Simplified formulae for the estimation of the positive-sequence resistance and reactance of three-phase cables for different frequencies

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Abstract-The installation of HVAC underground cables became more common in recent years, a trend expected to continue in the future. Underground cables are more complex than overhead lines and the calculation of their resistance and reactance can be challenging and time consuming for frequencies that are not power frequency. Software packages capable of performing exact calculations of these two parameters exist, but simple equations able to estimate the reactance and resistance of an underground cable for the frequencies associated to a transient or a resonance phenomenon would be helpful. This paper proposes new simplified formulae capable of calculating the positive-sequence resistance and reactance of a cable for frequencies associated to temporary overvoltages, slow-front overvoltages and resonance phenomena. The calculation of a cable's resistance and reactance is made using a simplified series impedance matrix. The proposed formulae are validated by means of sensitivity tests and comparisons with generic cables. The paper finishes with an example demonstrating the estimation of the resonance frequency associated to a cable energisation using the formulae previously developed.

Index Terms—Underground Cables, Electromagnetic Transient, Inductance, Resistance, Resonance

I. NOMENCLATURE

A: impedance of the conductor-ground loop
B: mutual impedance between the conductor-ground and the screen-ground loops
C: impedance of the screen-ground loop
D: mutual impedance between cables
Z_{Oute}: conductor outer series impedance
Z_{Cinse}: main insulation series impedance
Z_{inner}: screen inner-series impedance
Z_{Oute}: screen outer-series impedance
Z_{Cinse}: outer insulation series impedances
Z_{Mutual}: mutual impedance of two loops
Z_{Earth}: earth-return impedance
Z_{Earth, Mutual}: mutual impedance between cables
R_1: radius over the conductor
R_2: radius over the insulation
R_3: radius over the screen

R_e: outer radius of the cable
\mu_c: permeability of the conductor
\mu_s: permeability of the screen
\rho_c: conductor resistivity
\rho_s: screen resistivity
\rho_e: earth resistivity
h_{ij}: depth of the cable conductor
d_{ij}: distance between two conductors
CS: cross-section area of the conductor
Z_{Re}^+: Positive-sequence resistance
Z_{Im}^+: Positive-sequence reactance
Z': Positive-sequence impedance
Z_{Ph}: Phase impedance
Z_{Seq}: Sequence impedance
l: length of the cable
C_{w}: cable capacitance per unit of length
L(f): cable inductance per unit of length in function of frequency
L_{eq}: inductance of the equivalent network

II. INTRODUCTION

The author proposed in [1] formulae that simplified the loop impedances of three-phase single-core cables. These formulae are used in this paper to develop simplified expressions able to estimate the positive-sequence resistance and reactance of the cables. The new proposed equations have a complexity level alike the formulae used for estimating these parameters at power frequency, but are also able to correctly estimate these two quantities for a large range of frequencies.

III. POSITIVE-SEQUENCE IMPEDANCE

The impedance matrix of a three-phase single-core cable is a 6x6 matrix [3], [4], which can be simplified to (1) for trefoil formations.

\[
Z = \begin{bmatrix}
A & B & D & D & D & D \\
B & C & D & D & D & D \\
D & D & A & B & D & D \\
D & D & B & C & D & D \\
D & D & D & D & A & B \\
D & D & D & D & B & C \\
\end{bmatrix}
\]
The reactance (9) is calculated substituting the variables in (10) by the equations from [1]. Notice that the variable $X$ is several times smaller than the variable $Y$, for typical cables. As a result, $X$ may be removed and (10) simplifies to (11).

$$
Z_{in} = \frac{259 \omega_{\mu_e} \rho_e}{2000 \pi} + \left( \frac{d_j - R_j}{R_j} \right) + \frac{8 \omega_{\mu_e} (R_j - R_d)}{6 \pi (R_d + R_j)} - \frac{y_j}{\pi (R_i - R_j)}
$$

The positive-sequence impedance of (1) is calculated by rearranging the matrix (2), where $Z_{CoC}$, $Z_{CoSc}$, $Z_{CoSc}$, and $Z_{ScSc}$ are 3x3 sub-matrices. The values are transposed from the modal domain to the phase domain in (3) [6]. The positive, negative and zero-sequences are obtained using the Fortescue matrix (4), where (5) is the positive-sequence impedance of the conductor.

$$
[Z] = \begin{bmatrix}
A & D & B & D & D \\
D & A & D & B & D \\
D & D & A & D & B \\
B & D & D & C & D \\
B & D & D & D & C
\end{bmatrix} \\
\Rightarrow [Z] = \begin{bmatrix}
Z_{CoC} & Z_{CoSc} \\
Z_{CoSc} & Z_{ScSc}
\end{bmatrix}
$$

The reactance (9) is calculated by the equations from (1). Notice that the variables are separated into real and imaginary parts.

$$
Z_{in} = Z_{CoC} - Z_{CoSc} \cdot Z_{ScSc}^{-1} \cdot Z_{CoSc}
$$

The resistance (12) is calculated substituting the variables in (8) by the equations from (1). Notice the reactance, $X$ is several times smaller than $Y$ and (12) simplifies to (13).

$$
Z_{in} = \frac{R_j \omega_{\mu_e} \rho_e + 3 \omega \rho_j}{8 \pi R_j} \left( \frac{1.5 X + Y}{\pi (R_i - R_j)} \right)
$$

Equation (5) can be rewritten as (6), which becomes (7) when the variables are separated into real and imaginary parts.

$$
Z^* = -\frac{A - D - (B - D)^2}{C - D}
$$

$$
Z^* = A_x + jB_x - D_x - jD_x \left( \frac{B_x + jB_x - D_x - jD_x}{C_x + jC_x - D_x - jD_x} \right)
$$

IV. REACTANCE AND RESISTANCE

A. Reactance

The reactance (10) is calculated substituting the variables in (9) by the equations from (1). Notice that $X$ has an extra simplification for $R_2$ and $R_3$, explained in [1].

$$
Z_{in} = \frac{\omega_{\mu_e} \rho_e}{8 \pi R_j} + \frac{\omega \mu_e}{2 \pi} \left( \frac{d_j - R_j}{R_j} \right) + \frac{8 \omega_{\mu_e} (R_j - R_d)}{6 \pi (R_d + R_j)} - \frac{y_j}{\pi (R_i - R_j)}
$$

where, $X = \frac{\omega \mu_e}{2 \pi} \ln \left( \frac{d_j}{R_j} \right)$ and $Y = \frac{\omega_{\mu_e}}{2 \pi} \ln \left( \frac{d_j}{R_j} \right)$

B. Resistance

The resistance (12) is calculated substituting the variables in (8) by the equations from [1]. Notice the reactance, $X$ is several times smaller than $Y$ and (12) simplifies to (13).

$$
Z_{in} = \frac{R_j \omega_{\mu_e} \rho_e + 3 \omega \rho_j}{8 \pi R_j} \left( X + Y \right)
$$

C. Flat formations

The matrix (1) is different for cables installed in flat formation and some variables $B$ and $D$ are alike, allowing further simplifications.

$$
Z_{in} = A_x + jB_x - D_x - jD_x \left( \frac{B_x + jB_x - D_x - jD_x}{C_x + jC_x - D_x - jD_x} \right)
$$

The variable $X$ is several times smaller than the variable $Y$, for typical cables. As a result, $X$ may be removed and (10) simplifies to (11).
minimum, maximum and reference values for each of the tested parameters.

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>Max</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductor Resist. ($\rho_c$) [(\Omega\cdot m)]</td>
<td>(1\times10^4)</td>
<td>(4\times10^4)</td>
<td>(3.122\times10^4)</td>
</tr>
<tr>
<td>Screen Resist. ($\rho_s$) [(\Omega\cdot m)]</td>
<td>(1\times10^4)</td>
<td>(4\times10^4)</td>
<td>(1.72\times10^4)</td>
</tr>
<tr>
<td>Earth Resistivity ($\rho_e$) [(\Omega\cdot m)]</td>
<td>5</td>
<td>155</td>
<td>100</td>
</tr>
<tr>
<td>Conductor radius ($R_1$) [m]</td>
<td>(10\times10^4)</td>
<td>(40\times10^4)</td>
<td>(16.85\times10^4)</td>
</tr>
<tr>
<td>Insulation radius ($R_2$) [m]</td>
<td>(16.6\times10^4)</td>
<td>(14\times10^4)</td>
<td>(35.05\times10^4)</td>
</tr>
<tr>
<td>Screen radius ($R_3$) [m]</td>
<td>(16.6\times10^4)</td>
<td>(14\times10^4)</td>
<td>(35.05\times10^4)</td>
</tr>
<tr>
<td>Dist. flat form. (Dist) [m]</td>
<td>0.25</td>
<td>7.75</td>
<td>-</td>
</tr>
<tr>
<td>Outer insul. radius ($R_4$) [m]</td>
<td>(35.479\times10^{-3})</td>
<td>(41.5\times10^{-3})</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 1 and Fig. 2 show the error in percentage for the minimum and maximum values of the sensitivity test (Table I) for eight different frequencies between 1Hz and 10kHz. The value of the earth resistivity does not affect the results and it is not shown, instead the figures show the error for the reference cable. The figures only show two of the 31 tested cases. The errors of the other 29 are without exception between the two values shown in the figures. For a better visualisation the graphs are truncated and the error is presented numerically for some columns.

The relative error is small, typically inferior to 3% for the reactance and to 5% for the resistance. The exception is for large screens ($R_i$), which have large estimation errors. However, it is important to point out that the maximum $R_i$ value used in sensitivity tests is several times larger than the real value. I.e., it is likely that the errors are smaller for real cables, since that the individual change of the parameters leads to unrealistic cables. The next section addresses this issue.

B. Validation for generic cables

The previous section tested the proposal formulae for a reference cable. Each parameter was changed individually over a large range of values and became eventually too big or too small when compared with the remaining parameters, resulting in unrealistic cables. As a result, some of the simulations have no real meaning, mainly those for the extreme of the test intervals, which also presented the largest errors. Thus, the accuracy of the formulae may be better than indicated in the previous section when applied to real cables.

This section tests the proposed formulae for different nominal voltage and cross-section areas, using data from the ABB [8] and Nexans [9] catalogues. Eighteen cables are tested, 9 from [8] and 9 from [9]. The cables from [8] have a copper conductor, whereas the cables from [9] have an aluminium conductor.

The correction of the resistivity, for stranded conductors, and of the permittivity, due to the semi-conductor layers [2], is not made. The objective is to compare calculation methods and thus, the correction is not important. The temperature of the conductor is 20°C, the ground resistivity is $100\Omega\cdot m$ and the top cable is installed at a depth of 1m. The error is estimated for ten different frequencies from 1Hz to 50kHz. The following cables are used in the comparison:

- 66kV: 120mm²; 630mm² and 2000mm² [8]
- 150kV: 240mm²; 1200mm² and 2500mm² [8]
- 400kV: 630mm²; 1400mm² and 2500mm² [8]
- 66kV: 185mm²; 500mm² and 1600mm² [9]
- 150kV: 400mm²; 1000mm² and 2000mm² [9]
- 400kV: 500mm²; 1200mm² and 3000mm² [9]

Fig. 3-Fig. 6 show the error in percentage of the proposed formulae for the cables installed in trefoil formation, whereas Fig. 7 and Fig. 8 show the estimated inductance and resistance in function of the frequency, using both the reference and the proposed formulae for the 66kV cables.

Formulae for the estimation of a cable’s impedance at power frequency exist [2], [7]. One of the main objectives of this paper is to design formulae with a similar mathematical complexity, but accurate for more frequencies. Thus, the error obtained when using the power-frequency reference formulae is also presented in the figures and it can be seen that it is substantially large when compared with the proposed formulae. The results for flat formation are in Appendix and the values are similar to these.

The results confirm those of the previous section. The error for the reactance are typically very small, under 2% to all cable from [8] and below 2.5% for all cables from [9] with two exceptions, both for the 66kV cable from reference [9]. The error is larger for these two cables, because of the large cross-sections of the screen. As an example, the cross-section area of the screen of a 66kV-185mm² cable is 35mm² in reference [8] and 105mm² in reference [9]. Moreover, no information is given about the thickness of the semi-conductive layers in [9] and they are not considered in the simulations. Thus, the real thickness of the screen is smaller than the one used in this validation. As an example, the error at 50kHz of the cable with a screen cross-section area 105mm²
would reduce from 5.8% to 4.2% if there was considered the presence of two semi-conductive layers with a thickness of 1mm.

The error associated to the resistance is inferior to 4% for all cables up to 10kHz, becoming large for 20kHz and 50kHz. The error is positive up to 2kHz, meaning that the estimated resistance is larger than in the reference formulae and it becomes negative at 10kHz for the majority of the cables. This means that the proposed formulae are unable to accurately estimated the increase of the resistance due to skin and proximity effects for frequency larger than 10kHz. This result is expected as the simplifications made for the loop impedances were made for temporary overvoltages, slow-front overvoltages and resonances, phenomena with frequencies below 10kHz.

In summary, the comparisons show that the error is acceptable for the frequencies of interest. The reactance is accurately estimated for all frequencies and the resistance is accurately estimated up to 10kHz.
VI. RESONANCE CALCULATIONS

The proposed formulae can be used to estimate the resonance frequency of a cable during its energisation, a task often done with software help that requires building a model of the network. Reference [5] presents a method that can be used to estimate the resonance frequency using the reference formulae and it is, therefore, more exact, but also more complex than the method proposed in this paper.

The resonance frequency associated to the energisation of a cable is given by (14).

\[
f = \frac{1}{4\sqrt{C_0 \left( L(f) + \left( \frac{2\pi}{4} \right)^2 \cdot \frac{L_{eq}}{T} \right)}}
\]  

(14)

The resonance frequency of the energisation is calculated using equations (11) and (14) in an iterative process. The inductance \( L(f) \) in (14) is first calculated for 50Hz, or other random frequency, using (11) and a new frequency is calculated using (14). The inductance is recalculated for the new frequency and the process repeats itself until the values no longer change.

Fig. 9 compares the resonance frequencies estimated using this process versus simulations in PSCAD, for four different cables and five short-circuit powers. The first cable is a 400kV-40km cable, the second is a 150kV-20km cable, the third is a 400kV-5km cable and the last is a 20kV-5km cable. The estimation is accurate for all four cases and the error is inferior to 2% to nearly all short-circuit powers. Moreover, part of the error is not associated to the calculation of the inductance, but to (14), which does not considered the distributed nature of a cable’s impedance.

The resonance frequency is also estimated using the power frequency formulae in order to compare their accuracy with the accuracy of the formulæ proposed in this paper.

Fig. 10 shows the error of the two when compared with the PSCAD simulations. The results show that the error is substantially lower when using the proposed formulæ than the power frequency formulæ. The low error for low short-circuit powers is explained by the dominance of the equivalent network’s inductance.

Fig. 10. Error in percent when estimating the resonance frequency using the formulæ proposed in this paper (dark columns) and the formulæ used for calculations at power frequency (light columns). a) 400kV – 40km; b) 150kV – 20km; c) 150kV – 5km; d) 20kV – 5km
VII. ADVANTAGES AND DISADVANTAGES

The development of new formulae for the calculation of the positive-sequence impedance of three-phase single-core cables and the respective validation were made in the previous chapters. The discussion of the advantages and disadvantages of these formulae is relevant to show that in some situations it is beneficial to use them in substitution of the existing ones.

Advantages of the formulae:
- do not require several steps or the use of mathematical software;
- the mathematical complexity is at the same level of the formulae used for calculating the impedance of a cable at power frequency, with the advantage of being accurate over a large range of frequencies;
- the results are accurate for frequencies associated to temporary overvoltages, slow-front overvoltages and resonances;
- provide a fast estimation of the resonance frequency, which can be used to set up the target frequency of a Bergeron model or to verify the triggering of temporary overvoltages, for example;

Disadvantages of the formulae:
- are still long and difficult to solve in one step using a calculator, but can be subdivided into parcels that can be added and multiplied; the same issue exits for the standard formulae used for estimating the two parameters at power-frequency;
- have to be corrected for low frequencies and/or small cross-sections, introducing more complexity; however, this correction is not expected for the majority of real cases;
- do not calculate zero-sequence impedance;

In summary, the formulae are useful for cases that require the calculation of the positive-sequence impedance of a cable. They are less complex, easy to implement, can save time and reduce lapses.

Studies that require information on the modal velocities, modal attenuations or zero-sequence impedance should continue to use the reference formulae.

VIII. CONCLUSIONS

This paper presented new simplified formulae that can be used in the estimation of the positive-sequence resistance and reactance of three-phase single-core cables for the frequencies associated to typical electromagnetic transients and resonance phenomena.

The formulae were validated for different voltage levels and cross-sections, together with sensitivity tests. The accuracy of the simplified formulae is high and they can be used with a high degree of confidence in the estimation of the resistance and/or reactance of an underground cable.

IX. REFERENCES


X. APPENDIX

A. Error Flat Formation

Fig. 11 shows the error of the proposed formulae for cables installed in flat formation, with 1m separation. The error associated to flat formations is in the same range of the error associated to trefoil formations and the explanation of the results is the alike the one provided in the paper to the latter. The results also show that non-transposed flat cables can be considered as perfectly transposed without affecting the accuracy.

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**Fig. 11. Error in percent of reactance (up) and resistance (down) for the proposed formulae for the cables from [8] (left) and [9] (right) in flat formation. Solid line: 66kV; Dashed line: 150kV; Dotted-dashed line: 400kV; Cross-sections: Red: small; Blue: Medium; Black: Large**