

## **Bi-framelet systems with few vanishing moments characterize Besov spaces**

We study the approximation properties of wavelet bi-frame systems in  $L_p(\mathbb{R}^d)$ . For wavelet bi-frame systems the approximation spaces associated with best  $m$ -term approximation are completely characterized for a certain range of smoothness parameters limited by the number of vanishing moments of the generators of the dual frame. The approximation spaces turn out to be essentially Besov spaces, just as in the classical orthonormal wavelet case. We also prove that for smooth functions, the canonical expansion in the wavelet bi-frame system is sparse and one can reach the optimal rate of approximation by simply thresholding the canonical expansion. For twice oversampled MRA based wavelet frames, a characterization of the associated approximation space is obtained without any restrictions given by the number of vanishing moments, but at a price of replacing the canonical expansion by another linear expansion.