Enhancement in reliability-constrained unit commitment considering state-transition-process and uncertain resources

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Abstract
The high penetration of uncertain resources challenges the security of power system operation. By taking the impact of rescheduling under contingencies into consideration, reliability-constrained unit commitment (RCUC) is developed to address this challenge. Although several efforts have been made in modelling reliability constraints, the existing methods can only manage oversimplified low-order temporal-independent contingencies without considering wide-range contingencies or their state-transition-process issue. To quantify the impact of rescheduling on the normal-state scheduling process denoted by UC problem, this paper builds up a Bayesian inference method for encoding reliability constraints in wide-range temporal-dependent contingencies. Three predictors, for example, expected-generator-rescheduling-power, expected-energy-not-serviced and lost-of-load-probability, are selected to describe the possible corrective behaviours in rescheduling process and quantified by using Bayesian inference method. Then, these predictors are reformatted as a set of linearized constraints to be incorporated into UC. The proposed RCUC comprehensively considers the effect of rescheduling in wide-range temporal-dependent contingencies. Therefore, it can reveal the influence of generator rescheduling in wide-range contingencies and keep better reliability performance than those methods reported in previous RCUC studies. The modified IEEE 30-bus test system and IEEE 118-bus test system are used to show the proposed model’s effectiveness.

1 | INTRODUCTION

With the increasing penetration of uncertain resources, the secured operation of the power system faces increased more challenges involving various uncertain characteristics. In such situations, the fluctuation of these uncertain resources makes the system components operate at their physical limits and become more vulnerable, resulting in weak reliability margin for adequately handling contingencies [1]. Therefore, it is important for operators to set out a workable generator scheduling against contingencies [2, 3].

Motivated by such problems, the literature reported several reliability-constrained unit commitment (RCUC) models to ensure the reliability of rescheduling against contingencies. These reported RCUC studies, as illustrated in Table 1, can be divided into two categories: (1) events-driven RCUC and (2) index-driven RCUC.

Regarding event-driven RCUC, the reliability requirement is modelled as a set of corrective limits regarding each possible contingency. These preselected failure scenarios can be counted like $N - 1$, $N - k$, or randomly sampled through MCs. Especially, References [4–7] presented an RCUC considering the security of first-order contingencies. Reference [8] put forward a simplified RCUC considering $N - 1$ by using LODF information. Reference [9] carried out an RCUC including the probability and security of rescheduling under first-order contingencies. Reference [10] developed a robust RCUC including $N - k$ reliability constraints. Reference [11] proposed a generalized $N - \alpha k$ security criterion and then constructed a robust RCUC including $N - \alpha k$ security criterion. Reference [12] used a non-sequential MC to generate a limited number of contingencies and incorporate associated rescheduling under these failure scenarios into UC problems. Reference [13] extended the scheduling horizon of the RCUC from short term (24 h) to...
TABLE 1  Taxonomy table reviewing recent advances in RCUC

<table>
<thead>
<tr>
<th>Model</th>
<th>Uncertain resource</th>
<th>Preselected contingencies</th>
<th>RCUC model structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Events-driven</td>
<td>[3], [9], [13]</td>
<td>N - 1</td>
<td>[3], [9], [10]</td>
</tr>
<tr>
<td>RCUC</td>
<td>[4], [5], [6], [7], [8]</td>
<td>N - k</td>
<td>[11], [12]</td>
</tr>
<tr>
<td>Indexes-driven</td>
<td>[14], [15], [16], [18], [20]</td>
<td>MC</td>
<td>[8], [12]</td>
</tr>
<tr>
<td>RCUC</td>
<td>[14], [15], [16], [17], [18], [19], [20], [21]</td>
<td>Probabilities</td>
<td>[11], [13]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Two stage</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multi-stage</td>
<td></td>
</tr>
</tbody>
</table>

For the second category, the reliability constraint in RCUC is abstracted as a group of reliability index requirements, for example, expected-energy-not-serviced (EENS), loss-of-load-probability (LOLP). To enforce the reliability level of rescheduling processes. References [15, 16] put up with an RCUC including EENS constraint, References [17, 18] presented an RCUC including LOLP constraint and EENS constraint, References [19–21] incorporated the probabilistic PDF information of wind farms into UC and constructed an RCUC that aimed to minimize the sum of normal-state fuel-cost and potential corrective cost under contingencies. Reference [22] used a risk-constrained mean-variance method to assess the dependence of multiple wind farms and then reformulated it as confidence interval constraint participating into RCUC including EENS limit. Reference [23] extended a decentralized RCUC considering EENS constraint in restructured power environments. In the above studies, both limits of LOLP and EENS within existing index-driven RCUC might be almost attributed to the fluctuation of uncertain resources at normal state, where the discussion on how to reflect the impact of rescheduling with EENS/LOLP forms is relatively rare. As for this, it is essential to build up an efficient link between the normal-state scheduling and rescheduling under contingencies, and incorporate it with reformatted EENS/LOLP limits into UC studies.

To overcome the above limits, this paper proposes a new Bayesian inference-based framework to model reliability constraints and constructs an enhanced RCUC. This framework uses an off-line sequential MC to estimate rescheduling processes in possible temporal-dependent contingencies and abstracts them into three feature predictors, for example, LOLP predictor, EENS predictor, and a newly constructed expected-generator-rescheduling-power (EGRP) index predictor. These predictors are trained by utilizing the B-MLR method and reformatted into a set of linearized constraints to incorporate into UC. It is pertinent to note that: (1) the off-line simulation for generating high-fidelity data can be replaced with historical data, if available; (2) the STP in sampled contingencies is considered in the sequential MC simulation; (3) the predictors through B-MLR are correct because it is data-driven.

In all, the contribution of this paper can be described as: (1) A Bayesian-inference-based framework for encoding reliability constraints in temporal-dependent contingencies is presented. This proposed B-MLR method accurately depicts the relations between the scheduling result and possible rescheduling processes against contingencies. (2) An enhanced RCUC is developed in this paper. It fully considers the influence of generator rescheduling in wide-range temporal-dependent contingencies. (3) The proposed RCUC provides a customized and robust scheduling result with less impact on current industry practice. Therefore, it will be with better performance when implemented in real-life scenarios.

The rest of this paper is organized as follows: Section 2 describes the problem statement and then proposes a novel RCUC model framework. Section 3 presents three represented Bayesian-based predictors for rescheduling and provides an enhanced RCUC. Section 4 uses two modified test systems to check the validity of the proposed RCUC model. Section 5 concludes this work and discusses potential future research.

2 | PROBLEM STATEMENT AND MODEL STRUCTURE

The RCUC searches optimal scheduling results by coordinating the generator scheduling at normal-state and potential rescheduling against contingencies. By considering the influences of contingencies, it guarantees that the obtained scheduling results have a desired reliability level for any failure event from a predefined set $\mathcal{J}^c$ over the scheduling period $\mathcal{T}$. The RCUC form can be written as,

$$
\min C \left( u_r, p_c \right) \tag{1a}
$$

subject to:

$$
\alpha^c \left( u_r, p_c, d_r \right) = 0, \quad \beta^c \left( u_r, p_c, d_r \right) \leq 0, \forall t \in \mathcal{T} \tag{1b}
$$

$$
\alpha^c \left( p_c, d_r \right) = 0, \quad \beta^c \left( p_c, d_r \right) \leq 0, \forall t, j \in \mathcal{J}^c \tag{1c}
$$

where $\alpha^c$ and $\beta^c$, respectively, denote equality and inequality constraints at normal state. $u_r, p_c$, respectively, denote...
commitment variables and scheduling variables produced by all generators, and $d_c$ denotes the system load forecast. Likewise, $a_c$ and $b_c$ declare reliability constraints under contingencies set $\mathcal{C}_c$. $p_c$ and $d_c$ denote rescheduling variables of all generators and forced-load-curtailment variables for each contingency [4].

Since the number of reliability constraints (1c) is proportional to the number of contingencies $\mathcal{C}_c$, therefore, the set $\mathcal{C}_c$ is usually limited in a low-order range for preventing the RCUC from being too complex to be solved. Besides, these preselected contingencies are temporal independent, where the STP principle of contingencies is out of discussion. For distinguishing the difference between temporal-independent contingencies and temporal-dependent contingencies that consider STP principle, Figure 1 compares the difference of temporal-independent contingencies and temporal-dependent contingencies [23].

To quantify the impact of temporal-dependent contingencies, this paper builds up an RCUC framework as shown in Figure 2. The reliability constraints are no longer limited to a finite number of preselected temporal-independent contingencies but are trained by mining the effect of rescheduling within all possible temporal-dependent contingencies. For preventing RCUC from being too complex, we design three predictors (i.e. EENS predictor, LOLP predictor, and EGRP predictor) to estimate the reliability of rescheduling in any possible temporal-dependent contingencies.

### 3 | ENHANCED RCUC MODEL CONSIDERING UNCERTAIN RESOURCES AND STP

#### 3.1 | Data generation concerning rescheduling

Note that the component that failed in the current period cannot be used immediately in the following periods because it needs time to go back to its normal state, and a repair process is required. Keep in view this basic assumption of the temporal-dependent contingency, and a sequential-based MC is used to generate contingencies.

Since the MTTR of the failed component $i : i \in \{G, L\}$ is usually longer than the scheduling horizon (24 h). Thus, the FOR $\lambda_i$ of this component $i$ can be treated as a non-repairable failure process [14]. As for this, the STP issue of each component can be taken as a homogeneous Markov process. The FOR
\[ \lambda_i = \alpha_i \lambda_i^{\text{true}} + \beta_i \lambda_i^{\text{est}}, \forall i \in \{G, L\} \]  

(2a)

For the component \( i \), if it is failed, then \( u_i = 1 \), otherwise, \( u_i = 0 \). Assume that the PDF \( f_i(u_i, \Delta t^c_i) \) of the component \( i \) follows an exponential distribution, in combination with (2a), the \( f_i(u_i, \Delta t^c_i) \) can be expressed as,

\[ f_i(u_i, \Delta t^c_i) = \left(1 - \frac{1}{\lambda_i} \right) u_i \left(\frac{1}{\lambda_i}\right)^{1-u_i}, \forall i \in \{G, L\} \]  

(2b)

For the contingency \( \epsilon \) consisting of \( N \) components with their states and durations: \{\( u^c_1, \Delta t^c_1 \), \( u^c_2, \Delta t^c_2 \), ..., \( u^c_N, \Delta t^c_N \)\}, the related system state \( u^c_S \) and duration \( \Delta t^c_S \) can be calculated by merging the \{\( u^c_i, \Delta t^c_i \)\} of each component. That is, \( u^c_S = \{u^c_1, u^c_2, ..., u^c_N\} \) and \( \Delta t^c_S = \min(\Delta t^c_1, ..., \Delta t^c_N) \). On this basis, the PDF denoted by \( h(u^c_S, \Delta t^c_S) \) of the contingency \( \epsilon \) is,

\[ h_j(u^c_S, \Delta t^c_S) = \prod_{i \in \{G, L\}} f_i(\Delta t^c_i, u_i), \forall \epsilon \in J', i \in \{G, L\} \]  

(2c)

To generate sampled contingencies from (2c), a sequential state-transition sampling method [23] is used to sample contingencies. By implementing it, the chronological states of each component are sampled, and the system state is then synthesized. It is essential to note that if an individual component is failed at the period \( t \), the failure state of this component will be kept until the end of the scheduling stage.

Next, the state-analysis model [9, 14] is used to investigate the reliability of temporal-dependent contingencies. The related problem formulation is described below,

\[
\min \sum_{d \in D} V_d L_d^c \times \Delta f_{d,j}, \forall j \in J' \tag{2d}
\]

\[
0 \leq \Delta f_{d,j} \leq \Delta f_{d,j}, \forall d \in D, j \in J', t \in T \tag{2j}
\]

\[
\Delta f_{d,j} \in \mathbb{R}^+, f_{d,j} \in \mathbb{R}^+, \Delta f_{d,j} \in \mathbb{R}^+ \tag{2k}
\]

The model mentioned before aims to minimize the security violations by rescheduling generator output or shedding several loads in contingencies. The objective function (2d) limits the severity of forced-load-curtailment. Constraint (2e) manages the system power balance between generators and demands. Constraint (2i) guarantees that the transmission line runs within its limit in the linear power flow equation. Constraint (2g) restricts the lower/upper bound of rescheduled power produced by each generator. Constraint (2h) shows the minimum/maximum output of each generator. Constraint (2i) describes the lower/upper limit of uncertainty resource (here, we took a wind farm as an example, and similarly hereinafter). Constraint (2j) decides allowable amount of potential forced-load-curtailment. Constraint (2k) reinforces feasible solution of decision variables.

It is worth noting that both failure events of generators (e.g. thermal unit and wind) are directly reflected by \( u^c_i, \Delta t^c_i \). If a generator fails in an individual contingency, constraints (2g) and (2h) strictly guarantee that the failed generator will remain in its off-state and do not take part in rescheduling in the current contingency. In contrast, the failure of line, denoted by \( u^c_i \), is implicitly incarnated change of GSDF. It means that the GSDF records the change of network caused by any line’s failure event. For a quick calculation, the Wood–Bury Inversion algorithm [22] is used here to calculate the GSDF in such a line’s failure event.

\[
K^c = H^c K^s \left( E - \left(W^c + (M^c)^T B^{-1}_g M^c \right)^{-1} \right), \forall j \in J' \tag{2l}
\]

3.2 Feature selection and Bayesian inference

In terms of the rescheduling described by the state-analysis model (2d)–(2k), two critical corrective behaviours, including generators’ rescheduling power and forced-load-curtailment, are evaluated by EGRP and EENS. Besides, the LOLP is incorporated to reflect the probability of forced-load-curtailment. These indexes are selected and modelled as associated feature predictors to build the bridge between generating rescheduling under contingencies and the normal-state scheduling result.

For the note of convenience, let \( e^{\text{EGRP}} \), \( e^{\text{EENS}} \) and \( e^{\text{LOLP}} \) denote the EGRP predictor, EENS predictor, and LOLP predictor, respectively. Their values can be calculated as,

\[
e^{\text{EGRP}} = \frac{1}{N_t} \sum_{i \in J', t \in T} \left( \sum_{g \in G} \left| f_{g,i,t} - f_{g,i,t} \right| \Delta t^c_g \right), \forall j \in J', t \in T, g \in G \tag{2m}
\]
where the \( H(\Delta f_{dp_{ji}}) \) is an indicator function. If \( \Delta f_{dp_{ji}} \geq 0 \), then \( H(\Delta f_{dp_{ji}}) = 1 \); and \( H(\Delta f_{dp_{ji}}) = 0 \), otherwise.

Since the most part of modelling for these predictors is similar, a superscript \( \lambda \) is used to define the types of MLR process, where \( \lambda \in \{\text{ERGP, EENS, LOPL}\} \). The predictor \( \lambda \) can be calculated by utilizing (2m)–(2n) for any given scheduling result \( p_i = \{p_{ij}:\lambda \in \mathcal{G}\} \).

To build up the bridge between normal-state scheduling and rescheduling under contingencies, the B-MLR method, introduced in Reference [17], is implemented. Mathematically, the results solved by state-analysis model (2q)–(2k) contribute to all possible instances of rescheduling in temporal-dependent contingencies, and each instance can be represented by an indicator variable (or called measurable variable) \( x_i: x_i = \{p_{ij}:i \in \mathcal{J}\} \) and associated independent predictor (responsive variable) \( \gamma_i: \gamma_i = \{\lambda: \lambda \in \mathcal{E}\} \).

Let \( y = f(x): f(x) = \mathbf{i}^T x + \xi \) denotes the regression function between corrective behaviour \( y \) and normal-state scheduling result \( x \), where \( \mathbf{i} \) is the regression coefficient of \( x \), and \( \xi \) is a residual following Gaussian distribution \( N(\eta_\xi, \sigma_\xi^2) \): \( \eta_\xi = 0 \), \( \sigma_\xi^2 = \mathbf{T}^{-1}(a, b) \). To calculate the PDF of regression coefficients \( \mathbf{t} \) denoted by \( f(t) \), both records \( \{x_i, \gamma_i\} \) are generated in Section 3.1 and normalized by \( \mathbf{\hat{x}}_i = x_i - \mathbf{E}(x_i)/\mathbf{\sigma}(x_i); \mathbf{\hat{y}}_i = y_i - \mathbf{E}(y_i)/\mathbf{\sigma}(y_i) \): \( \forall i \in \mathcal{J} \). According to the Bayesian theorem, the \( f(t) \) can be derived as,

\[
f(t) = f(t|\mathbf{\hat{x}},\mathbf{\hat{y}}) = \frac{f(\mathbf{\hat{x}},\mathbf{\hat{y}}|t)}{f(\mathbf{\hat{x}},\mathbf{\hat{y}})} = \frac{f(\mathbf{\hat{x}},\mathbf{\hat{y}})}{f(\mathbf{\hat{x}},\mathbf{\hat{y}}|t)} dt
\]  

In (2p), \( f(\mathbf{y}|t, \mathbf{x}) \) is a likelihood function with its expression can be written as:

\[
f(\mathbf{y}|t, \mathbf{x}) = \prod_{i \in \mathcal{J}} N(\mathbf{x}_i, \mathbf{\hat{y}}_i; \mathbf{\hat{x}}_i; \mathbf{T}^{-1}(a, b)) \int f(\mathbf{y}|t, \mathbf{x}) f(\mathbf{t}) dt
\]

where \( \mathbb{E}(\mathbf{y}|\mathbf{\hat{x}}) = \mathbf{\hat{x}} \) is a Gaussian prior function for regression coefficients \( \mathbf{t} \), that is, \( \mathbf{g}(t) \propto N(0, \mathbf{\sigma}_t^2) \). Specially, \( \mathbf{\sigma}_t \) is the standard variance of prior function \( f(t) \). Since both likelihood function and prior function are Gaussian distributions, the posterior function of \( t \) is also a Gaussian distribution according to the conjugate characteristic of Gaussian function,

\[
f(t) = N(\mu_\lambda, \sigma_\lambda^2)
\]

\[
\propto f(\mathbf{y}|t, \mathbf{x}) g(t) = \left( \prod_{i \in \mathcal{J}} N(\mathbf{y}_i, \mathbf{\hat{x}}_i; \mathbf{T}^{-1}(a, b)) \right) N(0, \mathbf{\sigma}_t^2)
\]

(2q)

where \( \mu_\lambda, \sigma_\lambda^2 \) denote the mean and standard variance of these coefficients \( t \). By taking the logarithm of this posterior function and maximizing its value, we can get,

\[
tag\max f(\mathbf{y}^*|x^*; t, \xi) = \frac{1}{2} \sum_{i \in \mathcal{J}} ||\mathbf{y}_i - \mathbf{\hat{y}}_i||^2 + \lambda ||\mathbf{t}||^2
\]

where \( ||*|| \) denotes a 2-norm operator. It can be found that the maximum problem in (2q) can be equivalently reformed as:

\[
tag\min f(\mathbf{y}^*|x^*; t, \xi) = \sum_{i \in \mathcal{J}} ||\mathbf{y}_i - \mathbf{T}^{-1}(a, b)||^2 + \lambda \mathbf{t}^T \mathbf{t}
\]

(2r)

where \( \mathbf{E} \) is a unit matrix; \( \mathbf{X} \) is a composite matrix with each row consisting of response variables \( \mathbf{x} \); and \( \mathbf{Y} \) is a composite matrix with each row consisting of measurable variables \( \mathbf{y} \). Furthermore, the distribution of predictor \( \mathbf{y}^* = \mathbf{T} \mathbf{x}^* + \xi \) for any new variable \( \mathbf{x}^* \) is also a multivariate Gaussian distribution because of the conjugate characteristic of Gaussian function, therefore, the \( \mathbf{y}^* \) can be directly expressed as,

\[
f(\mathbf{y}^*|\mathbf{x}^*; t, \xi) = N(\mathbf{x}^*; \mu_\lambda, \sigma_\lambda^2)
\]

(2t)

After constructing the regression function \( f(\mathbf{y}^*|\mathbf{x}^*; t, \xi) \), a cross-validation is employed to check the validity of the regression function. Especially, the leave-one-out method, introduced in [23], is used for model evaluation. Each sampled data is considered and assessed by using it, which ensures the obtained function can effectively explain the mapping between rescheduling and the normal-state scheduling result.

### 3.3 Reliability constraint reformulation

As discussed before, the predictor \( \mathbf{y}^* \) through Bayesian method with any given data \( \mathbf{x}^* \) can be sampled from associated posterior distribution \( f(\mathbf{y}^*|\mathbf{x}^*; t, \xi) \) or replaced by calculating its expectation \( \mathbb{E}(\mathbf{y}^*|\mathbf{x}^*; t, \xi) = \mathbf{x}^*; t, \xi \). In such settings, the latter strategy is implemented to estimate the value of reliability predictors, that is: \( \mathbf{t}^* = (\mathbf{x}^*)^T \mu_\lambda : \lambda \in \{\text{ERGP, EENS, LOPL}\} \), in which, \( \mu_\lambda \) denotes coefficients of each predictor, which can be calculated by utilizing the posterior function (2t).

Again, these constructed predictors are reformulated into reliability constraints taking part in UC problems,

\[
\lambda_{\text{ERGP}} \leq \lambda_{\text{ERGP}}, \lambda_{\text{EENS}} \leq \lambda_{\text{EENS}}, \lambda_{\text{LOLP}} \leq \lambda_{\text{LOLP}}
\]

(2u)

where \( \lambda_{\text{ERGP}}, \lambda_{\text{EENS}} \), and \( \lambda_{\text{LOLP}} \), respectively, declare the upper bound of each predictor.
3.4 Enhanced RCUC model

To guarantee that the scheduling result has a well economic cost and adequate reliable abundance in wide-range temporal-dependent contingencies, the reliability limits under contingencies and stochastic realization of wind farms are incorporated into the initial UC. The realization of uncertainty resource output is based on a general positive-skewed PDF hypothesis (e.g. Weibull distribution). On this basis, 3000 scenarios are generated in MC and 30 represented scenarios through the backward scenario reduction method is selected [25–28]. Figure 3 shows the realization of wind farm output. Based on it, the below lists the formulation of the proposed RCUC,

\[
\begin{align*}
\min & \sum_{i \in T} \sum_{g \in G} \left( t_{s}^{p} \cdot x_{gi} + t_{s}^{up} \cdot x_{gi} + t_{s}^{dw} \cdot x_{gi} \right) \\
+ & \sum_{i \in T} \sum_{s \in S} \pi_{s} \left\{ \sum_{g \in G} \sum_{d \in D} \left( t_{s}^{p} \cdot p_{gi,d} + t_{s}^{up} \cdot p_{gi,d} + t_{s}^{dw} \cdot p_{gi,d} \right) \right\} \\
+ & \sum_{d \in D} \left( V_{a} L I_{d} \times \Delta p_{d,i} \right) \\
s.t., & \sum_{w \in M(t, U_{g} - 1)} x_{gw} \geq U_{d} \left( u_{gi} - u_{gi-1} \right) \\
& \sum_{w \in M(D-1, t)} \left(1 - u_{gw} \right) \geq U_{d} \left( u_{gi} - u_{gi-1} \right) \\
x_{gi} - x_{gi-1} \leq u_{gi}, \forall g \in G, i \in T \\
x_{gi-1} - x_{gi} \leq u_{gi} - v_{gi}, \forall g \in G, i \in T \\
\Delta R_{gi}^{+} + t_{s}^{max} \cdot x_{gi} \leq p_{gi} \leq -\Delta R_{gi}^{-} + t_{s}^{max} \cdot x_{gi} \\
\end{align*}
\] (3a)

\[
0 \leq \Delta R_{gi}^{+} \leq \Delta R_{gi}^{max}, \forall g \in G, i \in T \\
0 \leq \Delta R_{gi}^{-} \leq \Delta R_{gi}^{max}, \forall g \in G, i \in T \\
\sum_{g \in G} p_{gi,d} + \sum_{s \in S(g)} \left( p_{wi,j} - \Delta p_{wi,j} \right) = \sum_{d \in D} \left( p_{di,j} - \Delta p_{di,j} \right), \forall g \in G, i \in T \\
\sum_{g \in G} K_{g} \left( \sum_{g \in G} p_{gi,d} + \sum_{s \in S(g)} \left( p_{wi,j} - \Delta p_{wi,j} \right) - \sum_{d \in D} \left( p_{di,j} - \Delta p_{di,j} \right) \right) \leq \pi_{i}^{max}, \forall i, \forall i, i \in T \\
\left( p_{gi,j} + \Delta p_{gi,j} - p_{gi,j-1} \right) \leq RU_{d}^{max} + p_{max} \times (1 - u_{gi}) + SU_{d}^{max}, \forall g \in G, i \in S, i \in T \\
\left( p_{gi,j+1} + \Delta p_{gi,j} - p_{gi,j-1} \right) \leq RD_{d}^{max} + p_{max} \times (1 - u_{gi}) + SD_{d}^{max}, \forall g \in G, i \in S, i \in T \\
0 \leq \Delta p_{wi,j} \leq \Delta p_{wi,j}, \forall w \in W, i \in S, i \in T \\
0 \leq \Delta p_{di,j} \leq \Delta p_{di,j}, \forall d \in D, i \in S, i \in T \\
\sum_{g \in G} \pi_{s} \left( \sum_{i \in T} \sum_{g \in G} \left( \mu_{g}^{ERGP} \right) p_{gi,d} \right), \forall g \in G, i \in S, i \in T \\
\sum_{g \in G, d \in D, i \in S, i \in T} \left( \mu_{g}^{ERGP} + \Delta p_{di,j} \right), \forall g \in G, d \in D, i \in S, i \in T \\
\sum_{g \in G, d \in D, i \in S, i \in T} \left( \mu_{g}^{OLP} + \Delta p_{di,j} \right), \forall g \in G, d \in D, i \in S, i \in T \\
\end{align*}
\] (3r)
The proposed RCUC is based on the traditional UC (TUC) [24] and incorporates the influence of generator rescheduling in contingencies with a Bayesian inference-based learning process. The reliability constraint is trained and reformatted from a wide range of temporal-dependent contingencies, so it accurately depicts the correlation between scheduling and rescheduling processes. Compared with existing RCUCs, the characteristics of the proposed RCUC include,

1. **Tractability**: the superposition of reliability constraints in the previous RCUC leads to a too complicated model structure. It inevitably makes the model intractable, especially for large systems or wide-range contingencies. Conversely, the impact of rescheduling is reflected by trained predictors to provide reliability-related guidance. With this, the complexity of the RCUC model is reduced, which facilitates the solution.

2. **Integrity**: the preslected contingencies in previous studies are temporal independent, where the STP is ignored. The contingencies for training the inference function are temporal dependent, which makes it more applicable in real scenarios.

The next section will evaluate through experiments whether these characteristics are significant.

### 3.5 Solution method for proposed RCUC

For better describing the calculation process of proposed RCUC in this work, Table 2 illustrates the pseudo-code of the proposed RCUC model.

<table>
<thead>
<tr>
<th>Step 1: Off-simulation for data generating</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.1</strong> Initialize: Count: ( \epsilon = 0 ), ( c = 0 ); Contingency number ( N_c )</td>
</tr>
<tr>
<td><strong>1.2</strong> Generating the normal-state dispatching data ( g_t, p_t ) through traditional UC without contingencies considerations</td>
</tr>
<tr>
<td><strong>1.3</strong> For: ( \epsilon = 0 ), ( c &lt; N_c ), then do</td>
</tr>
<tr>
<td><strong>1.4</strong> Sampling possible temporal-dependent contingencies denoted by ( \mathbf{w}_t, \Delta \xi_t ) through (3d)–(3k)</td>
</tr>
<tr>
<td><strong>1.5</strong> Estimating corrective behaviour ( \mathbf{e}^{\text{ERGP}}, \mathbf{e}^{\text{EENS}}, \mathbf{e}^{\text{LOLP}} ) under contingency ( c ) through (2d)–(2t)</td>
</tr>
<tr>
<td><strong>1.6</strong> Saving rescheduling record for training in next stage</td>
</tr>
<tr>
<td><strong>1.7</strong> End For</td>
</tr>
</tbody>
</table>

**Step 2: Bayesian inference for MLR**

**2.1** Preparing training data \( \mathbf{e}^{\text{ERGP}}, \mathbf{e}^{\text{EENS}}, \mathbf{e}^{\text{LOLP}} \) ; \( \epsilon \in 1, 2, 3, \ldots, N_c \) |

| **2.2** | Multivariable regression through Bayesian inference (2m)–(2t) |
| **2.3** | Checking the accuracy of calculated B-MLR model |

**Step 3: Enhanced RCUC**

**3.1** Constructing reliability constraints (2m) and (2o) |

| **3.2** | Incorporating (2m) and (2o) into traditional UC and extending the proposed E-RCUC model |
| **3.3** | Callback commercial solver, i.e. Gurobi, CPLEX, etc., to calculate E-RCUC |

### 4 CASE STUDIES

Both the modified IEEE 30-bus test system and IEEE 118 bus test system are used to demonstrate the validity of the proposed RCUC in this work.

All experiments, including benchmarks and the proposed model, are coded in MATH WORKS MATLAB 2018(b) environment by calling IBM CPLEX 12.90 and then conducted on a personal Dell OptiPlex 7060 desktop with Intel(R) Core (TM) i7 – 8700 CPU and RAM 12.0 GB.
<p>4.1 Modified IEEE 30-bus test system</p>

This test system consists of 30 buses, 21 transmission lines, and 6 conventional generators. The total installed capacity of all generators is reached at 477.49 MW. The amount of peak load is 283.40 MW, which happens at the 12th scheduling period. Besides, the system has a total of three wind farms with each wind farm’s capacity assumed to be 50 MW. These wind farms are located on bus-3, bus-29, and bus-30, respectively. In such settings, the penetration ratio of wind farms with respect to overall demand for this system has reached 34.61%. The required data including all generators’ operation parameters, the network topology parameters, and the load curve are from https://labs.ece.uw.edu/pstca/. The output of wind farms is simulated through a MC simulation, which is illustrated in Figure 3.

Three models are implemented to compare the performance of different UC concerning scheduling cost and reliability level under contingencies.

1. Benchmark-1: a TUC that considers the stochastic characteristics of uncertain resources but does not consider the influence of rescheduling in contingencies. For the complete form of TUC refer to [24].

2. Benchmark-2: an RCUC introduced in [5] that considers the uncertain resources and the impact of rescheduling in all first-order contingencies. In [5], the size of reliability constraints is proportional to the number of contingencies (scenarios). So, in this case, the number of 1-order contingencies, including the single generator's failure (event) and the single line's failure (event), is $C_0^1 + C_1^1 = 27$, and the number of reliability constraints for each contingency is 127; thus, the total number of reliability constraint for all 1-order contingencies under each scenario $s$ is $27 \times 127 = 3429$.

3. The proposed model: the enhanced RCUC (E-RCUC) in this work that considers the influence of wide-range contingencies and uncertain resources. Its detailed form can refer to (3t)–(3v).

For simplification, both of three predictors are denoted by: $\epsilon_{EGRP} = t_1 x + e_1$, $\epsilon_{EENS} = t_2 x + e_2$, and $\epsilon_{LOLP} = t_3 x + e_3$ where $e_1, e_2, e_3$, respectively, denote the normalized residual of different predictors, and $t_1, t_2, t_3$ denote the regression coefficients of different predictors. Here, the EGRP predictor is taken as an example to check the performance of Bayesian inference method during the training process. Figure 4 illustrates the PDF curve of regression coefficient $t_1(\varphi)$ : $\varphi \in \mathcal{G}$ regarding the EGRP predictor, and Figure 5 illustrates the histograms of regression coefficient generated from obtained PDF function. It can be found that each coefficient’s distribution approximately follows Gaussian distribution given Gaussian prior assumption. As the increase of sampling numbers, both values of coefficients gradually approach their modal number, which approximately equals the mean of each regression coefficient. It declares that the proposed predictors through the B-MLR method can effectively reflect the correspondence between the scheduling result and possible rescheduling result under contingencies. Furthermore, both the MSE index and RMSE index are implemented to test normalized residuals between the sample data and predictors [29]. The function of these two indexes can be expressed as below,

$$RMSE = \sqrt{\text{MSE}}$$

$$\text{MSE} = \frac{1}{n} \sum_{i\in J_s} \text{MSE}_i : \text{MSE}_i = (x_i - \hat{x}_i)^2$$

Accordingly, Table 3 statistics the results of RMSE and MSE of different predictors. It can be found that both MSE and RMSE indexes of different predictors are almost ranged [1.25e-3, 2.7e-4] and [0.0036, 0.0162], respectively. As for this, it explains that the predictors in this work are asymptotically unbiased.

Next, to test whether the Gaussian prior fits our problem during the training process, Figure 6 statistics the histogram of normalized residuals; Figure 7 compares the difference between normalized residual and the theoretical Gaussian distribution. The histogram plot (in Figure 6) and quantile–quantile (q–q) plot (in Figure 7) show that the Gaussian prior is satisfied pretty well, which also demonstrates the stability and validness of these applied predictors.

Comparison of generator scheduling: Table 4 compares the costs and reserves of scheduling produced by different models. The scheduling costs and their reserve allocation through RCUC and E-RCUC are close but higher than TUC. The significant difference between scheduling cost and reserve lies in the safe disposal of generator rescheduling in contingencies. Since the RCUC and E-RCUC have considered the risk of generator rescheduling in contingencies, but the TUC only focuses on the ones at normal state; hence, the scheduling produced by RCUC and E-RCUC must spare more (spinning) reserve against unbalanced power supplements caused by failures of individual generator or transmission line. Thereby, the scheduling costs by using RCUC and E-RCUC are higher than those by using TUC.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Both MSE and RMSE indexes of different predictors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predictors</td>
<td>MSE</td>
</tr>
<tr>
<td>$\epsilon_{EGRP}$</td>
<td>1.26E-05</td>
</tr>
<tr>
<td>$\epsilon_{EENS}$</td>
<td>2.63E-04</td>
</tr>
<tr>
<td>$\epsilon_{LOLP}$</td>
<td>1.85E-04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Results of scheduling results produced by TUC, RCUC and E-RCUC in the 30-bus test system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>$\epsilon_{s}^{\text{gen/off}}$</td>
</tr>
<tr>
<td>---------</td>
<td>----------------</td>
</tr>
<tr>
<td>TUC</td>
<td>4.93</td>
</tr>
<tr>
<td>RCUC</td>
<td>6.71</td>
</tr>
<tr>
<td>E-RCUC</td>
<td>6.71</td>
</tr>
</tbody>
</table>
Comparison of rescheduling in temporal-dependent contingencies: the generator scheduling results are then treated as boundaries to calculate their reliability levels in wide-range contingencies. Three indexes are adopted, those are LOLP, EENS, and EGRP. The solution of these indexes above can be found in (2m) and (2o). It is worth noting that the contingencies are generated in a sequential MC simulation and not limited to the first-order range as did in benchmark 2. From this view, the post-evaluation results can comprehensively reflect the secure generator scheduling in a wide range of failure spaces. Table 5 declares results of reliability index concerning generator rescheduling in wide-range contingencies. The index through E-RCUC is lower than that through benchmarks, while these reliability indexes of scheduling result via TUC is the highest one among these models. Therefore, a conclusion can be made that the scheduling result through E-RCUC is the safest against contingencies compared with TUC and RCUC. This distinction could be attributed to the slight difference in generators’ power employment distribution and their reserve allocations at different generator buses and periods. Since the constructed predictors in Section 3.2 declare the importance of generators. Therefore, the proposed model tends to schedule the less reserves from critical generators’ location behind the bottlenecks. For this reason, the scheduling result through E-RCUC has a less corrective process in contingencies, thus keeping a higher reliability level.

**TABLE 5** Reliability results for rescheduling produced by TUC, RCUC and E-RCUC in the 30-bus test system

<table>
<thead>
<tr>
<th>Model</th>
<th>$\xi_{LOLP}$</th>
<th>$\xi_{EENS}$</th>
<th>$\xi_{EGRP}$ [×10^4 $]</th>
<th>[×10^3 $]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TUC</td>
<td>0.074</td>
<td>1.064</td>
<td>5.85</td>
<td></td>
</tr>
<tr>
<td>RCUC</td>
<td>0.018</td>
<td>0.222</td>
<td>2.30</td>
<td></td>
</tr>
<tr>
<td>E-RCUC</td>
<td>0.015</td>
<td>0.104</td>
<td>1.99</td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 4 The PDF curve of regression coefficients $\xi_1$ regarding constructed predictor $\hat{EGRP}$.
Comparison of coordination between scheduling at normal state and rescheduling in contingencies. Table 6 compares the total cost, $c_{\text{total}} = c_{\text{norm}} + c_{\text{cont}}$, that consists of generator scheduling cost $c_{\text{norm}} = c_{\text{on/off}} + c_{\text{prod}}$ and rescheduling cost $c_{\text{cont}} = EENS + EGRP$. The scheduling costs through RCUC and E-RCUC are close, as mentioned earlier, and higher than that via TUC. On the other hand, the total cost of scheduling result obtained by E-RCUC is the smallest compared with other benchmarks. It declares that the E-RCUC has a better trade-off between generator scheduling (economy oriented) and rescheduling process (reliability oriented).

Comparison of model structure and calculation time: both model structures and calculation times of different models are shown in Figure 8. It is found that TUC has the shortest calculation time, and RCUC has the longest one. Meanwhile, both the numbers of decision variables and constraints (including equality
constraint and inequality constraints) in TUC model are the smallest, and RCUC stays the largest. The ultra-complex model structure in RCUC is the critical bottleneck for its calculation time. Compared with RCUC, E-RCUC has a relatively simplified model structure, thus keeping a reduced calculation time. It is an obvious advantage if a wide-range contingencies are incorporated into UC. In RCUC, the higher the order of contingencies is considered, the more complex its model structure will be, and the more difficult it is to be solved. In contrast, the E-RCUC is independent of contingencies’ order because the influence of contingencies is abstracted to be predictors and reformatted into UC studies. As for this, the E-RCUC keeps a simplified model structure even if high-order contingencies are included.

### 4.2 Modified IEEE 118-bus test system

A modified IEEE 118-bus test system is applied to verify the scalable feasibility of the proposed model. The system consists of 54 conventional generators, 186 transmission lines, and 91 loads. The sum of installed capacity produced by generators is 7220 MW, and the peak load is 5516.08 MW. Besides, the system has four wind farms with each capacity assumed to be 500 MW. In such a setting, the percentage of wind power concerning the peak-load for this system is 20.79%.

### TABLE 6 Sum of scheduling cost at normal state and rescheduling cost in contingencies, produced by TUC, RCUC, and E-RCUC, in the 30-bus test system

<table>
<thead>
<tr>
<th>Model</th>
<th>$c_{\text{norm}}$ [$\times10^4$ $$/]</th>
<th>$c_{\text{cont}}$ [$\times10^4$ $$/]</th>
<th>$c_{\text{total}}$ [$\times10^4$ $$/]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TUC</td>
<td>1.78</td>
<td>1.65</td>
<td>3.43</td>
</tr>
<tr>
<td>RCUC</td>
<td>2.28</td>
<td>0.45</td>
<td>2.73</td>
</tr>
<tr>
<td>E-RCUC</td>
<td>2.28</td>
<td>0.30</td>
<td>2.58</td>
</tr>
</tbody>
</table>
TABLE 7  Results of scheduling results produced by RCUC and E-RCUC in the 118-bus test system

<table>
<thead>
<tr>
<th>Model</th>
<th>$c_{on/off}$</th>
<th>$c_{prod}$</th>
<th>$p^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[$\times 10^5$ $$/h]</td>
<td>[$\times 10^5$ $$/h]</td>
<td>[$\times 10^4$ $$/h]</td>
</tr>
<tr>
<td>RCUC</td>
<td>6.7112</td>
<td>1.6086</td>
<td>1.6111</td>
</tr>
<tr>
<td>E-RCUC</td>
<td>6.7112</td>
<td>1.6092</td>
<td>1.6111</td>
</tr>
</tbody>
</table>

TABLE 8  Reliability results for rescheduling produced by RCUC and E-RCUC in the 118-bus test system

<table>
<thead>
<tr>
<th>Model</th>
<th>$\epsilon_{LOLP}$</th>
<th>$\epsilon_{EENS}$</th>
<th>$\epsilon_{EGRP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[$\times 10^5$ $$/h]</td>
<td>[$\times 10^4$ $$/h]</td>
<td>[$\times 10^4$ $$/h]</td>
</tr>
<tr>
<td>RCUC</td>
<td>0.0177</td>
<td>2.2193</td>
<td>2.3020</td>
</tr>
<tr>
<td>E-RCUC</td>
<td>0.0151</td>
<td>1.0373</td>
<td>1.9973</td>
</tr>
</tbody>
</table>

To compare the coordination of scheduling at normal state and rescheduling in contingencies as well as their calculation time, TUC is out of discussion in this case. Tables 7 and 8, respectively, show the results of RCUC and E-RCUC concerning generator scheduling cost and reliability level in contingencies. Table 9 compares the total cost of scheduling at a normal state and rescheduling under contingencies computed by different models. Figure 9 compares the number of decision variables and constraints in different model structures.

It can be found in Tables 7 and 8 that the scheduling by using E-RCUC is slightly larger than that through RCUC, but associated reliability indexes are smaller. It again declares that the E-RCUC has a better reliability against contingencies. In terms of Table 9 that compares the coordination between the scheduling cost and rescheduling cost produced by RCUC and E-RCUC, the cost of E-RCUC is smaller, which reflects that it has an average more minor corrective cost regarding generator rescheduling and forced-load-curtailment in dealing with potential contingencies.

In calculation times for different models, the calculation time of RCUC is 29611.20 s, and E-RCUC is 5109.09 s. This difference lies in the model structure of two models, as shown in Figure 9. Since the number of decision variables and constraints in E-RCUC is significantly reduced compared with that in RCUC, therefore, E-RCUC keeps a simplified model structure conducive to solving. It declares the high efficiency of E-RCUC under wide-range temporal contingencies and large-scale systems.

TABLE 9  Sum of scheduling cost at normal state and rescheduling cost in contingencies, produced by TUC, RCUC and E-RCUC, in the 118-bus test system

<table>
<thead>
<tr>
<th>Model</th>
<th>$c_{norm}$</th>
<th>$c_{cont}$</th>
<th>$c_{total}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[$\times 10^5$ $$/h]</td>
<td>[$\times 10^5$ $$/h]</td>
<td>[$\times 10^5$ $$/h]</td>
</tr>
<tr>
<td>RCUC</td>
<td>2.2797</td>
<td>2.4495</td>
<td>4.7474</td>
</tr>
<tr>
<td>E-RCUC</td>
<td>2.2903</td>
<td>1.2370</td>
<td>3.5273</td>
</tr>
</tbody>
</table>

5 | CONCLUSION

This paper proposes an enhanced RCUC considering the influence of generator rescheduling in wide-range contingencies. Both contingencies’ state-transition-process and their probabilities are included. Compared with previous RCUC studies that emphasize low-range contingencies for preventing the unsolvable model, the proposed RCUC considers the effect of contingencies in wide range while keeping a relatively simplified model structure. Moreover, it has a better trade-off between generators’ scheduling at normal state and rescheduling in contingencies and stays an acceptable calculation time. Two case studies are carried out to verify the effectiveness of the proposed model. The results suggest that:

1. Compared with TUC studies, the RCUC tends to schedule more generators to supply more reserves for handling contingencies. For this reason, the cost of generator scheduling obtained by RCUC is higher than that by using the TUC model.

2. The influence of wide-range contingencies (especially the critical high-order contingencies that are behind bottlenecks) and their state-transition-process characteristics cannot be ignored. Compared with the generator scheduling through RCUC considering low-range contingencies, the proposed model has a better reliability level in contingencies.

3. The research for encoding reliability constraints in potential contingencies needs to be extended to prevent it from being too complex. It may result in an unacceptable calculation time against wide-range contingencies or large-size systems.

The modelling work for better coordination between generator scheduling and rescheduling in contingencies in the electrical markets could be further developed.
ACKNOWLEDGEMENTS
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CONFLICT OF INTEREST
There is no conflict of interest in submitting this manuscript, and all authors have approved this manuscript for publication.

Nomenclature
Abbreviations

- B-MLR: Bayesian-inference-based multivariable linear regression
- EENS: Expected-energy-not-supply
- EGRP: Expected-generator-rescheduling-power
- FOR: Failure-of-rate
- GSDF: Generation shift distribution factor
- LODF: Line-outage-distribution-factor
- LOLF: Lost-of-load-frequency
- LOLP: Lost-of-load-probability
- MAP: Maximum A posterior
- MCs: Monte Carlo simulation
- MSE: Mean square error
- MTTR: Mean time to repair
- PDF: Probabilistic distribution function
- RCUC: Reliability-constrained unit commitment
- RMSE: Root-mean-square error
- STP: State-transition-process
- VoLL: Value-of-lost-load

Parameters

\[ \Delta \rho_d,\gamma \] Forced-load-curtailment of load \( d \) under scenario \( \gamma \) at period \( t \) [MW]
\[ \Delta \rho_{\sigma g,\gamma} \] Forced-load-curtailment of load \( d \) at contingency \( \epsilon \) [MW]
\[ \Delta p_{\sigma g,\gamma} \] Power on the segment \( \epsilon \) of fuel-cost curve of unit \( g \) under scenario \( \sigma \) at period \( t \) [MW]
\[ S\rho_{\sigma g,\gamma}^\epsilon \] Up-forward/down-forward reserve of unit \( g \) at period \( t \) [MW/h]
\[ p_{\sigma g,\gamma}^\epsilon \] Power of generator \( g \) under scenario \( \sigma \) at period \( t \) [MW]
\[ p_{\sigma g,\gamma}^\epsilon \] Rescheduling power of generator \( g \) under contingency \( \epsilon \) [MW]
\[ p_{\sigma g,\gamma}^\epsilon \] Up-forward/down-forward reserve of generator \( g \) under scenario \( \sigma \) at period \( t \) [MW]
\[ \delta_{\epsilon} \] Shut-up decision for the generator \( g \) at period \( t \), equals to 1 if it is turned on at period \( t \), 0 otherwise
\[ \delta_{\epsilon} \] Shut-down decision for the generator \( g \) at period \( t \), equals to 1 if it is turned off at period \( t \), 0 otherwise
\[ \gamma_\sigma \] State of generator \( g \) at period \( t \), equals to 1 if it is online, 0 otherwise

Decision variables

\[ F_{\gamma} \] Slope of the \( \gamma \)th segment of fuel curve of generator \( g \)[$$/\text{MW}$$]
\[ N_c \] Number of sampled contingencies
\[ RU_{\gamma}, RD_{\gamma} \] Ramping-up/ramping-down rate of generator \( g \) [$$\text{MW}/\text{h}$$]
\[ S\psi_{\gamma}^\epsilon \] Maximum up-forward/down-forward (spinning) reserve provided by generator \( g \) [MW]
\[ UT_{\gamma}, DT_{\gamma} \] Minimum start-up/shut-down duration time of generator \( g \) [h]
\[ V_{\sigma d,\gamma} \] The VoLL of demand \( d \) in rescheduling processes at contingency \( \epsilon \)
\[ \rho_{\sigma d,\gamma}^\epsilon \] No-load cost, Shut-down cost, and start-up cost of generator \( g \)[$$
\[ \rho_{\sigma g,\gamma}^\epsilon \] Maximum/minimum output of generator \( g \) [MW]
\[ \rho_{\sigma g,\gamma}^\epsilon \] Capacity of transmission line \( l \) [MW]
\[ \rho_{\sigma g,\gamma}^\epsilon \] Maximum output of wind farm \( w \) [MW]
\[ \rho_{\sigma g,\gamma}^\epsilon \] States of conventional generator \( \{g, w\} \) and transmission line \( l \) at contingency \( \epsilon \), equals to 1 if it is normal, and, 0 if it is failed
\[ \rho_{\sigma g,\gamma}^\epsilon \] States of transmission line \( l \) at contingency \( \epsilon \), equals to 1 if it is normal, and, 0 if it is failed
\[ B_{\sigma} \] Node admittance matrix at normal state \( H \) Characteristic matrix with its row vector related to the normal lines \( \mathcal{L}_{\gamma; \epsilon} \) at the contingency \( \epsilon \)
\[ \psi_{\sigma g,\gamma}^\epsilon \] GSDF matrix at the contingency \( \epsilon \) and normal state \( n \)
\[ M', M'' \] Adjacent matrices between lines and buses at normal state \( 0 \) and contingency \( \epsilon \)
\[ W \] Characteristic matrix with its row vector related to failed lines \( \{\gamma_i \} \) at the contingency \( \epsilon \)
\[ \alpha_i, \beta_i \] Weight coefficient of component \( i \) in the normal-weather and severity-weather \( \gamma \) \( \alpha_i + \beta_i = 1 \)
\[ \lambda_i \] FOR of component \( i \) \( |\{\gamma_i \} \) of wind farms
\[ \alpha^{\text{nor}}, \alpha^{\text{sev}} \] FORs of \( i \) both at normal-weather and severity-weather condition
\[ \pi_{\gamma} \] Probability of scenarios \( \gamma \) of wind farms

Set and Indices

- \( B \) Set of bus nodes
- \( \mathcal{L} \) Set of transmission lines
- \( \mathcal{M}(i, j) \) Set of scheduling periods from \( i \) to \( j \)
- \( b \) Index of bus node
- \( c \) Index of individual contingency
- \( g, w, d \) Indexes of generator, wind farm, and demand
- \( l \) Index of transmission line
- \( s \) Index of stochastic scenario of wind output
- \( t \) Index of periods, \( t = 1 \) h, in this paper
- \( z \) Index of unit's production cost blocks
- \( D_{(b)} \) Set of demands located at bus \( b \)
- \( G_{(b)} \) Set of generators located at bus \( b \)
- \( G, W, D \) Sets of generators, wind farms, demands
- \( S \) Set of stochastic scenarios
- \( T \) Set of scheduling periods, \( T = \{1, 2, \ldots, 24\} \)
- \( J' \) Set of sampled contingencies
- \( W_{(b)} \) Set of wind farms located at bus \( b \)
- \( Z \) Set of generators’ production cost blocks
REFERENCES
