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GENERATION OF LONG WAVES USING NON-LINEAR DIGITAL FILTERS

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ABSTRACT

Transfer functions which enable 2nd order surface elevation or 2nd order paddle control signal to be calculated given the 1st order surface elevation have previously been presented by several authors. In the existing methods the 2nd order terms are calculated in the frequency domain from the Fourier transform of the 1st order surface elevation and subsequently inverse Fourier transformed. Hence, the methods are unsuitable for real-time applications, for example where white noise are filtered digitally to obtain a wave spectrum with built-in stochastic variability. In the present paper an approximative method for including the correct 2nd order bound terms in such applications is presented. The technique utilizes non-linear digital filters fitted to the appropriate transfer function and is derived only for bounded 2nd order subharmonics, as they in laboratory experiments generally are considered the most important. However, the technique can be modified to include the 2nd order superharmonics.

Introduction

In the last two decades 2nd order wave generation theory has been treated extensively by several authors, cf. Schäffer (1993) for a comprehensive historical summary. For irregular waves methods for calculating the correct 2nd order bounded sub and superharmonic terms in the surface elevation or paddle displacement signal given the 1st order surface elevation, $\eta^{(1)}$, have been presented:

Ottesen Hansen (1978) derived a transfer function which in the frequency domain enables a direct calculation of the 2nd order bounded subharmonic terms in the surface elevation. The transfer function was derived for the 2nd order bounded superharmonic terms by Sand and Mansard (1986). A general and compact form of the 1st order elevation to 2nd order elevation transfer function was rederived by Schäffer (1993).

Transfer functions enabling the 2nd order bounded subharmonic terms in the paddle displacement to be calculated were presented by Sand (1982) for a piston type wave maker. Sand and Mansard (1986) presented the corresponding transfer functions for the 2nd order bounded superharmonic terms. A general and compact form of the 1st order elevation to 2nd order paddle displacement transfer function was rederived by Schäffer (1993).

The present paper concentrates on the bounded subharmonic terms as they generally are considered to be the most important in practical applications. Because the formulations presented by Ottesen Hansen (1978) and Sand (1982), the latter especially after correcting the formula as described in Sand and Mansard (1986), generally are rather complex the formulations suggested by Schäffer (1993) are adopted herein.

Application of existing theory

The goal is to calculate the correct bounded 2nd order subharmonic terms in the surface elevation, $\eta^{(2)-}$, and the corresponding paddle displacement, $x^{(2)-}$. The calculations are performed using the 1st order surface elevation signal, $\eta^{(1)}$.

Discrete Fourier transform of $\eta^{(1)}$ decomposes the irregular surface elevation into, say N regular wavelets. Let y be the general 2nd order subharmonic signal, that is $y = \eta^{(2)-}$ when considering the surface elevation and $y = x^{(2)-}$ when considering the paddle displacement. The contribution to y by each pair of regular wavelets with complex amplitudes A_n and A_m and wave frequencies f_n and f_m, where $f_n > f_m$, can then be calculated. In the frequency domain:

$$Y(f) = \begin{cases} \frac{1}{2}K(f_n, f_m)A_n A_m^* & , f = f_n - f_m \\ \frac{1}{2}K^*(f_n, f_m)A_n^* A_m & , f = f_m - f_n \end{cases} \quad (1)$$

where Y is the discrete Fourier transform of y, * denotes complex conjugation and K for $y = \eta^{(2)-}$ equals the $\eta^{(1)}$ to $\eta^{(2)-}$ transfer function G^- derived by Ottesen Hansen (1978) and for $y = x^{(2)-}$ equals iF^- in which i is the imaginary unit and F^- is the $\eta^{(1)}$ to $x^{(2)-}$ transfer function derived by Schäffer (1993).

By adding the calculated y to the appropriate 1st order signal, $\eta^{(1)}$ or $x^{(1)}$, the surface elevation or paddle displacement correct to 2nd order, for linear and subharmonic components only, is obtained.

Transfer functions

Introducing the formulations by Schäffer (1993) the progressive part of the the $\eta^{(1)}$ to $\eta^{(2)-}$ transfer function G^- may be rewritten

$$G^-(f_n, f_m) = \frac{1}{g}\left\{(\omega_n - \omega_m)\frac{C_1}{C_2} - C_3\right\} \tag{2}$$

where

$$C_1 = (\omega_n - \omega_m)\left((-\omega_n\omega_m) - \frac{g^2 k_n k_m}{\omega_n\omega_m} + \frac{\omega_n^3 - \omega_m^3}{2}\right) - \frac{g^2}{2}\left(\frac{k_n^2}{\omega_n} - \frac{k_m^2}{\omega_m}\right) \tag{3}$$

$$C_2 = g(k_n - k_m)\tanh(k_n - k_m)h - (\omega_n - \omega_m)^2 \tag{4}$$

$$C_3 = \frac{1}{2}\left\{\frac{g^2 k_m k_m}{\omega_n\omega_m} + \omega_n\omega_m - (\omega_n^2 + \omega_m^2)\right\} \tag{5}$$

in which k is the wave number, ω is the cyclic wave frequency, g is the gravitational acceleration and h is the water depth.

Compared to G^- the complex $\eta^{(1)}$ to $x^{(2)-}$ transfer function, F^-, is more complicated, in general

$$F^- = (F_{11} + F_{12} + F_{13}) + i(F_{22} + F_{23} + F_{24}) \tag{6}$$

Each of the 6 functions eliminates free waves which otherwise would be emitted from the wave paddle due to interaction between two 1st order terms:

F_{11}	progressive wavelet and progressive wavelet,
F_{12}	component of paddle position and progressive wavelet,
F_{23}	component of paddle position and local disturbance wavelet,
F_{13} and F_{24}	progressive wavelet and local disturbance wavelet and
F_{22}	local disturbance wavelet and local disturbance wavelet

Cf. Schäffer (1993) for details. Sand (1982) showed that it is reasonable for laboratory applications, where only subharmonic components are considered, to omit 2nd order effects originating from any 1st order interaction with the local disturbance wavelets. Hence F^- reduces to

$$F_1^- = F_{11} + F_{12} \tag{7}$$

where

$$F_{11} = C_4\frac{k_n - k_m}{(k_n - k_m)^2 - k_{nm}^2}C_1 \tag{8}$$

$$F_{12} = C_4 g \left\{ \frac{\omega_n^2 - (\omega_n - \omega_m)^2}{c_m 2\omega_n} \frac{k_n^2}{k_m^2 - k_{nm}^2} + \frac{\omega_m^2 - (\omega_n - \omega_m)^2}{c_n 2\omega_m} \frac{k_m^2}{k_n^2 - k_{nm}^2} \right\} \quad (9)$$

$$C_4 = \frac{k_{nm}^2}{(\omega_n - \omega_m)^3} \quad (10)$$

in which k_{nm} is the solution to $(\omega_n - \omega_m)^2 = g k_{nm} \tanh k_{nm} h$ and c is the linear Biesel transfer function for the actual type of wave paddle. This simplified formulation is adopted herein.

Approximation

The exact method outlined in the previous section is efficient and straight forward to use and have been successfully implemented in several hydraulic laboratories. The method is, however, limited to applications where the 1st order elevation can be frequency analysed, or already is available in the frequency domain. This makes it inadequate for real-time applications, for example where the 1st order elevation is generated on-line by means of digital filtering of white noise, to produce a wave spectrum of a given shape but with built-in stochastic variability (non-deterministic spectral amplitude model).

The scope of the present paper is to present an approximative method for including the 2nd order subharmonic components in the surface elevation or paddle displacement in such applications. Two in principle different schemes can be considered: internal correction, where the approximative method is build into a real-time wave generation software, and external correction, where the analog 1st order paddle control signal is sampled from an existing wave generation system, manipulated to include the correct subharmonics and send to the wave paddle. In the following only the internal correction will be thoroughly described, but how to change it into an external correction will be briefly outlined.

The study took its offspring in an internal correction method build into the wave generating software in the Hydraulics & Coastal Engineering Laboratory at Aalborg University.

2nd order process

Consider a function z which is the sum of two regular wavelets with complex amplitudes A_n and A_m and wave frequencies f_n and f_m, respectively. Let $Z^{(2)}$ denote the discrete Fourier transform of z^2. According to the convolution theorem for Fourier transforms multiplication in the time domain corresponds to convolution in the frequency domain, and vice versa, hence $Z^{(2)}$ can be written

$$Z^{(2)}(f) = \begin{cases} \frac{1}{2}(A_n A_n^* + A_m A_m^*) & , f = 0 \\ \frac{1}{2}A_n A_m^* & , f = f_n - f_m \\ \frac{1}{4}A_m A_m & , f = 2f_m \\ \frac{1}{2}A_n A_m & , f = f_n + f_m \\ \frac{1}{4}A_n A_n & , f = 2f_n \end{cases} \tag{11}$$

Keeping in mind that $Z^{(2)}(-f) = Z^{(2)*}(f)$ it is seen that all phases and frequencies in Equation 11 correspond, except for the off-set ($f = 0$), to the subharmonics in Equation 1 ($f = f_n - f_m$ and $f = f_m - f_n$), the superharmonic components from Stokes 2nd order regular wave theory ($f = 2f_m$ and $f = 2f_n$) and the superharmonic 2nd order components from wave-wave interaction as described by Sand and Mansard (1986) ($f = f_m + f_n$ and $f = -f_m - f_n$).

<u>Hilbert transform</u>

The Hilbert transform relates the real and imaginary part of an analytic function. That is, the imaginary part is the Hilbert transform of the real part, and vice versa. Hence, in the frequency domain the Hilbert transform, $\mathcal{H}$, is defined by

$$H(f) = \begin{cases} -i & , f > 0 \\ 0 & , f = 0 \\ i & , f < 0 \end{cases} \tag{12}$$

Now consider the function $z^{(2)-}$

$$z^{(2)-}(t) = \frac{1}{2}\left(z^2(t) + \mathcal{H}^2[z(t)]\right) \tag{13}$$

in which z is given in the previous section. The discrete Fourier transform of $z^{(2)-}$, $Z^{(2)-}$, is then

$$Z^{(2)-}(f) = \begin{cases} \frac{1}{2}A_n^* A_m & , f = f_m - f_n \\ \frac{1}{2}(A_n A_n^* + A_m A_m^*) & , f = 0 \\ \frac{1}{2}A_n A_m^* & , f = f_n - f_m \end{cases} \tag{14}$$

By comparing Equation 14 to Equation 1 it is evident that $z^{(2)-}$, except for a linear transfer function and an off-set equals the 2nd order subharmonic function y when considering interaction between two regular wavelets.

Filter approach

Assume that the transfer functions G^- and F_1^- can be approximated by $\mathcal{G}^-$ and $\mathcal{F}_1^-$, respectively, which both can be separated into two real functions, H_1 and H_2, in the following manner (in the following only the approximation of F_1^- by $\mathcal{F}_1^-$ is discussed, but the method equally applies to G^-):

$$\mathcal{F}^-(f_n, f_m) = H_1(f_n)H_2(f_n - f_m)H_1(f_m) \tag{15}$$

where $H_2(0) = 0$. The Fourier transform of y, Y may then be approximated by Y'

$$Y'(f) = \begin{cases} \frac{1}{2}H_1(f_n)H_2(f)H_1(f_m)\delta A_n A_m^* & , f = f_n - f_m \\ \frac{1}{2}H_1(f_n)H_2(f)H_1(f_m)\delta^* A_n^* A_m & , f = f_m - f_n \end{cases} \tag{16}$$

where $\delta = 1$ for $y = \eta^{(2)-}$ and $\delta = i$ for $y = x^{(2)-}$. Hence the inverse Fourier transform of Y', y', will approximate y. Using the convolution theorem for Fourier transforms and Equations 13 and 14, y' may be written:

$$y'(t) = \frac{1}{2}h_2 * \left\{(h_1 * \eta^{(1)})^2 + (h * h_1 * \eta^{(1)})^2\right\} \tag{17}$$

where h, h_1 and h_2 are filters defined by their Fourier transforms: $H(f)$, $H_1(f)$ and $\delta H_2(f)$, respectively.

Hence, $\eta^{(1)}$ may be filtered digitally to give $\eta^{(2)-}$ or $x^{(2)-}$. Using discrete FIR filters of equal odd finite length, say M, the delay between the last calculated or sampled 1st order surface elevation and the calculated 2nd order elevation or paddle displacement will be $3(M-1)/(2f_s)$ in which f_s is the frequency by which the surface elevation is calculated or sampled. The scope of the present paper is not to discuss the choice of filter length, tapering etc., reference is made to existing literature on the subject.

If calculation time is a problem it may be decided only to generate 2nd order bound subharmonic waves below a certain frequency, say the lowest 1st order wave frequency. In this case there is no need to include the Hilbert filter h in Equation 17, because h_2 will act as a low-pass filter, removing any super harmonic components. Hence the calculation time will be reduced by 33 %, if the filter lengths are unchanged.

From Equations 2 to 10 it is obvious that the variables in the theoretical transfer functions generally cannot be separated as suggested in Equation 15 which means that in general $\mathcal{F}_1^- \neq F_1^-$. Only when considering a surface elevation consisting of wavelets with frequencies that ensure that the corresponding subharmonic components have different frequencies will the approximation be exact. However, it is possible, using a steepest descent fitting method as outlined below, to calculate a $\mathcal{F}_1^-$ that makes the filter approach generally applicable as will be shown.

The filters are fitted by minimizing the merit function, χ^2

$$\chi^2 = \sum_{m=1}^{n-1} \sum_{n=2}^{N} \left\{ F_{\overline{1}}^{-}(f_n, f_m) - H_1(f_n) H_2(f_n - f_m) H_1(f_m) \right\}^2 \tag{18}$$

in which N is the number of frequency components, $N = (M+1)/2$, by successive calculations of the gradient to χ^2, $\nabla\chi^2$, in each point on the $n-m$ plane and subsequent adjustment of $H_1(f_n)$, $H_2(f_n - f_m)$ and $H_1(f_m)$ by a small amount down this gradient, until χ^2 converges.

To take into account the actual distribution of wave energy in the 1st order surface elevation and the actual shape of the transfer function a weighting function, W, is introduced. W is chosen as the relative long wave energy induced by each pair of wavelets in the irregular 1st order wave spectrum S_η, that is:

$$W(f_n, f_m) = \frac{S_\eta(f_n) S_\eta(f_m) (G^-(f_n, f_m))^2}{|S_\eta(f_n) S_\eta(f_m) (G^-(f_n, f_m))^2|_{max}} \tag{19}$$

in which max denotes the maximum value. Hence the small step down the gradient is chosen as $\Delta W(f_n, f_m)(F_{\overline{1}}^{-}(f_n, f_m) - H_1(f_n) H_2(f_n - f_m) H_1(f_m))$, in which Δ is sufficiently small to avoid instability.

To evaluate the quality of fitting, the relative long wave error induced by each pair of wavelets, $\varepsilon(f_n, f_m) = (1 - \mathcal{F}_{\overline{1}}^{-}(f_n, f_m)/F_{\overline{1}}^{-}(f_n, f_m)) W(f_n, f_m)$ and the sum of ε relative to the total long wave energy, ε_{tot}, are calculated. In Figure 1 ε is shown for a JONSWAP type wave spectrum. As observed the overall error is quite small, $\varepsilon_{tot} =$ 2.3 %, and $\mathcal{F}_{\overline{1}}^{-}$ only differs slightly from $F_{\overline{1}}^{-}$ in this case. Fitting the corresponding $\mathcal{G}^-$ to G^- leads to $\varepsilon_{tot} = 3.0$ %. It is in fact the general observation that $\mathcal{F}_{\overline{1}}^{-}$ fits better to $F_{\overline{1}}^{-}$ than $\mathcal{G}^-$ does to G^-.

Example

Two examples of applying the presented approach to a JONSWAP type wave spectrum and piston type wave maker are described in this section.

Figure 2 shows the 1st order paddle displacement signal $x^{(1)}$ and the corresponding 2nd order subharmonic signal, $x^{(2)-}$, calculated using the filter approach and the existing theory. From the figure it is seen that the overall agreement between the filter approach and the existing theory is very good. But because multiple frequency combinations induce bounded long waves on equal frequencies there will be some differences. From the figure it appears that these differences mainly are on the subharmonic components with relative high frequencies.

In Figure 3 the measured 1st and subharmonic 2nd order surface elevation, $\eta^{(1)}$ and $\eta^{(2)-}$, are shown for the paddle displacement calculated using the filter approach, the existing theory and without including the 2nd order terms. As for the paddle displacements in Figure 2 differences between the filter approach and the existing theory mainly are on the subharmonic components with relative high frequencies. But still the overall agreement is very good. Furthermore the figure clearly indicates the problems when not including the bound long wave correction: The bounded long waves will be formed, but freely propagating long waves will be generated and the phase and amplitude of the observed long wave bounded to the wave group will vary along the flume.

To change the internal correction method, described above, into an external correction method, $x^{(1)}$ is calculated from the sampled linear paddle control signal and filtered through an inverse Biesel filter, b^{-1}, defined by its Fourier transform $B^{-1}(f)$, to obtain $\eta^{(1)}$. For a piston type wave maker:

$$B^{-1}(f) = \frac{\sinh kh \cosh kh + kh}{2 \sinh^2 kh} \tag{20}$$

Equation 17 may then be rewritten

$$y'(t) = \frac{1}{2} h_2 * \left\{ (h_1 * b^{-1} * x^{(1)})^2 + (h * h_1 * b^{-1} * x^{(1)})^2 \right\} \tag{21}$$

The filters b^{-1}, h_1 and h_2 of course need to be calculated according to the actual wave parameters.

Closure

A method has been presented for filtering a 1st order surface elevation to obtain the 2nd order bound subharmonic surface elevation or corresponding paddle displacement. The method has been compared in simulations and physical experiments to the existing theory. The filter approach gives exact 2nd order subharmonic components when only considering the interaction between two regular wavelets. For irregular wave spectra the filter approach gives estimates which differs slightly from the existing theory especially for relative high subharmonic frequencies. For the low subharmonic frequencies, which generally are the most important as far as long wave phenomena are concerned, only insignificant differences are observed. Hence, the method is suitable for applications were bounded subharmonics otherwise cannot be included using existing theory.

In addition a real-time scheme for manipulating the 1st order paddle control signal to include the 2nd order subharmonic components has been outlined.

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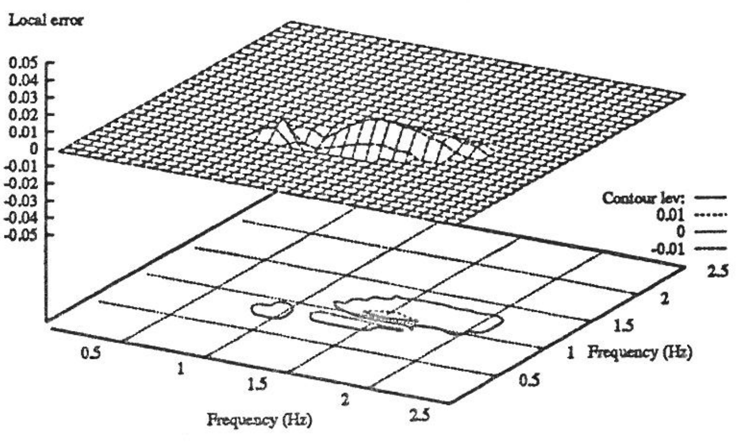

Figure 1: Induced relative long wave error ε when fitting $\mathcal{F}_1^-$ to F_1^-, $\varepsilon_{tot} = 2.3$ %. JONSWAP spectrum, peak frequency, $f_p = 1.0$ Hz, $\gamma = 10$ and $h = 0.5$ m.

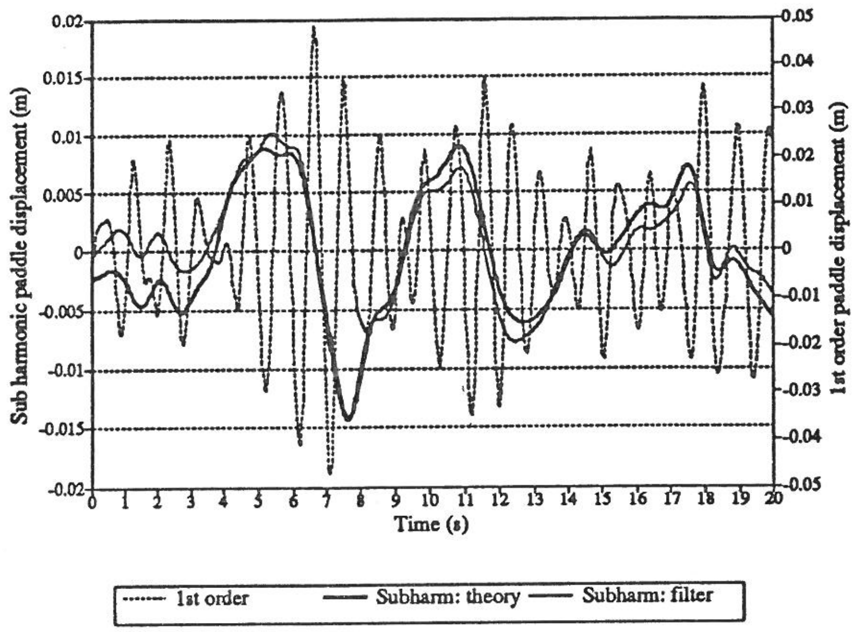

Figure 2: Calculated 1st and 2nd order piston displacement, $x^{(1)}$ and $x^{(2)-}$. JONSWAP spectrum, peak frequency, $f_p = 1.0$ Hz, $\gamma = 10$ and $h = 0.5$ m. 2nd order subharmonic components calculated using filter approach (filter) and existing theory (theory).

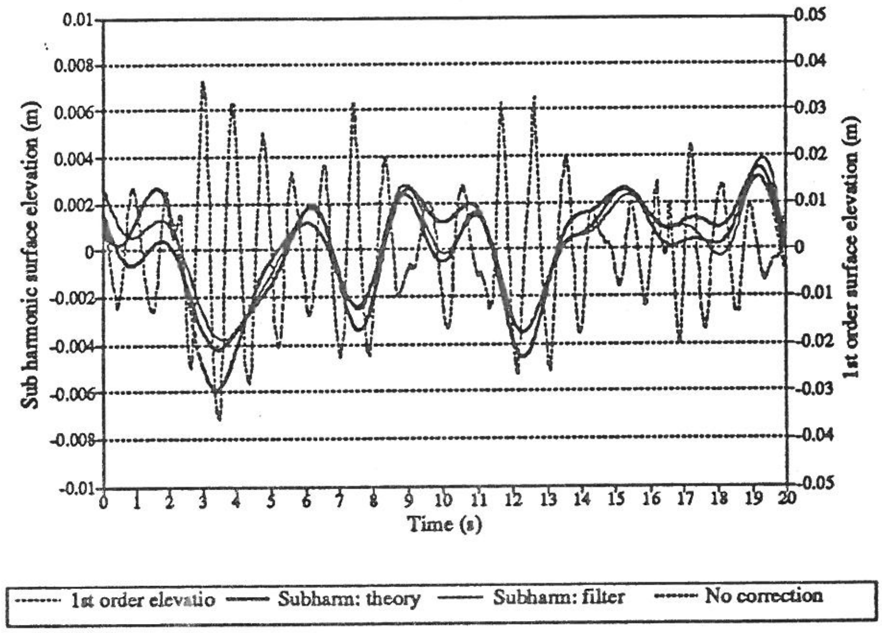

Figure 3: Measured 1st and 2nd order surface elevation, $\eta^{(1)}$ and $\eta^{(2)-}$. JONSWAP spectrum, peak frequency, $f_p = 1.0$ Hz, $\gamma = 10$ and $h = 0.5$ m. Wave generation not including (no correction) and including 2nd order subharmonic components, calculated using filter approach (filter) and existing theory (theory).