A Small-signal Stability Study for Open-loop I-f Control of Permanent Magnet Synchronous Machine Drives

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Abstract—This paper deals with stability analysis of open-loop I-f control for permanent magnet synchronous machine (PMSM) drives. The nonlinear system dynamics is linearized at steady state working point for analyzing the local stability of open-loop I-f control for PMSM. The analysis shows that the open-loop I-f control is poorly damped but can achieve stable operation in a wide speed range without the mid-frequency instability issue that commonly exists for the open-loop V/f control. In fact, the open-loop I-f control can operate stably from zero to rated speed under different load conditions. Extensive simulation results are presented to verify the theoretical analysis.

Keywords—Stability, I-f control, PMSM, scalar control

1. INTRODUCTION

Sensorless control of permanent magnet synchronous machines (PMSM) drives is of great interest for cost reduction and reliability improvement and has been studied in detail for a few decades. While many high-performance sensorless schemes based on field-oriented-control (FOC) have been reported [1]. When PMSM are used for applications, such as fans and pumps that do not require high dynamic performance and normally operate in medium to high speed range, simpler schemes like scalar control (V/f and I-f) are also popular [2].

The open-loop V/f control for PMSM without damper windings is shown to have instability issue after the machine goes over a certain speed [3]. To solve this “mid-frequency instability” problem, stabilization by adjusting the frequency command using the high-frequency component of input power has been proposed in [3], [4]. Following that, modern versions of V/f control introduce an additional stabilization loop that adjusts the voltage command in addition to the frequency adjustment loop for achieving improved stability and efficiency [5]. However, because there is no inherent current control, over-current problems can occur even with stabilization control [3], [6].

Unlike the V/f control, the I-f control has closed-loop current regulation and is less likely to experience the overcurrent problem. The I-f control has been implemented in two ways: (1) as a startup method for PMSM drive [7]–[14]; (2) as a standalone control scheme [15]–[17]. In the first approach, I-f control is used to start the machine and after the machine reaches a sufficient speed, the control is switched to FOC mode. In the second one, similar stabilization loops used for V/f control are incorporated into the I-f control for achieving better performance. While there have been many works reported on I-f control, there is not yet detailed stability analysis for the simple open-loop control scheme, which can be important to understand for utilizing its full potential in real applications.

In this paper, we study the stability characteristics of open-loop I-f control for PMSM drives. The stability analysis is carried out by means of linearization of the nonlinear dynamics at steady state working points. Instead of using the simplified model that neglects the current dynamics, in this work, the current loop dynamics including the PI controller is considered in the linearization. The stability analysis reveals that the open-loop I-f control is poorly damped but can run stably in a wide speed range without the mid-frequency instability that commonly exists for V/f control of PMSM.

The rest of the paper is organized as follows. Section II deals with the model of the machine. In Section III, the basic operation principle of the open-loop I-f control is introduced and then the stability analysis is carried out by means of linearization. In Section IV, simulation results are presented to verify the stability characteristics obtained through the small-signal analysis. Conclusions are given in Section V.

II. PMSM MODEL

For simplicity, the PMSM considered in this work is a nonsalient one. The electrical subsystem dynamics of the PMSM under the well-known synchronous dq reference frame is given by

$$\begin{align*}
\dot{i}_d &= -\frac{R_\alpha}{L}i_d + \omega_Li_q + \frac{1}{L}v_d \\
\dot{i}_q &= -\omega_Li_d - \frac{R_\alpha}{L}i_q - \frac{\lambda_m}{L}\omega_r + \frac{1}{L}v_q
\end{align*}$$

(1)

(2)

where $i_d$, $i_q$, $v_d$ and $v_q$ are $d$- and $q$-axes currents and voltages; $R_\alpha$ is the phase winding resistance; $L$ is the $d$- and $q$-axes inductance; $\lambda_m$ is the flux linkage by permanent magnet and $\omega_r$ is the electrical rotor angular speed.

The I-f control is realized in the $d'q'$ reference frame defined in Fig. 1, where the angle $\delta$ is the difference between the $q'$-axis and the machine $d$-axis and $\omega_e$ is the angular velocity of the $d'q'$ reference frame. For analyzing the I-f control, the electrical
regulating the stator current. The basic working principle is as follows: The current references are chosen to be

\[ i_{d}^{ref} = 0 \]  \hspace{1cm} (8)
\[ i_{q}^{ref} = I_0 \]  \hspace{1cm} (9)

where the constant \( I_0 \) is chosen to be a large value, e.g. the rated value. The \( q^* \)-axis is initially aligned with the machine \( d \)-axis and \( \delta = 0 \), therefore there is no torque output according to (6). As the \( q^* \)-axis rotates, \( \delta \) will increase and torque will be generated, and the machine will start rotating when \( T_e > T_l \). For a more detailed explanation of the operation principle, refer to [8].

**B. Stability analysis**

When the machine is under the I-f control, in addition to the dynamics in (3)-(7), the PI controllers bring additional dynamics, which can be expressed as

\[ v_{d^*} = K_p (i_{d^*}^{ref} - i_{d^*}) + K_i w_1 \]  \hspace{1cm} (10)
\[ \dot{w}_1 = i_{d^*}^{ref} - i_{d^*} \]  \hspace{1cm} (11)
\[ v_{q^*} = K_p (i_{q^*}^{ref} - i_{q^*}) + K_i w_2 \]  \hspace{1cm} (12)
\[ \dot{w}_2 = i_{q^*}^{ref} - i_{q^*} \]  \hspace{1cm} (13)

where \( K_p, K_i \) are the gains for the PI controllers and \( w_1, w_2 \) are two auxiliary state variables. The system dynamics including the controller and the machine is governed by (3)-(7) and (10)-(13) can be written in a compact form as

\[ \dot{x} = f(x, u) \]  \hspace{1cm} (14)

where \( x = [i_{d^*}, i_{q^*}, \omega_r, \delta, w_1, w_2]^T \) is the system state vector and \( u = [i_{d^*}^{ref}, i_{q^*}^{ref}, \omega_e]^T \) is the input vector and \( f \) is the nonlinear vector function of the state \( x \) and input \( u \).

Stability analysis is to be carried out by means of linearization, for this, first the steady state working point needs to be calculated. Substituting the input \( u_0 = [0, I_0, \omega_e]^T \) to (14) and solving \( \dot{x} = 0 \), the steady state working point can be obtained as

\[ x_0 = \begin{bmatrix} 0 \\ \omega_0 \\ \frac{\omega_0}{2 \lambda m} \\ \left[ \frac{\omega_0}{2 \lambda m} \right] \\ \arcsin \left( \frac{\omega_0}{2 \lambda m} \right) \\ \frac{\omega_0}{2 \lambda m} \sin \left( \frac{\omega_0}{2 \lambda m} \right) \end{bmatrix} \]  \hspace{1cm} (15)

The linearized model can be obtained as

\[ \Delta x = A \Delta x + B \Delta u \]  \hspace{1cm} (16)

where \( \Delta x, \Delta u \) denote the small deviation from the steady state value, i.e. \( \Delta x = x - x_0, \Delta u = u - u_0 \), and matrix \( A \) interested in for stability analysis can be obtained as (17).

The eigenvalues of \( A \) govern the local stability for the system. Fig. 3 shows the eigenvalue loci of the system matrix \( A \) under different speed and load conditions. Three load levels
\[ A = \begin{bmatrix} -\frac{R_a}{L} - \frac{K_p}{L} & \omega_{e0} \\ -\omega_{e0} & -\frac{R_a}{L} - \frac{K_p}{L} \\ -\frac{3\cos(\delta)\lambda_m n_{pp}^2}{2J} & -\frac{3\sin(\delta)\lambda_m n_{pp}^2}{2J} \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \]

(17)

**Fig. 3.** Loci of eigenvalues for matrix A under different load and speed conditions. Operation conditions: three load levels, i.e. no load, 50% and 100% rated load are chosen and for each load condition the speed is varied from 0 to 100% rated speed. (a): Loci of all eigenvalues. (b): Loci of dominant eigenvalues, due to the symmetric nature only the upper part of the loci is shown.

As can be seen, the dominant eigenvalues are close to the imaginary axis especially at zero and low speed under heavy load condition, which indicates poor damping. Though poorly damped, the dominant eigenvalues stay in the left side of the complex plane for all the chosen conditions, which means the simple method can operate stably from zero to rated speed under different load conditions. For the results obtained in Fig. 3, the data used for machine is given in Table I the d’q-axes current PI controllers are chosen as \( K_p = 10.6 \) and \( K_i = 1921 \). The \( q^* \)-axis current reference is set as 10 A.

**IV. SIMULATION RESULTS**

The stability analysis above indicates that the open-loop I-f control can run stably from zero to rated speed under different load conditions. In this section, the analysis is verified by means of Matlab/Simulink simulation. The data for the machine used in the simulation is given in Table I the parameters for the PI controller is \( K_p = 10.6 \) and \( K_i = 1921 \) and the \( q^* \)-axis current reference is chosen as 10 A.

Figure 4 shows the speed and dq-axes current of the machine under open-loop I-f control under no load condition. As can be seen the machine can accelerate to rated speed stably. This property is different from that of open-loop V/f control, which is shown to be unstable after the machine reaches a certain speed [3].

Figure 5 shows the performance of the open-loop I-f control with load disturbance at rated speed. Load step of 50% rated load is applied to the machine at 4 s and 4.5 s as shown in Fig. 5(c), the machine can operate stably against the load torque and can run stably at rated speed with rated load.
Figure 4. Open-loop I-f control from zero to rated speed under no load condition. (a): Rotor speed (b): Dq-axes currents.

Figure 6. Open-loop I-f control from zero to 0.3% rated speed under no load condition. (a): Rotor speed (b): Dq-axes currents.

Figure 5. Open-loop I-f control from zero to rated speed, with load disturbance at rated speed. (a): Rotor speed (b): Dq-axes currents (c): Load profile.

Figure 7. shows the performance of the open-loop I-f control in extreme low speed with no load. The speed reference is chosen to be 0.3% of rated speed or 15 rpm. The open-loop control operates stably at this speed range; however, the drive is poorly damped, and it takes a long time for the speed to settle. This is expected according to the previous small-signal analysis which shows that the dominant poles of the linearized model get closer to the imaginary axis when speed is low.

Figure 7 shows the performance of the open-loop I-f control with load disturbance at low speed. The load is slowly increased from zero to rated load as shown in Fig. 7(c), and the machine can operate stably with rated load torque at this extreme low speed.

V. CONCLUSION

In this paper, the stability characteristics of the simple open-loop I-f control is analyzed. The analysis is carried out by means of linearization. The stability is analyzed by studying the eigenvalue of the system matrix under different load and speed conditions. Matlab/Simulink simulation is used to verify the small-signal analysis. The study shows that open-loop I-f control is poorly damped but can operate stably in a wide speed range without the well-known mid-frequency instability issue that exists for the open-loop V/f control. Although for standalone operation, the simple open-loop I-f control is not suitable due to the poor damping and low efficiency. The good stability and ease of implementation supports the usage of open-loop I-f control as a startup method for PMSM for applications such as fans and pumps that do not require high dynamic performance at low speed range.
Extensive simulations are carried out to verify the analysis and experiments are underway.

Fig. 7. Open-loop I-f control from zero to 0.3% rated speed, with load disturbance. (a): Rotor speed (b): Dq-axes currents (c): Load profile. After speed settles at 15 rpm, ramp load is applied.

REFERENCES


