Abstract — Stability is of vital importance in the operation of microgrids and it is dependent on various factors. Many methods have thus been developed to facilitate the modeling and evaluation of stability of microgrids, seen from various aspects. However, there are many shortcomings to generalize the methods. Thus, in this paper, the modeling complexity and conditions of typical modeling methods are first presented and compared. The variations of key characteristics behind the methods are attributed based on the size, system order, and modeling accuracy of microgrids, and then mapped for a microgrid system. In this way, these modeling methods can be selected properly according to applications. Furthermore, this mapping will also serve as the first step for the stability validation of microgrids. To illustrate the significance of the mapping, system-level simulations on a small multi-converter system and the CIGRE Low-Voltage (LV) benchmark are conducted as study cases.

Keywords — mapping, stability analysis, modeling methods, microgrids

I. INTRODUCTION

Power electronics have been the key enabling technology for distributed power generation and flexible power transmission. With increasing controllability and efficiency of power electronic devices, renewable power generation from wind and solar can be flexibly integrated into power grids [1]. In this case, controlling and scheduling of the grid is performed in a hierarchical way to be adapted to the high complexity, and the giant power grids are partitioned into smaller regional and relatively autonomous blocks, i.e., microgrids.

Stable and reliable operation of power grids is always of concern. One of the well-known examples is the 2003 blackout in the USA, which had affected over 50 million people and costed over 6 billion dollars [2], [3]. In microgrids, the inertia is reducing due to increasing power electronic converters with fast dynamics [4], [5], which still remains a big concern. To figure out the cause of instability and be more focused on the fragile or sensitive links, there have been many discussions on the modeling, evaluation and enhancement of stability in microgrids. These researches aim to form methodologies [6] or address the impact of various factors on the stability, such as system topologies, voltage/current/power controllers [7], [8], phase locked-loops [9], and even cybersecurity issues [10].

Generally, the stability of microgrids can be divided into control system stability and power balance stability [11], where the power balance stability normally involves voltage stability and frequency stability. Intuitively, the ways to evaluate the stability of microgrids differ from the operation conditions: steady states (e.g., high-order voltage/current harmonics in [7] and low-frequency oscillations in [8]), or transient states (e.g., fault ride-through [9]), but the methods are not always suitable for every circumstance. For example, Bode plots in [8] can reflect the frequency of harmonics, but they are not as simple as the phase portraits (δ–δ plot) in [9] for angle-related stability. Therefore, it will be meaningful to characterize and organize the analysis of microgrid stability in a systematic manner. In a similar context, a framework for stability analysis proposed in [12] is coherent and instructive for developing a stable microgrid system. The analysis can go step by step accordingly. However, with more and more nonlinear applications in microgrids, the analysis could grow varied and complex. When we are dealing with the increasingly diverse modeling methods, the framework can be upgraded: the modeling methods can be distinguished and classified in more dimensions, and candidates of modeling methods can be provided in a more targeted way for specific applications.

In light of the abovementioned research gaps, this paper aims at summarizing the existing modeling methods for stability in microgrids, and arranging them in terms of different forms and applications. The modeling methods are evaluated and mapped to microgrids based on the size, order of microgrids and the accuracy, which can be a guideline or a tool in stability characterization of microgrids. The rest of the paper is organized as following: Section II presents a summary and general classifications of typical modeling methods for the microgrid stability analysis. Then, they are analyzed and synthesized for microgrid applications in Section III. Section IV introduces the case study for verifications. Concluding remarks are given in Section V.

II. MODELING METHODS FOR STABILITY IN MICROGRIDS

A. Domains of Stability Modeling in Microgrids

The Fourier transformation has enabled dynamic systems to be modelled in the frequency domain. In microgrids, the modeling conducted in frequency and time domains are usually described as impedance-based modeling and state-space-based modeling, respectively. For non-linear links in microgrids such as pulse-width modulation (PWM) and control delays, they can be averaged, linearized near particular operation points [6] (i.e., small-signal stability), or linearly approximated through the Padé approximation [13], [14].

Impedance-based modeling is a classical modeling tool in microgrids, where the converters are usually modelled as Thevenin circuits with two parts: the converters as voltage sources and the filters as impedances. In certain cases, the controllers can also be modeled as equivalent impedances [15]. Then, microgrids can be modeled as a combination of sources and impedances, and the stability can be characterized by...
analyzing the Bode/Nyquist plots [16] or pole-zero maps. The impedance-based modeling is more suitable for linearized single-input-single-output (SISO) systems. It is straightforward and intuitive to reveal the coupling among converters (controllers, filters, transmission lines, etc.), and to quantify the frequency-domain behavior of microgrids in response to regular inputs or disturbances. Alternatively, the modeling process can be approximated by means of frequency scanning where the system is taken as a black box [12].

State-space-based modeling is specially developed for multiple-input-multiple-output (MIMO) systems. The state variables of the target system should be determined, and then, the systems are modelled via matrices. Eventually, the stability of the entire system can be characterized by either performing an eigenvalue analysis of the entire system matrices or using the Lyapunov functions [17]. State-space-based modeling is well applicable to systems with multiple state variables, without specially focusing on the coupling among state variables. In state-space-based modeling, it is not necessary to consider all nodes, and the minor ones can be laid aside by properly selecting the state variables. Additionally, according to the Lyapunov stability theorem, it is also possible to characterize non-linear systems without linearization or approximation when the state-space-based modeling is adopted [18]. But in many cases, the Lyapunov functions can be difficult to construct when the number of state variables increases and the interactions are not easy to neglect.

The boundary between impedance-based and state-space modeling is not absolute, and normally they can be transferred from one to the other. Taking the simple but typical grid-connected converter in Fig. 1 as an example, Eq. (1) and (2) are two impedance-based modeling approaches, and \( Z_L \) is the impedance matrix of the inductor. Eq. (3) is a typical state-space-based modeling approach, where \( A \) and \( B \) are respectively the state and input matrix in the state space. In (2), it is also a commonly-used approach to plot the eigenvalue portraits especially when the number of state variables increases [12], which normally accords with the poles of the system.

![Fig. 1. Three-phase grid-connected converter with an inductor filter, where \( u_L, i_L \) refer to the averaged voltage and current of inductor, and \( L_L, R_L \) refer to the inductance and resistance of the inductor per phase, assuming that there is no mutual inductance among phases. \( \omega_L \) is the angular frequency of grid.](image)

\[
V_{L_a}(s) = \begin{bmatrix} R_f + sL_f \\ R_f + sL_f \\ o_L R_f \\ a_L R_f \end{bmatrix} \begin{bmatrix} I_{L_a}(s) \\ I_{L_d}(s) \\ I_{L_q}(s) \end{bmatrix} \quad i = a, b, c \tag{1}
\]

\[
\begin{bmatrix} V_{L_d}(s) \\ V_{L_q}(s) \end{bmatrix} = \begin{bmatrix} R_f + sL_f & -o_L R_f & o_L R_f & R_f + sL_f \\ -o_L R_f & R_f + sL_f & R_f + sL_f & -o_L R_f \end{bmatrix} \begin{bmatrix} I_{L_d}(s) \\ I_{L_d}(s) \\ I_{L_q}(s) \end{bmatrix} \tag{2}
\]

\[
\frac{d}{dt} \begin{bmatrix} i_{L_d} \\ i_{L_q} \end{bmatrix} = \begin{bmatrix} -\frac{R_f}{L_f} & \frac{o_L}{L_f} \\ -\frac{o_L}{L_f} & -\frac{R_f}{L_f} \end{bmatrix} \begin{bmatrix} i_{L_d} \\ i_{L_q} \end{bmatrix} + \frac{1}{L_f} \begin{bmatrix} V_{L_d} \\ V_{L_q} \end{bmatrix} \tag{3}
\]

\[
\begin{bmatrix} V_s \\ V_f \end{bmatrix} = \begin{bmatrix} 1 & j \omega_L \\ 1 & -j \omega_L \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix} \tag{5}
\]

B. Coordinate Frames of Stability Modeling in Microgrids

With the Clarke and Park transformations, microgrids can be modelled in the \( a\beta \) and \( dq \) frames, respectively. The \( a\beta \) frame, the voltage and current will be DC quantities, which makes it easier to tune the controllers of power converters. The \( a\beta \) frame is similar to the basic \( abc \) frame with one fewer variable to control, when zero-sequence components can be neglected. When focusing on AC components such as harmonics and oscillations, the modeling in the \( a\beta \) frame can be more straightforward, and a common reference will not be required among different converters [19].

A newly developed idea in this field is the complex vector [\((V_{d}+jV_{q})\) or \((V_{d}+jV_{q})\)], or in other words, the sequence domain [15], [19], [20], [23]. For example, in [15], the coupling between \( d \)- and \( q \)- components is removed from the mathematical model after the transformation of (5), and \( V_{d} \) and \( V_{q} \) denote the positive- and negative-sequence components respectively. If the \( d \)-to-\( q \) and \( q \)-to-\( d \) coupling are symmetric, there will be no coupling between \( V_{d} \) and \( V_{q} \). Based on this transformation, the model can be reduced into an SISO system. The evaluation of the system stability is then simplified: the controlled sources reflecting the \( dq \) coupling will not appear in the equivalent circuits, and the passive impedances will be a better choice to show the instable source of the system.

\[
\begin{bmatrix} V_{d} \\ V_{q} \end{bmatrix} = \begin{bmatrix} 1 & j \omega_L \\ 1 & -j \omega_L \end{bmatrix} \begin{bmatrix} V_{d} \\ V_{q} \end{bmatrix} \tag{5}
\]

The small-signal modeling is based on linearization near particular operation point, but the nonlinearity of components or transients cannot be easily neglected in large-signal modeling. Different from the impedance-based and state-space-based modeling approaches, indirect evaluation of stability can also be an alternative solution, especially for the transient stability analysis. Phasor diagram is utilized in [9] for the modeling, and the stability is evaluated with phase portraits (\( \delta-\delta \) plots). This approach can avoid the conflict between the operation point variation and small-signal linearization, and is easy and straightforward to conduct. However, instead of showing stability margin and resonance points, it is more specially used in the study of synchronization stability, for the design of a phase-locked loop (PLL) and the boundary of fault clearance.

C. Comparison of the Modeling Methods

Typical examples of the modeling methods mentioned above are compared in Table I, in terms of complexity and accuracy. The target of these methods can be divided into static stability (usually small-signal modeling) and transient stability (large-signal modeling needed). The complexity includes three aspects: the complexity of converter modeling (e.g. controllers), the dimension of matrix for system-level evaluation (e.g. impedance matrix or state matrix reflecting the system topology), and the simplicity of stability evaluation. The accuracy is more about the assumptions applied to each modeling approach. According to Table I, impedance-based modeling approaches normally need the Thevenin or Norton equivalence of converters, but the impedance or admittance matrix can be smaller. State-space-based modeling can reduce the complexity for converter modeling, but it will lead to a larger matrix. It should be pointed that Lyapunov functions can be applicable to both static and transient stability, but the complexity will be much higher for higher accuracy. Hence,
The denotation of the functions with the same order of magnitude is crucial such as Thevenin equivalence with neglecting the feedforward terms, which is similar to impedance-based modeling.

Besides the methods in Table I, there are also advanced modeling methods such as constructing a harmonic-domain Toeplitz matrix [28]. These methods are proposed for specific applications, but they are usually much more complicated for general purpose. Therefore, these methods will not be discussed in detail in this paper.

III. MAPPING OF THE MODELING METHODS

Based on the features of the abovementioned methods, an evaluation of impedance-based and state-space-based modeling of microgrids can be concluded as Table II. The two types of modeling methods are compared in three aspects: size of microgrids, order of microgrids and modeling accuracy. The impedance-based modeling is usually easier to perform, while state-space-based modeling also has its advantage in terms of the possibility to focus on major state variables.

Table I. Comparison of Typical Modeling Methods of Stability in Microgrids

<table>
<thead>
<tr>
<th>Type of stability</th>
<th>Typical modeling methods</th>
<th>Complexity of modeling</th>
<th>Potential causes of inaccuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Modeling of converters</td>
<td>Dimension of matrix</td>
</tr>
<tr>
<td>Static stability</td>
<td>Modeling by Thevenin (impedance) or Norton (admittance) equivalence in dq or αβ frame, e.g. [16][21][22]</td>
<td>+</td>
<td>$[O(2n)]^2$</td>
</tr>
<tr>
<td></td>
<td>Equivalent SISO modeling by complex vectors, e.g. [15][19][20][23]</td>
<td>+</td>
<td>$[O(n)]^2$</td>
</tr>
<tr>
<td></td>
<td>State-space-based small-signal modeling, e.g. [6][24][25]</td>
<td>−</td>
<td>$[O(m+n+p)]^2$</td>
</tr>
<tr>
<td>Transient stability</td>
<td>State-space-based modeling with Lyapunov functions, e.g. [26][27]</td>
<td>+</td>
<td>$[O(n)]^2$</td>
</tr>
<tr>
<td></td>
<td>Modeling of transient stability by droop curve, phasor diagram and related tools such as phase portraits, e.g. [9]</td>
<td>+</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Note: 1: For the dimension of matrix, $m$ is the number of converters, $n$ is the number of nodes, and $p$ is the number of loads. To be more general, $O(n)$ is used to denominate the functions with the same order of $n$, and $O([n])^2$ means that the matrix is in the scale of $O(n^2)$. Note 2: State-space-based modeling with eigenvalue loci is also sometimes used in transient stability, but normally by modeling the stability before/after the nonlinear transients separately. This case is more suitable for qualitative analysis. Note 3: N/A = not applicable. The modeling of transient stability by phase portraits is based on local perspective, thus might additionally need the modeling of rest parts into an infinite bus with an impedance.

Table II. Evaluation of Impedance-Based and State-Space-Based Modeling

<table>
<thead>
<tr>
<th>Size of Microgrids</th>
<th>Impedance-Based Modeling</th>
<th>State-Space-Based Modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Suitable for systems with small number of nodes (converters or buses), especially for SISO systems.</td>
<td>Suitable for systems with small number of state variables (not necessarily required for each node).</td>
</tr>
<tr>
<td>Order of Microgrids</td>
<td>The number of poles will increase for high-order systems. Bode plots are not influenced so much.</td>
<td>The number of eigenvalues will increase for high-order systems. Lyapunov approaches are not influenced.</td>
</tr>
<tr>
<td>Modeling Accuracy</td>
<td>Suitable for linear or linearized systems. Neglecting the coupling of impedances will cause inaccuracy.</td>
<td>Not only suitable for linear systems, but also for non-linear systems. The coupling need not be neglected.</td>
</tr>
</tbody>
</table>

Table III. Complexities of the modeling methods

<table>
<thead>
<tr>
<th>Converter Level</th>
<th>Sub-System Level</th>
<th>System Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>E.g., 1-2 units</td>
<td>E.g., 3-5 units</td>
<td>E.g., over 10 units</td>
</tr>
<tr>
<td>Impedance-Based Modeling</td>
<td>State-Space-Based Modeling</td>
<td></td>
</tr>
<tr>
<td>Bode/Nyquist Plots</td>
<td>Lyapunov Approaches</td>
<td>N/A</td>
</tr>
<tr>
<td>Zeros &amp; Poles / Eigenvalue Loci</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency Domain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time Domain</td>
<td>N/A</td>
<td></td>
</tr>
</tbody>
</table>

(a) Stability Modeling and Size of Microgrids

<table>
<thead>
<tr>
<th>Frequency Domain</th>
<th>(around 50/60 Hz) (3rd, 5th, 7th,… ) (kHz or more)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dq Frame</td>
<td>a/β Frame</td>
</tr>
<tr>
<td>Frequency Domain</td>
<td>N/A</td>
</tr>
<tr>
<td>Time Domain</td>
<td></td>
</tr>
</tbody>
</table>

(b) Stability Modeling and Frequency Under Study

Fig. 2. Prior-art modeling methods mapping to microgrids for stability analysis: (a) mapping to size of target microgrids and (b) mapping to frequency under study, where N/A means not suitable for the methods in corresponding rows.

Accordingly, a mapping relationship can be summarized in Fig. 2, as a guideline to properly apply the modeling methods to stability validation and analysis of microgrids. The two perspectives for mapping are (a) the size of microgrids and (b) the frequency under study, respectively. In Fig. 2(a), the units can be converters or buses (in a mesh microgrid). Impedance-based modeling is normally used in smaller systems with one or more buses due to its simplicity, and state-space-based modeling is more suitable for relatively larger systems by cutting the number of state variables. However, for systems with a large number of state variables, the Lyapunov
approaches are also not suitable, due to the complicated Lyapunov functions. Fig. 2(b) shows the mapping with respect to the frequency under study. The $dq0$ frame is better for modeling in the fundamental frequency, but when it comes to harmonics, the $aβ0$ frame will be a good choice.

IV. CASE STUDIES

A. Necessity of Modeling by Multiple Approaches

A microgrid system in [6] is selected as the first study case, as shown in Fig. 3. The system is an isolated AC microgrid with three three-phase converters, controlled by droop controllers with the same droop gains. In this case, the DC sources are assumed to be ideal. Simulations are conducted in PLECS, to show the necessity of stability modeling by multiple approaches, and the mapping of these approaches.

The system is firstly modelled by state-space-based method, and the active droop gain increases from 0.05% to 0.1%. The eigenvalue loci are shown in Fig. 4, where $λ_{ij}$ is the eigenvalue related to converter $i$ and $j$. With the state-space-based method, it can be concluded from the eigenvalue loci that when $m_p$ increases, the system will go towards instability.

The boundary can be found, and the droop gain can be designed accordingly.

![Graph showing eigenvalue loci](image)

The eigenvalue loci in Fig. 4 can always be preferable when the focus is the static stability such as the design of control parameters. In this case, the system goes unstable when $m_p > 1.9 \times 10^{-4}$, and in Fig. 5 (a) and (b), the boundary indicated by the eigenvalue loci is shown to be practical. The droop coefficients can be designed accordingly.

In Fig. 5 (c), resonance occurs, which is also type of static instability. With improper design of control parameters or insufficient damping, both low-order harmonics and sideband harmonics may appear. The harmonics can be at the level of $10^2$–$10^3$ Hz (low-order harmonics) or sideband harmonics. With these harmonics, the system might not immediately collapse, but it is not stable considering the operating point, and the voltage can show similar order of harmonics to current. When focusing on transient stability or the harmonics / oscillations, the impedance-based modeling will be a better choice. Harmonic instability is easier to be identified by Bode plots (resonance peaks in magnitude-frequency responses), or the real parts of low-frequency poles / eigenvalues.

![Graph showing voltage and current waveforms](image)

Fig. 5. Voltage and current waveforms of Converter 3 in Fig. 3: (a) under normal operating condition, where the control parameters are the same as those in [6], (b) and (c) with static stability issues caused by improper design of control parameters. In (b), the droop coefficient $m_p$ is increased, and in (c), $K_{PC}$ of current controller is decreased.
However, the multi time-scale dynamics, which are also critical performances and are highly dependent on the bandwidth of the controllers, cannot be revealed from the eigenvalue loci. Electromechanical parts in microgrids normally have large inertia, and have slower dynamics than the electrical parts, which is related to both static and dynamic stability. In Fig. 6, a 4-kW electric motor is connected to the system in parallel with Load 2 in Fig. 3. The motor is first accelerated, and there is a step increase of the mechanical load afterwards. The oscillation in the transient process is much slower (less than 10 Hz) than the resonance with the frequency being 500-600 Hz in Fig. 5. Unfortunately, this performance cannot be reflected by either Bode plots or eigenvalue loci. As a result, the stability investigation does not fully comply with the time-domain simulations due to inaccuracy in modeling-

![Fig. 6. (a) active / reactive power and (b) frequency / voltage (amplitude) waveforms of Converter 3 in Fig. 3 under load dynamics. An 4-kW electric motor (in parallel with Load 2) is first accelerated to maximum rotational speed (1800 rpm) until $t = 1$ s, and its mechanical load is increased by 30% at $t = 1.5$ s.](image)

The case in Fig. 6 is similar for renewable generations such as wind turbines and photovoltaic (PV) arrays. The multi-time-scale dynamics will have impact on the power flow in the microgrid system. An alternative approach is to linearize the longer transients in shorter time-scales, and average or integrate the shorter variations in longer time-scales.

### B. Other Types of Stability in General Microgrids

A microgrid benchmark in [29] is further adopted as another study case to illustrate the importance of mapping, as shown in Fig. 7. The system can be operated in the islanded mode or the grid-connected mode. There are distributed generations in the system, including a wind turbine, two PV arrays, a fuel cell and a battery. For conducting simulations, the benchmark is simplified, where the distributed generations are replaced by ideal sources and the electric motor is replaced by a constant load. The grid-connected controllers are droop controllers with the same droop parameters.

![Fig. 7. A CIGRE Low-Voltage (LV) benchmark for case study. The breaker is assumed to be open in this case (islanded operation).](image)

Fig. 8 shows the active and reactive power of three of the converters (Converter 1, 4, and 7) when there is a step variation of load (Load 4, a 10-mH inductive load connected in parallel). The load variation can lead to oscillations (or overshoots) in power, and thereby variations in frequency and voltage. Ideal sources are used in this case, but in practical microgrids, the distributed generations have more complicated transients, which can be the cause of harmonics and instability. Also, the power capabilities of distributed generations are finite and much lower compared to ideal sources or utility grids. This will induce transient stability issues when the dynamic process with overshoots cannot be well supported.

Additionally, there are also other types of potential instability issues in a general grid, such as:

1. The static stability related to the mission profiles and the power capability of distributed generations.
2. The stability related to interactions among distributed generations or new types of loads.
3. The transient stability related to faults of lines, loads, generations or the connections to grids.
4. The transient stability related to cyber security or other kinds of failures in the upper layers.

These performances also need respective approaches for modeling, evaluation and validation.
In this paper, certain modeling methods for the stability analysis in microgrids are presented and summarized, including different domains and frames. The modeling methods are evaluated and mapped to different microgrids according to the system size or the frequency of interest. Simulations are presented, illustrating the importance of the mapping and strategic usage of the modeling methods. The mapping will be helpful to guide the application of modeling methods to stability analysis, and to lay a foundation for the validation of microgrids with more power electronics.

V. CONCLUSIONS

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