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Generation of Long Waves using Non-linear Digital Filters

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Abstract

The importance of reproducing the correct second order bound terms in irregular laboratory waves has been recognized for some years. Several authors have addressed the topic and highly non-linear transfer functions, that enable a calculation of the second order terms in the elevation signal or the control signal for a wave generator given the 1st order elevation signal, have been presented.

In the methods published the 1st order elevation signal is Fourier transformed to the frequency domain where the correct 2nd order signal is calculated using the derived transfer functions. When performing inverse Fourier transform back to the time domain the control or elevation signal correct to 2nd order is available.

Such a method is efficient and straight forward to use and has been successfully implemented in several hydraulic laboratories. The method is, however, limited to applications where the 1st order elevation signal can be frequency analysed, which makes it inadequate in real-time applications, for example where random noise on-line is filtered digitally, to produce a wave spectrum of a given shape but with build-in stochastic variability. Such stochastic wave generation techniques are especially useful in experiments of long duration, where it is important not to repeat the wave sequences and where the length of the elevation signal makes it impossible, or too time consuming, to perform a frequency analysis.

In the present paper an approximative method for including the correct 2nd order terms in such applications will be described. The technique is based on a use of non-linear digital filters and is derived for bounded long waves, or sub-harmonics, as they in laboratory environment generally are considered the most important of the 2nd order terms. However, the technique is straight forward to modify to include the 2nd order super-harmonics.

Approach

The general approach when generating long waves is to define a transfer function which relate the 2nd order signal y, (elevation: $y = \eta^{(2)}$; wave board displacement: $y = e^{(2)}$) to the 1st order elevation signal, $\eta^{(1)}$. Consider $\eta^{(1)}$ to be the sum of two regular waves with the complex amplitudes A_n and A_m and the wave frequencies f_n and f_m, where $f_n > f_m$. In the frequency domain y may then be written

$$Y(f) = \begin{cases} \frac{1}{2} K_{nm} A_n A_m^* & f = f_n - f_m \\ \frac{1}{2} K_{nm} A_n^* A_m & f = f_m - f_n \end{cases} \tag{1}$$

where * denotes the complex conjugate and the transfer function K_{nm}, for $y = \eta^{(2)}$, equals the real function G_{nm} derived by Ottesen Hansen (1978) and Sand (1982), whereas K_{nm}, for $y = e^{(2)}$, equals the complex function $i/h \cdot F_{nm}^-$, in which i is the imaginary unit and h

the water depth, derived by Sand (1982) and Schäffer (1993). For practical applications Sand suggests that only the real parts of F_{nm}^{-}, that is $F_{1,nm}$, are used.

Let $H(\eta)$ denote the Hilbert transform of η. The frequency representation of $x = \eta^2 + H^2(\eta)$ is then

$$X(f) = \begin{cases} \frac{1}{2}A_n A_m^* & f = f_n - f_m \\ \frac{1}{2}(A_n A_n^* + A_m A_m^*) & f = 0 \\ \frac{1}{2}A_n^* A_m & f = f_m - f_n \end{cases} \quad (2)$$

By comparing Equations 2 to 1 it seen that the phases in x equal those in the theoretical 2nd order signal, y. Hence, an approximative method for calculating the 2nd order signal, y_a, would be to use non-linear digital filters on the 1st order elevation signal:

$$y_a = h_2 * \left((h_1 * \eta^{(1)})^2 + (h * (h_1 * \eta^{(1)}))^2\right) \quad (3)$$

where $*$ denotes convolution and h, h_1, and h_2 are digital filters: h is the hilbert transform filter and h_1 and h_2 are the transfer filters defined by their Fourier transforms, H_1 and H_2, which are fitted to the theoretical transfer function, K_{nm}, by minimizing the equation

$$\left(H_1(f_n) \cdot H_2(f_n - f_m) \cdot H_1(f_m) - K_{nm}\right)^2 \quad (4)$$

From Equation 2 it is seen that squaring $\eta^{(1)}$ leads to an off-set error, $X(0) \neq 0$. To eliminate this error h_2 must act as a notch filter, that is $H_2(0) = 0$.

Example

Consider an irregular wave field generated as a JONSWAP wave spectrum with peak period, $T_p = 1.3$ s, significant wave height, $H_s = 0.1$ m, and water depth, $h = 0.7$ m. Specifying a piston type wave generator, the transfer function $F_{1,nm}$ may be calculated and by minimizing Equation 4 a fitted transfer function, $\mathcal{F}_{nm} = H_1(f_n) \cdot H_2(f_n - f_m) \cdot H_1(f_m)$ is obtained. The fitted and the theoretical transfer function are compared by calculating a relative amplitude error, $\epsilon_{nm} = |\mathcal{F}_{1,nm} - F_{1,nm}| \cdot a_{nm}/a_{max}$, where a_{nm} is the amplitude of the long wave induced by the 1st order wave components at frequencies f_n and f_m, and a_{max} is the amplitude of the highest generated long wave.

In Figure 1 $F_{1,nm}$ and ϵ_{nm} are shown as a function of the 1st order wave frequencies, f_n and f_m. The figure clearly shows that the fitted transfer function quite closely follows the theoretical transfer function in the presented example, where the average amplitude error, $\bar{\epsilon} = 0.4\%$. Substituting $F_{1,nm}$ with the elevation transfer function, G_{nm}, the presented method leads to $\bar{\epsilon} = 3.2\%$ for the example described above.

Closure

The method presented in the paper is a simple and very efficient way to implement a nearly-correct generation of bounded long waves in a laboratory environment. The method may be implemented directly in a real-time wave generating system or on a stand-alone basis by filtering the 1st order signal from a wave synthesizer to include the correct long wave control signal before passing the signal to the wave generator.

The method has been successfully implemented in the wave generating system in the Hydraulics & Coastal Engineering Laboratory at Aalborg University. Results from experiments in the laboratory will be presented in the final paper.

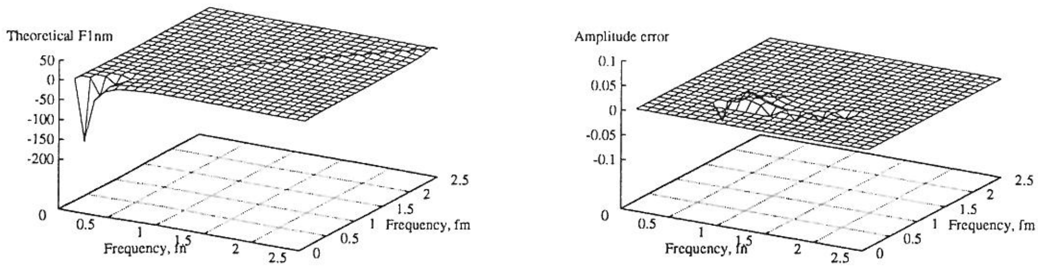

Figure 1: Theoretical long wave transfer function, $F_{1,nm}$, for piston type wave generator (left) and corresponding relative amplitude error, ϵ_{nm}, when using the method presented in the paper (right).

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Suggested topic area:
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