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Clavier, Laurent; Pedersen, Troels; Rodriguez, Ignacio; Egan, Malcolm

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Laurent Clavier
IMT Nord Europe
Univ. Lille, CNRS, UMR 8520 - IEMN
Lille, France
laurent.clavier@imt-nord-europe.fr

Troels Pedersen, Ignacio Rodriguez Larrad
Dept. Electronic Systems
Aalborg University
Aalborg, Denmark
troels@es.aau.dk; irl@es.aau.dk

Malcolm Egan
INRIA, CITI
University of Lyon, INSA Lyon
Lyon, France
malcom.egan@inria.fr

***Abstract*—The increase in connected device, linked with the evolution of IoT and cellular networks with 5G and towards 6G makes crucial the limitation of the interference impact. Devices in networks (like LoRa, SigFox and even WiFi) or in different networks are not necessarily coordinated to optimize their communication scheme and reduce interference. As a consequence, it is essential to characterize the interference statistics in order to assess its impact on the network performance and define the appropriate access policies. With a theoretical approach, a large number of studies have tackled this characterization question. However, much less is available from an experimental point of view. In this paper, we address this key gap. Measurements were performed in Aalborg, Denmark, in the unlicensed $863-870$ MHz. We show that the heavy tailed behaviour of the interference, predicted by theory, is indeed appropriate. We also show that the α-stable distribution can be a good model for interference in IoT networks.**

***Index Terms*—IoT, Interference, α-stable distributions**

I. Introduction

The coming years should see an ever increasing density of wirelessly connected devices as the Internet of Things (IoT) emerges. This increase implies several facts:

- To cope with the expected lifetime requirements of devices, signaling has to be minimized and it is difficult to envision a scheduled channel access protocol. Grant free access are consequently an option to be explored - it is nowadays an option chosen by LoRa and SigFox.
- To meet the expected high number of devices, non orthogonal multiple access schemes are to be implemented.
- And to face the growing application demand, several solutions will be available implying a high heterogeneity in both devices and networks (architectures and protocols).

These facts will result in highly uncoordinated transmissions with a large number of devices sharing the same frequency band. It has been shown in theory and experimentally [1] that the generated interference will, in such situations, exhibit an impulsive nature. It is essential to take into account the interference statistical properties, for instance to design robust receivers [2].

Characterizing the interference is a non-trivial issue. In this paper, a complement to [1], we further analyze measurements of interference in the 864 to 870 MHz band in Aalborg, Denmark, first reported in [3]. We previously showed that the empirical distribution of the interference from the measurement data is heavy tailed. In this work, we investigate the specific family of statistical models known as α-stable distributions. It arises as the interference distribution when interferers are distributed according to an homogeneous Poisson point process with no guard zone. We provide evidence that suggests these models are consistent with the measurement data and give indications about adequate parameters.

II. Theoretical Interference Models

We consider a transmission model where the received signal $\mathbf{Y} \in \mathbb{R}^K$ is

$$\mathbf{Y} = s\mathbf{h} + \mathbf{I} + \mathbf{N}, \tag{1}$$

where s is the unknown transmitted symbol, $\mathbf{h} \in \mathbb{R}^K$ is the block fading channel coefficients, $\mathbf{I} \in \mathbb{R}^K$ is the interference and $\mathbf{N} \in \mathbb{R}^K$ is the thermal noise with its elements $N_k \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2)$.

Specifying the Probability Density Function (PDF) of the interference is an important issue, for instance when deriving the likelihood for designing an optimal receiver. In many previous papers, it has been shown that the interference term is not adequately modelled with a simple Gaussian distribution assumption.

This is the case in the works from Middleton [4], [5] who obtained quite general expressions based on series expansions assuming Poisson distributed interference sources. This popular model remains however challenging to work with due to the infinite sums. Approximation models have been proposed, considering only the most significant terms leading to a Gaussian mixture [6] or the ϵ-contaminated noise [7] if only two terms are considered. In this case the interference PDF is $\mathbb{P}(x) = (1-p)\mathcal{N}\left(0, \sigma^2\right) + p\mathcal{N}\left(0, \kappa\sigma^2\right)$, where p denotes the probability to have an impulse, distributed from a Normal with variance $\kappa\sigma^2$ while $(1-p)$ gives the probability to only have the Gaussian noise with variance σ^2. The Class B model can also be approximated by an α-stable distribution [5].

More recently, many works concerning Time Hopping Ultra Wide Band (TH-UWB) [8] introduced some empirical distributions justified by simulations, observations of the estimated PDF and/or gains in BER: Gaussian-Laplace mixture [9], Generalized Gaussian [10], Gaussian mixtures [11] or Cauchy-Gaussian mixture [12]. In this last paper it is mentioned that the heavier tail of the Gaussian Mixture allows better performance than the Laplace approach.

Another class of model of direct relevance to interference modelling is the α-stable. It has often been used in the UWB context [13], [14]. But on the contrary to the previously discussed approaches, it relies on a theoretical derivation, finding its foundation in stochastic geometry [15]–[17].

In a network, we can express interference as

$$I = \sum_{i \in \Omega} l(d_i).\mathcal{Q}_i, \tag{2}$$

where d_i is the distance between interferer i and the destination and $l(d)$ the attenuation as a function of the distance; a classical model is $l_{\gamma,\epsilon}(d) = d^{-\gamma} 1_{\mathrm{r} \geq \epsilon}, \mathrm{d} \in \mathbb{R}^+$ where γ is the channel attenuation coefficient; ϵ is a guard zone, meaning no interferer can be closer than ϵ from the receiver; Q_i includes the propagation effects (multipath, shadowing) and the physical layer characteristics; Ω is the set of interferers. If applied in an *ad hoc* network, an unbounded received power assumption makes the interference fall in the attraction domain of a stable law. This unbounded assumption means taking the limit as $\epsilon \to 0$;

Interference power: in the case of the interference power, a detailed study has been carried out by Haenggi and Ganti [18]. In particular for interferers located according to a Poisson point process, they showed that the distribution is heavily dependent on the path loss attenuation coefficient γ. For example, also shown by Win and Pinto [15], in a network with infinite radius and no guard zone ($\epsilon = 0$), the interference power has the totally skewed α-stable distribution, where α depends on γ.

III. Statistical representation of impulsiveness

A. Heavy tailed distributions

Impulsiveness is characterized by large values that rarely appear. This can be modeled with heavy tail distributions, meaning distribution having tails heavier than the exponential distribution. Such a behavior can be related to the moment generating function (MGF) as one often considers heavy tailed models as those with non-finite mean or variance or higher order moments. Hence, one may characterize heavy tailed distributions or processes with impulsive realizations as those distributions which have tails which fail to satisfy the following bound on the complementary cumulative distribution function $\overline{F}(x) = \mathbb{P}(X > x)$ [19]: for some positive real numbers M and t, $\overline{F}(x) \leq M \exp(-tx)$, $\forall x > 0$. For instance the Middleton or the α-stable can be characterized as sub-families of the sub-exponential class on the entire real line.

B. Non-Gaussian α-Stable Distributions

The α-stable distributions are a special case of heavy tailed distributions with infinite variance (fat tail distributions) when $0 < \alpha < 2$. The Gaussian case also belongs to this family and is obtained with $\alpha = 2$. The distribution function of an α-stable random variable is described by four parameters: the characteristic exponent $0 < \alpha \leq 2$; the scale parameter $\gamma \in \mathbb{R}_+$; the skew parameter $\beta \in [-1, 1]$; and the shift parameter $\delta \in \mathbb{R}$. As such, a common notation for an α-stable random variable X is $X \sim S_\alpha(\gamma, \beta, \delta)$. In general, α-stable random variables do not have closed-form porbability Density Functions (PDFs), but are usually represented by their characteristic function given by [20, Eq. 1.1.6].

$$\mathbb{E}[e^{i\theta X}] = \begin{cases} \exp\left\{-\gamma^\alpha |\theta|^\alpha (1 - i\beta (\mathrm{sign}\theta) \tan \frac{\pi\alpha}{2}) + i\delta\theta\right\}, \alpha \neq 1 \\ \exp\left\{-\gamma|\theta|(1 + i\beta \frac{2}{\pi} (\mathrm{sign}\theta) \log|\theta|) + i\delta\theta\right\}, \alpha = 1 \end{cases} \tag{3}$$

As noted in Section II, the α-stable distributions arise as the interference distribution for Poisson point process models with no guard zones ($\epsilon = 0$).

The parameters of the α-stable can be estimated using classical methods, such as fitting the four parameters to the empirical characteristic function estimated from data as proposed by Koutrouvelis [21] The quality of the fit between the measurement data and the estimated α-stable distribution can be evaluated by generating samples from an α-stable distribution based on the estimated parameters. The quantiles of the measurement samples are then plotted against those of the generated samples, i.e. a quantile-quantile (QQ) plot. If the measurement data and the generated samples are from the same distribution, the QQ plot will be close to the line $y = x$.

IV. Measurement Data Analysis

A. Measurements

First reported in [3], received power measurements were performed at five distinct locations within Aalborg: three downtown measurements in a shopping area, a business park with office buildings, and a hospital complex with multiple, large hospital buildings and some single family houses; an industrial area consisting of industrial production facilities and office buildings; and a residential area with single-family houses. All measurements were performed while static in a parking lot or at the roadside; that is, at street level. The measurement data set consists of power measurements on a frequency grid from 863 MHz to 870 MHz with 7 kHz bins. The sampling time was 200 ms and measurements were conducted over a period of two hours. Further details of the setup and measurements can be found in [3].

All samples falling in the chosen time-frequency window—corresponding to non-overlapping windows of 200 ms and 126 kHz (to fit a LoRa scheme)—give a sequence of interference samples $\mathcal{I}_{1,1}, \ldots, \mathcal{I}_{N_t,N_f}$, with N_t and N_f being the number of time and frequency samples, respectively. One example in a given frequency band as a function of time is presented in Fig. 1.

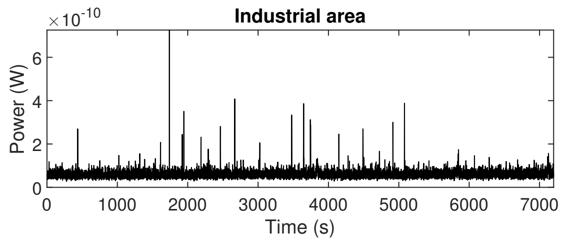

Fig. 1. Example of interference samples measured in the industria area.

B. Heavy tail

In [1] we tested the heavy tail properties of the data, firstly with the converging variance test, secondly with a log-tail test. In this second approah, given observations $\mathcal{I}_1, \ldots, \mathcal{I}_n$, to test for a subexponentially decaying tail the empirical distribution function $\hat{F}$ is first estimated via

$$\hat{F}(x) = \frac{1}{n} \sum_{k=1}^{n} \mathbf{1}_{\{\mathcal{I}_k \leq x\}}. \tag{4}$$

The logarithm of the empirical survival function is then given by $\log\left(1 - \hat{F}(x)\right)$ and plotted as a function of $\log x$. For sub exponentially decaying $\hat{F}$, the curve is a straight line with slope $-\frac{1}{\gamma}$, while for exponentially decaying distributions γ will be 0 leading to an abrupt decrease in the curve as $\log x$ increases. Both tests gave evidence og the heavy tail behaviour of the interference, as illustrated in Fig. 2 for the shopping area with the log-tail test.

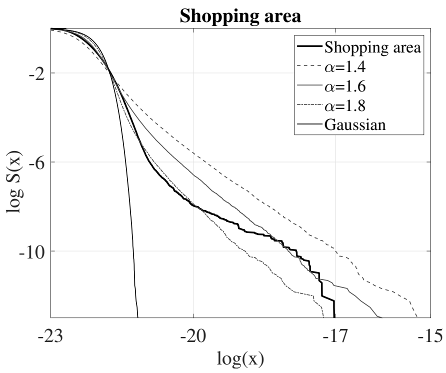

Fig. 2. Survival function as a function of $\log x$ in the case of the shopping area. Comparison with Gaussian and α-stable distributions.

C. Alpha-Stable

The parameters of α-stable distributions can be estimated using classical methods, such as fitting the four parameters to the empirical characteristic function estimated from data as proposed by Koutrouvelis [22].

The quality of the fit between the measurement data and the estimated α-stable distribution can be evaluated by generating samples from an α-stable distribution based on the estimated parameters. The quantiles of the measurement samples are then plotted against those of the generated samples, known as a quantile-quantile (QQ) plot. If the measurement data and the generated samples have the same underlying distribution, the QQ plot will be the line $y = x$.

Fig. 3 to 7 illustrate the fit between the measurement data (power samples over time/frequency intervals) and the α-stable samples generated from the estimated model. The estimated parameters value for the α-stable distributionsin the different cases are given in Tab. I.

First we mention that in all cases but one, β is estimated to one, which is expected due to totally skewed distributions. Further investigation is needed to understand the value found in the business scenario. However the QQ plots tend to show that the stable assumption is reasonable in most cases.

We also notice that the estimated α can significantly vary.

- It is found between one and two when the homogeneous Poisson point process (without guard zone) leads to an α

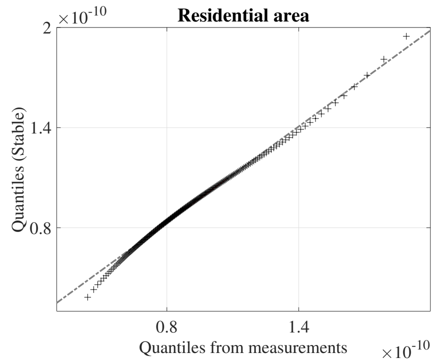

Fig. 3. Quantile-quantile plot of interference samples from the residential area.

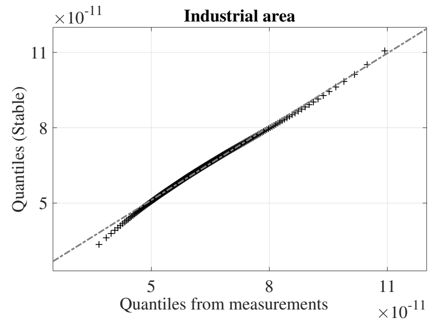

Fig. 4. Quantile-quantile plot of interference samples from the industrial area.

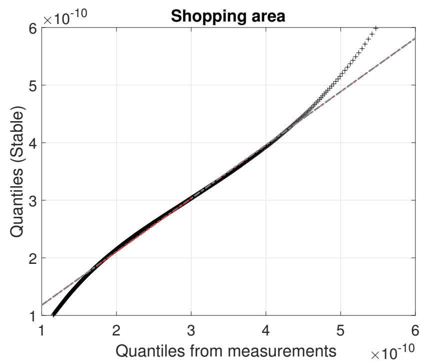

Fig. 5. Quantile-quantile plot of interference samples from the shopping area.

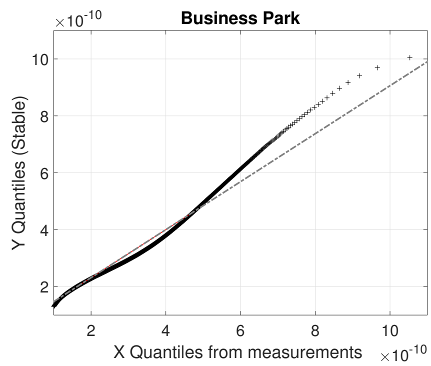

Fig. 6. Quantile-quantile plot of interference samples from the shopping area.

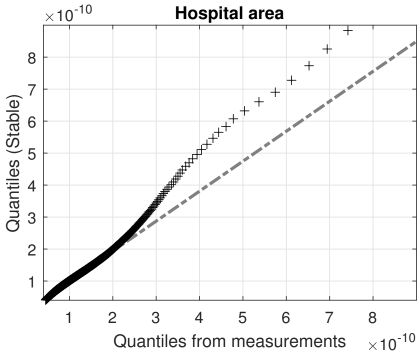

Fig. 7. Quantile-quantile plot of interference samples from the shopping area.

TABLE I
THE α-STABLE DISTRIBUTION PARAMETERS ESTIMATED IN THE DIFFERENT AREAS.

Area	α			
Residential	1.80	1	$0.13.10^{-10}$	$0.88.10^{-10}$
Business	1.82	0.4	$0.12.10^{-9}$	$0.36.10^{-9}$
Shopping	1.78	1	$0.069.10^{-9}$	$0.33.10^{-9}$
Industrial	1.84	1	$0.098.10^{-10}$	$0.69.10^{-10}$
Hospital	1.25	1	$0.05.10^{-9}$	$0.26.10^{-9}$

smaller than 1. However, this difference is in accordance with the results in [23]. The presence of a guard zone around the receiver, where no transmitter can be, impact the value of α. The larger this guard zone, the closer to 2 is the α meaning that we are getting closer to a Gaussian distribution. The study was made fo the amplitude statistics but is probably also valide for the power.

- It is close to 1.8 in four of the five cases and significantly lower (1.25) in the last one. Again a further analysis is needed to really understand this fact. It can come from the environmental specificity (and especially the radio channel attenuation coefficient). But it can also be due to the presence of interferers at a closer distance or to the presence of strong interferers in the considered area.

V. CONCLUSIONS

While there is an abundance of theoretical studies of interference statistics, the measurements in Aalborg are—to the best of our knowledge—the first to clearly validate the heavy tailed nature of the interference in the context of IoT communications. Interference models are key to design efficient coding and decoding strategies as well as efficiently adapting channel access and network topology. As such, the measurement data suggests that there is a need to reconsider the utility of Gaussian models in network design for the IoT.

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