u_3^{T,E} = \frac{D_0}{2}\left(1 - \frac{4\Delta V - \pi D_0^2 u_1}{8 V_0}\right) - \sqrt{\frac{V_0 - \Delta V}{\pi(H_0 - u_1)}} \tag{11}$$

Fig. 3 shows how e varies with respect to the two deformation measurements (most often) collected during triaxial tests. As expected the figure shows that the magnitude of e increases as the axial displacement increases. Moreover, the figure shows how the volume change influences e.

The use of method E can at moderate to high levels of deformation produce significant errors. So the method E should only be applied in cases where deformations are truly small.

2.3.2. Natural strain

If the natural strain definition is applied, the radial deformation can be determined on the basis of (6) and (9):

$$u_3^{N} = \frac{D_0}{2} - \sqrt{\frac{V_0 - \Delta V}{\pi(H_0 - u_1)}} \tag{12}$$

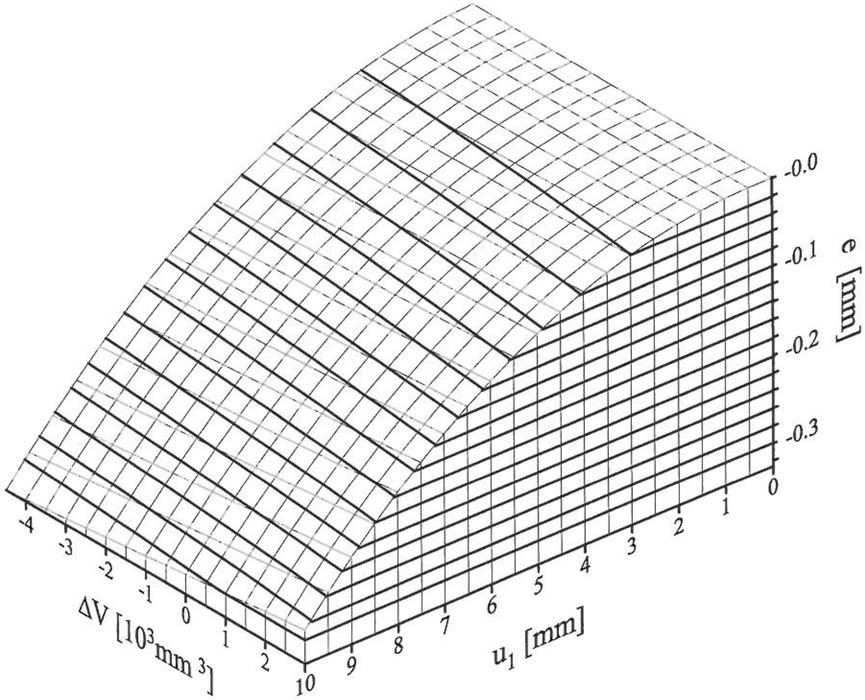

Fig. 3. Error due to the use of method E, $H_0 = D_0 = 70$ mm.

As this expression is seen to be identical to (4), it appears that the error e, and the inconsistency caused by mixed finite and infinite strain measures, can be eliminated by adopting the natural strain.

3. Effect on some key geotechnical parameters

As described in the preceding sections, precipitate analysis of triaxial tests can lead to erroneous results. The following reveals an investigation of how the strain measures affect the description of the behaviour of geomaterials, firstly by performing a simple analysis of a conventional triaxial compression test and secondly by calibrating a constitutive model which may be applied in more complex boundary value problems.

3.1. Analysis of a conventional triaxial compression test

The analysis of the conventional triaxial compression test is performed in two parts, firstly following an analysis based on the exact displacement field and secondly by the three methods (T, N and E). The conventional triaxial compression test is performed on Eastern Scheldt Sand deposit with a relative density of 72.5%. The initial sample size was measured to $H_0 = 71.5$ mm and $D_0 = 69.5$ mm. More details concerning the sand and test procedures are found in Jakobsen and Praastrup [12]. The specimen was isotropically consolidated to an isotropic state of stress of 160 kPa and subsequently sheared at a constant confining pressure.

Fig. 4 shows the deviator stress $q = \sigma_1 - \sigma_3$ versus the directly measured axial displacement u_1.

The graph shows a typical stress–displacement curve for a medium dense sand, performed under the above-mentioned stress levels. The initial slope of the stress–displacement curve is steep and flattens out as the specimen hardens until failure and progresses into softening hereafter. The cross-sectional area of the specimen determines, indirectly, the axial stress applied onto the specimen during shear.

As the calculation of the cross-sectional area in each of the three methods is identical, the axial stress remains unaffected by the applied methods. The radial stress in triaxial tests is measured directly and is, therefore, unaffected by the methods. Geotechnical parameters that solely depend on stresses are hence unaffected by the three methods. Therefore, the two strain measures do not affect the geotechnical parameters that are determined solely on the basis of the stresses. The friction angle φ' is an important geotechnical parameter that is solely based on stresses. This parameter is unaffected by the strain measure. The secant friction angle for the test shown in Fig. 4 is $\varphi' = 40.3°$.

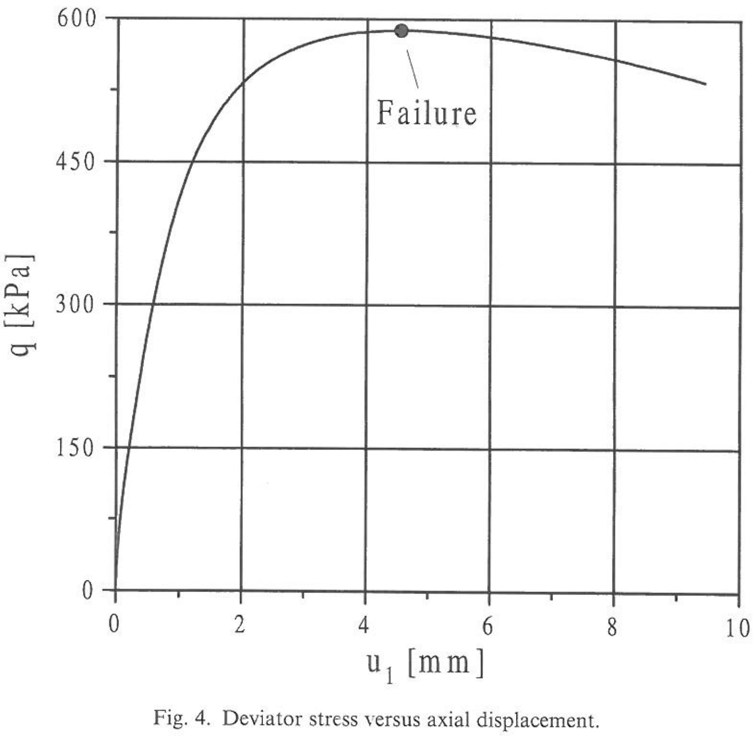

Fig. 4. Deviator stress versus axial displacement.

Geotechnical parameters that are based solely on strains, or both stresses and strains, will, however, be affected by the method and the strain measure used. Fig. 5 shows the measured volume change versus the measured axial displacement.

The figure shows that the specimen initially compresses and subsequently dilates. The effect on a particular strain-dependent parameter depends on how the parameter is determined and in particular on the strain level. Parameters determined at low strain levels are less affected by the chosen method than parameters determined at high strain levels. The initial tangent modulus of a stress–strain curve is for practical purposes unaffected as the parameter is determined in the beginning of the shearing process [7]. Geotechnical parameters such as the angle of dilation and strain to failure are affected more significantly. Parameters determined by strain increments will, however, be less affected than parameters determined by total strains.

In Section 2.1 it was mentioned that the volumetric behaviour is greatly influenced by the choice of strain measure. This effect is illustrated in Fig. 6, where the volumetric behaviour of the specimen is presented in terms of strains and plotted versus both the axial engineering and natural strains. The three curves representing each of the three methods diverge significantly as the axial strain increases. At failure the relative difference is as high as 18%. This difference affects the angle of dilation, which is an important parameter in the description of the volumetric behaviour of

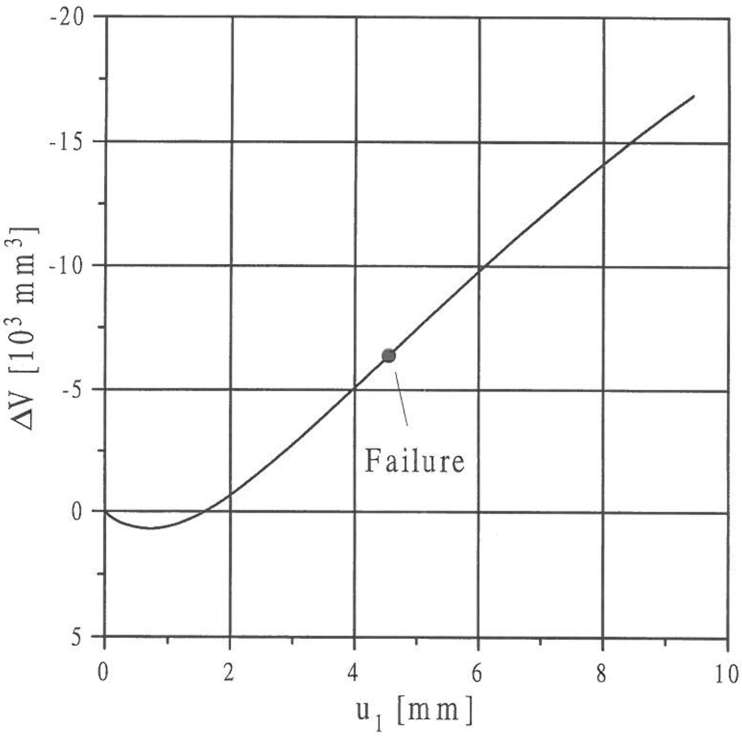

Fig. 5. Volume change versus axial displacement.

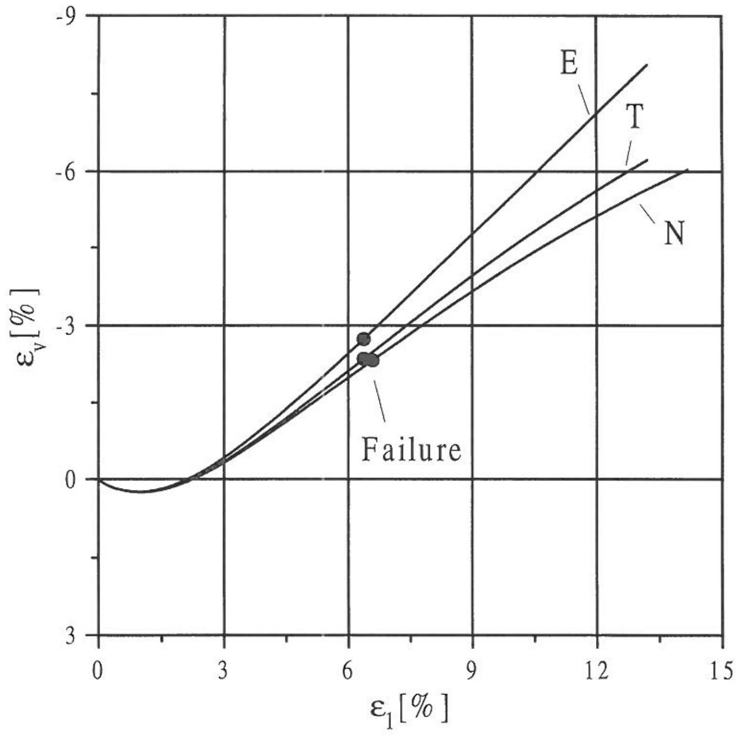

Fig. 6. Volumetric strain versus axial engineering and natural strain.

soils. The value of the angle of dilation is calculated to 12.7, 13.6 and 18.1° based on the methods N, T and E, respectively.

3.2. The single hardening model

In this section it is demonstrated how the three methods affect a particular constitutive relation, which is found important for the description of soil behaviour. The single hardening model is adopted for this purpose [8–11]. The single hardening model is an advanced constitutive model for frictional materials such as soils, concrete and rock. The single hardening model is an elasto-plastic constitutive model. The model consists, as do many other elasto-plastic models, of a failure criterion, a yield criterion, a plastic potential, a hypoelastic model and a hardening relation. The failure criterion determines the maximum load that a soil element can withstand. The yield criterion controls whether plastic deformations occur. The plastic potential controls the direction of the plastic strain increments and the elastic model determines the elastic behaviour of the material.

The single hardening model follows a non-associated flow rule because the yield criterion and the plastic potential are described by different functions. The model can in addition handle stress–strain behaviour in the softening regime, but cannot in the form used herein handle large stress reversals. The model is, furthermore, restricted to model the stress–strain behaviour of isotropic materials. The single hardening model has as many as 12 material parameters, but they are all easily determined. For simplicity, it has been decided not to show any of the expressions involved in the model and just refer to the relevant articles and use an identical parameter representation.

The parameters listed in Table 1 are calibrated on the basis of the six conventional triaxial compression tests performed on the sand mentioned in the previous section and deposited with the same relative density. The specimens were sheared under constant confining pressures ranging from 80 to 800 kPa [12].

Material parameters fitted solely on the basis of stresses are independent of the three methods as explained above. Poisson's ratio ν is set to a constant value of 0.2 due to significant scatter in the test results [11]. The variation among the parameters associated with the elastic behaviour of the material is small. The parameters for T and E are identical. A minor change in the elastic parameters can barely be observed on a monotonic stress–strain curve as the elastic contribution is small compared to the plastic contribution for a normally consolidated sand as none of the specimens has been presheared. Minor changes among the parameters included in the plastic potential and the yield function have a more pronounced effect on the overall stress–strain behaviour. The effect is illustrated by a prediction of the test described in the previous section. The prediction has been limited to show the relationship between volume change and the axial displacement as the effect of the three methods is most pronounced for the volume change. However, it should be mentioned that the three

Table 1
Material parameters

Parameter	Method T	Method E	Method N
Elastic behaviour			
ν	0.20	0.20	0.20
M	477.65	477.65	458.45
λ	0.4142	0.4142	0.4081
Failure criterion			
a[a]	0.00	0.00	0.00
m[a]	0.2879	0.2879	0.2879
η_1[a]	70.19	70.19	70.19
Plastic potential			
ψ_1[a]	0.00754	0.00754	0.00754
ψ_2	−3.1375	−3.1118	−3.1540
μ	1.9862	1.7814	2.0611
Yield function			
10^{-4}C	1.3101	1.3101	1.2748
p	1.6188	1.6188	1.6078
h	0.6416	0.6476	0.6166
α	0.5613	0.5726	0.5525

[a] Strain-independent parameters.

predicted deviator–displacement curves all capture the deviator–displacement curve shown in Fig. 4 quite accurately. Fig. 7 shows that the choice of the strain calculation method has considerable impact on the prediction relationship between the volume change and the axial displacement. It is, moreover, observed that none of the predictions captures the compressive portion of the measured soil response. This may be a shortcoming of the single hardening model itself and has nothing to do with the three methods.

Fig. 7 shows further that the difference between the three methods at small levels of axial displacement is insignificant. It is also observed that method E fails in predicting the soil response at moderate to high displacements levels. This is a consequence of limitations associated with this method.

Methods N and T capture the soil response equally well. As method N is theoretically consistent and method T is theoretically inconsistent, the correct choice is to use method N in the analysis of triaxial tests.

A comparison of the graphs in Figs. 6 and 7 reveals that the single hardening model is very robust and that it can be used independently of the two strain measures. So the variation among the parameters in Table 1 reflects the difference between the three methods, thus allowing the model to capture the soil response using different strain measures.

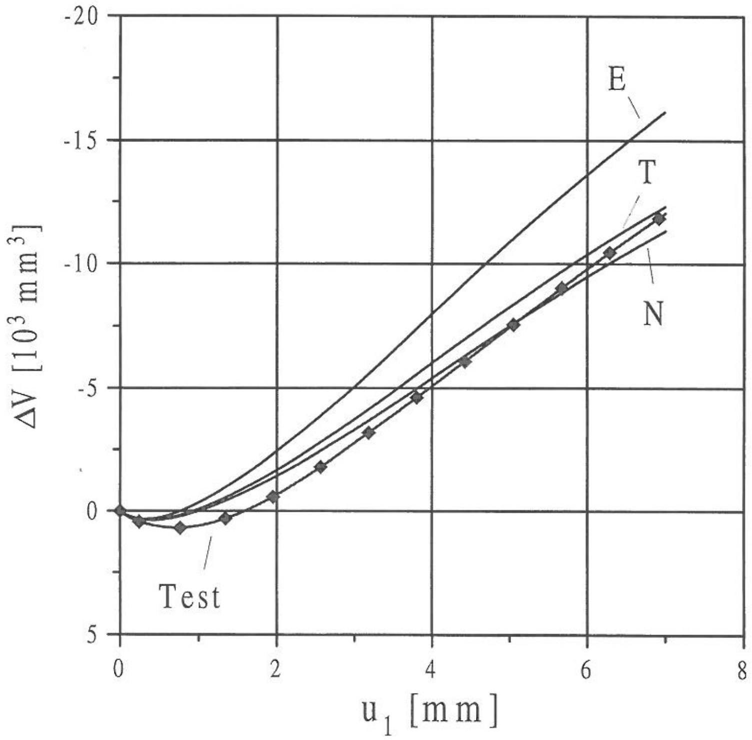

Fig. 7. Predicted and measured volume change versus axial displacement.

4. Conclusion

Within the scope of this work, which concerns the choice of strain measures in geomechanics, the conclusion that can be drawn from the results presented in this paper is that the chosen strain measure has a considerable effect on the volumetric strain. The effect on the volumetric strain affects strain-dependent geotechnical parameters, while parameters solely determined on the basic of stresses are unaffected. The traditional analysis of triaxial tests, denoted method T, has been found to be theoretically inconsistent as an infinitesimal strain measure is mixed with a finite strain measure. Therefore, two theoretically consistent methods, denoted N and E, were proposed and examined. Method N is based on the natural strain measure, whereas method E is based on the engineering strain measure. Method E has been found to produce erroneous results within the normal range of deformations in triaxial tests. Therefore, the authors suggest that method N is used in the analysis of triaxial tests. Method E could, however, apply in situations where deformations are reasonably small and where the material with reasonable justification could be modelled as a purely elastic material. The recommended procedure for analysing triaxial tests is outlined below:

- Establish the exact displacement field using both measured volume change and axial displacement.
- Establish all Cauchy stress components using the current cross-sectional area.
- Establish all strain components using the natural strain measure, i.e. method N.
- Finally display the results using the same diagrams as used in method T.

It has been shown that the traditional method of analysing triaxial tests has severe shortcomings and may result in erroneous calculations of soil parameters that depend on strains, whereas employment of natural strains results in correct calculation of strain-dependent soil parameters.

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